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RESEARCH ARTICLE

Buckling strength of uniaxially compressed orthogonally stiffened plates

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Abstract

The bucking strength calculation includes the determination of global buckling strength, the effective plate width and the torsional buckling strength of open section stiffeners. In each calculation the classic strength formula should be modified considering the effect of initial imperfections and residual welding stresses. The calculation method of Det Norske Veritas design rules and the Eurocode 3 are compared to each other.

Keywords

stiffened plates · buckling strength · welded structures · effective width · effect of initial imperfections

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1 Introduction

Stiffened plates are widely used in various steel structures such as bridges, roofs, platforms, gates, towers, pilons etc. In our previous studies [1, 2] it has been found that the orthogonally stiffened plates are more economic than the longitudinally stiffened ones. The available design rules relate more to longitudinally stiffened plates, thus, it can be interesting to summarize the buckling strength calculation of orthogonally stiffened plates.

For the buckling strength calculation of plate structures the design rules of Eurocode 3 [3] and Det Norske Veritas [4, 5] can be used. In the present paper the calculation methods of these two design rules are used and compared to each other.

In the buckling strength calculation the main problem is to consider the initial imperfections as well as residual welding stresses and deformations, which decrease the buckling strength significantly. Therefore the classic buckling strength formulae should be modified.

The well known modification is related to the buckling of welded steel compressed bars, since the Euler formula does not take into account the effect of initial manufacturing imperfections and compressive residual stresses due to shrinkage of welds [6]. This modification should be carried out for other cases of buckling, namely also for stiffened plates [7]. For this modification various methods are used in different design rules.

In the case of stiffened plates the buckling strength calculation includes three parts as follows: (1) global buckling strength, (2) effective plate width, (3) torsional buckling of stiffeners. Our study investigates these three calculation parts.

The investigated plates are rectangular, simply supported and halved rolled I-section stiffeners are used welded to the base plate in both directions (Fig. 1).

2 Global buckling strength

The classic buckling stress is derived from the Huber's differential equation. This stress is modified taking into account the effect of residual welding stresses and initial imperfections.

The Huber's differential equation for deflection w of uniaxially compressed orthotropic plates is given by

$$B_x w'''' + 2H w''\cdot\cdot + B_y w\cdot\cdot\cdot\cdot + N_x w'' = 0 \quad (1)$$

where the prime (') and dot (·) superscripts denote partial derivatives with respect to x and y respectively. The corresponding bending stiffnesses are defined as

$$B_x = \frac{E_1 I_y}{a_y};\ B_y = \frac{E_1 I_x}{a_x};\ E_1 = \frac{E}{1-\nu^2} \quad (2)$$

where E is the elastic modulus, I_y and I_x are the moments of inertia of stiffeners including the effective width of the base plate, ν is the Poisson ratio. The solution of Eq.(1) yields the classic buckling formula for critical force of a simply supported rectangular plate

$$N_E = \frac{\pi^2}{b_0^2}\left(B_x \frac{b_0^2}{a_0^2} + 2H + B_y \frac{a_0^2}{b_0^2}\right) \quad (3)$$

For open section stiffeners the torsional stiffness H can be neglected. It can be seen from Eq.(3) that the bending stiffness of the transverse stiffeners (B_y) has a significant effect on the critical buckling force. This effect causes the fact that the orthogonally stiffened plates are more economic than the longitudinally stiffened ones.

The global buckling strength according to DNV [4] is

$$\sigma_{cr} = \frac{f_{y1}}{\sqrt{1+\lambda^4}},\ f_{y1} = \frac{f_y}{1.1} \quad (4)$$

$$\lambda = \sqrt{\frac{f_{y1}}{\sigma_E}},\ \sigma_E = \frac{N_E s_y}{A_{ey}},\ N_E = \frac{\pi^2}{b_0^2}\left(B_x \frac{b_0^2}{a_0^2} + B_y \frac{a_0^2}{b_0^2}\right) \quad (5)$$

Effective cross-sectional areas ($i = x, y$)

$$A_{ei} = \frac{h_{1i} t_{wi}}{2} + b_i t_{fi} + s_{ei} t,\ s_y = \frac{b_0}{n_y},\ s_x = \frac{a_0}{n_x}. \quad (6)$$

It can be seen that, according to DNV the classic buckling strength σ_E is modified by a buckling factor of

$$\rho = \frac{1}{\sqrt{1+\lambda^4}}. \quad (7)$$

This factor is proposed in DNV 1995 [4] for plate buckling, but in DNV 2002 [5] it is replaced by another formula, which is similar to the column buckling factor and is used for buckling of longitudinal stiffeners. Eq.(7) is proposed in DNV 2002 for shells. Since DNV and EC [3] do not propose any modifying formula for global buckling of orthogonally stiffened plates, Eq.(7) can be used.

3 Effective plate width

Three methods are compared to each other.

(1) Method of DNV 1995 [4]

$$s_e = \left(\frac{1.8}{\beta} - \frac{0.8}{\beta^2}\right) s, \quad (8)$$

$$\beta = \frac{s}{t}\sqrt{\frac{f_y}{E}} \quad \text{if} \quad \beta_{\geq} 1 \beta_{=} 1 \text{if} \beta_{<} 1 \quad (9)$$

(2) Method of DNV 2002 [5]

$$\frac{s_{e2}}{s} = \frac{\bar\lambda_p - 0.22}{\bar\lambda_p^2} \quad \text{if} \quad \bar\lambda_p \leq 0.673 \quad (10)$$

$$\frac{s_{e2}}{s} = 1 \quad \text{if} \quad \bar\lambda_p \geq 0.673 \quad (11)$$

$$\bar\lambda_p = 0.525 \frac{s}{t}\sqrt{\frac{f_y}{E}} \quad (12)$$

(3) Method of EC 3 [3]

$$\frac{s_{e3}}{s} = \frac{\bar\lambda_{p1} - 0.22}{\bar\lambda_{p1}^2} \quad \text{if} \quad \bar\lambda_{p1} \geq 0.673 \quad (13)$$

$$\frac{s_{e3}}{s} = 1 \quad \text{if} \quad \bar\lambda_{p1} \leq 0.673 \quad (14)$$

$$\bar\lambda_{p1} = \frac{s}{t}\frac{1}{28.4\varepsilon\sqrt{k_\sigma}}, \quad \varepsilon = \sqrt{\frac{235}{f_y}}, k_\sigma = 4 \quad (15)$$

Multiplying and dividing $\bar\lambda_{p1}$ by $\sqrt{E}$ and calculating with $E = 2.1\text{x}10^5$ MPa for steels, it can be seen that $\bar\lambda_{p1} = \bar\lambda_p$, thus the (2) and (3) methods are identical. It can be concluded that all the three methods for the effective plate width calculation are identical.

These methods take into account the effect of initial imperfections and residual welding stresses, since they produce smaller values than the classic Karman formula [6]

$$\psi = \frac{1}{\bar\lambda_p},\ \psi \leq 1 \quad (16)$$

as it can be seen from the Table 1.

Tab. 1. Effective plate widths according to different calculation methods

β	1.182	2	2.5	3
$\bar\lambda_p$	0.673	1.050	1.3125	1.5750
s_e/s	1	0.8504	0.6342	0.5463
Ψ	1	0.9500	0.7600	0.6333

4 Torsional buckling of stiffeners

4.1 The DNV method

The classic torsional buckling stress for a centrally compressed bar is

$$\sigma_{crT} = \frac{G I_T}{I_P} + \frac{E I_\omega}{L^2 I_P} \quad (17)$$

where $G = E/2.6$ is the shear modulus, I_T is the torsional moment of inertia, I_P is the polar moment of inertia, I_ω is the warping constant, L is the distance between lateral restraint points. The derivation of Eq. (17) can be found in [6].

DNV 1995 [4] and 2002 [5] propose the same method as follows:

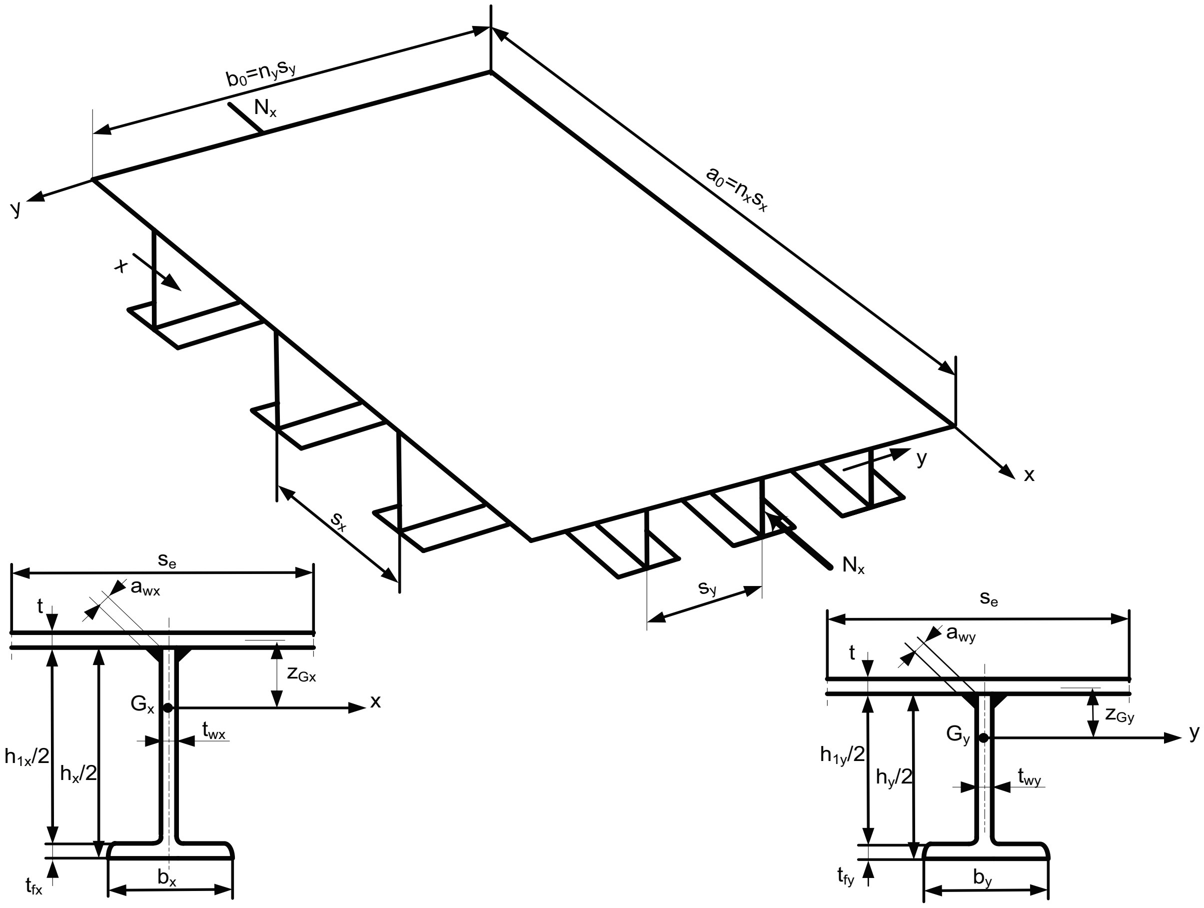

Fig. 1. Orthogonally stiffened rectangular plate

The classic torsional buckling stress for halved rolled I-section stiffeners is expressed by

$$\sigma_{ET} = \frac{A_{wy} + A_{fy}\left(\frac{t_{fy}}{t_{wy}}\right)^2}{A_{wfy}} G \left(\frac{2t_{wy}}{h_{1y}}\right)^2 + \frac{3x2.6\pi^2 E I_z}{A_{wfy} s_x^2} \quad (18)$$

where

$$A_{wy} = \frac{h_{1y} t_{wy}}{2}, A_{fy} = b_y t_{fy}, A_{wfy} = A_{wy} + 3A_{fy},$$

$$I_z = \frac{b_y^3 t_{fy}}{12} \quad (19)$$

The classic buckling stress given by Eq.(18) is modified considering the initial imperfections and residual welding stresses by

$$\lambda_T = \sqrt{\frac{f_y}{\sigma_{ET}}} \quad (20)$$

$$\sigma_T = \frac{f_{y1}}{\phi_T + \sqrt{\phi_T^2 - \lambda_T^2}}, \phi_T = 0.5\left(1 + \mu_T + \lambda_T^2\right) \quad (21)$$

$$\mu_T = 0.007\,(\lambda_T - 0.6) \quad (22)$$

For the verification the condition

$$\frac{N_x}{n_y A_{ey}} \le \sigma_T \quad (23)$$

is used.

4.2 The Eurocode 3 method

Eurocode 3 [3] gives only a very simple criterion of

$$\frac{I_T}{I_p} \ge 5.3 \frac{f_y}{E} \quad (24)$$

This criterion is unrealistic, since it does not contain the effect of the distance of lateral restraint points (in our case s_x).

It should be noted that another method for the calculation of torsional buckling strength of open section stiffeners is used in [7].

5 Conclusions

Two design rules are considered, namely the DNV and Eurocode 3 (EC) prescriptions. In each buckling calculation the effect of initial imperfections and residual welding stresses should be taken into account.

The buckling strength calculation includes three parts as follows.

1 The global buckling strength should be calculated using the classic solution of the Huber's differential equation for orthotropic plates modified by a proposed buckling factor. Since the two design rules do not propose any method for the buckling strength calculation of orthogonally stiffened plates, the proposed buckling factor can be used.

2 The formulae for the effective plate width given by DNV and

EC are identical and consider the effect of initial imperfections and residual welding stresses.

3 For the calculation of torsional buckling stress of open section stiffeners the formulae of DNV can be used. The EC criterion is unrealistic, since does not take into account the distance of transverse restraint points.

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