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SHEARING TESTS WITH CONTINUOUSLY INCREASING NORMAL STRESS

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Abstract

In the conventional shearing tests the normal force or the normal stiffness are kept constant during the experiment. Sliding under constant normal load occurs in slopes if the environment is not changing during the process. The goal of this paper is to investigate the influence of the continuously changing (i.e. increasing) normal load for the continuously increasing shearing load. Cement mortar specimens with triangular teeth that have the same inclination (regular triangular teeth) were used to carry out this research. Both the geometry of the specimens and the mechanical constants were well-known. The "Continuous Failure State" (CFS) shear tests were carried out according to CFS-triaxial tests, with different starting normal stresses. According to the investigations, the behaviour of the material in case of CFS-shearing tests was always different as in case of CFS-triaxial tests. Linear and nonlinear theoretical models are proposed for the measured curves.

Keywords: shearing test, continuous failure state, sliding.

1. Introduction

For triaxial tests to determine the peak and the residual strength of rock materials the International Society for Rock Mechanics (ISRM) suggested the "Continuous Failure State" triaxial test (CFS-triaxial test) [3]. According to this method confining pressure and axial stress are applied so as to cause the test specimen to be permanently in a state of failure. In this way it is possible to obtain at least parts of the failure envelope for both the peak and the residual strength with the aid of a single specimen. KOVÁRI et al. [2] explained this method and realized that to keep the material in a pre-failure state the best is to chose the rate of the increasing confining pressure so that the slope of the axial stress–axial displacement curve is equal to the Young's modulus of the sample. Because the slope of the stress-strain curve of a damaged material decreases with the axial stress, one can avoid an early failure and the measured strength will have some reserves.

TISA and KOVÁRI [7] showed that CFS-triaxial test might be directly adapted to the direct shear test, because on comparing the results of conventional triaxial tests with those of direct shear tests on joints or planes of weakness a considerable similarity may be observed. The curves representing the relationship between axial stress and axial strain in the triaxial test exhibit basically the same form as those for shear deformation and shear force in the direct shear test, including the characteristics for peak and residual strength (*Fig. 1*). After several investigations they realized that using the CFS direct shear test the determination of the residual shear strength is exact. However, for determining the peak strength envelope of rough surfaces (or with teeth) only with decreasing normal load is correct [7] This material investigation for determining the shearing constants of jointed rocks is also used in the practice, mostly in Switzerland.

According to the results of TISA and KOVÁRI [7] using continuously increasing normal load as CFS direct shear test for rough surfaces or those with teeth on cement mortar and brick specimens the slopes of shear stress–normal stress curves were always under the curve of the exact failure envelope. An example of the exact envelope of shear failure with single specimens under constant normal load can be seen on *Fig. 1*.

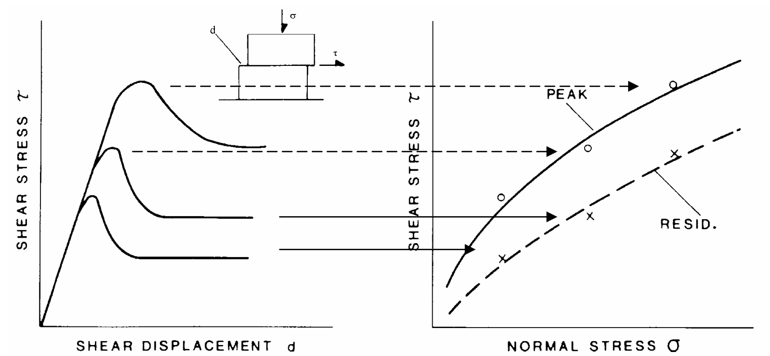

Fig. 1. Characteristic results of direct tests with constant normal loading on rock joints indicating similarity to triaxial test results [7]. The arrows indicate some related residual and peak shear stress values of the two figures.

Shearing tests with specimens of regular triangular teeth were carried out with increasing normal load from the maximal shearing stress point according to the research of TISA and KOVÁRI [7] with nine different starting normal loads and the relationship between the shear and the normal stress was described.

2. Relationships between the Critical Shear Stress and Normal Stress

PATTON [5] was the first who performed a series of constant load stress direct shear tests on rock with regular teeth inclination (i), at varying normal stresses. From these tests he established a bilinear failure envelope – failure from an asperity sliding and asperity shearing mode. The equation for the first part of the two portions of the failure envelope is:

$$\tau = \sigma_n \tan(\phi_\mu + i), \tag{1}$$

if the σ_n normal stress is less than the σ_T transition stress, the boundary between the different modes of failure. Here $\tau$is the shear stress and ϕ_μ denotes the sliding friction angle. If the normal stress equal or exceeds the transition stress ($\sigma_n \geq \sigma_T$), the shear stress is:

$$\tau = c + \sigma_n \tan \phi_r, \tag{2}$$

where c is the cohesion and ϕ_r is the angle of the internal friction. Generally it can be assumed that $\phi_\mu \approx \phi_r$. The theoretical background of the Patton failure envelope supposes rigid asperities in the sliding region and failure according to the Coulomb-Mohr criteria in the shearing region above the transition stress.

Later LADANYI and ARCHAMBAULT [4] extended the Patton *Eq.* (1), considering natural rock joints with irregular (but rigid) asperities. They proposed the following equation for the sliding part of the failure envelope if the shear area is one:

$$\tau = \sigma_n \tan(\phi_\mu + \nu), \tag{3}$$

where $\tan \nu = \dot{v}$ is the rate of dilation at failure. If the asperities are supposed to be rigid and regular then $\nu = i$ therefore *Eq.* (3) reduces to *Eq.* (1). For the shear part they proposed a fitting to the Coulomb-Mohr failure criteria as well as to the Fairhurst criteria, extending *Eq.* (3) with additional terms.

SEIDEL and HABERFIELD [6] argued in favor of the original Patton equation (1) for the sliding part. They claimed to consider elastic teeth and their theoretical model was supported by constant normal stiffness (CNS) experiments.

VÁSÁRHELYI [8] investigated the dependence of the constant normal load on the rate of dilation at failure with Constant Normal Load (CNL) experiments, too. According to his results the Ladanyi and Archambault's equation (3) can be extended the whole curve (including the shearing part) and it is better then Patton's (1), then correct until the teeth (or irregularities) are not shorn off. Therefore *Eq.* (3) is valid far beyond the transition stress and gives a unified approach to the two seemingly independent failure modes. The practical disadvantage of the approach that in this case the rate of dilation ($\dot{v}$) is a variable to be measured.

The real measurements are better approximated by a smooth curve instead of the bilinear one of Patton. One of the simplest generalizations was suggested by JAEGER [1] on purely phenomenological reasoning:

$$\tau = c\left[1 - e^{-b\sigma_n}\right] + \sigma_n \tan \phi_\mu \tag{4}$$

where:

b: empirical constant;
c: cohesion,
σ_n normal stress
ϕ_μ : basic friction angle
q_u: uniaxial compressive strength

This equation is asymptotically equal with the bilinear one of Patton when the normal load σ_n goes to infinity. Jaeger's basic equation is shown with Patton's equation on *Fig. 2*.

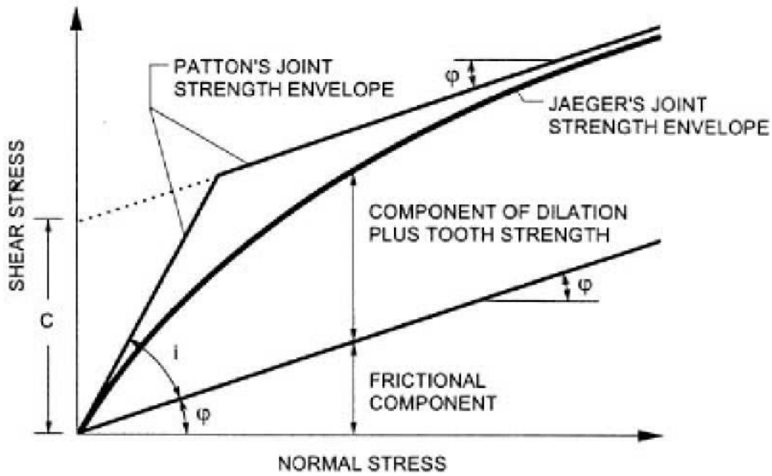

Fig. 2. Comparing the rock joint strength envelopes (Patton's bilinear model and Jaeger's strength envelope)

3. Material Description and Experiment

Cement mortar was selected as test material because has rock-like properties. *Table 1* presents the physical properties of the cement mortal. Basic shearing constants were determined in the previous researches of TISA and KOVÁRI [7] and VÁSÁRHELYI [8].

All specimens were 150 mm wide and 140 mm long with four regular teeth where the distance between the teeth was kept constant at 20 mm. The teeth were 5 mm high, 15 mm thick in the bottom and the inclination angle (i) of the teeth was 26.6°.

The CNL equipment was designed and built in the Rock Mechanics Laboratory of the Swiss Federal Institute of Technology, Zurich [7]. The force transmission to the specimen was ensured by casting it in epoxy resin in the usual manner. The shear box fits exactly between two massive L-shaped steel pieces (*Fig. 3*). The normal force (N) and shear force (S) were applied through one of these L-shaped steel pieces. The other L-shaped steel piece provided the reactions, which were the lower loading plates of the servo-controlled press. This was achieved by resting the L-shaped steel piece against the front side of the frame as well as on a cylindrical

Table 1. Physico-mechanical properties of the applyied cement-mortal

Physico-mechanical properties	Index	Unit	Value
Modulus of elasticity	E	GPa	17.5
Poissons ratio	ν	–	0.1
Maximal stress	σ	MPa	60
Tensile stress	σ_t	MPa	6.5
Basic friction angle	Φ_u	degree	33.64
Asperity sliding angle	α	degree	59.00
Cohesion	c_j	N/mm^2	2.13
Transition stress	σ_T	N/mm^2	2.10
Density	γ	g/cm^3	2.2

bearing box with rests on the lower loading plate. Using this machine the normal load could be changed manually during the research.

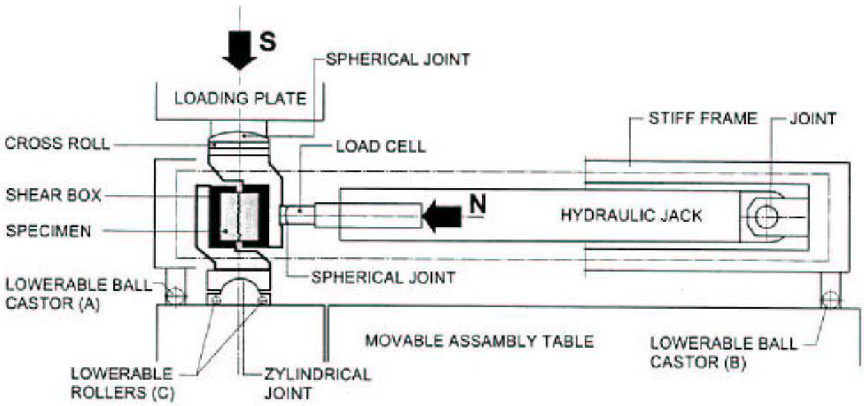

Fig. 3. Schematically view of the shear apparatus used in combination with a loading machine [7]

The test monitoring arrangement is shown schematically in *Fig. 4.* It is necessary that the shear stress, normal stress and shear displacements were monitored continuously. The two $x - y$ recorders allow the separate recording of the shear stress – shear displacement development and that of the corresponding stress path. Every research was carried out at constant 0.003 mm/sec shear displacement rate.

Till the maximal shearing stress (which was determined by single tests and using the previous results, as well) the normal load was constant. Measurements

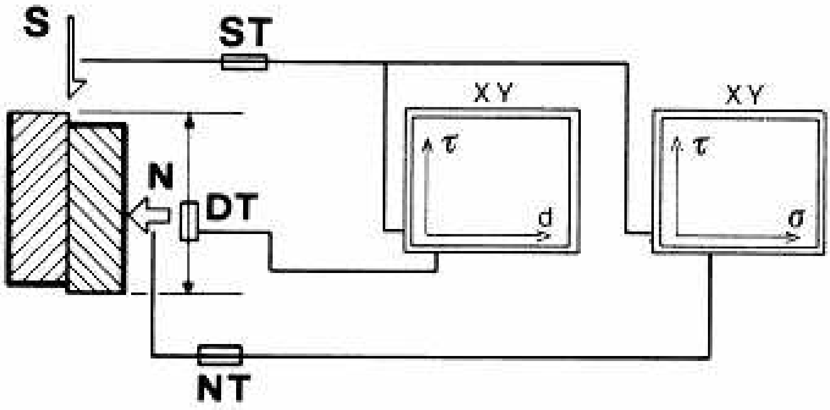

Fig. 4. Schematic diagram of the test monitoring arrangement (Tisa and Kovári, 1984). S: Shear force, ST: Shear load transducer, N: Normal force, NT: Normal load transducer, DT: Shear displacement transducer, XY Recorders

have been carried out starting with the following constant normal load: 0.3; 0.6; 1.0; 1.5; 2.0; 2.5; 3.0; 3.5 and 4.0 N/mm^2. At the maximal shearing stress the normal load was increased manually keeping the slope of the shear stress – shear displacement curve constant which were equal to the slope of this curve at constant normal load, according to the shearing model of TISA and KOVÁRI [7]. In this case the specimen was maintained in a state of permanent sliding.

4. Evaluation of the Experimental Results

The measured shear stress – normal stress curves are shown in *Fig. 5*. We can observe more or less linear shear stress – normal stress curves beyond the maximal shear stress and expect a tendency to the residual line. With low normal stress the changing of the direction of the curve is longer (i.e. big curvature of the curve before the second linear part) but beyond 1.5 N/mm^2 normal stress the curves have very small curvature. Remarkable that the slope of measured curves is similar to the Ostwald curves for shear stress and shear rate [9]. The linear part of the graphs is quasi parallel with each other both before and after the transition stress, thus they are not influenced by the starting normal stress. Both linear and non-linear model was written for the curves.

4.1. Linear Model

The slopes of the shear stress – normal stress lines (*Fig. 6*) are independent on the starting normal stress and it was approximately 25°. This slope can depend on the mechanical behavior, shear displacement rate and the roughness of the rock. Therefore a linear equation of this curve is:

$$\tau = \tau_s + (\sigma_{nx} - \sigma_s)\tan\gamma. \tag{5}$$

where τ_s is the maximal shearing stress at the starting normal load (σ_s) and σ_{nx} is the continuously increasing normal stress and $\tan\gamma$ is the average slope of the normal stress – shear stress line. *Fig. 6* compares the measured result with the linear equation in case of 2 N/mm^2 starting normal stress.

Accepting the validity of *Eq.* (3) for both modes of failure (supported by the experiments of VÁSÁRHELYI (1998)) [8] we can get that τ_s linearly depends on the σ_s starting normal stress

$$\tau_s = \sigma_s \tan(\phi_\mu + \nu) \tag{6}$$

In case of planar surface $\nu = 0$ and the shear stress – normal stress curve is equal to the failure envelope ($\tan\gamma = \tan\phi_\mu$) thus $\tau_s = \sigma_s \tan\gamma$.

Eq. (5) can be reduced with *Eqs.* (1) and (2). If the starting normal stress is below the transition stress ($\sigma_s < \sigma_T$) the rate of the dilation is equal (or nearly equal) to the teeth angle ($\nu = i$):

$$\tau = \sigma_s(\tan(\phi_\mu + i) - \tan\gamma) + \sigma_{nx}\tan\gamma. \tag{7}$$

If the starting normal stress is equal or above the transition stress

$$\tau = c + \sigma_s(\tan\phi_\mu - \tan\gamma) + \sigma_{nx}\tan\gamma, \tag{8}$$

where the material constants are the same as in case of *Eq.* (2).

4.2. Non-linear Model

The previous linear model is not entirely satisfactory because according to *Eq.* (5) in case of large normal stresses the shear stress can go below the residual strength (see *Fig. 6*). Therefore a nonlinear modification of the linear equation above should satisfy the following requirements

- the curve starts from the same point as the linear one, that is $\tau(\sigma_s)=\tau_s$;
- the slope of the curve at $\sigma_{nx} = \sigma_s$ equals to $\tan\gamma$;
- the asymptote at $\sigma_{nx} \to \infty$ is the residual strength envelope ($\sigma_{nx}\tan\phi_\mu$);
- the tangent of the curve is positive.

In the spirit of Jaeger equation (4) we can suggest an exponential interpolation and the following non-linear equation to get a correct asymptotic behaviour

$$\tau = (\tau_s - \sigma_s \tan\phi_\mu e^{b(\sigma_{nx}-\sigma_s)} + \sigma_{nx}\tan\phi_\mu, \tag{9}$$

where $b = \dfrac{\tan\gamma - \tan\phi_\mu}{\tau_s - \sigma_s \tan\phi_\mu}$ and τ_s is given in *Eq.* (6). The linear approximation of *Eq.* (5) gives back *Eq.* (5) for small normal stresses and goes to the residual strength envelope ($\sigma_{nx}\tan\phi_\mu$) if σ_{nx} tends to infinity, for large normal stresses. Let us remark that *Eq.* (9) does not contain any parameters to adjust. *Fig. 6* compares the measured curve in case of 2.0 N/mm^2 starting normal stress, the linear and the non-linear equations. Let us remark that the non-linear *Eq.* (9) does not consider the concave part of the normal stress - shear stress curve, which is typical if the starting normal stress is under the critical value, therefore the initial slope of the non-linear curve on *Fig. 5* is under the initial slope of measured line.

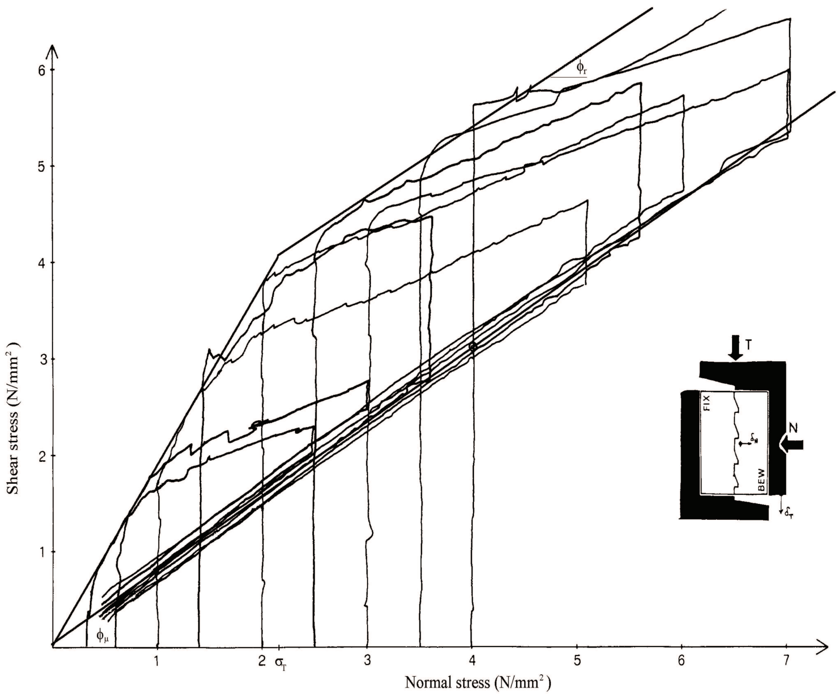

Fig. 5. Measured shear-stresses as functions of the continuously increasing normal stress

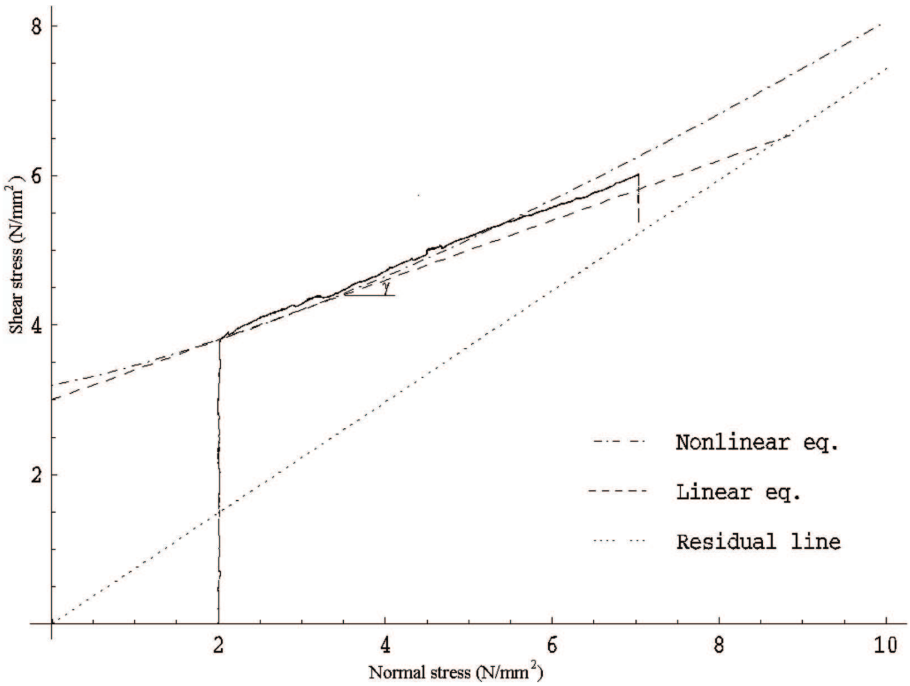

Fig. 6. Comparing the linear equation, the non-linear equation and the measured normal stress-shear stress curve in case of 2.0 N/mm^2 starting normal stress.

5. Conclusion

Shearing tests of specimen with triangular teeth were carried out similarly to the Continuous Failure State (CFS) triaxial test with different starting constant normal stress. The measured shear stress–normal stress curves were analyzed where the normal stress was increased continuously from the maximal shear stress. In this case, according to the measured results, the specimen is in "Continuous Sliding State" (CSS). The CSS is not equal to the "Continuous Failure State", as it was supposed before TISA and KOVÁRI, [8] thus the traditional CFS-triaxial testing-method can be not used for shearing tests.

The slopes of the measured shear stress – normal stress curves are independent on the starting constant normal stress. Both linear and non-linear equations were suggested to explain the experimental results. The slope of this line depends on the roughness, on the mechanical behavior of the rock and on the ratio of the shear and normal stress rate.

6. Acknowledgement

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