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CONVERSION BETWEEN AUSTRIAN AND HUNGARIAN MAP PROJECTION SYSTEMS

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Abstract

Conversion between Austrian and Hungarian map projection systems is presented here. The conversion may be performed in two steps: first any kind of map projection systems should be transformed into WGS-84 ellipsoidal co-ordinates in one country, and then from WGS-84 ellipsoidal co-ordinates should be transformed into the desired system for the other country. A computer programme has been developed to carry out all the possible transformations between the two countries. Using our method and software the transformation between Austrian and Hungarian map projection systems can be performed with a few centimeters accuracy for a few ten kilometers range of common border.

Keywords: map projection systems, transformation, WGS-84 ellipsoidal co-ordinates, GPS, Gauss-Krüger projection, conversion between Austrian and Hungarian systems.

1. Introduction

Map projection systems and their reference surfaces, as well their triangulation networks differ in each countries. Conversions between countries are necessary if somebody wants to use the own special map projection system in the neighbouring country.

It is possible to make exact conversions between two map projection systems with closed mathematical expressions in cases only when both projection systems have the same reference surface and points of the same triangulation network coming from the same adjustment, represented in both projection systems.

A more precise and secure conversion can be made using the so-called *mixed method,* when the transformation can be performed in two steps: first the distortions of projection and then the discrepancies of triangulation networks can be eliminated. In the first step we suppose that the two map projection systems have the same reference surface and the same triangulation network, and we perform the computation by the *co-ordinate method* using closed mathematical expressions (VARGA 1986). So in the first step we get approximated plane co-ordinates in the second projection system. Then in the second step we perform a transformation by polynomials using common points. The common points for determining the coefficients of these transformation polynomials should be the points which have both the previously computed approximated values and the original plane co-ordinates in the second projection system. We can use transformation polynomials having lower degrees in the second step of transformation to eliminate discrepancies of the different triangulation networks, unlike the case when we do the conversion in only one step using power series.

2. Conversion between Austrian and Hungarian Systems

Conversion between Hungarian and Austrian map projection systems can not be executed by the *co-ordinate method* using closed mathematical expressions because the position and orientation of the reference surfaces are slightly different, and the triangulation networks had been adjusted one by one – although there is the Bessel's ellipsoid as a reference surface of projection systems which is applied in Hungary and Austria too, and there are some common points of different triangulation networks. So the conversion between the two countries can only be performed by transformation polynomials using common points.

Map projection systems of neighbouring countries can be generally expanded only for a few ten kilometers range from the common border because common points can always be found only in this region. GPS is the most powerful tool for making common points anywhere, because determining of X, Y, Z spatial geocentric Cartesian, or WGS-84 co-ordinates of points of triangulation network, we can create such a system of common points which is very suitable for conversion of map projection systems between the countries.

Having enough common points made by GPS makes it possible to make a conversion between map projection systems of Hungary and Austria. So it is all the same, to transform co-ordinates between map projection systems of Hungary and Austria with different reference surfaces (Bessel's ellipsoid in Austria, and Bessel's, Krassovky's or IUGG-67 ellipsoids in Hungary) and different meridian of origin (prime meridian of Ferro for Austria and prime meridian of Greenwich for Hungary).

Transformations between all existing Hungarian map projection systems were completed earlier (VÖLGYESI at all, 1996) and there are very precise transformations from all Hungarian map projection systems into WGS-84 or X, Y, Z spatial geocentric Cartesian systems (VÖLGYESI, 1997). If we want to convert co-ordinates between Hungary and Austria, the next important task is to make transformations between WGS-84 and the other map projection systems used in Austria.

3. Practical Solution

Conversion between co-ordinates in *Table 1* is performed by the conversion programme in the area of Hungary and Austria in 213 combinations as they are enlisted in *Table 2*.

Table 1. Hungarian and Austrian map projection systems

VTN	System without projection in Hungary
BES	Bessel's Ellipsoidal
SZT	Budapest Stereographic Projection
KST	Hungarian Military Stereographic Projection
HER	Hungarian North Cylindrical System
HKR	Hungarian Middle Cylindrical System
ABE	Austrian Bessel's Ellipsoidal
AGK	Austrian Gauss-Krüger Projection
IUG	Hungarian IUGG-67 Ellipsoidal
EOV	Hungarian Unified National Projection
KRA	Hungarian Krassovsky's Ellipsoidal
GAK	Hungarian Gauss-Krüger Projection
WGS	WGS-84 Ellipsoidal /GPS/
XYZ	Spatial Cartesian Geocentric /GPS/
UTM	Universal Transverse Mercator

South cylindrical projection system (HDR) and Budapest city stereographic projection (VST) are not to be found on the above list because the regions where these two Hungarian map projection systems are used, are not neighbouring to Austria and using these two systems there is no practical need to make conversion between Hungary and Austria.

Table 2 conveys us information on the possibility and accuracy of conversions very simply.

Double lines in *Table 2* separate map projection systems belonging to different reference surfaces. (By reference surface the ellipsoid is meant, though the fact should be acknowledged that the approximating /Gaussian/ sphere serves also as a reference surface for those map projection systems where a double projection is applied and an intermediate sphere is the reference surface at the second step of the projection to get co-ordinates on a plane or on a plane developable surface. Co-Ordinates on this approximating sphere have no practical role for users.)

Plus "+" signs at the intersection fields of rows and columns indicate that an exact conversion between the two map projection systems is possible using closed mathematical formulas found in reference works of (Hazay, 1964) and (Varga, 1981, 1986) for transformation. In this case the accuracy of transformed co-ordinates is the same as the accuracy of co-ordinates to be transformed.

Table 2. Combination of transformations

	VTN	BES	SZT	KST	HER	HKR	ABE	AGK	IUG	EOV	KRA	GAK	WGS	XYZ	UTM
VTN	–	×	×	×	×	×	×	×	×	×	×	×	×	×	×
BES	×	–	+	+	+	+	×	×	×	×	×	×	×	×	×
SZT	×	+	–	+	+	+	×	×	×	×	×	×	×	×	×
KST	×	+	+	–	+	+	×	×	×	×	×	×	×	×	×
HER	×	+	+	+	–	+	×	×	×	×	×	×	×	×	×
HKR	×	+	+	+	+	–	×	×	×	×	×	×	×	×	×
ABE	×	×	×	×	×	×	–	+	×	×	×	×	×	×	×
AGK	×	×	×	×	×	×	+	!+!	×	×	×	×	×	×	×
IUG	×	×	×	×	×	×	×	×	–	+	×	×	×	×	×
EOV	×	×	×	×	×	×	×	×	+	–	×	×	×	×	×
KRA	×	×	×	×	×	×	×	×	×	×	–	+	×	×	×
GAK	×	×	×	×	×	×	×	×	×	×	+	!+!	×	×	×
WGS	×	×	×	×	×	×	×	×	×	×	×	×	–	+	+
XYZ	×	×	×	×	×	×	×	×	×	×	×	×	+	–	+
UTM	×	×	×	×	×	×	×	×	×	×	×	×	+	+	!+!

Cross "×" signs in *Table 2* indicate the impossibility of transformation between the two map projection systems with closed mathematical formulas and the conversion – according to rules found in [2] is performed using polynomials as of a finite (maximum five) degree with limited accuracy (VÖLGYESI at all, 1996; VÖLGYESI, 1997).

Minus "–" signs in *Table 2* are reminders of the fact that an identical (transformation into itself) conversion has no meaning except the Gauss-Krüger and UTM projection systems where the need of conversion between different zones frequently arises. Hence a "!+!" sign indicates that it is possible to make exact conversions between different zones of the Gauss-Krüger and UTM map projection systems.

The conversion logic between the different map projection systems can be overviewed on *Fig. 1*.

Transformation paths - and their directions - between different systems are pictured by arrows. It can be seen that it is possible to convert between both WGS-84 ↔ Unified National Projection (EOV) and WGS-84 ↔ Gauss-Krüger systems only through other intermediate systems. E.g. if a conversion between WGS and EOV systems is needed, then WGS-84 co-ordinates first have to be converted into a so-called auxiliary system (AUX) and finally they should be converted from this AUX system into EOV co-ordinates; or e.g. if a conversion between GAK and WGS systems is needed, then Gauss-Krüger co-ordinates first have to be converted into an auxiliary system (AUX) and finally they should be converted into the WGS-84 ellipsoid.

If any two systems in *Fig. 1* are connected through a hexagonal block, then between these two systems only an approximately accurate conversion could be

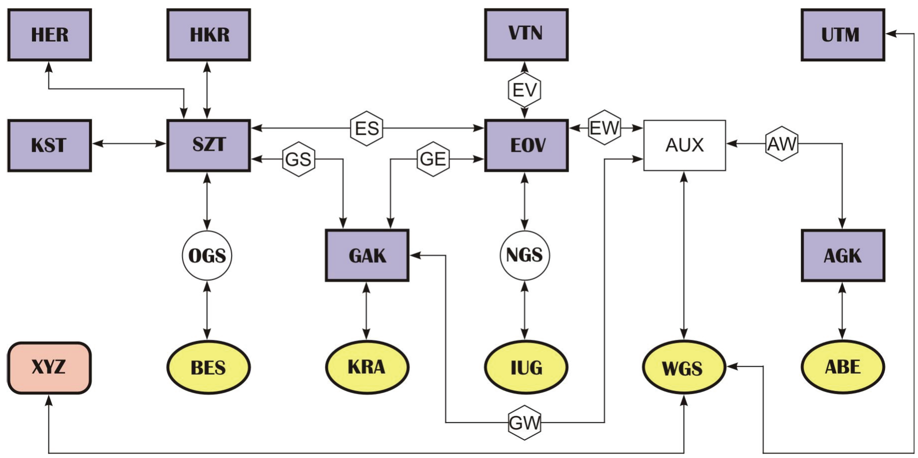

Fig. 1. Conversion flow between different map projection systems

made by transformation polynomials. In *Fig. 1* the two-letter abbreviations in hexagonal blocks show which data files, containing transformation polynomials, have to be used to convert between the two neighbouring systems. If any two systems in *Fig. 1* are connected by a continuous line, then an exact conversion by the co-ordinate method, i.e. through closed mathematical expressions can be made.

Since it may cause problems even for experts to apply correct methods of conversion between a multitude of map projection systems, we worked out such a software by which conversions can be made between Hungarian and Austrian map projection systems and their reference co-ordinates in all combinations, the usage of which can cause no problem even for users having no deep knowledge in map projections.

4. Initial Data

In cases of any two systems in *Fig. 1* connected through a hexagonal block the conversion could only be made by transformation polynomials using common points, e.g. in the case of the Austrian Gauss-Krüger and Spatial Cartesian Geocentric /XYZ/ or WGS-84 systems.

Between the Austrian Gauss-Krüger and Spatial Cartesian Geocentric /XYZ/ systems 64 common points were used to determine the coefficients of transformational polynomials for the complete area of Austria. The X, Y, Z Spatial Cartesian Geocentric co-ordinates supplied by GPS measurements refer to *ITRF94* (for 1993 epoch).

5. Transformation between WGS-84 and Austrian Gauss-Krüger Systems

A simple conversion is possible by closed mathematical formulas, between Spatial Cartesian Geocentric (XYZ) and WGS-84 systems. GPS can provide both XYZ and WGS-84 co-ordinates. Transformation between WGS-84 and Austrian Gauss-Krüger systems can be completed in two steps: first WGS-84 co-ordinates have to be converted into an auxiliary plain system (AUX), and the next step is the conversion from this auxiliary plain system into the Austrian Gauss-Krüger system using polynomials - as it can be seen in *Fig. 1*. The first step can be computed by simple closed mathematical formulas (VARGA, 1986), but the second step can be completed by maximum five-order polynomials depending on the number of common points [2]. For example, the connection between x, y co-ordinates of the projection system I. and x', y' co-ordinates of the projection system I. is established by the polynomials

$$\begin{aligned} x' = {} & A_0 + A_1x + A_2y + A_3x^2 + A_4xy + A_5y^2 + A_6x^3 + A_7x^2y + A_8xy^2 \\ & + A_9y^3 + A_{10}x^4 + A_{11}x^3y + A_{12}x^2y^2 + A_{13}xy^3 + A_{14}y^4 + A_{15}x^5 \\ & + A_{16}x^4y + A_{17}x^3y^2 + A_{18}x^2y^3 + A_{19}xy^4 + A_{20}y^5 \end{aligned} \tag{1.a}$$

$$\begin{aligned} y' = {} & B_0 + B_1x + B_2y + B_3x^2 + B_4xy + B_5y^2 + B_6x^3 + B_7x^2y + B_8xy^2 \\ & + B_9y^3 + B_{10}x^4 + B_{11}x^3y + B_{12}x^2y^2 + B_{13}xy^3 + B_{14}y^4 \\ & + B_{15}x^5 + B_{16}x^4y + B_{17}x^3y^2 + B_{18}x^2y^3 + B_{19}xy^4 + B_{20}y^5 + \dots \end{aligned} \tag{1.b}$$

Coefficients $A_0 - A_{20}$ and $B_0 - B_{20}$ (altogether 42 coefficients) can be determined by using common points suitably through an adjustment process.

An important question is to determine the optimal degree of the polynomial. By considering a simple way of reasoning one could arrive at the conclusion that the higher the degree of the polynomial, the higher the accuracy of the map projection conversions. Quite the opposite, it could be proved by our tests that the maximum accuracy was resulted by applying five-degree polynomials. No matter whether the degree was decreased or increased, the accuracy of transformed co-ordinates was lessened alike (more considerably by decreasing, less considerably by increasing - while the biggest discrepancies could be found at the edges of the networks).

6. Accuracy of Conversion

It is possible to convert through closed mathematical expressions between certain map projection systems. In these cases the accuracy of transformed plane co-ordinates is equal to the accuracy of initial co-ordinates (1 mm or 0.0001"). These conversions are referred to in *Table 2* with "+" and "!+!" signs or these systems are connected by continuous lines (arrows) in *Fig. 1*.

In all other cases when the transformation path between any two systems passes through a hexagonal block (or blocks), the accuracy of transformed co-ordinates depends on, on the one hand, how accurately the control networks of these systems fit into each other; and on the other hand, how successful the determination of transformation polynomial coefficients was. It follows also from these facts that no matter how accurately these transformation polynomial coefficients were determined, if the triangulation networks of these two systems do not fit into each other accurately – since there were measurement, adjustment and other errors during their establishment − then certainly no conversion of unlimited accuracy can be performed (in other terms, conversions between two map projection systems can only be accurate to such an extent that is allowed by the determination errors or discrepancies of these control networks). This fact, of course, does not mean that you, should not be very careful when the method of transformation is selected or – when the polynomial method is applied – the coefficients are determined.

So accuracy of transformation can be described by the following logic: Coefficients of transformation polynomials (1) should be first computed based on co-ordinates of common points y_i , x_i and y_i', x_i' in systems *I* and *II*, respectively. Then y_i , x_i co-ordinates in system *I* can be transformed into co-ordinates ty_i' , tx_i' in system *II* by using these coefficients. Finally, the standard error characteristic to conversion,

$$\mu = \sqrt{\frac{\sum_{i=1}^{n}(ty_i' - y_i')^2 + \sum_{i=1}^{n}(tx_i' - x_i')^2}{n}} \tag{2}$$

can be determined, where

$$\begin{aligned} \Delta y_i &= ty_i' - y_i' \\ \Delta x_i &= tx_i' - x_i'. \end{aligned} \tag{3}$$

Using polynomial method and applying expression (2) standard errors are summarized between Hungarian systems for the complete area of Hungary in *Table 3*.

With a view to transformation between Austrian and Hungarian map projection systems the two most important Hungarian transformations are EOV–WGS-84 and the Hungarian Gauss-Krüger-WGS-84. The contour line map of standard errors defined by Eq. (2) for these two systems can be seen in *Fig. 2* and *Fig. 3*, respectively.

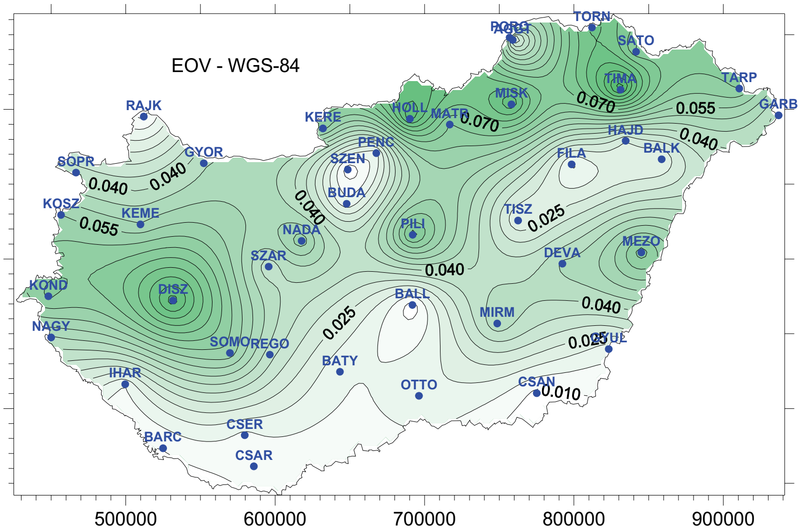

Fig. 2. Standard errors of EOV-WGS-84 transformation (contour labels in [m])

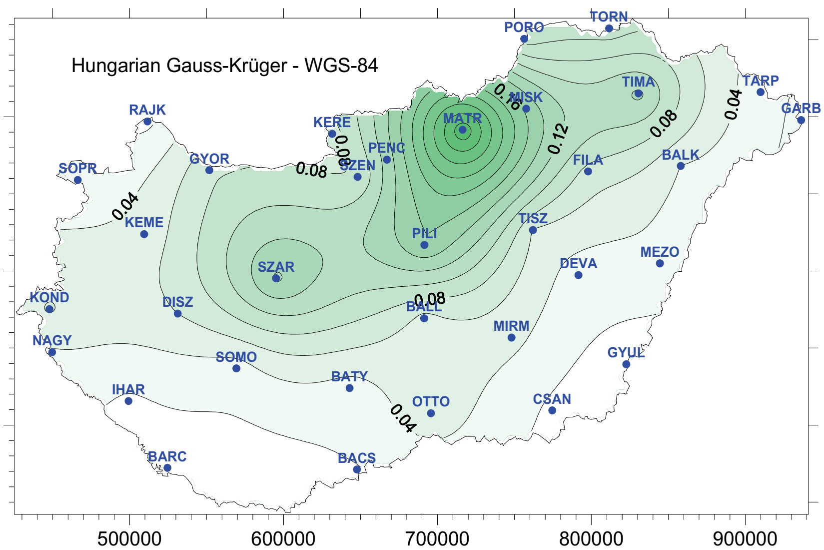

Fig. 3. Standard errors of Gauss-Krüger-WGS-84 transformation (contour labels in [m])

Table 3. Standard errors of polynomial method

Hungarian systems	Number of common points	Standard error
EOV–SZT	162	±0.247m
EOV–WGS	43	±0.050m
EOV–GAK	79	±0.102m
EOV–VTN	27	±0.046m*
GAK–WGS	34	±0.084m
GAK–SZT	184	±0.046m

* valid only for territory *Baranya*

Our experience shows that although the accuracy can be somewhat increased by increasing the number of common points within the polynomial method, the accuracy of conversion can not be increased beyond a certain limit even with this method since there is a difference between the two triangulation networks. In certain cases, however, an improvement could be gained when transformation polynomial coefficients are not determined for the complete area of the country but only for a smaller region common points are given and transformation polynomial coefficients are determined. In such cases conversions, of course, must not be made outside the sub-area where the coefficients of transformation polynomials were determined, and the junction of these regions is not a simple problem.

The next question is the accuracy of the transformation between the Austrian Gauss-Krüger and WGS-84 systems. We summarized the results of our test computations in *Table 4*. There are Δy and Δx differences between the original and the transformed co-ordinates in five different versions for each common point computed by (3), and the standard error characteristic to different versions of conversion computed by (2) is in the last row of *Table 4*.

In the case of *version 1* all the given 64 common points between the Austrian Gauss-Krüger and WGS-84 systems were used for the complete area of Austria for determining the coefficients of transformation polynomials (1). Using these coefficients, WGS-84 co-ordinates were transformed into Gauss-Krüger system, and the differences of the original and the transformed Gauss-Krüger co-ordinates are listed in the 2nd and 3rd columns of *Table 4*. There are surface views of these differences in *Fig. 4*, the 'surface heights' are $\sqrt{\Delta y^2 + \Delta x^2}$ in the figure. There are 3 points (*EBRI*, *LEND* and *OBWG*) in which very big errors (a few hundred meters differences) can be found. The standard error characteristic to transformation of *version 1* is ±68.328 m. Probably the GPS stations were not set up correctly to the places where Gauss-Krüger co-ordinates are referred. So these three points were cancelled from the next versions of computations.

In *version 2* the remaining 61 common points were used for determining the coefficients. Using these values for transformation the differences of the co-

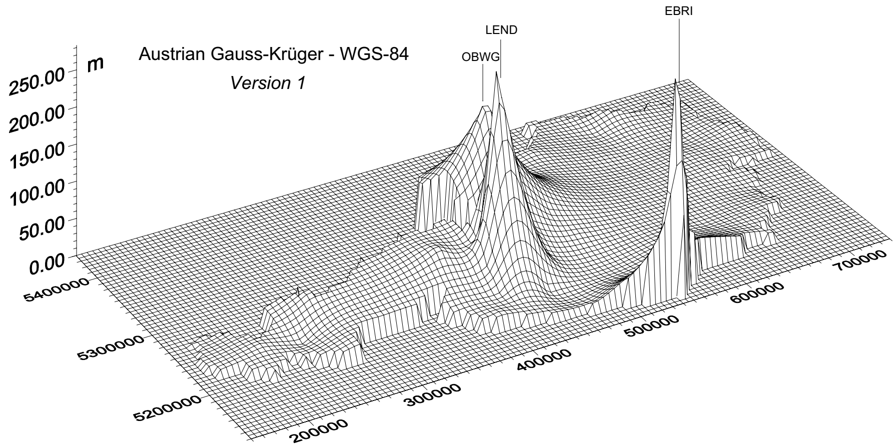

Fig. 4. Surface view of differences between the original and the transformed co-ordinates in *version 1* of the Austrian Gauss-Krüger-WGS-84 transformation

ordinates are listed in the 4th and 5th columns of *Table 4* and the surface view of these differences can be seen in *Fig. 5*. In *version 2* there is 1 point (*ASTN*) in which a too big error, *50 m* difference can be found. The standard error of *version 2* is ±11.616 m. So the point *ASTN* was cancelled from the next versions of computations.

In *version 3* the remaining 60 common points were used for determining the coefficients. Using these coefficients for transformation the differences of co-ordinates are listed in the 6th and 7th columns of *Table 4* and the surface view of these differences can be seen in *Fig. 6*. In *version 3* there was 1 point (*GUBG*) in which nearly 15 m difference could be found. The standard error of *version 3* is ±2.530 m. So this point was canceled from the next versions of computations.

In *version 4* the remaining 59 common points were used for determining the coefficients. Using these coefficients for transformation the differences of co-ordinates are listed in the 8th and 9th columns of *Table 4* and the surface view of these differences can be seen in *Fig. 7*. In *version 4* there were 2 points (*HAID* and *TEIA*) in which a few meters differences could be found, and the standard error of *version 4* was ±1.251 m. These two points were cancelled from the last version of computations.

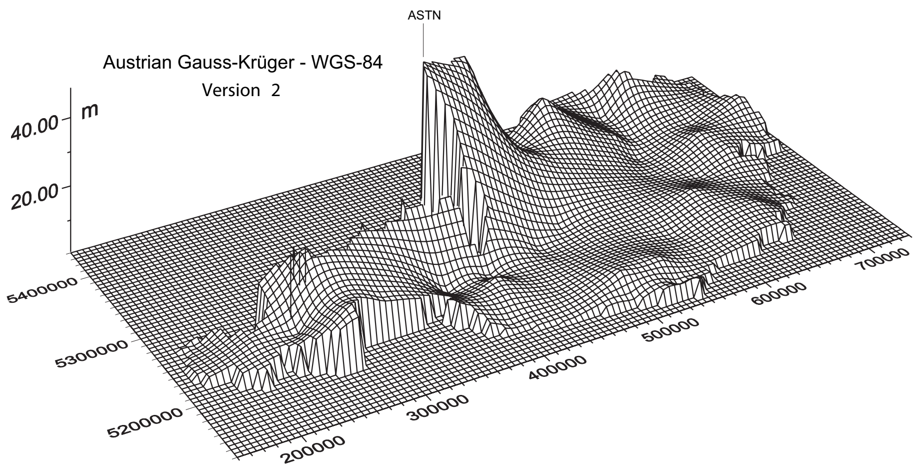

Fig. 5. *S*urface view of differences between the original and the transformed co-ordinates in *version 2* of the Austrian Gauss-Krüger-WGS-84 transformation

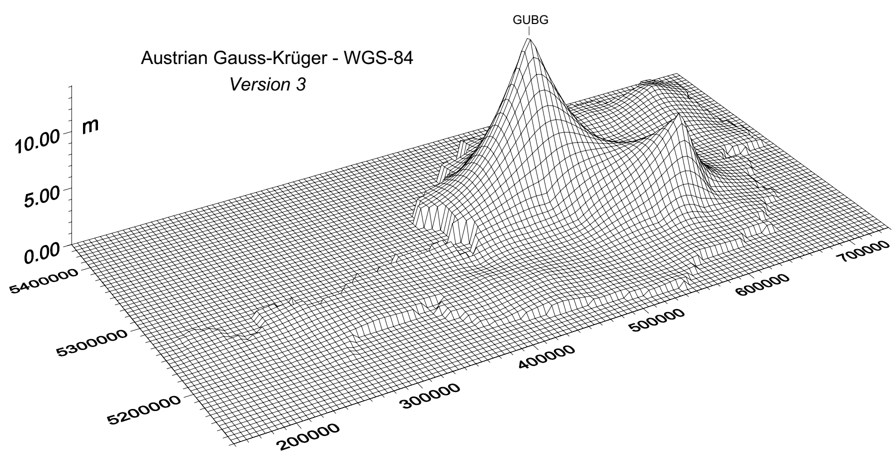

Fig. 6. Surface view of differences between the original and the transformed co-ordinates in *version 3* of the Austrian Gauss-Krüger-WGS-84 transformation

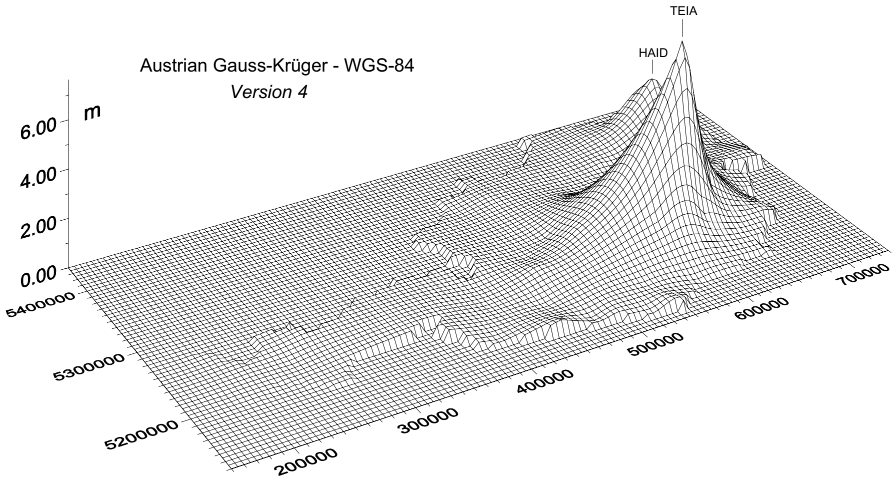

Fig. 7. Surface view of differences between the original and the transformed co-ordinates in the *version 4* of the Austrian Gauss-Krüger-WGS-84 transformation

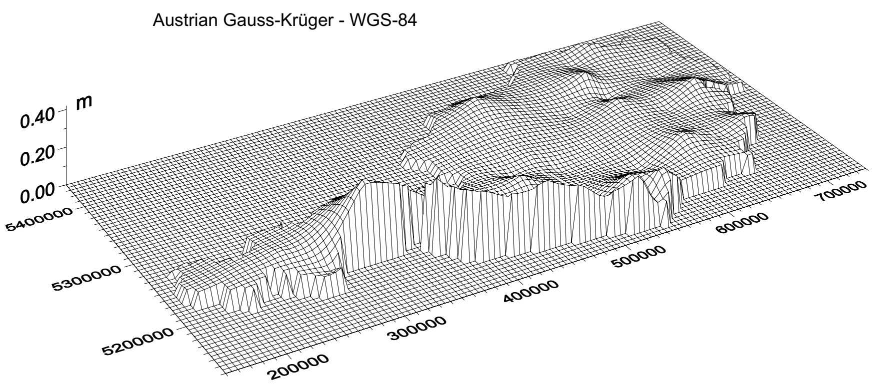

Fig. 8. Surface view of differences between the original and the transformed co-ordinates in *version 5* of the Austrian Gauss-Krüger-WGS-84 transformation

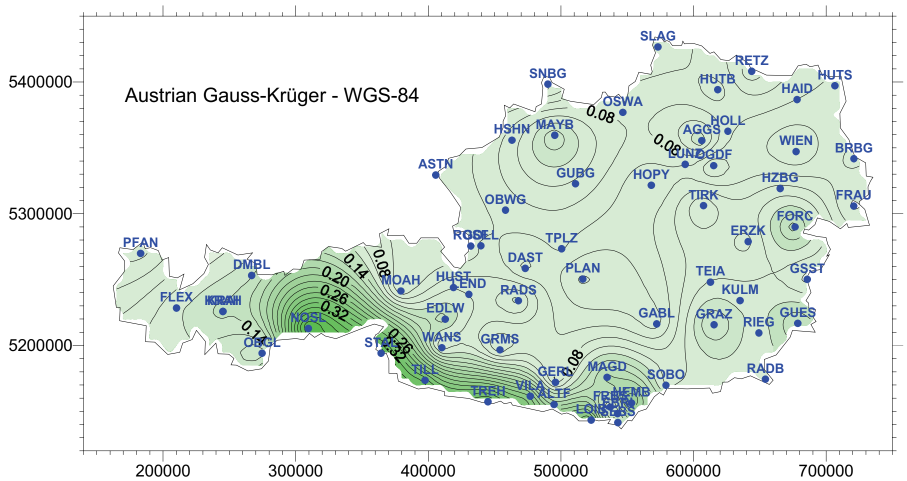

Fig. 9. Standard errors of Austrian Gauss-Krüger-WGS-84 transformation (contour labels in [m])

In the case of *version 5* the remaining 57 common points were used for determining the coefficients of transformation polynomials. Using these coefficients, for the complete area of Austria, in general, a few centimeters, maximum 4 decimeters differences could be found, and the standard error of transformation was ± 0.152 m between the Austrian Gauss-Krüger and WGS-84 systems. The differences of the co-ordinates are listed in the 10th and 11th columns of *Table 4*. The surface view and the contour line map of these differences can be seen in *Fig. 8* and *Fig. 9* respectively.

Using the coefficients of transformation polynomials of *version 5*, the discrepancies of the original and the transformed Gauss-Krüger co-ordinates of the seven cancelled common points are listed in *Table 5*.

The gross errors in the first four points (*EBRI, LEND, OBWG, ASTN*) indicate that the GPS stations were not set up correctly to the places where Gauss-Krüger co-ordinates are referred, and it may be justified to omit them from the common points.

The explanation of discrepancies in the remaining three points in *Table 5* is uncertain. It may be important to investigate whether the problems are local or refer to bigger surroundings of points *GUBG, TEIA* and *HAID* - so the GPS measurements should be controlled and repeated here.

If the problem is local, the reason might be the same as in the case of the first four points, and it may be justified to omit them from the common points too - or to replace them by exact new values.

If the problems refer to bigger surroundings of these three points, the reason might come from not too precise earlier triangulation measurements and/or wrong

Table 4. Δy and Δx differences between the original and the transformed co-ordinates in transformation.

	Version 1		Version 2		Version 3		Version 4		Version 5	
Point	**Δ y**	**Δ x**	**Δ y**	**Δ x**	**Δ y**	**Δ x**	**Δ y**	**Δ x**	**Δ y**	**Δ x**
AGGS	−5.207	4.165	−3.633	6.321	−0.579	−0.295	0.376	0.256	0.108	0.089
BRBG	−6.819	12.798	−4.732	10.556	0.043	0.212	0.010	0.193	−0.059	−0.029
ERZK	0.736	−7.803	−0.093	−2.773	−1.514	0.304	−1.748	0.169	−0.046	0.054
FORC	7.819	−5.195	2.906	−6.926	−0.515	0.484	−0.948	0.234	−0.006	0.171
FRAU	6.314	−1.885	1.786	−3.630	0.332	−0.482	0.613	−0.320	0.028	−0.022
GRAZ	−4.945	4.354	−3.388	5.033	−1.166	0.219	−1.380	0.095	0.109	−0.128
GSST	−0.263	−1.652	0.329	−1.654	−0.368	−0.145	−0.313	−0.113	−0.045	−0.057
GUES	−11.192	−3.557	−0.656	1.775	0.242	−0.171	0.680	0.081	0.139	0.083
HAID	2.387	−6.658	4.778	−7.263	2.256	−1.800	0.904	−2.580	–	–
HOLL	−3.696	0.008	−0.671	3.013	0.456	0.573	0.256	0.457	0.063	0.021
HUTB	−2.492	1.381	−0.822	4.500	0.716	1.169	−0.313	0.576	−0.033	0.033
HUTS	−3.814	3.785	−4.268	5.488	−1.750	0.034	−0.394	0.816	0.045	0.034
HZBG	5.299	−7.787	3.811	−7.711	0.034	0.470	−0.474	0.177	0.032	−0.081
KULM	−5.728	2.507	−3.557	4.856	−1.456	0.305	−1.800	0.107	−0.030	−0.107
LUNZ	−3.894	4.493	−4.432	4.425	−1.783	−1.314	0.273	−0.128	0.027	−0.051
OGDF	−2.425	−2.234	−1.503	0.564	−0.996	−0.535	0.088	0.090	−0.011	−0.010
RADB	11.245	−1.456	2.479	−4.221	0.480	0.109	0.118	−0.100	−0.068	−0.072
RETZ	11.835	−14.820	7.900	−15.073	0.487	0.985	−0.246	0.563	−0.089	−0.069
RIEG	−4.770	9.798	−3.473	5.625	−0.884	0.018	−0.554	0.209	−0.078	0.072
SLAG	−3.826	11.965	−4.148	7.290	−0.472	−0.672	0.190	−0.290	0.047	0.007
TEIA	3.054	−6.412	7.154	2.046	8.398	−0.650	7.881	−0.949	–	–
TIRK	−1.868	−5.679	−1.814	−1.327	−2.074	−0.763	−0.841	−0.052	−0.122	−0.014
WIEN	1.318	−2.255	3.533	−3.912	1.137	1.278	−0.050	0.594	0.011	−0.013
ALTF	20.783	12.659	0.752	−1.984	0.000	−0.356	0.284	−0.192	−0.014	−0.188
ASTN	−14.986	64.319	−20.994	45.474	–	–	–	–	–	–
DAST	10.985	41.737	−1.093	−0.392	−1.086	−0.406	−0.283	0.057	0.023	0.025
EBRI	−276.222	−217.304	–	–	–	–	–	–	–	–
EDLW	8.759	40.935	−1.969	5.797	0.614	0.201	0.062	−0.118	−0.045	−0.145
FRBS	47.547	35.478	0.743	−1.740	0.077	−0.295	0.400	−0.109	0.089	−0.043
GABL	−7.337	−6.390	−1.309	1.542	−0.905	0.665	−1.644	0.239	0.020	0.018
GERL	6.382	−6.952	3.329	−6.577	0.337	−0.096	0.344	−0.092	0.036	−0.044
GOLL	25.205	50.856	4.215	−13.209	−1.445	−0.948	0.336	0.080	−0.002	0.065
GRMS	−3.47	1.010	1.514	−1.297	0.695	0.475	−0.112	0.010	−0.038	0.008
GUBG	25.054	38.135	12.896	7.122	12.781	7.370	–	–	–	
HEMB	48.714	36.038	0.844	−2.321	−0.192	−0.076	0.202	0.151	−0.093	0.239
HOPY	−2.026	11.027	−5.740	3.999	−2.924	−2.101	0.170	−0.317	0.017	−0.034
HSHN	22.765	−28.274	16.657	−40.446	−1.582	−0.940	0.121	0.042	0.002	0.089
HUST	18.270	56.332	−0.152	0.529	0.027	0.141	0.071	0.167	−0.074	0.066
LEND	−75.005	−274.003	–	–	–	–	–	–	–	–
LOIB	58.347	54.196	−2.034	5.042	0.288	0.012	0.073	−0.112	0.244	−0.171
MAGD	13.985	0.638	2.949	−6.055	0.154	−0.002	0.307	0.086	0.152	0.110
MAYB	6.714	3.420	0.761	−9.518	−2.726	−1.965	−0.020	−0.405	−0.016	−0.157
MOAH	21.044	57.190	0.238	0.387	0.382	0.075	0.359	0.062	0.028	−0.027
OBWG	−63.438	−172.519	–	–	–	–	–	–	–	–
OSWA	−10.794	19.040	−10.905	18.385	−1.971	−0.968	−0.399	−0.061	−0.051	0.056
PLAN	−1.958	14.444	−2.103	2.029	−1.199	0.072	−1.173	0.086	0.016	0.003
RADS	4.508	29.264	−0.665	1.998	−0.002	0.562	−0.573	0.233	−0.052	0.119
ROSF	27.099	50.343	5.512	−15.563	−1.251	−0.915	0.412	0.044	0.038	0.002
SEBS	76.846	69.086	−2.347	5.853	0.126	0.495	−0.493	0.138	−0.023	0.107
SNBG	−0.734	−16.842	4.014	−4.059	1.598	1.174	−0.058	0.219	−0.008	−0.004
SOBO	31.811	24.058	0.484	−2.563	−0.511	−0.408	0.059	−0.079	−0.088	−0.032
STAL	−1.955	15.111	−3.081	7.389	0.389	−0.127	0.475	−0.077	0.399	0.044
TILL	−14.967	−11.343	−0.556	1.191	−0.041	0.076	0.017	0.109	0.311	0.115
TPLZ	7.231	33.067	−2.600	−0.191	−2.142	−1.184	−0.354	−0.153	0.023	−0.060
TREH	−7.356	−6.040	−0.999	0.826	−0.696	0.168	−0.573	0.239	−0.336	0.138
VILA	5.886	−2.320	1.369	−3.535	−0.176	−0.189	0.015	−0.079	−0.255	−0.062
WANS	−1.300	16.044	−1.414	4.411	0.560	0.135	0.090	−0.136	−0.025	−0.074
DMBL	−1.571	−38.326	9.777	−19.402	0.570	0.539	−0.353	0.007	0.105	−0.085
FLEX	−2.687	16.559	−2.769	6.142	0.082	−0.033	0.093	−0.027	0.055	−0.102
KRAH	−2.837	−0.005	−2.512	5.179	−0.112	−0.020	0.027	0.060	0.014	0.142
KRAI	−2.813	0.013	−2.503	5.158	−0.117	−0.010	0.023	0.070	0.008	0.151
NOSL	5.528	24.739	−5.320	9.943	−0.512	−0.472	−0.158	−0.268	−0.413	−0.112
OBGL	6.798	−23.023	7.162	−15.510	−0.042	0.095	−0.111	0.055	−0.038	−0.072
PFAN	2.124	−0.255	0.369	−1.038	−0.074	−0.080	0.038	−0.015	−0.032	0.006
	± 68.328 m		*± 11.616 m*		*± 2.530 m*		*± 1.251 m*		***± 0.152 m***	

Table 5. Discrepancies of original and transformed Gauss-Krüger co-ordinates of the seven canceled common points

	$\Delta y\ [m]$	$\Delta x\ [m]$	$\sqrt{\Delta y^2 + \Delta x^2}$
EBRI	−316.45	−290.78	429.76
LEND	−65.29	−329.96	336.36
OBWG	−73.62	−212.24	224.65
ASTN	−76.20	136.73	156.53
GUBG	15.80	11.49	19.54
TEIA	10.16	−0.80	10.19
HAID	1.18	−3.53	3.72

adjustment of Gauss-Küger control network points.

In this case a denser net of common points should be made in the vicinity of few ten kilometers of points *GUBG, TEIA* and *HAID*, and it would be necessary to determine new coefficients of transformation polynomial for the surroundings of these 3 points one by one. So, the transformation for the whole country will not be damaged by the points *GUBG, TEIA* and *HAID*, but the co-ordinates could be transformed with a suitable accuracy in the vicinity of these points, at the same time, using the local coefficients of transformation polynomial.

Table 6. Accuracy of conversion in common points

AGK - WGS			**GAK - WGS**			**EOV - WGS**		
Point	Δy	Δx	Point	Δy	Δx	Point	Δy	Δx
FRAU	0.028	−0.022	RAJK	−0.016	0.003	RAJK	−0.001	0.008
FORC	−0.006	0.171	SOPR	0.022	−0.013	SOPR	−0.023	−0.029
GSST	−0.045	−0.057	KOND	−0.060	0.014	KOSZ	0.045	0.031
GUES	0.139	0.083				KOND	−0.046	−0.048
±0.124			±0.039			±0.045		

Concerning the transformation between Austrian and Hungarian map projection systems, there is a remarkable accuracy of conversion for a few ten kilometers range of the common border. Accuracy of the conversion between the two countries can be characterized based on the accuracy of the conversion of points in the vicinity of the common border. Accuracy of the conversion of common points next to the border is summarized in *Table 6*. It can be seen that mean error of the conversion between the Austrian Gauss-Krüger and WGS-84 systems based on 4 points next to the Hungarian border is ±0.124m, mean error of the conversion between the Hungarian Gauss-Krüger and WGS-84 systems based on 3 points next to the Austrian border is ±0.039m, and mean error of the conversion between Hungarian EOV and WGS-84 systems based on 4 points next to the Austrian border is ±0.045m. So the final conclusion may be that using our method and software for the given common points, the transformation between the Austrian and the Hungarian map projection systems can be performed with a few centimeters accuracy for a few ten kilometers range of the common border.

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