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COVERING GRANDSTANDS BY CABLE STRUCTURES

Lajos KOLLÁR

Budapest University of Technology and Economics
Department of Building Construction
H-1521 Budapest, POB. 91. Hungary

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Abstract

Grandstands can be expediently covered by cable structures which are light and do not transmit horizontal forces to the supports. The paper shows how its seemingly unstable nature turns out to be stable, after performing deformations larger than usual.

Keywords: grandstands, covering, cable structures, stability.

1. Introduction

Larger sport stadiums (e.g. football fields) are generally not completely covered. A roof only over the grandstands will be erected in order to protect the spectators from precipitation. This results in a ring-shaped structure shown in *Fig. 1*, which must not contain any columns along the inner periphery, which would disturb the free outlook of the spectators.

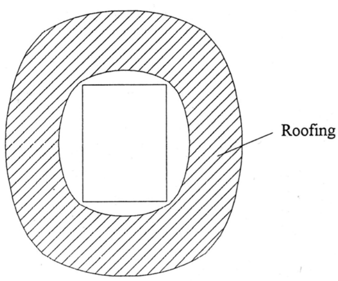

Fig. 1.

Thus, the structure can be supported along its outer periphery, resulting in a cantilever-like arrangement, protruding towards the playing field. However, if the structure consisted simply of a row of independent cantilevers, these would have rather large dimensions resulting in high expenses and clumsy appearance. Thus the spatial arrangement of the structure has to be taken into consideration in order to reduce the dimensions. By taking spatial action into account, a light structure can be built, where also cables can be used.

Most grandstand coverings erected so far have been built with the structure sketched in *Fig. 2* [1]. However, at first sight it is not quite clear how this structure works statically and why it is stable. Our engineering sense, educated on conventional structures, suggests that the two parts of this roof tilt to the inside and collapse.

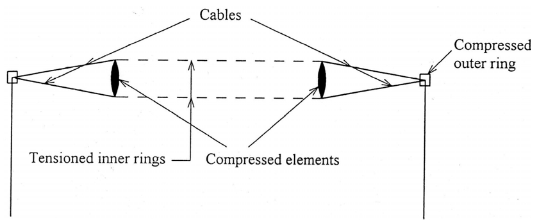

Fig. 2.

In the following we intend to give a clear explanation of the static behaviour of the roof structure shown in *Fig. 2* and to elucidate why it is stable.

2. Some Basic Remarks on the Static Behaviour of Cable Structures

Cable structures have to be prestressed in order to prevent slacking of the cables when external loads cause compressive forces in them. Consequently, prestressed cable structures can be treated as common trusses. (To enable prestressing, the truss has to be statically at least once indeterminate.) The main difference between the cable structure and the common truss is that from cables we can construct 'kinematically indeterminate' structures which become stable after performing large deformations. This phenomenon will be elucidated on a simple example.

Let us consider a single horizontal cable, stressed by a force H_0 (*Fig. 3*) between two fixed supports, subjected to a vertical load P. The structure can be considered as a 'truss' consisting of two bars of the length $l/2$, which is – according to the Theory of Structures – kinematically once indeterminate, since its middle point is capable to move vertically. We can write the equilibrium equation in the vertical direction on the deformed shape.

Due to the deflection the cable elongates. The new length $s/2$ of half of the cable is:

$$\frac{s}{2} = \sqrt{\left(\frac{l}{2}\right)^2 + w^2} \approx \frac{l}{2} * \left[1 + \frac{1}{2} * \left(\frac{w}{\frac{l}{2}}\right)^2\right]$$

where w is the deflection of the middle point.

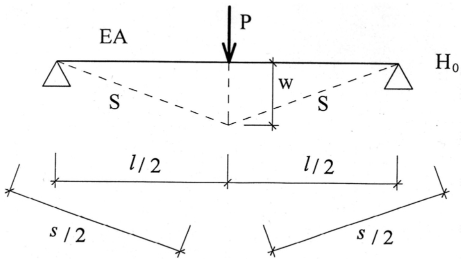

Fig. 3.

The second term in the square bracket is the elongation e, which, multiplied by the tesile stiffness EA of the cable, yields the cable force. Since the prestressing force H_0 also acts in the cable, the total cable force after deflection is:

$$S = 2 * EA * \frac{w^2}{l^2} + H_0$$

and the external load, P, is equilibrated by the vertical components of the two forces S arising in the cable:

$$P = 2 * S * \frac{w}{\frac{l}{2}} = 8 * EA * \frac{w^3}{l^3} + 4 * H_0 * \frac{w}{l}$$

The relationship between load P and the deflection w is depicted in *Fig. 4.* It can be seen that the prestressed straight cable has a finite initial stiffness (in the vertical direction) which – due to the elongation of the cable during deflection – gradually increases. As the two cable forces and their inclination increase, after a certain deflection they will equilibrate the external force P. This is the peculiarity of this kind of cable structures, as contrasted to other, common structures: every structure deforms under the applied loads, but other structures can equilibrate the loads in the undeformed state, while cable structures only in the deformed state.

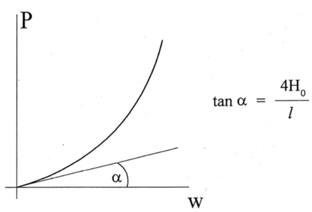

Fig. 4.

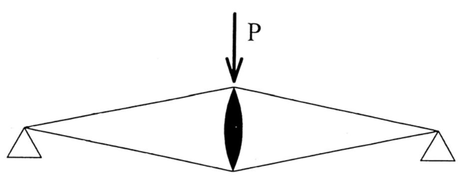

Fig. 5.

It can also be seen that the straight cable is not capable to equilibrate a fairly small external force either in its undeformed shape, although it has a finite initial stiffness in the direction of force P. Cable structures of this kind will thus be characterized by the fact that they ’change their shape’ (they can bear external loads only when changing shape), as contrasted e.g. with the structure shown in *Fig. 5*, which ’keeps its shape’, i.e. it behaves (when prestressed) as a common truss, and is able to carry loads also in its undeformed shape. (The bars which can withstand also compressive forces will be represented in the following as the middle vertical bar in *Fig. 5*.)

3. A Simple Plane Model of the Cable Structure Covering the Grandstand

Before analyzing the cable structure of the grandstand roof, let us investigate the model depicted in *Fig. 6*, which can also be considered as a simplified skeleton of the roof structure. Supports e and f are unmovable; the structure is stressed between them. This model either keeps or changes its shape, depending on whether it is subjected to symmetric or to antisymmetric loads.

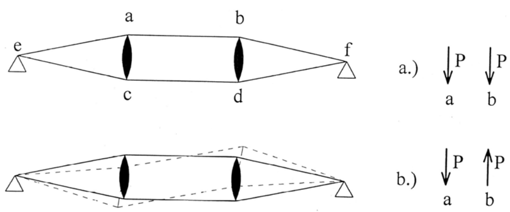

Fig. 6.

Let us first investigate the symmetric loads (*Fig. 6*) which act in points a and b. From the rectangle a, b, c, d of the structure a diagonal bar is ’missing’, consequently it is kinematically once indeterminate. However, from the symmetric loads no force would arise in this ’missing’ diagonal bar and, hence, it makes no difference whether this diagonal bar exists or not. The structure behaves as a common truss, keeping its shape.

The situation is different if in points a and b an antisymmetric load system acts (*Fig. 6b*). Now the diagonal bar would be needed, and since it is not there, the rectangle a, b, c, d distorts to a rhomboid.

In the following we assume the bars e-a, e-c, b-f, d-f as inextensible. Hence the distortion of the rectangle a, b, c, d to a rhomboid causes elongation only in bars ab and cd, which causes – like the structure shown in *Fig. 3* – steadily growing bar forces in these two bars, which become more and more inclined. This deformation continues until the increasing bar forces, becoming more and more inclined, get into equilibrium with the two applied forces P. Thus, under this loading case the structure changes its shape and after a suitably large deformation equilibrium is achived.

Essentially the same result will be obtained if bars *ea*, *ec*, *bf*, *df* are not considered as inextensible, with the difference that the deformation of the structure becomes larger.

4. The Behaviour of the Cable Structure of the Grandstands

After this preparation we can investigate the static behaviour of the grandstand roof (the cross-section of which is represented in *Fig. 2*) under symmetric and antisymmetric load. First let us assume that the ground plan of the roof has the shape of a circular ring (*Fig. 7*).

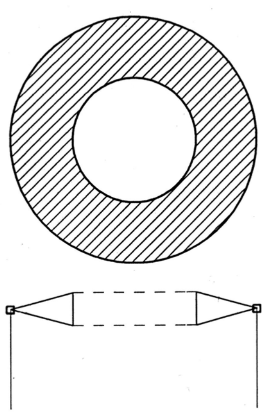

Fig. 7.

Under symmetric load the structure behaves as the model of *Fig. 6*: the role of bars *a*-*b* and *c*-*d* will be taken by the two inner rings, and the structure carries the load so that it keeps its shape. (Unlike *Fig. 6*, no fixed supports are needed, since the outer, compressed ring acts as a fixed support.)

The problem is caused by the antisymmetric load. (Asymmetric loads, e.g. snow load, can be decomposed, as usual, into a symmetric and an antisymmetric part.) The two inner rings then get inclined, 'points' *a*, *b*, *c*, *d* come to the position denoted in the figure by dashed lines and – if we consider the radial cables as inextensible – the inner rings are compelled to elongate in the same way as bars *a*-*b* and *c*-*d* of the model in *Fig. 6*. The structure thus carries the antisymmetric load by changing its shape. As a consequence, the radial cables also elongate (the outer, compressed ring can be considered as inextensible, since its cross-section is much larger than those of the cables). We emphasize that the essential fact is that the cables of the structure have to get inclined and, consequently, have to elongate during the deformation caused by the antisymmetric load. The arising forces, increasing and getting more and more inclined, will equilibrate the external loads after a certain deformation, as we have seen on the model in *Fig. 6*.

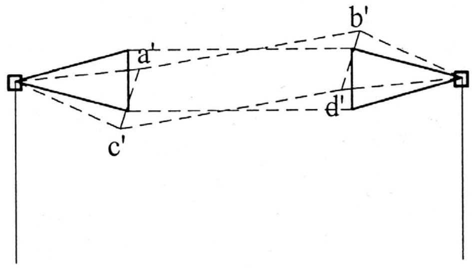

Fig. 8.

The above reasoning shows that the roofing structure (although not in its original shape, but in a deformed position) is capable to equilibrate the antisymmetric load. As a consequence, the cable structure undergoes much larger deformations than usual, however, in the case of a grandstand roof, this is acceptable.

The cable structure behaves essentially in the same way if its ground plan is not a circular ring, but has a 'distorted' shape, approaching a rectangle (Fig. 1). Some requirements have to be fulfilled (the contour cannot have straight sections; the shapes of the outer and inner rings have to be affine etc. [1]).

The deformations of the roof structure are much larger than those of a traditinal structure, so care should be taken with drainage: the rain water should not flow to the inside of the roof and on the spectators.

It has to be stressed that the structure does not transmit horizontal forces to the supporting columns because these forces equilibrate each other in any case.

5. Summary

This brief description shows why it is possible to erect a roof of a light cable structure over grandstands, which takes all the loads, does not transmit horizontal forces to the supporting columns, and exerts only vertical support forces.

References

[1] GÓPPERT, K.: Weitgespannte Leichtbaukonstruktionen für Sportstadien – Entwurf und Konstruktion. Berichte der Fachtagung Baustatik-Baupraxis 8, pp. 257-270. Herausgeber: D. Dinkler. Institut für Statik, Beethovenstr. 51, Braunschweig. (2002)