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MODELLING OF REINFORCED SOIL

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Abstract

Geosynthetics are being used increasingly to improve soil properties and to obtain better performance from soft or weak foundation sites. It is well-known that the overall stability is improved and that the movement of the structures founded on the reinforced soil reduces because of the mobilization of tensile forces in geosynthetics. This force is well quantified in the analysis of stability of structures constructed with reinforced soil. However, studies on modelling aspects are not many. In this paper, models are considered to quantify the effect of tensile force. Solutions for an embankment (strip) type of load are presented.

Keywords: reinforced soil, geosynthetics, mechanical subgrade models, tensile force, settlement.

1. Introduction

One of the fundamental problems in the analysis of shallow foundations is the estimation of settlements of the subgrade. So far a number of subgrade models have been developed for obtaining better approximations to the rational behaviour of foundations under static loads. A comprehensive review of all these subgrade models was given by KERR [4].

Two models have been chosen to the problem under investigation. In both of the cases geosynthetics are assumed smooth and hence under tension with certain force per unit length. All of the model parameters have been defined in terms of elastic parameters of the subgrade and the results are obtained in terms of non-dimensional quantities.

2. Mechanical Models

The simplest but most widely known subgrade model is the WINKLER (1867) model (*Fig. 1*). It assumes the subgrade as a series of linear, discrete, independent, closely spaced springs. The relation between applied surface pressure (q) and vertical surface deflection (w) can be expressed as

$$q = kw, \tag{1}$$

where $k =$ the modulus or coefficient of subgrade reaction.

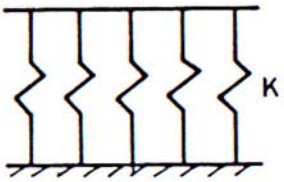

Fig. 1. Winkler model

FILONENKO–BORODICH (1940) considered a stretched elastic smooth membrane over the Winkler springs. This membrane imparts interaction among the springs. The resulting differential equation for this kind of model (*Fig.* 2) with tensile force (T) in the membrane is

$$q = kw - T\nabla^2 w, \tag{2}$$

where ∇^2 = Laplace operator of 2nd order.

HETENYI [1] proposed a model incorporating an elastic plate in the case of three-dimensional problems or an elastic beam in the case of two-dimensional problems to show the interaction between the independent, linear spring elements (*Fig.* 2). The response equation for this model is given by

$$q = kw - D\nabla^4 w, \tag{3}$$

where $D\left[= Eh^3/12\left(1-\nu^2\right)\right]$ is the flexural rigidity of the plate or beam and ∇^4 is the biharmonic operator.

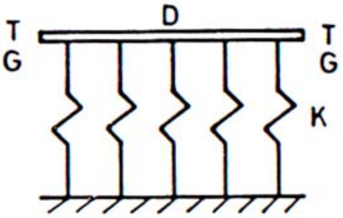

Fig. 2. Filonenko–Borodich model (T); Hetenyi model (D); Pasternak model (G)

PASTERNAK (1954) suggested a layer of incompressible vertical elements over Winkler springs (*Fig.* 2) and the elements deform in transverse shear only. The response equation for this model with characteristic constant G of the shear layer is

$$q = kw - G\nabla^2 w. \tag{4}$$

KERR [4] modified the Pasternak's model by incorporating another Winkler medium over the shear layer (*Fig.* 3). RHINES [6] considered an elastic, perfectly plastic constitutive law for the shear layer and solved the differential equation analytically. The governing expression for this model with c and k as top and bottom spring constants respectively and G as shear layer constant is

$$\left(1+\frac{k}{c}\right)q-\frac{G}{c}\nabla^2 q=kw-G\nabla^2 w. \tag{5}$$

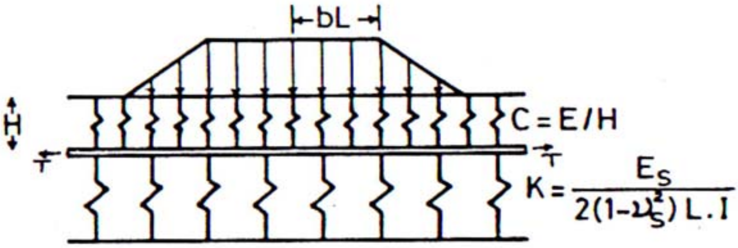

Fig. 3. 3-Parameter subgrade model

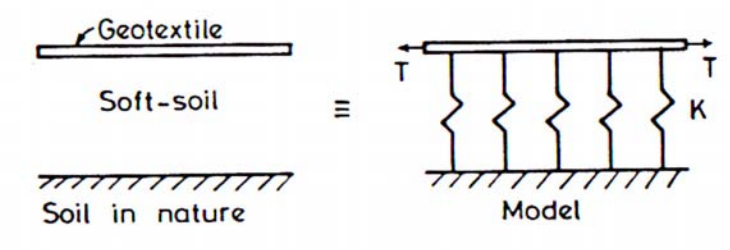

Fig. 4. Model for geosynthetic reinforced soils

3. Analysis

Most of the mechanical models described above were derived initially for the distributed load q, acting directly on the subgrade. The use of thin plate theory (HETENYI, [1]) allows the combined effect of foundation element and subgrade. Moreover, two-parameter models are analoguous to the isolated shallow footing on soft subgrade (*Figs.* 4–5) where as three-parameter models are assumed to be granular base underlain by soft subbase.

An embankment (strip) type of loading is considered as follows:

$$q(x) = q_0(x) \qquad \text{for} \quad |x| \le bL \tag{6}$$

$$q(x) = q_0(x)\left[1 - \frac{(x - bL)}{(L - bL)}\right] \qquad \text{for} \quad bL < |x| < L \tag{7}$$

$$q(x) = 0 \qquad \text{for} \quad |x| > L, \tag{8}$$

where $b =$ top/bottom width of embankment.

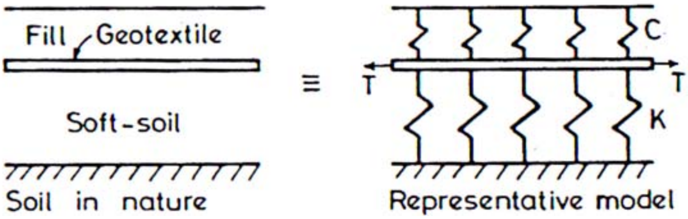

Fig. 5. Model for geosynthetic reinforced soils

The boundary and continuity conditions are

$$\frac{dw_1}{dx} = 0 \qquad \text{at} \quad x = 0 \tag{9}$$

$$\frac{dw_1}{dx} = \frac{dw_2}{dx} \text{ and } w_1 = w_2 \qquad \text{at} \quad x = bL \tag{10}$$

$$\frac{dw_2}{dx} = \frac{dw_3}{dx} \text{ and } w_2 = w_3 \qquad \text{at} \quad x = L \tag{11}$$

and

$$w_3 = 0 \qquad \text{at} \quad x \to \infty$$

where w_1, w_2, w_3 are the vertical deflection in the range $|x| \le bL$, $bL < |x| < L$, $|x| > L$ respectively.

With the help of *Eqs.* (6) to (11) the expressions for vertical deflection from equation (2) are

$$W_1 = \frac{\left[\left(e^{-t} - e^{-tb}\right)\cosh(tX) + (1 - b)\,t\right]}{(1 - b)\,t} \qquad \text{for} \quad |X| \le b \tag{12}$$

$$W_2 = \frac{\left[e^{-t}\cosh(tX) - \cosh(tb)\,e^{-tX} + (1 - b)\,t\left(1 - \frac{(X - b)}{1 - b}\right)\right]}{(1 - b)\,t} \qquad \text{for} \quad b < |X| < 1 \tag{13}$$

$$W_3 = \frac{[\cosh(t) - \cosh(tb)]\,e^{-tX}}{(1 - b)\,t} \qquad \text{for} \quad |X| > 1, \tag{14}$$

where $W = \dfrac{w}{w_0}$, $w_0 = \dfrac{q_0}{k}$, $X = \frac{x}{L}$, $t = \sqrt{\dfrac{k}{T}}$. *Eq.* (5) can be conveniently split into the form (when $T = G$)

$$\frac{d^2 w_2}{dx^2} - \frac{k+c}{T} w_2 = -\frac{c}{T} w \tag{15}$$

$$\frac{d^2 w_2}{dx^2} - \frac{k}{T} w_2 = \frac{q}{T} \tag{16}$$

$$w = \frac{q}{c} + w_2 \tag{17}$$

So for embankment type of loading on a three-parameter foundation model, the *Eq.* (16) is identical with *Eq.* (2) and total deflection at the surface can be obtained by using *Eqs.* (12) to (14) in the *Eq.* (17).

4. Results

Results are obtained for embankment type of loading on both two- and three-parameter subgrade model. The two-parameter is closer to the behaviour of a geosynthetic one over a soft soil while the three-parameter model corresponds to the geosynthetic one that is overlain by a fill and underlain by a soft soil. The models account for the main characteristics of the constituents, the compressibility of soft soil and fill materials, and the tensile resistance of the geosynthetic. In the simple models discussed in this paper, the interaction between the various constituents is not accounted for.

5. Conclusions

The response of an embankment type of loading on a geosynthetic-reinforced soil is studied. A two-parameter model is proposed to represent a geosynthetic-soft soil system while a three-parameter model corresponds to a fill-geosynthetic-soft soil system. A parametric study might bring out the importance of membrane tension, the thickness and stiffness of the fill, on the surface settlements. These models need improvement by considering the effect of interaction between different constituents.

References

[1] HETENYI, M., *Beams on Elastic Foundation*, The University of Michigan Press, Ann Arbor, Mich., 1946.

[2] HORVATH, J. S., New Subgrade Model Applied to Mat Foundations, *Journal of Geotechnical Engineering*, **109**, No. 12, December, (1983), pp. 1567–1587.

[3] KERR, A. D., Viscoelastic Winkler Foundation with Shear Interactions, *Journal of the Engineering Mechanics Division*, ASCE, **87** No. EM3, June (1961), pp. 13–30.

[4] KERR, A. D., Elastic and Viscoelastic Foundation Models, *Journal of Applied Mechanics*, **31** (1964), Transactions of the American Society of Mechanical Engineers, Series E, Sept. pp. 491–498.

[5] MADHAV, M. R. – GHOSH, C., Modelling for Settlement Analysis of Reinforced Soil, *First Indian Geotextiles Conference on Reinforced Soil and Geotextiles*, 1988, pp. 33–39.

[6] RHINES, W. J., Foundation Models for Continuously Supported Structures, thesis presented to New York University, Bronx, New York, in partial fulfilment of the requirements for the degree of Doctor of Philosophy, 1965.