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LOCAL GEOID DETERMINATION USING VARIABLE SURFACE DENSITIES

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Abstract

Local gravimetric geoid determination was carried out in the test region of Sóskút. In this solution 1610 point gravity data as well as a high resolution DTM were used. The latest Hungarian gravimetric geoid solution of the BUTE was chosen as a reference for the local geoid determination. From the point gravity anomalies and the elevation data a surface density model has been derived. Afterwards the surface density data were used for the calculation of the terrain corrections and the indirect effect. The calculations were made with constant density values too. The difference between the two solutions reached the maximum of 2 mm.

Keywords: gravimetric geoid, surface density, terrain correction.

1. Introduction

In the previous years many investigations were carried out in the test region of Sóskút concerning gravity field modelling and geodynamics. Therefore we have collected various data, e.g. gravity anomalies, high resolution DTM, elevations from spirit levelling and GPS measurements, etc. These data sets allowed us to calculate a high-resolution local geoid solution (TÓTH and RÓZSA, 2000b).

At that time, a constant density value was used for the calculations. The aim of this recent experiment was to investigate the effect of variable surface densities on the calculation of terrain corrections and on the geoid solution too. In order to achieve the goals a surface density model was derived from the available gravity anomalies and elevation data. The density values were determined using a simple regression model between the free-air gravity anomalies and the elevation.

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2. Input Data

2.1. *Digital Elevation Data*

In order to achieve a high resolution geoid, a high resolution DTM was purchased for the test region from the Mapping Agency of the Hungarian Army. The original DTM has a resolution of 10 meters, however, a lower resolution DTM was used for the calculation of terrain corrections. The distance between the grid points of this DTM was 60 meters. The DTM covered the area between $613860 \leq Y \leq 653760$ and $209590 \leq X \leq 249490$ meters in the Unified Hungarian Projection System, where Y and X mean Easting and Northing, respectively. The statistics of the DTM can be seen in *Table 1*, whereas *Fig. 1* shows the DTM itself.

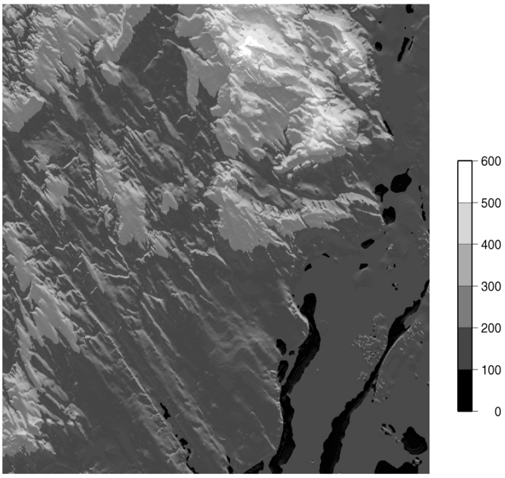

Fig. 1. The DTM of the Sóskút area [units in meter]

2.2. *Gravity Anomalies*

Within the boundaries of our test area, the national gravity dataset had been densified by the Loránd Eötvös Geophysical Institute. In this way we could use 1610 point free-air gravity anomalies. Out of these 1459 point anomalies were in the original dataset. Although height information is also needed for the derivation of the density model, the elevation for these point values was not included in the dataset. Therefore for those gravity data, for which the elevation was unknown, the height data were interpolated using the original 10-meter-resolution DTM.

Table 1. The statistics of the elevation data from DTM [m]

Min	Max	Mean	Std.dev
85	558	168.7	72.0

3. Derivation of the Surface Density Model

When we investigate the statistical correlation between the free-air anomalies and the elevation we usually get a strong correlation. If we neglect the topographic effect, we can use a linear regression model to describe the relationship between the free-air anomalies and the elevation. From the gradient of the regression line, the mean density of the surface masses (ρ), and its standard deviation can be calculated (PAPP, 1996):

$$\overline{\rho} = \frac{b}{2\pi G}, \tag{1}$$

where b is the Bouguer coefficient (the gradient of the regression line), and G is the gravitational constant. The calculation of the standard deviation is analogous to the calculation of mean density. As we can determine the standard deviation of b, dividing it with the constant of $2\pi G$, we can derive the standard deviation of the calculated mean density value. However, we have to emphasize that this standard deviation is linked to the statistical investigations only.

The density calculations have been done according to the following method. At first the whole investigated area was divided into 256 blocks (16-16 in both directions) with a side of about 1.3 km. After this step, a regression model was determined for each block, using the gravity data in the given block. From these regression models it became possible to calculate the mean density values according to (1). The regression model of one of the blocks can be seen in *Fig*2.

Unfortunately the distribution of the gravity measurements is only rarely homogeneous. In our case there were many measurements in the north-western part of the area, while in the eastern part we had less point gravity data. Due to the inhomogeneous distribution of the data, it was not possible to calculate an acceptable density value for all of the blocks. In this investigation ‘acceptable’ means a density value between 1600 and 2900 kg/m^3. With the regression model itself it was possible to calculate the mean density value of 83 blocks only, which is 32.4% of the whole area. However, more than 40% of the gravity data was measured within these blocks. The average number of gravity data within these blocks was 8. Based on the calculations it can be stated that the more data within the block are, the bigger the chance is for calculating the mean density value from (1). On the other hand this does not mean that increasing the size of the blocks is a solution for this problem, because the Bouguer-plate simplification becomes invalid for bigger areas, due to the topography, and the local surface deviations. Therefore the resolution of the

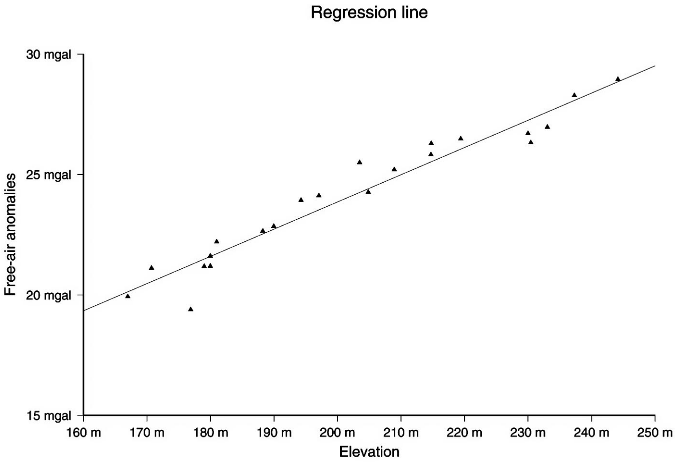

Fig. 2. The regression model of a given block

gravity data set should be increased in the future, in order to be able to derive a more realistic density model.

In the last step of the creation of surface density values, the mean density values of the rest of the blocks were estimated using the mean density values of the aforementioned 83 blocks. For this purpose an interpolation technique was used.

In such a way a surface density model was derived for the area between $625380 \leq Y \leq 644610$ and $221320 \leq X \leq 238990$. The density model can be seen in *Fig. 3*.

Due to the fact that the gravity dataset was given for a smaller area than the DTM, the value of 2670 kg/cm^3 was used for the neighbouring territories, which were not covered with gravity data.

4. Determination of the Gravimetric Geoid

In order to determine the gravimetric geoid the remove-restore technique was used (HEISKANEN–MORITZ, 1967). We have chosen the previous gravimetric geoid solution for Hungary as a reference for the calculations (TÓTH–RÓZSA, 2000a). Therefore the contribution of the new data to this solution was calculated. At first the terrain corrections were determined with the derived variable density according to the method described by TZIAVOS et al. (1996). The statistical properties of the terrain corrections can be seen in *Table 2*.

Due to the fact that our previous national geoid solution was used as a reference, from the calculated terrain corrections the terrain corrections applied in our reference solution were to be subtracted. Afterwards we derived a grid of gravity anomalies from the 1610 point gravity data, and we subtracted from these values the

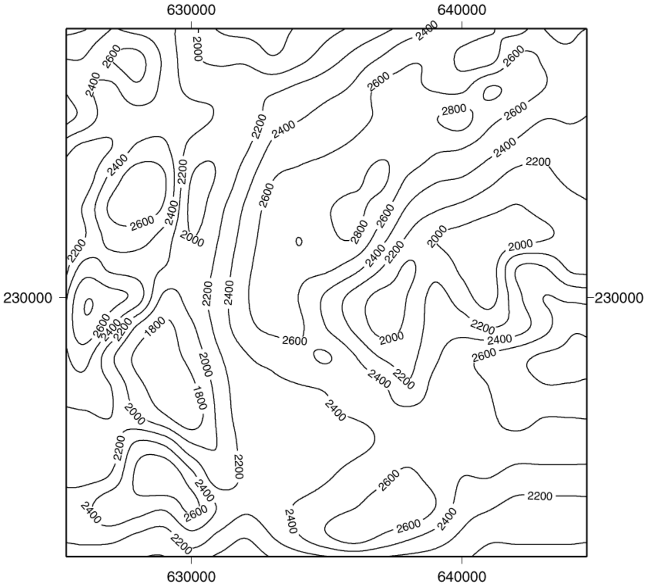

Fig. 3. The surface density model (units in kg/m^3)

Table 2. The statistical properties of terrain corrections [mgal]

	Min	Max	Mean	Std.dev.
Variable densities	0.003	6.206	0.221	0.349
Constant densities	0.003	6.222	0.226	0.351

anomaly values, that were used for the calculation of our reference solution. From the residual gravity anomalies and terrain corrections the contribution of the new datasets to the reference solution was calculated using FFT technique, and planar approximation.

In the restore step the indirect effect was calculated using the density model and the high resolution DTM. Due to the fact that the indirect effect was taken into account in the national solution too, we had to subtract from the calculated indirect effect the quantity of the indirect effect applied in our reference solution.

At last we added up the three components:

$$N = N_{\mathrm{HGTUB1999}} + N_{\Delta g} + N_{\mathrm{top.i.e}}, \tag{2}$$

where $N_{\mathrm{HGTUB1999}}$ is the geoid undulation from the regional model, $N_{\Delta g}$ is the contribution of the densified gravity dataset (the part of the gravity field, which was not taken into account during the calculation of the national geoid solution),

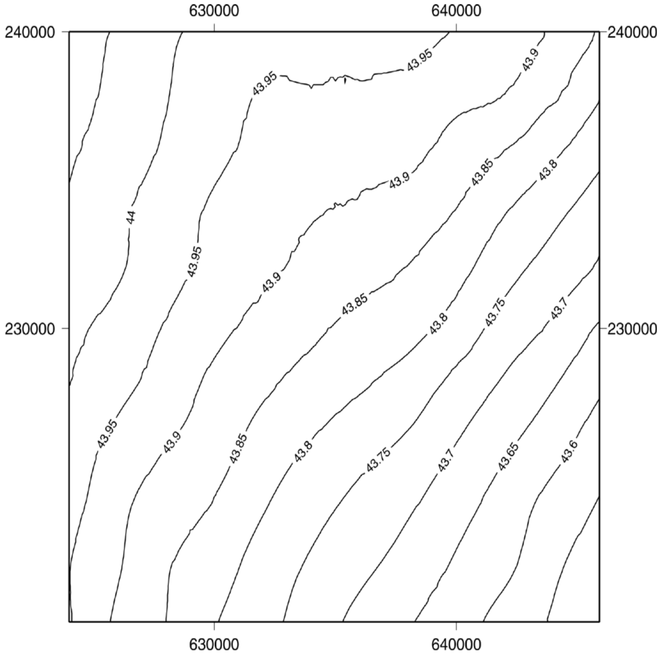

Fig. 4. The final gravimetric geoid solution in the Sóskút area using variable surface densities (units in m, contour line interval 5 cm)

while $N_{\text{top.i.e}}$ is the indirect effect of the topography calculated by the dense DTM. Of course as it was mentioned before, the indirect effect applied in the national solution was subtracted from these calculated values. Therefore Δg and $N_{\text{top.i.e}}$ can be written as:

$$\Delta g = \Delta g_{\text{Faye}} - \Delta g_{\text{Faye}}^{\text{HGTUB}}, \tag{3}$$

$$N_{\text{top.i.e}} = N_{\text{top.i.e}}^{DTM} - N_{\text{top.i.e}}^{\text{HGTUB}}. \tag{4}$$

5. Results and Conclusions

According to (2) the final local geoid solution could be obtained. The solution can be seen in *Fig. 4*, while the statistics of the geoid solution is in *Table 3*. In order to evaluate the effect of using variable densities on the geoid undulations, the calculations were carried out parallel with constant and with variable surface densities. The statistics of the difference of the two solutions can be seen in *Table 3*, too. *Fig. 5* shows the distribution of the differences.

Table 3. The statistical properties of gravimetric geoid solutions [m]

	Min	Max	Mean	Std.dev.
Constant density	43.500	44.079	43.841	0.124
Variable density	43.500	44.079	43.841	0.124
Difference (c. - v.)	–0.002	0.002	0.000	0.001

Due to the fact that the terrain correction is not very significant in this area the effect of surface density variations reaches only the level of few millimeters. From the results it can be stated that the implementation of surface density variations in the geoid determination did not cause a significant improvement in the final solution in this particular area. However, further investigations should be made to evaluate the effect of density variations in areas with rougher topography.

On the other hand, the investigation showed that for calculating the surface density values with the Nettleton method, a denser gravity dataset would be needed in the area. The position of the calculated mean density values correlates well with the distribution of the gravity data. Therefore it would be useful to densify the gravity dataset in the eastern part of the investigated area too.

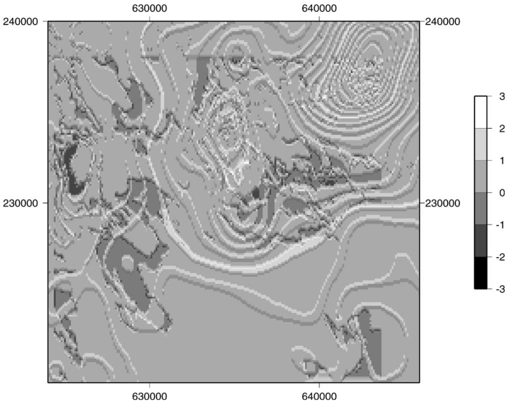

Fig. 5. The distribution of the differences (m)

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