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PREDICTION BY EÖTVÖS' TORSION BALANCE DATA IN HUNGARY

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Abstract

Possible geodetic applications of Eötvös' torsion balance gravity gradient data were investigated in the present study. As a practical example gravity data have been predicted for two test areas in Hungary with the method of least squares collocation. By evaluating the results with gridded data interpolated from gravity measurements the standard deviation of differences $\pm 1 - 1.7$ mGal was found which proves the usability of such gradient data for geodetic applications.

Keywords: Eötvös' torsion balance, least squares collocation.

1. Introduction

Nowadays there is an increasing interest in measuring gravity gradients of the earth's gravity field. Mainly these measurements are intended to be made on mobile (air, marine and space mounted) platforms (BELL et al., 1997; PAWLOWSKI, 1998). Undoubtedly an advantage of these gradient measurements is their relative insensitivity to small platform accelerations which constitutes a principal problem for aerial gravimetry, for example. Moreover, for space gradiometric devices (e.g. GOCE, VISSER, 1999) the exponential gravity signal decrease for high altitudes is counteracted by the signal amplification at shorter wavelengths due to the double differentiation for second-order gradients of the gravity field (RUMMEL et al., 1993).

Beside the planned satellite mission it is a historical fact that gravity gradiometric measurements for the first time were successfully taken at the surface of the earth due to Loránd Eötvös' terrestrial gradiometric device, the torsion balance. This means an advantageous position for Hungary since a great deal of gradiometric measurement data has been collected here. Even in time of Eötvös these data have been used for geodetic tasks as well (see HOMORÓDI, 1966). In our present paper we would like to highlight some ideas about the application of this kind of data to geodesy by taking into account not only modern computing facilities but also recent theoretical developments in the solution of the Geodetic Boundary Value Problem (GBVP).

At first we introduce shortly the principle and history of measurements taken by Eötvös' torsion balance device. Then we continue with Hungarian torsion balance measurements highlighting the data collected at ELGI (Eötvös Loránd Geophysical Institute). Then a short discussion will present how it is possible to use such data in solving the GBVP. Finally a prediction experiment by the method of least squares collocation will show how to get gravity values from torsion balance gradient measurements.

2. Measurement Principle of Eötvös' Torsion Balance and a Short History of Measurements

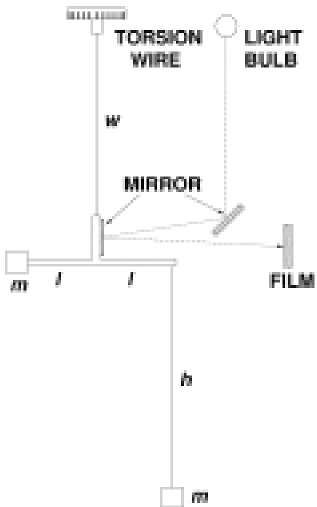

Fig. 1. Principle of Eötvös' torsion balance

The principle of measurement with the Eötvös' torsion balance is shown in *Fig. 1*. This device was called by Eötvös horizontal variometer, and through gravity change between the two masses m a force arises which exerts a small moment on the horizontal swinging arm of length $2l$. This moment is counteracted by the moment exerted by the thread w, and the corresponding angle position can be read or photographically registered on film for certain kind of torsion balance devices.

As of the original torsion balance, at each site the readings should be taken in 5 different azimuths in order to determine the corresponding 5 unknowns (reading for the torsion-free position, two gradient and two curvature parameters). Later in a more advanced design two such balances were mounted at opposite position to each other and hence readings had to be taken at only 3 azimuths to compute the 6 unknowns. (twin/double balances). We would like to mention that recently

DOROBANTU (1999) has done research at the University of Munich to equip the device with automatic electronic reading.

The first measurements with torsion balance were led by Eötvös himself, first at the foot of Gellért Hill (1889), then at Sághegy near Celldömölk (1891), then on the ice sheet of Lake Balaton (winters of 1901–1903). The measured gravity gradients were in agreement with lake bathymetric data. After the first successful tests for resource exploration at Morvamező near Egbell (Gbely) in 1916, the torsion balance device gained an increasing field of application for gravity exploration in Germany, Hungary and Bohemia. The first hydrocarbon reservoir in the USA was detected by the device in 1924 (Nash Dome).

The introduction of astatized spring gravimeters by the end of the thirties made an end to the torsion balance measurements in the United States. The reason behind this was not only the large field work required by the torsion balance (the terrain around the field point had to be carefully levelled along eight directions for terrain correction computations), but also the interpretation of the gradient and curvature data was not so straightforward as for gravimetric Δg data. An assertion to this is the fact that the curvature data measured were simply discarded for resource exploration due to its interpretation difficulties. Torsion balance measurements, however, are continued in Europe; for example in Hungary even in the sixties it was used for field work.

3. Torsion Balance Measurements and Data in Hungary

In the external gravity field three elements of the gradient tensor **E** (Eötvös or Marussi tensor), and the difference of two other elements, altogether 4 data from the 5 independent elements can be determined from torsion balance gradiometry. If

$$\mathbf{E} = \mathbf{grad}\, g = \begin{bmatrix} W_{xx} & W_{xy} & W_{xz} \\ W_{xy} & W_{yy} & W_{yz} \\ W_{xz} & W_{yz} & W_{zz} \end{bmatrix}$$

denotes the elements of the Eötvös tensor, then W_{xz}, W_{yz}, W_{xy} and $W_{\Delta} = W_{yy} - W_{xx}$ can be measured by the balance. It is customary to call the first two of them, (W_{xz}, W_{yz}) components of gradients, and the other two, (W_{xy} and W_{Δ}) components of curvatures since $W_{xz} = g_x$, $W_{yz} = g_y$. are components of the horizontal gravity gradient vector and W_{xy} and W_{Δ} are in connection with the curvature of the level surface.

Conventionally the gradient values are illustrated by vectors and curvatures by line segments, respectively, as illustrated in *Fig. 2*. The following equations

$$\tan\alpha = \frac{W_{yz}}{W_{xz}}, \qquad W_{sz} = \sqrt{W_{xz}^2 + W_{yz}^2}$$

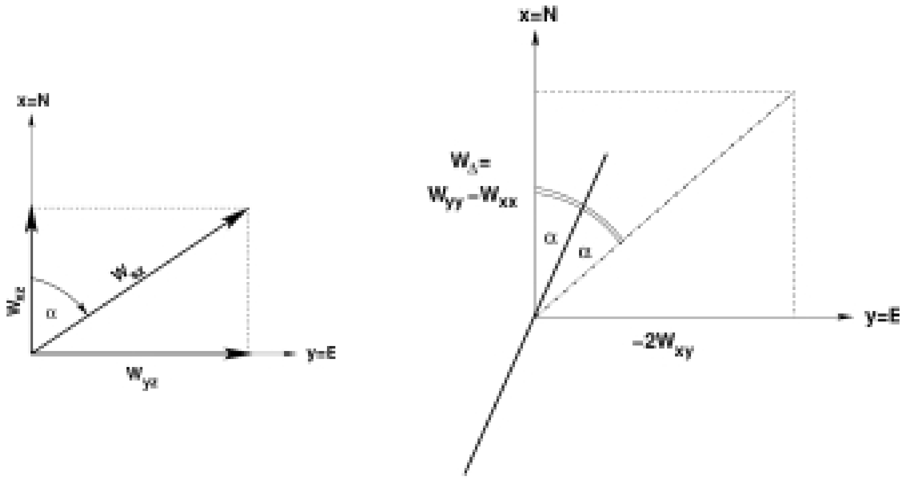

Fig. 2. Gradient and curvature components

can be used to compute the azimuth and magnitude of gradient vectors, and $\kappa = W_{sz}/W_{sz}g$ connects the magnitude with curvature κ of the plumb line. The difference of main curvature parameters

$$R = g\left(\frac{1}{r_{\min}} - \frac{1}{r_{\max}}\right) = \sqrt{W_\Delta^2 + (2W_{xy})^2}$$

can be expressed, as well as the azimuth of maximal curvature from the expression

$$\tan 2\alpha = -\frac{2W_{xy}}{W_\Delta}.$$

This may also be plotted on a map as a line segment of azimuth α and length R.

Original files of measurements with the torsion balance are stored at the Loránd Eötvös Geophysical Institute (ELGI). In the past this material has been treated somewhat carelessly henceforth some original files have been lost and they cannot be recovered. During the years 1995-2000 in the framework of a research project 11795 point gradient and curvature data have been saved to computer files and given to Technical University of Budapest by ELGI. These data mainly cover the central part of Hungary (data distribution is shown in *Fig.3*). This figure also shows two test areas chosen (the larger one, ‘A’ is mostly plain area while the other one, ‘B’ is a hilly area with moderate topography). These areas were chosen for extensive tests and these will be described in much more detail later on in the present study.

A statistical analysis was performed on the above test data including gradient and curvature parameters and topographic effects, also contained in the original files. These statistics are in *Table 1* below.

Table 1. Statistics of gradients and curvatures for test areas A and B (number of data: 752 and 691 resp.; Eötvös Unit, 1E.U. $= 10^{-9}$ s^{-2}), t – topographic effect, and the linear correlation coefficients of a specific gradient/curvature parameter and its topographic correction

Area A	W_{xz}	W_{yz}	$2W_{xy}$	W_Δ	t_{xz}	t_{yz}	$2t_{xy}$	t_Δ
Min.	–16.3	–22.9	–29.1	–43.2	–10.3	–11.4	–36.7	–28.2
Max.	42.5	27.6	32.3	33.7	8.5	13.3	22.4	16.2
Mean	8.5	0.6	4.3	0.5	0.0	–0.3	0.2	–0.1
Std.dev.	±6.9	±6.4	±8.2	±10.0	±2.3	±2.6	±5.0	±4.8
Corr.coeff.	0.300	0.419	0.488	0.455				
Area B	W_{xz}	W_{yz}	$2W_{xy}$	W_Δ	t_{xz}	t_{yz}	$2t_{xy}$	T_Δ
Min.	–38.4	–65.8	–175.3	–324.3	–42.3	–38.4	–167.9	–292.6
Max.	75.9	67.4	299.7	216.0	35.1	38.9	257.7	163.1
Mean	13.1	–8.6	2.1	–7.8	1.5	–2.4	–1.4	–5.5
Std.dev	±15.1	±17.3	±51.6	±65.9	±7.9	±8.4	±44.6	±56.4
Corr.coeff.	0.649	0.708	0.927	0.918				

The high correlation is very pronounced with topography for area B of gradients and especially of curvatures. This confirms the well-known fact that the torsion balance measurements are very sensitive to nearby topographic masses, something must be noted for geodetic applications as well.

4. Possible Utilization of Torsion Balance Measurements in Geodesy

Torsion balance measurements have already been used for geodetic purposes. The mathematical tool used was line integration of gradients and curvatures, which allowed to express differences of gravity and deflection of the plumbline between any two points. Following this technique it became possible to interpolate gravity and deflections of the vertical in a whole measurement network when some points have known gravity or deflection values (VÖLGYESI, 1998).

A great theoretical achievement in the solution of GBVPs can be thanked to GELDEREN and RUMMEL (2001). They for the first time derived certain combinations of the Eötvös tensor that can be used to solve the GBVP and the corresponding kernel functions that may be used in surface integrals for the solution, just as the Stokes' integral for gravity anomalies. By using these integrals, combinations of W_{xz}, W_{yz} and $2W_{xy}$, W_Δ of torsion balance measurements will be boundary values for the determination of the outer gravity field (parametrized e.g. by the disturbing potential). The practical application of these integrals is an open field for research.

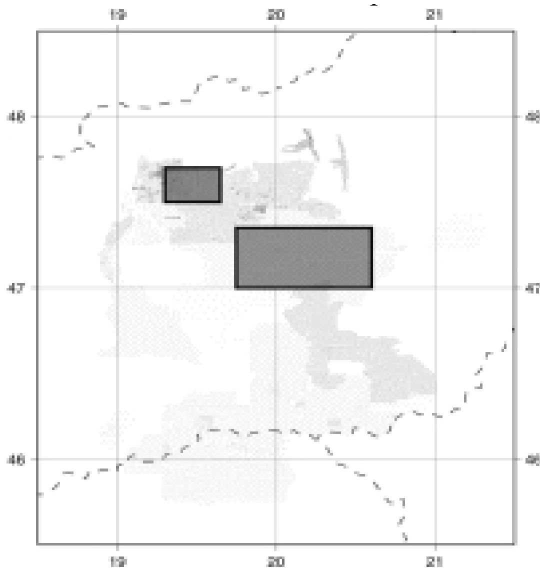

Fig. 3. Distribution of torsion balance measurement sites. The upper rectangle shows test area 'B', the other shows 'A'. The coordinate system is GSR80, geodetic coordinates

A more traditional approach in physical geodesy is the least squares collocation (TSCHERNING, 1994), the advantage of which lies in the combination of different kinds of gravity measurements in the gravity field determination process, and this of course includes torsion balance data as well. Another advantage is that any other parameter of the gravity field can be obtained in one computational step. An example can be found in literature for its application for the GGSS gravity gradient data (JEKELI, 1993) processing (ARABELOS and TZIAVOS, 1992; ARABELOS and TSCHERNING, 1999). A known limitation of the method is that the number of measurement points cannot go beyond several thousand due to the size of the equations to be solved during collocation. This method was chosen for the present study to predict gravity anomalies for the two test areas by using only (W_{xz}, W_{yz}) gradients measured by the torsion balance.

5. Prediction of Gravity Anomalies by Least Squares Collocation

Gravity anomalies have been used for prediction on the one hand since it can serve a good starting point for numerous geodetic and geophysical applications (geoid determination, gravity field interpretation, inverse problems), and on the other hand a grid of interpolated Faye anomalies for the two test areas was available and these data served well for evaluation purposes. We have concentrated on using gravity gradients although usage of curvature components in collocation poses no problem or even the combination of the two is easily manageable.

The first step in gravity field modelling by collocation is to remove all possible short and long wavelength effects (trends) from our data in order to make the residuals more smooth, easily interpolatable, and with zero mean. The so called normal effect (VÖLGYESI, 1982) has been removed for the purpose as well as long-wavelength gradient components of the EGM96 geopotential model and the above mentioned topographic effect.

The normal effect and the geopotential model are only parts of the trend removal. Since there is no way to remove a localized trend effect by applying these corrections only, we decided to take into account a local trend in reduced (W_{xz}, W_{yz}) gradients before determining empirical covariance functions. This process was similar to the one used by HEIN and JOCHEMCZYK (1979) for the torsion balance data in Germany when modelling local covariance functions from torsion balance gradients. Two kinds of such local trend models – as functions of the position – were applied to reduced gradient components:

- Linear trend function
- Quadratic trend function

In the next step an analytic covariance model has to be fitted to the empirical covariance function through an essentially iterative process. Then parameters of the analytic covariance function of Tscherning–Rapp Model 2 with $B = 4$ in the denominator (see TSCHERNING, 1994, Eq. 19) were yielded.

The last step was to compile the job file for GEOCOL program written by Tscherning in Fortran. The collocation step predicted residual gravity anomalies which do not contain the gravity effects of the geopotential model in addition to the normal effect and topography.

Finally these gravity predictions were compared with interpolated $1' \times 1.5'$ residual Faye anomalies corrected also to the effect of the EGM96 geopotential model. This comparison is valid since both values are topographically reduced ones. Statisics of the differences for the two test areas are summarized in *Table2* for the cases of linear and quadratic trend models.

The standard deviation of differences is seemingly better for linear trend model and for the plain area, what of course, was something to be expected. The ± 1 mGal standard deviation of differences for the plain area can be said to be very low especially if we take into account the fact that according to PAPP (1993) the same prediction error of gravity anomalies is to be expected by using the point gravity anomalies themselves!

Our results are in harmony with the study of VÖLGYESI (1998) as well, who noted differences around $\pm 0.60'' - 0.65''$ in deflections at evaluation points by interpolating these data from curvatures measured by the torsion balance. At the same time according to TÓTH et al. (2000) the standard deviation of differences between astronomically measured and gravimetrically predicted deflection components is about the same value, $\pm 0.5'' - 0.6''$. These results confirm the conclusion that

Table 2. Statistics of the differences of residual gravimetric and (W_{xz}, W_{yz})-predicted gravity anomalies for test areas A and B. (units are mGal). $T2$ denotes linear, $T3$ quadratic trend, respectively, and F indicates that a least squares fitting plane has been removed from the differences

	A–T2	A–T2–F	A–T3	A–T3–F	B–T2	B–T2–F	B–T3	B–T3–F
Min.	–2.52	–4.00	–2.14	–4.54	–11.95	–3.37	–12.99	–3.61
Max.	3.66	2.21	4.77	2.09	3.31	4.08	1.50	4.95
Mean	1.30	0.00	1.38	0.00	–5.31	0.00	–6.07	0.00
Std.dev.	±1.10	±1.02	±1.36	±1.05	±3.96	±1.67	±3.82	±1.77

torsion balance measurements may be used for the determination of certain gravity field parameters as well as gravity anomalies.

6. Conclusions and Outlook

Our studies for two selected test areas have demonstrated the fact that the Hungarian torsion balance measurements can be advantageously applied for gravity field determination. This means that these data can be combined with gravity anomalies for more reliable gravity field determination. However, they in themselves may be appropriate to solve the GBVP. Because of the great number of such data the collocation method meets numerical difficulties, hence we propose to apply other numerical methods as well (e.g. FFT-based numerical integration methods).

Besides the gradient components, the application of curvature parameters needs more studies, since according to the work of ARABELOS and TZIAVOS (1992) this may further improve the prediction. Also a detailed comparison with the results of line integration methods is planned.

It can be told that Hungary has a very advantageous position as regards terrestrial gradiometry and it would be pity to let these valuable gradient measurements go unused that was collected (although for resources exploration) through laborious field work during decades. The present study wants to point out that practical work with this exceptional kind of data may yield fruitful results in the future for geodesy.

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