PERIODICA POLYTECHNICA SER. CIV. ENG. VOL. 43, NO. 1, PP. 33–42 (2000)

CALCULATION OF IMPACT OF ELASTIC STRUCTURES WITH INTERNAL DAMPING

József GYÖRGYI

Department of Structural Mechanics
Faculty of Civil Engineering
Technical University of Budapest
H–1521 Budapest, Hungary
Phone: (36-1)-463-1432
Fax: (36 -1)-463-1099

Received: December 20, 1998

Abstract

There are some types of structures impact problems where we have to calculate with elastic structures. One of these accidental situations is the impact (drop) of different heavy loads on the existing reinforced concrete slabs and walls of the reactor building during transportation. The other situation is the impact of neighbour building parts of nuclear buildings in the case of earthquake. In these cases it is very important for the reactor hermetic containment to stay in elastic region. It is well known that dynamic stress values are influenced by external and internal damping in the dynamic analysis of elastic structures. There are some adequate numerical methods for the analysis of structures with several degrees of freedom under external damping. An algorithm has been presented in this paper for the analysis of dynamic excess displacements of structures by modal analysis, for cases the effects of internal friction have to be reckoned with. The developed algorithm and numerical method have been tested on examples. The mentioned factors showed important effects, justifying to be reckoned with in the analysis of real structures.

Keywords: internal damping, modal analysis, impact of structures.

1. Introduction

An important problem of the dynamic analysis of structures is to determine displacements and stresses in a structure in the case of external and internal damping. Generally the external damping is proportional to speed and the internal damping is frequency independent. In [1] a method is given for calculation of simultaneous effect of external and internal damping. It will be used in this paper for impact problem of elastic structure vibration. The general numerical method to solve the matrix differential equation of vibration is the numerical integration. For an important problem of the dynamic analysis of structures is to determine displacements by this method we have to give the equivalent damping matrix of internal friction and for that we have to solve an eigenvalue problem. Using the eigenvectors from that calculation during the modal analysis we can solve the matrix differential equation in shorter time. We will analyse two impact problems. The first problem is the drop of body for a structure and the second is the impact of neighbour parts of building in the case of horizontal support vibration.

2. Taking a Proportional Internal Damping into Consideration

The second-order linear differential equation

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{q} \tag{1}$$

describing the displacement of structures expresses the dynamic equilibrium at any time in the considered time range. Forces of inertia are expressed by $\mathbf{M}\ddot{\mathbf{x}} = \mathbf{f}_I(t)$, damping forces by $\mathbf{C}\dot{\mathbf{x}} = \mathbf{f}_D(t)$, stiffness forces by $\mathbf{K}\mathbf{x} = \mathbf{f}_E(t)$, while $\mathbf{q}(t)$ is the vector of external forces. ($\mathbf{M}$ mass matrix, $\mathbf{C}$ damping matrix and $\mathbf{K}$ stiffness matrix are of order n). The dynamic analysis is intended to solve the matrix differential equation under initial conditions $\mathbf{x}_0, \dot{\mathbf{x}}_0$ at a time t_0, and in knowledge of displacements, to compute the dynamic stresses.

For an external damping, the $\mathbf{C}$ damping matrix of the structure can be assembled in knowledge of single damping elements related to the structure. For a damping due to frequency-independent internal friction, the matrix of equivalent external damping – for different damping parameters of single structural units – may be assumed in knowledge of complex stiffness matrix $\mathbf{K}_u + i\mathbf{K}_v$, in form $\mathbf{C} = \mathbf{MV}\left\langle \frac{1}{\omega_{0ru}} \right\rangle \mathbf{V}^T\mathbf{K}_v$ using eigenvectors normed to $\mathbf{M}$ of the eigenvalue problem $\mathbf{K}_u\mathbf{v}_r = \omega_{0ru}^2\mathbf{M}\mathbf{v}_r$ [1].

For structural units with the same damping parameters (proportional damping) the equivalent damping matrix $\mathbf{C} = v\mathbf{MV}\left\langle \frac{1}{\omega_{0ru}} \right\rangle \mathbf{V}^T\mathbf{K}$ and $\mathbf{K}_u = u\mathbf{K}$ are relevant, where

$$v = \frac{4\gamma}{4+\gamma^2}, \quad u = \frac{4-\gamma^2}{4+\gamma^2}, \quad \omega_{0ru} = \frac{\omega_{0r}}{\sqrt{1+\frac{\gamma^2}{4}}} \quad \text{and} \quad \gamma = \frac{\vartheta}{\pi}.$$

Here ϑ is the logarithmic decrement of damping, ω_{0r} may be obtained from the r-th eigenvalue of the eigenvalue problem $\mathbf{K}\mathbf{v} = \omega^2\mathbf{M}\mathbf{v}$ for the undamped case, while $\mathbf{V}$ is a matrix containing eigenvectors normed for $\mathbf{M}$.

In the case of structural damping $\gamma^2 \ll 1.0$ therefore $v \approx \gamma$, $u \approx 1$, $\omega_{0ru} \approx \omega_{0r}$ and the equivalent damping matrix in practical calculation

$$\mathbf{C} = \mathbf{MV}\left\langle \frac{1}{\omega_{0r}} \right\rangle \mathbf{V}^T\mathbf{K}. \tag{2}$$

Obviously, in the case of internal damping, the direct integration problem has to be preceded by solving an eigenvalue problem. All these argue for taking it into consideration in selecting the solution method of the dynamic problem, and to try to apply modal analysis.

3. Impact of a Falling Body on a Supported Elastic Structure

Now the task is to solve the matrix differential equation without damping

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = q \tag{3}$$

under initial conditions $\mathbf{x}_0 = \mathbf{0}$, $\dot{\mathbf{x}}_0$. Applying the modal analysis solution is wanted in form $\mathbf{x} = \mathbf{V}\mathbf{y}$, in knowledge of eigenvalues and eigenvectors normed to $\mathbf{M}$ ($\mathbf{V}^T\mathbf{M}\mathbf{V} = \mathbf{E}$) of the eigenvalue problem

$$\mathbf{K}\mathbf{v} = \omega_{0r}^2\mathbf{M}\mathbf{v}. \tag{4}$$

After substitution and multiplying from the left by transposed matrix $\mathbf{V}^T$:

$$\mathbf{V}^T\mathbf{M}\mathbf{V}\ddot{\mathbf{y}} + \mathbf{V}^T\mathbf{K}\mathbf{V}\mathbf{y} = \mathbf{V}^T\mathbf{q}. \tag{5}$$

Due to orthogonality, theoretically, n single-unknown equations may be considered. It is known that in solving real technical problems, in the solution computed on the basis of eigenvectors it is sufficient to involve a certain number ($m < n$) of eigenvectors, computable by convenient procedures (e.g. subspace iteration) even for extended systems. Equation r-th of n single-unknown equations:

$$\ddot{y}_r(t) + \omega_{0r}^2 y_r(t) = \mathbf{v}_r^T\mathbf{q} = f_r. \tag{6}$$

The solution will be

$$y_r(t) = a_r \cos\omega_{0r}t + b_r \sin\omega_{0r}t + \frac{f_r}{\omega_{0r}^2}. \tag{7}$$

Constants a_r and b_r can be computed from the initial conditions

$$\mathbf{y}_0 = \mathbf{V}^{-1}\mathbf{x}_0 = 0 \quad \text{resp.} \quad \dot{\mathbf{y}}_0 = \mathbf{V}^{-1}\dot{\mathbf{x}}_0 = \mathbf{V}^T\mathbf{M}\dot{\mathbf{x}}_0.$$

After substitution we get:

$$a_r = -\frac{f_r}{\omega_{0r}^2}, \qquad b_r = \frac{1}{\omega_{0r}}\mathbf{v}_r^T\mathbf{M}\dot{\mathbf{x}}_0.$$

Afterwards:

$$y_r(t) = \frac{f_r}{\omega_{0r}^2}(1 - \cos\omega_{0r}t) + \frac{1}{\omega_{0r}}\mathbf{v}_r^T\mathbf{M}\dot{\mathbf{x}}_0 \sin\omega_{0r}t. \tag{8}$$

In accordance with the relationship $\mathbf{x}(t) = \mathbf{V}\mathbf{y}(t)$ we obtain:

$$\underline{x}(t) = \sum_{r=1}^{n}\mathbf{v}_r\mathbf{v}_r^T\left[\frac{1}{\omega_{0r}^2}\mathbf{q}(1 - \cos\omega_{0r}t) + \frac{1}{\omega_{0r}}\mathbf{M}\dot{\mathbf{x}}_0 \sin\omega_{0r}t\right]. \tag{9}$$

For proportional structural damping the matrix differential equation is

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{M}\mathbf{V}\left\langle \frac{\gamma}{\omega_{0r}} \right\rangle \mathbf{V}^T\mathbf{K}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{q}. \tag{10}$$

Applying the modal analysis

$$\mathbf{V}^T\mathbf{M}\mathbf{V}\ddot{\mathbf{y}} + \mathbf{V}^T\mathbf{M}\mathbf{V}\left\langle \frac{\gamma}{\omega_{0r}} \right\rangle \mathbf{V}^T\mathbf{K}\mathbf{V}\dot{\mathbf{y}} + \mathbf{V}^T\mathbf{K}\mathbf{V}\mathbf{y} = \mathbf{V}^T\mathbf{q}, \tag{11}$$

$$\ddot{y}_r(t) + \gamma\omega_{0r}\dot{y}_r(t) + \omega_{0r}^2 y_r(t) + \mathbf{v}_r^T\mathbf{q} = f_r. \tag{12}$$

The solution will be

$$y_r(t) = e^{-\frac{\gamma}{2}\omega_{0r}t}\left(a_r \cos\omega_{0r}^* t + b_r \sin\omega_{0r}^* t\right) + \frac{f_r}{\omega_{0r}^2}. \tag{13}$$

Here

$$\omega_{0r}^* = \omega_{0r}\sqrt{1 - \frac{\gamma^2}{4}} \approx \omega_{0r}. \tag{14}$$

From the initial conditions $a_r = -\frac{f_r}{\omega_{0r}^2}$, $b_r = \frac{1}{\omega_{0r}}\left(\mathbf{v}_r^T\mathbf{M}\dot{\mathbf{x}}_0 - \frac{\gamma}{2}\frac{f_r}{\omega_{0r}}\right)$.
Afterwards:

$$y_r(t) = e^{-\frac{\gamma}{2}\omega_{0r}t}\left[\frac{f_r}{\omega_{0r}^2}(1 - \cos\omega_{0r}t) + \frac{1}{\omega_{0r}}\left(\mathbf{v}_r^T\mathbf{M}\dot{\mathbf{x}}_0 - \frac{\gamma}{2}\frac{f_r}{\omega_{0r}}\right)\sin\omega_{0r}t\right]. \tag{15}$$

In accordance with the relationship $\mathbf{x}(t) = \mathbf{V}\mathbf{y}(t)$:

$$\mathbf{x}(t) = \sum_{r=1}^{n}\mathbf{v}_r\mathbf{v}_r^T e^{-\frac{\gamma}{2}\omega_{0r}t}\left[\frac{1}{\omega_{0r}^2}\mathbf{q}(1 - \cos\omega_{0r}t) + \frac{1}{\omega_{0r}}\left(\mathbf{M}\dot{\mathbf{x}}_0 - \frac{\gamma}{2}\frac{1}{\omega_{0r}}\mathbf{q}\right)\sin\omega_{0r}t\right]. \tag{16}$$

Approximately we can see the influence of structural damping if we calculate the value of $\mu = e^{-\frac{\gamma}{2}\omega_{0r}t}$ in the case of $\omega_{0r}t = \pi$. For reinforced concrete ($\gamma = 0, 1$) the result is: $\mu = 0.855$. The influence of the structural damping for the displacement is about 15%.

4. Impact of the Neighbour Building Parts

Fig. 1 displays a stiff building part and a flexible frame on a common base. Horizontal support vibration of the mechanical system is $x_R(t)$ and further displacements of the flexible frame are contained in vector $\mathbf{x}_s(t)$.

Matrix differential equation containing the flexible displacements (between both parts of the building) is as follows:

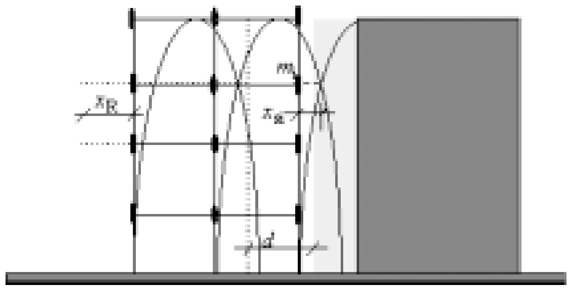

Fig. 1. The model of the impacting structure at support movement

$$\mathbf{M}\ddot{\mathbf{x}}_s(t) + \mathbf{K}\mathbf{x}_s(t) = -\mathbf{M}\mathbf{j}_s\ddot{x}_R(t) = -\mathbf{m}_s\ddot{x}_R(t)\,. \tag{17}$$

$\mathbf{j}_s$ is here a signal vector showing in which direction of the displacement's degree of freedom the rigid-body-like displacement $x_R(t)$ is observed.

If displacement x_{si} of any material point of the model next to the rigid body reaches the prescribed distance between both structures, impact occurs in the point given. Now, to the i^{th} component of the principal diagonal in the stiffness matrix of the flexible structure, the spring stiffness k_{1i} computed from deformation of the impacting structures in the point of impact is added, and afterwards the differential equation

$$\mathbf{M}\ddot{\mathbf{x}}_s(t) + (\mathbf{K} + \mathbf{K}_1)\,\mathbf{x}_s(t) = -\mathbf{m}_s\ddot{x}_R(t) + \mathbf{K}_1\mathbf{x}_d \tag{18}$$

should be solved for the initial conditions obtained from the previous vibration phase. Only one single element in the principal diagonal of matrix $\mathbf{K}_1$ given in *Eq.* (18) differs from zero (its value is k_{1i}), while elements of vector $\mathbf{x}_d$ are equal to d. We note here that damping elements proportional to the velocity and filling the distance d between both structures can be placed in points of the presumed impact.

In this case, the differential equation

$$\mathbf{M}\ddot{\mathbf{x}}_s(t) + \mathbf{C}\dot{\mathbf{x}}_s(t) + (\mathbf{K} + \mathbf{K}_1)\,\mathbf{x}_s(t) = -\mathbf{m}_s\ddot{x}_R(t) + \mathbf{K}_1\mathbf{x}_d \tag{19}$$

should be solved instead of differential equation (18).

A more sophisticated task, however, in principle treatable by the method described, arises when damping elements do not connect both structures, and work only when the flexible structure moves in direction of the stiff structure. In this case, only that element in the C diagonal damping matrix will differ from zero where the horizontal displacement is smaller than the given distance d. This means that in examinations further vibration phases described by deviating differential equations should be distinguished.

For solution of matrix differential equations (17) and (18) the modal analysis can be applied where the differential equation breaks into differential equations with one unknown, and also structural damping can be taken into consideration. Direct integration may provide a solution, too. In our experience, application of the

Wilson-θ method suggested by WILSON (1976) may be advantageous for solution of matrix differential equation (19).

If we want to take also the internal damping of the structure into account together with external damping, knowing the eigenvectors belonging to the undamped case, an equivalent damping matrix can be generated. Applying the method introduced in the previous point, structural damping can be taken into consideration even without generating an equivalent damping matrix.

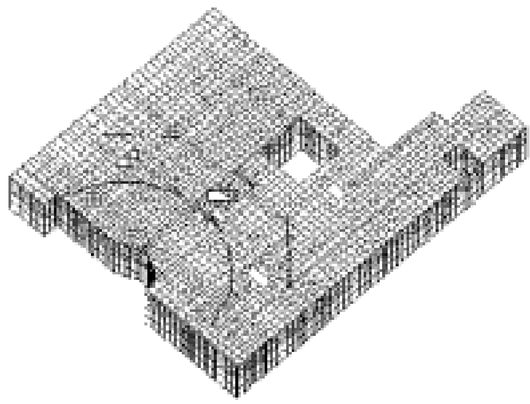

Fig. 2. The finite element model of structure

Now, the solution of *Eq.* (18) will be shown by modal analysis considering structural damping. Introducing notations of

$$\tilde{\mathbf{K}} = \mathbf{K} + \mathbf{K}_1, \qquad \mathbf{q}(t) = -\mathbf{m}_s \ddot{x}_R(t) \quad \text{and } \mathbf{q}_d = \mathbf{K}_1 \mathbf{x}_d,$$

and knowing eigenvectors normed for $\mathbf{M}$ from the eigenvalue task

$$\tilde{\mathbf{K}}\mathbf{v} = \tilde{\omega}_0^2 \mathbf{M}\mathbf{v}, \tag{20}$$

$$\mathbf{V}^T \tilde{\mathbf{K}} \mathbf{V} = \langle \ldots \tilde{\omega}_{0r}^2 \ldots \rangle; \qquad \mathbf{V}^T \mathbf{M} \mathbf{V} = \mathbf{E}. \tag{21}$$

Seeking for the solution in form of

$$\mathbf{x}_s(t) = \sum_{r=1}^{n} \mathbf{v}_r y_r(t) \tag{22}$$

after substitutions and multiplication by $\mathbf{V}^T$, the system parts into one-degree-of-freedom differential equations. r^{th} differential equation containing also the structural damping is:

$$\ddot{y}_r(t) + \gamma \tilde{\omega}_{0r} \dot{y}_r(t) + \tilde{\omega}_{0r}^2 y_r(t) = f_r(t) + f_{dr}. \tag{23}$$

If support vibration is harmonic

$$\ddot{x}_R\,(t) = z\cos\alpha t,$$

then

$$f_r\,(t) = -\mathbf{v}_r^T\,\mathbf{q}\,(t) = -\mathbf{v}_r^T\,\mathbf{m}_s z\cos\alpha t = f_r\cos\alpha t,$$

and

$$f_{dr} = \mathbf{v}_r^T\,\mathbf{q}_d = \mathbf{v}_r^T\,\mathbf{K}_1\mathbf{x}_d.$$

General solution of the generated one-degree-of-freedom system is:

$$y_r(t) = a_r e^{-\frac{\gamma}{2}\tilde{\omega}_{0r}(t-t_d)}\cos\tilde{\omega}_{0r}^*\,(t-t_d) + b_r e^{-\frac{\gamma}{2}\tilde{\omega}_{0r}(t-t_d)}\sin\tilde{\omega}_{0r}^*\,(t-t_d) + \\ + \frac{1}{\tilde{\omega}_{0r}^2}f_{dr} + \frac{1}{\tilde{\omega}_{0r}^2}f_r\frac{1}{\sqrt{\left(1-\frac{\alpha^2}{\tilde{\omega}_{0r}^2}\right)^2 + \gamma^2\frac{\alpha^2}{\tilde{\omega}_{0r}^2}}}\cos\left(\alpha t - \beta_r\right). \qquad (24)$$

Here

$$\beta_r = \mathrm{arctg}\,\frac{-\gamma\alpha\tilde{\omega}_{0r}}{\tilde{\omega}_{0r}^2-\alpha^2}, \qquad \omega_{0r}^* = \omega_{0r}\sqrt{1-\frac{\gamma^2}{4}} \approx \omega_{0r}.$$

Constants a_r, b_r can be calculated from the initial conditions y_{dr}, $\dot{y}_{dr}$ belonging to the one-degree-of-freedom system. These initial conditions can be determined from the displacement and velocity vectors belonging to the moment of impact by the relationships

$$y_{dr} = \mathbf{v}_r^T\mathbf{M}\mathbf{x}_d, \quad \dot{y}_{dr} = \mathbf{v}_r^T\mathbf{M}\dot{\mathbf{x}}_d. \qquad (25)$$

Having calculated integration constants, solution of the one-degree-of-freedom systems can be described in function of time. Afterwards, solution belonging to the given vibration phase of the structure can be computed using (22).

5. Numerical Results

5.1. Impact of Transport Flask on Thick Reinforced Concrete Slabs

One of the accidental situations is the impact (drop) of the spent fuel transport flask on the existing reinforced concrete slabs and walls of the reactor building during transportation. The task was to assess the safety of the existing structures strength against such an impact, and choose the optimal transport route. The initial data consisted of the transport route of the flask with drop points, the drop heights, the weight and geometry of the flask, the material properties and geometry of structures, including the amount of reinforcement. The 10 dropping points were analysed. The finite element model is shown in *Fig. 2* and *Fig. 3*.

Different (usually 0.5 and 0.2 m) drop heights were included and several energy absorbing measures were taken into account.

Two types of flasks (91 and 116) were taken into account with flask diameter of 2 m on the impact surface. The thickness of slab above the containment was 1.5 m and outside of the containment it was 0.4–0.6 m. For calculation the first 100 eigenvectors were used. The displacements and internal forces were determined in the interval of 0–1 sec by 0.001 sec steps. The detailed results and experience are in [3] and [4].

5.2. *Numerical Experience for Impact of the Neighbour Building Parts*

Fig. 3 shows a planar steel frame located on a reinforced concrete structure that can be regarded as stiff.

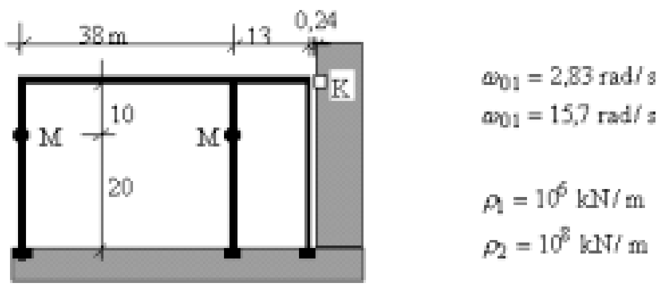

Fig. 3. Data of the structure tested

$$\omega_{01} = 2.83 \text{ rad/s}$$
$$\omega_{01} = 15.7 \text{ rad/s}$$
$$\rho_1 = 10^6 \text{ kN/m}$$
$$\rho_2 = 10^8 \text{ kN/m}$$

One of the side wings of the reinforced concrete structure is of the same height as the steel frame. Mass of the frame structure also comprises the weight of the covering in addition to its own weight. On two posts of the steel hall also masses M, equivalent of a crane's weight, appear. Between both structural units there is an expansion gap. On effect of support movement, the frame regarded as flexible will hit the structure in point K.

The smallest natural circular frequency of the system is ω_{01}, while in the case of a system supported in point $K : \hat{\omega}_{01}$. Spring stiffness computed from the data of the reinforced concrete structure in the point of impact makes ρ_1. Changes of impact force were calculated with a stiffer spring ρ_2, too. Horizontal support movement was $\ddot{x}_R(t) = 0.4 \text{ g} \cos 5t$. The displacement of point K without impact is in *Fig. 4*. We can see the effect of internal damping. In this case, horizontal vibration amplitude of the frame in point K is 0.257 m, i.e. impact occurs. Displacements of point K can be seen in *Fig.5*.

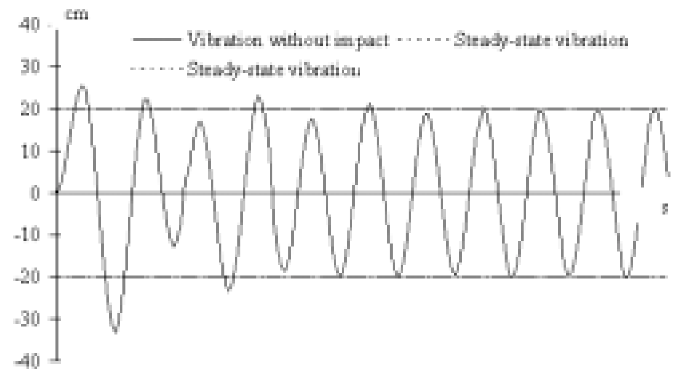

Fig. 4. Impact force in the case of different spring stiffness

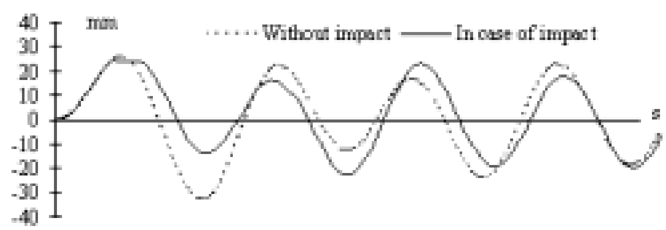

Fig. 5. Displacements of point K at impact and without impact

Displacements next to the impact are displayed on a larger scale in *Fig. 6*.

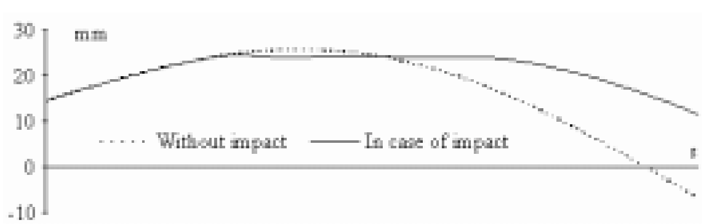

Fig. 6. Displacements of point K on a larger scale

Change of impact force with time is given in *Fig. 7/a*. With a larger spring stiffness, a much larger impact force and a quite different change with time can be observed as it is shown in *Fig. 7/b*. The detailed results and numerical experience are contained in [5].

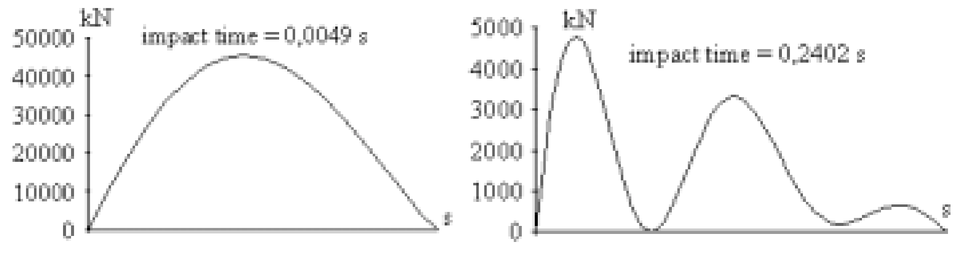

Fig. 7. Impact force in the case of different spring stiffness

6. Summary

An algorithm has been presented for computing dynamic excess displacements of structures, if effects of internal damping are to be taken into consideration during the impact. We analysed the effect of dropping body and presented a computation method taking impacts in various neighbouring building parts in the vibration process into consideration. The developed algorithm and the numerical results have been tested on actual problems. The task is solved by building up the vibration process from phases, and this allows computation of the impact force, too. It may be stated that the mentioned factors have an important effect, justified to be reckoned with in the analysis of real structures.

Acknowledgement

The author thanks for supporting the research by the Hungarian Scientific Research Fund (OTKA 11499-009) and Higher Education Research and Development Program (FKFP 0397/97).

References

[1] GYÖRGYI, J.: Viscous and Hysteretic Damping in Vibration of Structures. *Periodica Polytechnica, Ser. Civil Engineering*, Vol. 29, No. (1–2), pp. 23–31, 1985.
[2] BATHE, K. J. – WILSON, E. L.: Numerical Methods in Finite Element Analysis, Prentice-Hall, New Jersey, 1976.
[3] LENKEI, P. – GYÖRGYI, J. – DOBÓ, J. – RÁTKAI, S. (1995): Drop Impact Study of Spent Fuel Transport Flask on Structures, *Proceedings of the Int. Conf. on Dynamic Behaviour of Concrete Structures*, pp. 335–338, Kosice, Slovakia, 1995.
[4] GYÖRGYI, J. – LENKEI, P.: Impact of Transport Flask on Thick Reinforced Concrete Slab. In: Structures Under Shock and Impact IV, Edited by N. Jones, C. A. Brebbia, A. J. Watson, 181–190, Computational Mechanics Publications, Southampton and Boston, 1996.
[5] GYÖRGYI, J.: Impact of Neighbour Building Parts in Case of Earthquake. In: Structures Under Shock and Impact V, Edited by N. Jones, D. G. Talaslidis, C. A. Brebbia, G. D.. Manolis, 365–374, Computational Mechanics Publications, Southampton and Boston, 1998.