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NUMERICAL ANALYSIS OF INEXTENSIONAL, KINEMATICALLY INDETERMINATE ASSEMBLIES

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Abstract

This paper gives a summary of numerical experience of analysis of continuous motion of finite mechanism. The procedure can calculate finite displacement of assemblies composed of rigid bars and pin joints with one degree of freedom. At the beginning of or during the motion, change of degree of freedom, that is bifurcation, and limit point can occur. The algorithm can solve these problems quite easily. We will show the usage of this algorithm in some examples.

Keywords: mechanism, kinematically indeterminate, exact equation.

Nomenclature

j	number of the internal joints
b	number of the bars
h_i, k_i	sign of the starting and end point of the i^{th} bar, $i = 1, 2, .., b$
l_i	length of bar i.
μ	dimension of physical space (plane: $\mu = 2$, space $\mu = 3$)
$\mathbf{r}$	vector of joints co-ordinates
$r_{g,\nu}$	the ν^{th} component of the position vector of node g, $\nu = 1, .., \mu$
$\mathrm{d}\mathbf{r}$	$\mu \cdot j$-dimensional vector of infinitesimal co-ordinate increments
$\delta_{g\nu}$	finite increment of the ν^{th} co-ordinate of node g
$\mathbf{d}$	$\mu \cdot j$-dimensional vector of finite co-ordinate increments
$\mathbf{t}$	vector of compatibility error of bars
t_i	compatibility error of i^{th} bar
$\rho(\mathbf{B})$	rank of matrix $\mathbf{B}$
$\bullet$	scalar product
$()^T$	transpose

1. Introduction

In practice structural engineers have become more and more interested in assemblies with pin-jointed bars forming finite mechanisms, like deployable and foldable bar structures. These assemblies with inextensional deformation undergo finite motions. We will use vector-matrix method to the description of this motion. With this technique we can examine assemblies of arbitrary large number of bars, in contrast with geometric construction, that was widely used before. The basis of our procedure is the exact compatibility [1]. With the help of exact equations we can find all possible positions, all the motion path of the assemblies, what is very important for instance near the bifurcation point.

2. The Exact Equation of the Finite Change of State of the Assembly

If we change the exact co-ordinates of the initial state ($r_{k_i,\nu}, r_{h_i,\nu}$) with finite co-ordinate increments ($\delta_{k_i,\nu}, \delta_{h_i,\nu}$), then we will get a compatible state, provided the finite co-ordinate increments ($\delta_{k_i,\nu}, \delta_{h_i,\nu}$) satisfy the following equation system:

$$\frac{\sum_{\nu=1}^{\mu}\left(\left(r_{k_i,\nu}+\frac{\delta_{k_i,\nu}}{2}\right)-\left(r_{h_i,\nu}+\frac{\delta_{h_i,\nu}}{2}\right)\right)\left(\delta_{h_i,\nu}-\delta_{k_i,\nu}\right)}{\ell_i}=0$$

$$i=1,2,\ldots,b\,. \tag{1}$$

Eq. (1) is the exact equation of the finite change of state of the assembly, which is identical to a system of equations consisting of b non-linear equations of $\mu \cdot j$ variables.

The linear approximation of the system of equation (1) is

$$\mathbf{B} \bullet \mathbf{d} = 0, \tag{2}$$

where $\mathbf{B}$ is the transpose of the equilibrium matrix ($\mathbf{B} = \mathbf{G}^T$) of the assembly, and it is called compatibility matrix.

The number of the linearly independent infinitesimal mechanism is $\mu \cdot j - \rho(\mathbf{B})$. A finite mechanism at the same time is an infinitesimal mechanism, so if we want to give a one or more-degree-of-freedom mechanism, we have to carry out the necessary condition: $\mu \cdot j - \rho(\mathbf{B}) \geq 1$. During the displacement of one-degree-of-freedom structure $\mu \cdot j - \rho(\mathbf{B}) > 1$ can occur, so more than one linearly independent infinitesimal mechanism can occur. We call right these positions bifurcation points, because as many independent displacements are possible as the degree of freedom. We can choose optionally one of the paths we are going on with the motion. After the bifurcation point the examined structures have become again a one-degree-of-freedom finite mechanism.

3. Description of the Numerical Algorithm

The main steps of the analysis of continuous motion of the one-degree-of-freedom finite mechanism:

a) We know an initial compatible state of the bar-joint structure.
b) We make the compatibility matrix ($\mathbf{B}$). $\mathbf{B}$ is the transpose of the equilibrium matrix of the assembly.
c) $\mathbf{B}$ is not a full-column-rank matrix, so we can partition it into blocks $\mathbf{B} = [\mathbf{B}_{11}\mathbf{B}_{12}]$ so, that $\mathbf{B}_{11}$ is the largest-absolute-value determinant submatrix (by pivoting).
d) The *Eq.* (2) takes the form

$$\mathbf{B} \bullet \mathbf{d} = [\mathbf{B}_{11}\mathbf{B}_{12}] \bullet \mathbf{d} = \mathbf{B}_{11} \bullet \mathbf{d}_1 + \mathbf{B}_{12} \bullet \mathbf{d}_2 = 0. \tag{3}$$

Here, the dependent co-ordinate increments are in vector $\mathbf{d}_1$, and the only independent co-ordinate increment is in $\mathbf{d}_2$. Prescribing the value of the independent co-ordinate increment $\mathbf{d}_2$, the dependent co-ordinate increments in $\mathbf{d}_1$ can be determined. Let us prescribe $\mathbf{d}_2$ with finite value in the motion region. According to *Eq.* (3) the 0th approximation of $\mathbf{d}_1$ is

$$\mathbf{d}_1^{(0)} = -\mathbf{B}_{11}^{-1} \bullet \mathbf{B}_{12} \bullet \mathbf{d}_2. \tag{4}$$

e) We calculate the compatibility error of all bars

$$t_i^{(n)} = \frac{\sum_{\nu=1}^{\mu} \left(\left(r_{k_i,\nu} + \frac{\delta_{k_i,\nu}^{(n-l)}}{2} \right) - \left(r_{h_i,\nu} + \frac{\delta_{h_i,\nu}^{(n-l)}}{2} \right) \right) \left(\delta_{h_i,\nu}^{(n-l)} - \delta_{k_i,\nu}^{(n-l)} \right)}{\ell_i}$$

$$i = 1, 2, \ldots, b \qquad n = 1, 2, \ldots, \infty. \tag{5}$$

f) The next step is the determination of the new dependent co-ordinates:

$$\mathbf{B}_{11}\mathbf{d}_1^{(n)} + \mathbf{B}_{12}\mathbf{d}_2 + \mathbf{t}^{(n)} = 0, \qquad \mathbf{d}_1^{(n)} = -\mathbf{B}_{11}^{-1}\mathbf{B}_{12}\mathbf{d}_2 - \mathbf{B}_{11}^{-1}\mathbf{t}^{(n)}. \tag{6}$$

g) Let us continue the calculation from point (e). If the value of maximum norm of vector $t^{(n)}$ is not greater than the specified error tolerance $\varepsilon > 0$,

$$|\mathbf{t}^{(n)}| \leq \varepsilon, \tag{7}$$

then the procedure may be considered finished in the n^{th} step. If the procedure is divergent, then the independent co-ordinate increment is not correct (out of the motion region) or the assembly is not a finite mechanism.
h) The vector of the co-ordinates in the new compatible state is: $\mathbf{r}^{\text{new}} = \mathbf{r}^{\text{old}} + \mathbf{d}^{\text{last}}$.

4. Continuous Motion and Bifurcation

Fig. 1 shows a simple one-degree-of-freedom finite mechanism. The size of the compatibility matrix is 3×4, whose rank is $\rho(\mathbf{B}) = 3$. After a small displacement, the rank is unchanged, so the mechanism indeed is finite. During the continuous motion the assembly gets to a nearly horizontal position. At this point the rank seems to decrease, because the absolute value of the third pivot element is near zero. In the perfectly horizontal position the compatibility matrix is:

$$\mathbf{B} = \begin{array}{c} \begin{array}{cccc} 11 & 12 & 21 & 22 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ (b1 \\ (b2 \\ (b3 \end{array}$$

Here $\rho(\mathbf{B}) = 2$, that is the rank has decreased by 1. The change of rank indicates

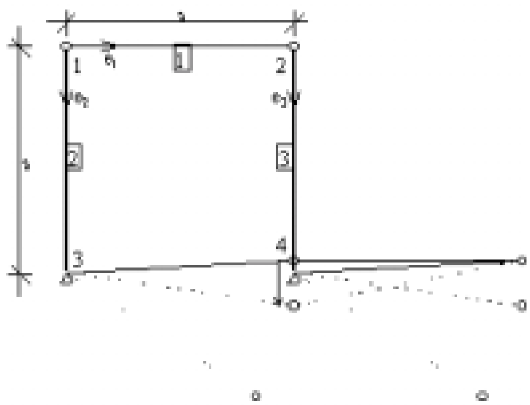

Fig. 1.

that in this position the motion has a bifurcation. Locally, the assembly behaves like a two-degree-of-freedom infinitesimal mechanism; that is, it has two different independent infinitesimal motions. We will show that the bifurcation does not cause any difficulty in our numerical procedure.

Firstly, let us prescribe $\delta_{12} = -\ell/10$, and keep $\delta_{22} = 0$. Let us see the compatibility matrix of this fictitious one-degree-of-freedom mechanism.

$$\mathbf{B} = \begin{array}{c} \begin{array}{cccc} 11 & 12 & 21 & 22 \end{array} \\ \begin{bmatrix} 0.995037 & 0.0995037 & -0.995037 & -0.0995037 \\ -0.995037 & 0.0995037 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ (b1 \\ (b2 \\ (b3 \end{array}.$$

Let us partition **B** so that δ_{12} is the independent co-ordinate increment:

$$\mathbf{B}_{11} = \begin{array}{c} \begin{array}{ccc} 11 & 21 & 22 \end{array} \\ \left[\begin{array}{ccc} 0.995037 & -0.995037 & -0.0995037 \\ -0.995037 & 0 & 0 \\ 0 & -1 & 0 \end{array}\right] \end{array} \begin{array}{l} (b1 \\ (b2\,, \\ (b3 \end{array}$$

$$\mathbf{B}_{12} = \begin{array}{c} 12 \\ \left[\begin{array}{c} 0.0995037 \\ 0.0995037 \\ 0 \end{array}\right] \end{array}$$

After the matrix calculation we get:

$$\mathbf{B}_{11}^{-1} = \begin{array}{c} \begin{array}{ccc} 11 & 21 & 22 \end{array} \\ \left[\begin{array}{ccc} 0 & -1.005 & 0 \\ 0 & 0 & -1 \\ -10.05 & -10.05 & 10 \end{array}\right] \end{array} \begin{array}{l} (b1 \\ (b2\,, \\ (b3 \end{array} \qquad \mathbf{B}_{11}^{-1}\mathbf{B}_{12} = \begin{array}{c} 12 \\ \left[\begin{array}{c} -0.1 \\ 0 \\ -2 \end{array}\right] \end{array}$$

The initial value of the finite co-ordinate increment vector is:

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \end{bmatrix}, \quad \mathbf{d}_1^{\langle 0\rangle} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{l} (11 \\ (21\,, \\ (22 \end{array} \quad \mathbf{d}_2 = \left[-\frac{\ell}{10}\right] (12.$$

The calculation of the dependent co-ordinate increments is as follows:

$$\Delta\mathbf{d}_1^{\langle 0\rangle} = -B_{11}^{-1}B_{12}\mathbf{d}_2 = \begin{bmatrix} -\ell/100 \\ 0 \\ -\ell/5 \end{bmatrix}, \quad \mathbf{d}_1^{\langle 1\rangle} = \mathbf{d}_1^{\langle 0\rangle} + \Delta\mathbf{d}_1^{\langle 0\rangle} = \begin{bmatrix} -\ell/100 \\ 0 \\ -\ell/5 \end{bmatrix}.$$

The compatibility state of the assembly is now the perfectly folded state. The position vector of the nodes has the form:

$$\mathbf{r}^T = \begin{array}{c} \begin{array}{cccccccc} 11 & 12 & 21 & 22 & 31 & 32 & 41 & 42 \end{array} \\ \left[\begin{array}{cccccccc} \ell & 0 & 2\ell & 0 & 0 & 0 & \ell & 0 \end{array}\right] \end{array}.$$

After the first approximation of the finite co-ordinate increments we must calculate the vector of compatibility errors (*Eq.* (5)):

$$t_1 = \frac{\left[\left(r_{21} + \frac{\delta_{21}}{2}\right) - \left(r_{11} + \frac{\delta_{11}}{2}\right)\right](\delta_{11} - \delta_{21}) + \left[\left(r_{22} + \frac{\delta_{22}}{2}\right) - \left(r_{12} + \frac{\delta_{12}}{2}\right)\right](\delta_{12} - \delta_{22})}{\ell}$$

$$t_2 = \frac{\left[r_{31} - \left(r_{11} + \frac{\delta_{11}}{2}\right)\right]\delta_{11} + \left[r_{32} - \left(r_{12} + \frac{\delta_{12}}{2}\right)\right]\delta_{12}}{\ell},$$

$$t_3 = \frac{\left[r_{41} - \left(r_{21} + \frac{\delta_{21}}{2}\right)\right]\delta_{21} + \left[r_{42} - \left(r_{22} + \frac{\delta_{22}}{2}\right)\right]\delta_{22}}{\ell},$$

$$t_1^{\langle 1\rangle} = \frac{\left[\left(2\ell + \frac{0}{2}\right) - \left(\ell - \frac{\ell}{200}\right)\right]\left(-\frac{\ell}{100} - 0\right) + \left[\left(0 - \frac{\ell}{10}\right) - \left(0 - \frac{\ell}{20}\right)\right]\left(-\frac{\ell}{10} + \frac{\ell}{5}\right)}{\ell}$$

$$= -\frac{\ell}{66.44},$$

$$t_2^{\langle 1\rangle} = \frac{\left[0 - \left(\ell - \frac{\ell}{200}\right)\right]\left(-\frac{\ell}{100}\right) + \left[0 - \left(0 - \frac{\ell}{20}\right)\right]\left(-\frac{\ell}{10}\right)}{\ell} = \frac{\ell}{202},$$

$$t_3^{\langle 1\rangle} = \frac{\left[\ell - \left(2\ell + \frac{0}{2}\right)\right]0 + \left[0 - \left(0 - \frac{\ell}{10}\right)\right]\left(-\frac{\ell}{5}\right)}{\ell} = -\frac{\ell}{50}.$$

Let us modify the dependent co-ordinate increments:

$$\mathbf{t}^{\langle 1\rangle} = \begin{bmatrix} -\ell/66.44 \\ \ell/202 \\ -\ell/50 \end{bmatrix}, \qquad \Delta\mathbf{d}_1^{\langle 1\rangle} = -\mathbf{B}_{11}^{-1}\mathbf{t}^{\langle 1\rangle} = \begin{bmatrix} \ell/201 \\ -\ell/50 \\ \ell/10.15 \end{bmatrix},$$

$$\mathbf{d}_1^{\langle 2\rangle} = -\mathbf{d}_1^{\langle 1\rangle} + \Delta\mathbf{d}_1^{\langle 1\rangle} = \begin{bmatrix} -\ell/199 \\ -\ell/50 \\ \ell/9.85 \end{bmatrix}.$$

The last two steps continued until the largest absolute value of vector $\mathbf{t}^{\langle n\rangle}$ will be sufficiently small.

$$\mathbf{t}^{\langle 2\rangle} = \begin{bmatrix} \ell/67.28 \\ \ell/80002 \\ \ell/68.27 \end{bmatrix}, \qquad \Delta\mathbf{d}_1^{\langle 2\rangle} = -\mathbf{B}_{11}^{-1}\mathbf{t}^{\langle 2\rangle} = \begin{bmatrix} \ell/79604 \\ \ell/68.27 \\ \ell/330.7 \end{bmatrix},$$

$$\mathbf{d}_1^{\langle 3\rangle} = \mathbf{d}_1^{\langle 2\rangle} + \Delta\mathbf{d}_1^{\langle 2\rangle} = \begin{bmatrix} -\ell/199.5 \\ -\ell/186.8 \\ -\ell/10.15 \end{bmatrix}.$$

$$\mathbf{t}^{\langle 3\rangle} = \begin{bmatrix} \ell/2960 \\ \ell/1.410^9 \\ \ell/2060 \end{bmatrix}, \quad \Delta\mathbf{d}_1^{\langle 3\rangle} = -\mathbf{B}_{11}^{-1}\mathbf{t}^{\langle 3\rangle} = \begin{bmatrix} \ell/1.410^9 \\ \ell/2060 \\ -\ell/685 \end{bmatrix},$$

$$\mathbf{d}_1^{\langle 4\rangle} = \mathbf{d}_1^{\langle 3\rangle} + \Delta\mathbf{d}_1^{\langle 3\rangle} = \begin{bmatrix} -\ell/199.5 \\ -\ell/205.5 \\ -\ell/10 \end{bmatrix},$$

$$\mathbf{t}^{\langle 4\rangle} = \begin{bmatrix} -\ell/6822 \\ 0 \\ -\ell/7019 \end{bmatrix}, \quad \Delta\mathbf{d}_1^{\langle 4\rangle} = -\mathbf{B}_{11}^{-1}\mathbf{t}^{\langle 4\rangle} = \begin{bmatrix} 0 \\ -\ell/7019 \\ -\ell/20620 \end{bmatrix},$$

$$\mathbf{d}_1^{\langle 5\rangle} = \mathbf{d}_1^{\langle 4\rangle} + \Delta\mathbf{d}_1^{\langle 4\rangle} = \begin{bmatrix} -\ell/199.5 \\ -\ell/199.6 \\ -\ell/10 \end{bmatrix}.$$

By adding the last vector **d** to the vector **r** we have:

$$\mathbf{r} = \begin{array}{l} \begin{bmatrix} \ell - \ell/199.5 \\ -\ell/10 \\ 2\ell - \ell/199.6 \\ -\ell/10 \\ 0 \\ 0 \\ \ell \\ 0 \end{bmatrix} \end{array} \begin{array}{l} (11 \\ (12 \\ (21 \\ (22 \\ (31 \\ (32 \\ (41 \\ (42 \end{array} .$$

After the first displacement path we analyse the second displacement possibility (*Fig. 2*). The initial state is the folded state again.

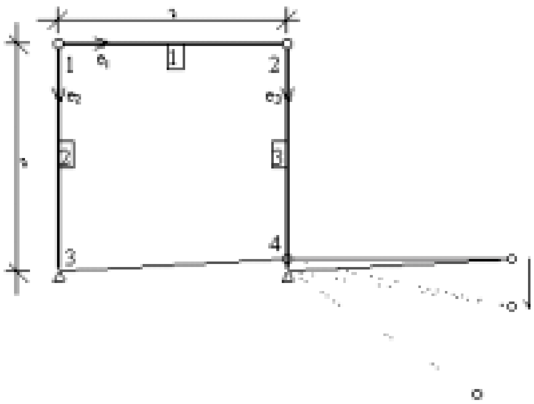

Fig. 2.

$$\mathbf{r}^T = \begin{array}{c} \begin{array}{cccccccc} 11 & 12 & 21 & 22 & 31 & 32 & 41 & 42 \end{array} \\ \begin{bmatrix} \ell & 0 & 2\ell & 0 & 0 & 0 & \ell & 0 \end{bmatrix} \end{array}$$

Let us prescribe $\delta_{22} = -\ell/10$, and keep $\delta_{12} = 0$.

The compatibility matrix of this fictitious one-degree-of-freedom mechanism is:

$$\mathbf{B} = \begin{array}{c} \begin{array}{cccc} 11 & 12 & 21 & 22 \end{array} \\ \begin{bmatrix} 0.995037 & -0.0995037 & -0.995037 & 0.0995037 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -0.995037 & 0.0995037 \end{bmatrix} \end{array} \begin{array}{l} (b1 \\ (b2 \\ (b3 \end{array} .$$

Let us partition **B** so that δ_{22} is the independent co-ordinate increment:

$$\mathbf{B}_{11} = \begin{array}{c} \begin{array}{ccc} 11 & 12 & 21 \end{array} \\ \begin{bmatrix} 0.995037 & -0.0995037 & -0.995037 \\ -1 & 0 & 0 \\ 0 & 0 & -0.995037 \end{bmatrix} \end{array} \begin{array}{l} (b1 \\ (b2 \\ (b3 \end{array} ,$$

$$\mathbf{B}_{12} = \begin{array}{c} 22 \\ \begin{bmatrix} 0.0995037 \\ 0 \\ 0.0995037 \end{bmatrix} \end{array}.$$

After the matrix calculation we get:

$$\mathbf{B}_{11}^{-1} = \begin{array}{cc} \begin{array}{ccc} 11 & 12 & 21 \end{array} & \\ \begin{bmatrix} 0 & -1 & 0 \\ -10.05 & -10 & 10.05 \\ 0 & 0 & -1.005 \end{bmatrix} & \begin{array}{l} (b1 \\ (b2 \\ (b3 \end{array} \end{array}, \qquad \mathbf{B}_{11}^{-1}\mathbf{B}_{12} = \begin{array}{c} 22 \\ \begin{bmatrix} 0 \\ 0 \\ -0.1 \end{bmatrix} \end{array}.$$

The initial value of the finite co-ordinate increment vector is:

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \end{bmatrix}, \quad \mathbf{d}_1^{\langle 0 \rangle} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{l} (11 \\ (12 \\ (21 \end{array}, \quad \mathbf{d}_2 = \left[-\frac{\ell}{10} \right] (22.$$

The calculation of the dependent co-ordinate increments is as follows:

$$\Delta\mathbf{d}_1^{\langle 0 \rangle} = -\mathbf{B}_{11}^{-1}\mathbf{B}_{12}\mathbf{d}_2 = \begin{bmatrix} 0 \\ 0 \\ -\ell/100 \end{bmatrix}, \quad \mathbf{d}_1^{\langle 1 \rangle} = \mathbf{d}_1^{\langle 0 \rangle} + \Delta\mathbf{d}_1^{\langle 0 \rangle} = \begin{bmatrix} 0 \\ 0 \\ -\ell/100 \end{bmatrix},$$

Let us calculate the vector of compatibility errors (*Eq.* (5)):

$$\mathbf{t}^{\langle 1 \rangle} = \begin{bmatrix} \ell/202 \\ 0 \\ \ell/202 \end{bmatrix}, \qquad \Delta\mathbf{d}_1^{\langle 1 \rangle} = -\mathbf{B}_{11}^{-1}\mathbf{t}^{\langle 1 \rangle} = \begin{bmatrix} 0 \\ 0 \\ \ell/201 \end{bmatrix},$$

$$\mathbf{d}_1^{\langle 2 \rangle} = \mathbf{d}_1^{\langle 1 \rangle} + \Delta\mathbf{d}_1^{\langle 1 \rangle} = \begin{bmatrix} 0 \\ 0 \\ -\ell/199 \end{bmatrix},$$

$$\mathbf{t}^{\langle 2 \rangle} = \begin{bmatrix} \ell/78841 \\ 0 \\ \ell/78841 \end{bmatrix}, \qquad \Delta\mathbf{d}_1^{\langle 2 \rangle} = -\mathbf{B}_{11}^{-1}\mathbf{t}^{\langle 2 \rangle} = \begin{bmatrix} 0 \\ 0 \\ \ell/78450 \end{bmatrix},$$

$$\mathbf{d}_1^{\langle 3 \rangle} = \mathbf{d}_1^{\langle 2 \rangle} + \Delta\mathbf{d}_1^{\langle 2 \rangle} = \begin{bmatrix} 0 \\ 0 \\ -\ell/199.5 \end{bmatrix},$$

$$\mathbf{t}^{\langle 3 \rangle} = \begin{bmatrix} \ell/1.410^{9} \\ 0 \\ \ell/1.410^{9} \end{bmatrix}, \qquad \Delta\mathbf{d}_1^{\langle 3 \rangle} = -\mathbf{B}_{11}^{-1}\mathbf{t}^{\langle 3 \rangle} = \begin{bmatrix} 0 \\ 0 \\ \ell/1.410^{9} \end{bmatrix},$$

$$\mathbf{d}_1^{\langle 4 \rangle} = \mathbf{d}_1^{\langle 3 \rangle} + \Delta\mathbf{d}_1^{\langle 3 \rangle} = \begin{bmatrix} 0 \\ 0 \\ -\ell/199.5 \end{bmatrix}.$$

By adding the last vector **d** to the vector **r** we have:

$$\mathbf{r} = \begin{bmatrix} \ell \\ 0 \\ 2\ell - \ell/199.5 \\ -\ell/10 \\ 0 \\ 0 \\ \ell \\ 0 \end{bmatrix} \begin{array}{l} (11 \\ (12 \\ (21 \\ (22 \\ (31 \\ (32 \\ (41 \\ (42 \end{array} .$$

After the two independent displacements we get two different compatibility states, which are one-degree-of-freedom finite mechanisms (the rank of **B** is again $\rho(\mathbf{B}) = 3$).

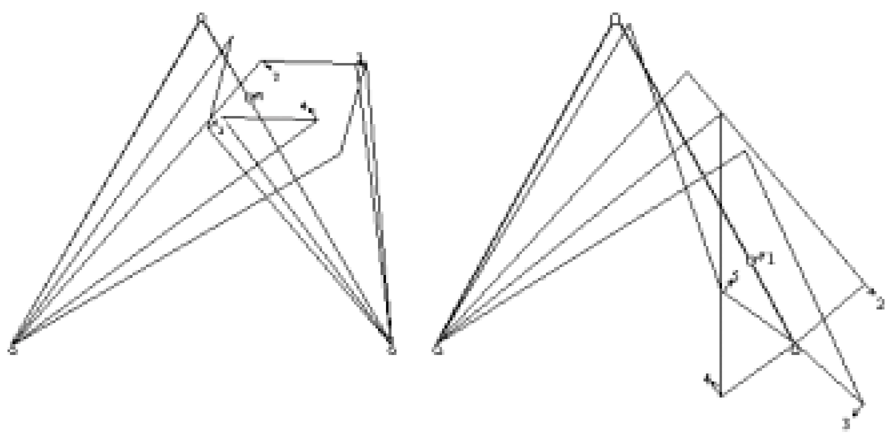

Fig. 3.

5. Limit Point at the Motion Path

During motion, along its path some node can reach a limit point, and it can turn back. This is not a problem in the algorithm, because the independent co-ordinate always has the largest velocity during motion. This is the result of the pivoting technique. The independent co-ordinate with the largest velocity is never extreme, so the continuous motion is always possible.

6. Unfolding the Foldable Assemblies

Fig. 4

Fig. 5

In *Fig. 4* a foldable structure is shown. The four inner nodes and the eight bars form an infinitesimal mechanism, because the inner nodes cannot move from their place without elongation of bars. If we remove a bar (*Fig. 5*), we get a four-degree-of-freedom infinitesimal mechanism ($\mu \cdot j - \rho(\mathbf{B}) = 4$). After the four linearly independent finite displacements we will get a one-degree-of-freedom finite mechanism (we can displace the inner nodes without elongation of bars). *Fig. 6* shows two unfolding shapes, symmetrical pairs of these are the other two different shapes.

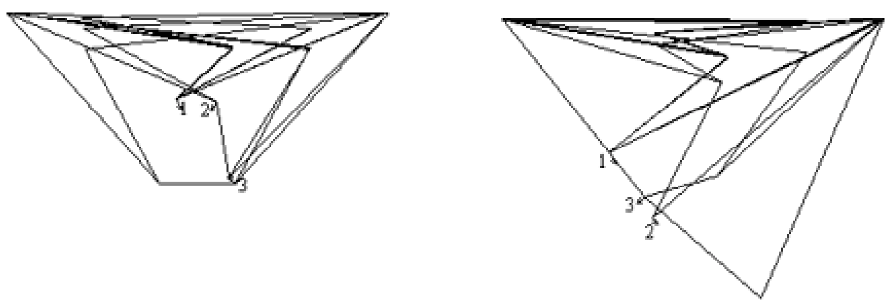

Fig. 6.

If we remove another bar from the infinitesimal mechanism (*Fig. 7*) we get an analogous case as previously. We can unfold the assembly in four different ways (*Fig. 8* shows two cases).

Fig. 7.

In the algorithm we have to prescribe the vertical co-ordinate increments of the four inner nodes (four-degree-of-freedom) with small finite values. The vertical co-ordinate component of the largest displacement node wills the independent co-ordinate increments, so we can control the four different unfolding shapes.

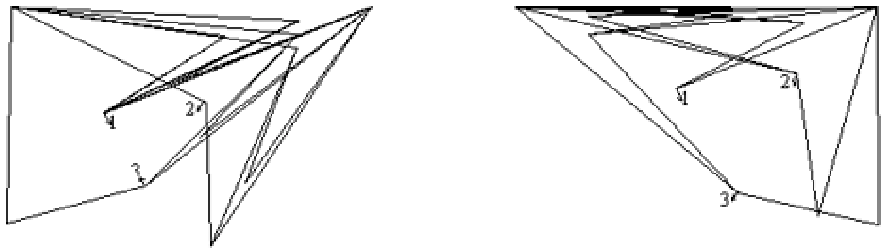

Fig. 8.

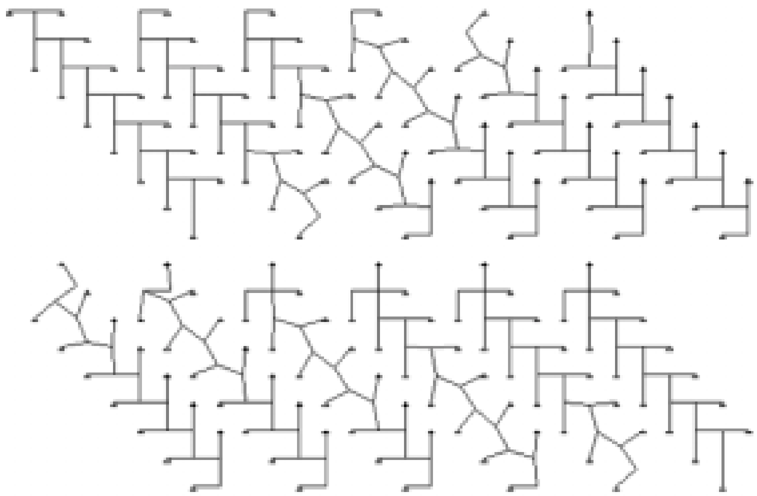

Fig. 9.

7. Higher Order Infinitesimal Mechanisms

The next example, shown in *Fig. 9*, is a higher order infinitesimal mechanism (TARNAI, 1989). The order is $2^{14} - 1$. Removing one bar we obtain a one-degree-of-freedom finite mechanism.

8. Compound Mechanisms

In the next example (*Fig. 10*) there are two mechanisms connected by a horizontal bar (8 inner nodes and 15 bars) (CONNELLY, 1993). In the symmetrical configuration (initial state) the rank of the compatibility matrix **B** is less than the number of columns of **B** by 2 (and it is less than the number of rows of **B** by 1; $\mu \cdot j - \rho(\mathbf{B}) = 2$). The assembly at this point behaves like a two- degree-of-freedom infinitesimal mechanism. We can move infinitesimally one of the end points of the connecting bar downwards and the other end point is unchanged. The other displacement possibility is the symmetrical pair. After the finite displacement the mechanism behaves like a one-degree-of-freedom finite mechanism (the **B** is a full row-rank matrix: $\mu \cdot j - \rho(\mathbf{B}) = 1$). The interest of this assembly is that we cannot displace the nodes in the upward direction, because there is no compatible state of the assembly in a small neighbourhood of the initial state in the upward direction. *Fig. 11* shows the trajectories of the end points of the connecting bar, along which the points travel twice during a period of motion.

9. Bar-and-Joint Assemblies in the Three-Dimensional Space

The aim of this research was the analysis of pin-jointed space structures in the post-critical regime. In initial stage statically indeterminate space lattices with one-parameter loads can lose their bars because of buckling or breaking. These bars fall out of further bearing of load, so it will be decrease the indeterminate degree. The load cannot increase when the structure will be mechanism. *Figs. 12–13* show space structures (17 inner joints and 50 bars). The compatibility matrix is a full-row-rank matrix, and $\mu \cdot j - \rho(\mathbf{B}) = 3 \cdot 17 - 50 = 1$, so the assembly has one finite degree of freedom. During the continuous motion (*Fig. 14*) a bifurcation can occur, when all bars meeting at a node will be in coplanar position. A fictitious increase in the degree of freedom cannot cause any problem, as we have seen above in examples of planar assemblies.

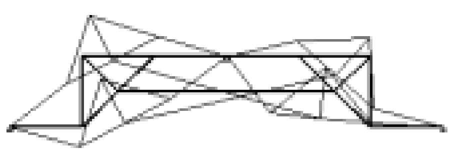

Fig. 10

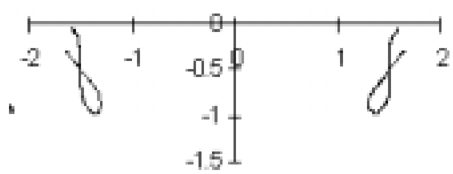

Fig. 11

10. Conclusions

The presented numerical procedure is suitable for determining all compatible states of a one or more-degree-of-freedom mechanism consisting of rigid bars and pin joints. The computer program can show graphically the paths of any nodes during the full motion period. At the bifurcation points in an interactive way we can choose any displacement possibility. The algorithm is suitable for finding equilibrium forms of kinematically indeterminate space structures under given load patterns, and if the bars are elastic, then it may be used for describing the behaviour of the assembly in the post-critical region where the bar-and-joint structure starts to behave like a finite mechanism. Our vector-matrix method is no more complicated if there are many bars and joints in a mechanism in contrast to the analysis based on the geometric constructions.

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References

[1] TARNAI, T. – SZABÓ, J.: On the Exact Equation of Inextensional, Kinematically Indeterminate Assemblies (manuscript).

[2] CONNELLY, R. – WHITELEY, W. (1996): Second Order Rigidity and Pre-Stress Stability for Tensegrity Frameworks. *SIAM J. Disc. Math.*, Vol. 9, pp. 453–491.

[3] CONNELLY, R. – SERVATIUS, H. (1994): Higher Order Rigidity - What is the Proper Definition? *Discrete & Computational Geometry*, Vol. 11, pp. 193–200.

[4] CONNELLY, R. (1993): Rigidity. Chapter 1.7 in Handbook of Convex Geometry (ed.: P.M. Gruber and J. M. Wills), Elsevier Sci. Pub., pp. 223–271.

[5] TARNAI, T. (1989): Higher Order Infinitesimal Mechanism. *Acta Technica Acad. Sci. Hung.*, Vol. 102, pp. 363–378.

[6] GÁSPÁR, ZS. – TARNAI, T. (1994): Finite Mechanisms Have No Higher-Order Rigidity. *Acta Technica Acad. Sci. Hung.*, Vol. 106, pp. 119–125.