PERIODICA POLYTECHNICA SER. CIV. ENG. VOL. 44, NO. 1, PP. 127–133 (2000)

ACTIVELY CONTROLLED COMPOSITE LOAD-BEARING STRUCTURES

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Received: December 20, 1998

Abstract

This study deals with the application of the active control of high-rise buildings with composite load-bearing structures. After introducing it, the theoretical solution is described, and then a numerical example shows the practical application of the method.

Keywords: load-bearing structures.

1. Preface

Actively controlled structures are a new strain of structural system and their application plays a dominant part of nowadays vibration control of high-rise buildings. The study deals with the problem of active control in the case of composite load-bearing structures. It means that the building to be analyzed consists of horizontal floors and supporting (load-bearing) vertical walls, columns, frameworks, or some combination. If vertical structural load-bearing members are other than bisymmetrical, the storeys vibrate in a 'coupled' mode (simultaneous torsional and bending vibrations) in their plane. In the following the study gives theoretical and practical instructions for the application of active control of the mentioned buildings.

2. The Differential Equation System of Motion

Complying with linear-elastic system with viscous damping, using a co-ordinate system *x*, *y*, *z* in the same direction for any mass, and with origins along the same vertical line, the matrix differential equation of motion becomes [1], [2]:

$$\mathbf{A}\ddot{d} + \mathbf{C}\dot{a} + \mathbf{K}d = \mathbf{D}\boldsymbol{u}(t) + \mathbf{E}\,\boldsymbol{p}(t). \tag{1}$$

Accordingly, terms in the matrix differential equation system (1), properly partitioned, are hypermatrices:

$$\mathbf{A} = \begin{bmatrix} \mathbf{M} & \mathbf{O} & -\mathbf{Y_s M} \\ \mathbf{O} & \mathbf{M} & \mathbf{X_s M} \\ -\mathbf{Y_s M} & \mathbf{X_s}M & \mathbf{I}_0 \end{bmatrix}$$

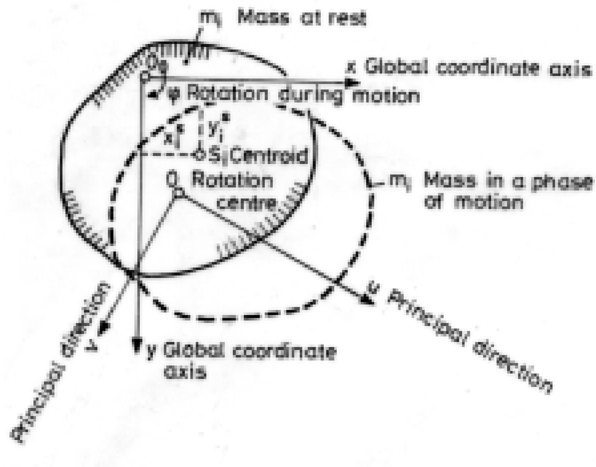

Fig. 1.

$\mathbf{M} = \langle m_1, m_2, \ldots, m_n \rangle$ diagonal matrix including masses concentrated in each storey plane;

$I_0 = \langle I_{01}, I_{02}, \ldots, I_{0n} \rangle$ diagonal matrix including mass moments of inertia of masses concentrated in each storey plane, referred to the origin of the coordinate system; $\left.\begin{matrix} \mathbf{Y}_s = \langle y_1^s, y_2^s, \ldots, y_n^s \rangle \\ \mathbf{X}_s = \langle x_1^s, x_2^s, \ldots, x_n^s \rangle \end{matrix}\right\}$ diagonal matrices including mass centroid coordinates (see *Fig.* 1).

Element of **K** and **C** are the stiffnes and damping matrices,

$$\mathbf{d} = \begin{bmatrix} \boldsymbol{X} \\ \boldsymbol{Y} \\ \varphi \end{bmatrix} \qquad \text{hypervector of } 3n \text{ dimensions, describing displacements:}$$

$$\boldsymbol{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad \varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_n \end{bmatrix} \quad \text{displacement ordinate;}$$

$$p(\mathbf{t}) = \begin{bmatrix} p_x(t) \\ p_y(t) \\ m(t) \end{bmatrix} \quad \text{and} \quad p_x(t) = \begin{bmatrix} p_{1x}\ (t) \\ p_{2x}(t) \\ \vdots \\ p_{nx}(t) \end{bmatrix} \quad \text{etc.}$$

representing applied load or external excitation at the originate of the coordinate system, and $\mathbf{u}(t)$ is the m dimensional control force vector. The $n \times m$ matrix **D** and $n \times n$ matrix **E** are location matrices which define locations of the control force and exitations, and 3 Control Algorithms, respectively.

To facilitate discussions, let us use *Eq.* (1) to represent the structure under consideration which, using the state – place representation, can be written in the form [3]; [4]:

$$\dot{\mathbf{z}}(t) = \mathbf{L}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{H}\mathbf{p}(t), \quad \mathbf{z}(o) = \mathbf{z}_o, \tag{2}$$

where

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{bmatrix}$$

is the $2n$ dimensional state hypervector

$$\mathbf{L} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{A}^{-1}K & -\mathbf{A}^{-1}\mathbf{C} \end{bmatrix} \tag{3}$$

is the $2n \times 2n$ system hypermatrix and

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{A}^{-1}\mathbf{D} \end{bmatrix} \quad \text{and} \quad \mathbf{H} = \begin{bmatrix} \mathbf{I} \\ \mathbf{A}^{-1}\mathbf{E} \end{bmatrix} \tag{4}$$

are $2n \times m$ and $2n \times n$ location hypermatrices specifying the locations of controllers and external exitations in the state-space, respectively. **0** and **I** denote the null matrix and the identity matrix of appropriate dimensions, respectively.

When the control vector is regulated by the state vector one has

$$\dot{\lambda} = \mathbf{L}^T\lambda + 2\mathbf{Q}z, \tag{5}$$

$$u = -\mathbf{R}^{-1}B^T\lambda \tag{6}$$

and

$$\lambda(t) = \mathbf{P}(t)z(t) \qquad \text{(Closed-loop control)}. \tag{7}$$

The matrices **Q** and $\mathbb{R}$ referred to as weighting matrices, whose magnitudes are assigned according to the relative importance attached to the state variables and to the control forces in the minimization procedure [3], [4].

The unknown matrix $\mathbf{P}(t)$ can be determined by substituting *Eq.* (7) into *Eqs.* (2,6,7). One can show that it is satisfied if $\mathbf{p}(t)$ is zero:

$$\mathbf{P}(t) + \mathbf{P}(t)\mathbf{L} - \frac{1}{2}\mathbf{P}(t)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}(t) + 2\mathbf{Q} = 0. \tag{8}$$

The optimal control theory, *Eq.* (8) is referred to as the matrix Riccati equation and $\mathbf{P}(t)$ is the Riccati matrix. Methods for solving the matrix Riccati equation are well documented in the literature [5], [6].

The substitution of *Eq.* (7) into *Eq.* (6) shows that the control vector $\mathbf{u}(t)$ is linear in $\mathbf{z}(t)$. The linear optimal control law is

$$\mathbf{u}(t) = \mathbf{G}\mathbf{z}(t) - \frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}(t)z(t) \tag{9}$$

where

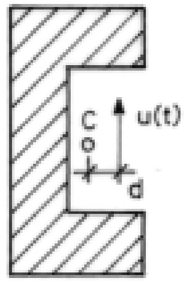

Fig. 2.

$$\mathbf{G}(t) = \frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{P}(t).$$

Upon substituting *Eq.* (9) into *Eq.* (2) the behaviour of the optimally controlled structure is described by

$$\dot{z}(t) = (\mathbf{L} + \mathbf{BG})z(t) + \mathbf{H}p(t) \qquad z(o) = z_o. \tag{10}$$

3. Example

Most of the environmental loads, such as wind and earthquakes, to which civil engineering structures are subjected are random in nature. Hence, the analysis of the behaviour of an actively controlled as well as an uncontrolled structure is based on the theory of random vibrations, we will use the following example to demonstrate some steps involved in such an analysis using [7]. By using a two degree of freedom structural system (contemporary elongation and torsion), relative merits of several different control configurations can be examined in an elementary way. The reader is referred to [8], [9] for a review of some basic principles in random vibration analysis.

Consider a one-storey building with the storey layout shown in *Fig.* 2, which is excited by an earthquake-type ground acceleration $\ddot{X}_o(t)$. In this example, $\ddot{X}_o(t)$ is modelled by a nonstationary Gaussian shot noise with

$$\ddot{X}_o(t) = \psi(t)W(t) \tag{11}$$

in which $W(t)$ is a stationary zero-mean Gaussian white noise and $\psi(t)$ is a deterministic modulating function of the form

$$\psi(t) = g(e^{\alpha t} - e^{\beta t})^2 h(t),$$

where $h(t)$ is the unit step function and g, α and β are constants. Accordingly the mean of $\ddot{X}_o(t)$ is zero and its covariance is

$$E[\ddot{X}_o(t)\ddot{X}_o(s)] = g^2(e^{-\alpha t} - e^{-\beta t})^2 h(t) D\delta(t - s),$$

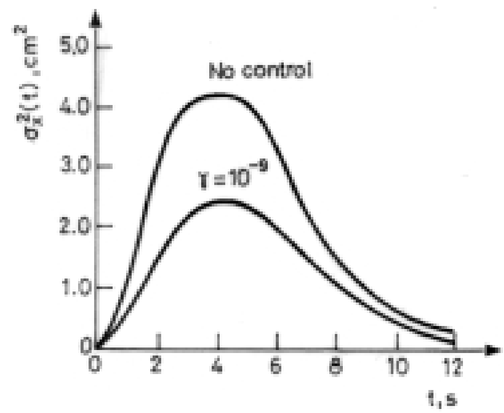

Fig. 3.

where D is the power spectral density of $W(t)$. For numerical calculations we will set

$$\alpha = 0.25 \text{ s}^{-1}, \quad \beta = 0.63 \text{ s}^{-1}, \quad g = 3.00 \quad \text{and} \quad D = 0.04 \text{ m}^2\text{s}^{-4}.$$

Since the excitation is random, the structural response is random and the control as determined from *Eq.* (9) is also random. (These random quantities will be written in capital letters.)

The state-space equation in this case has the form

$$\underline{\dot{z}}(t) = \mathbf{Lz}(t) + \mathbf{Bu}(t) + \mathbf{h}\ddot{X}_0(t), \qquad \mathbf{z}(0) = 0,$$

with

$$\mathbf{z}(t) = \begin{bmatrix} X(t) \\ \varphi(t) \\ X(t) \\ \dot{\varphi}(t) \end{bmatrix}; \quad \mathbf{h} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{u}(t) = \begin{bmatrix} u(t) \\ u(t)d = M(t) \end{bmatrix}.$$

$X(t)$ and $\varphi(t)$ is the displacement of the centroid in x direction and φ the torsion, respectively. For numerical computations, the following structural parameter values are used: $m = 4000$ kg,

$$I_0 = 1000 \text{ kg m}^2 = \text{g m}^2, \quad \xi = \frac{C}{C_{kr}} = 0 \cdot 0.2,$$

$$\omega_{01} = \left(\frac{k_x}{m}\right)^2 = 2^{\text{Hz}}, \qquad \omega_{02} = \left(\frac{k_\varphi}{I_0}\right)^2 = 1.5^{\text{Hz}}.$$

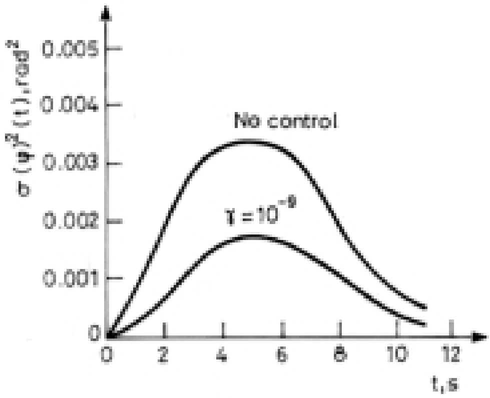

Fig. 4.

Finally the weighting matrices are assumed.

$$\mathbf{Q} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad \mathbf{R} = \gamma \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

where γ is a parameter representing the relative importance between the covariances of the response and those of the control forces. With the optimal control determined from *Eq.* (9), the mean of the controlled structural response is zero, and its covariance matrix at $t = s$, defined by

$$\mathbf{R}_{zz}(t) = E\left[(\mathbf{z}(t)\mathbf{z}^T(t)\right]$$

satisfied the first order matrix differential equation [6]

$$\dot{\mathbf{R}}_{zz}(t) = (\mathbf{L} + \mathbf{BG})\mathbf{R}_{zz}(t) + \mathbf{R}_{zz}(t)(\mathbf{L} + \mathbf{BG})^T + 2\mathbf{h}\mathbf{R}_{\ddot{X}_0\ddot{X}_0}(t)\mathbf{h}^T$$

with initial condition

$$\mathbf{R}_{zz}(o) = \mathbf{0}.$$

The covariance matrix of the control vector at $t = s$ can be obtained from *Eq.* (9) as

$$\mathbf{R}_{nn}(t) = \mathbf{G}\mathbf{R}_{zz}(t)\mathbf{G}^T. \tag{12}$$

The variance, $\sigma_x^2(t)$, of the relative displacement of the centroid between the foundation and the floor under optimal control is plotted in *Fig.* 3 if $\gamma = 10^{-10}$. The case is also plotted in *Fig.* 3 if $\sigma_x^2(t)$ is without control. The variance $\sigma_\varphi^2(t)$ of the relative torsion between the foundation and the floor is shown in *Fig.* 4 together with the no control case.

Acknowledgement

The present study was supported by the National Development and Research Foundation (OTKA).

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