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A NOTE ON THE ROLE OF INTERNAL STRUCTURE OF ROCKS IN THE DEVIATIONS OF COMPRESSION STRENGTH RESULTS

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Abstract

Our paper focuses on the uniaxial compression failure strength of granular rocks. The results of laboratory tests usually show a rather high scatter that has mostly been explained by the small random errors when doing the experiments. Our aim was to supervise this problem from microstructural point of view and to point out that the geometrical randomness of microstructure has a significant role in the differences between the results of individual tests.

Keywords: petrophysics, rock mechanics, compressive strength, rupture mechanism.

1. Introduction

Uniaxial compression failure strength is a qualifying characteristic for building stones as for most building materials. When trying to decide whether an analysed rock is suitable for a given purpose or not, the first question usually is 'How much is its compressional failure strength?' Determination of this important parameter is an everyday task in any laboratories analysing the behaviour of building stones. The Rock-Physical Laboratory of the Department of Geology at TU Budapest has wide experience not only about Hungarian rocks but also about building stones from all over the world.

Measurement techniques of compression strength tests have been standardised for many decades. As early as in 1885, the Department of Mechanics of the Hungarian Royal József University of Polytechnics was entrusted with testing the inland building materials. Its mechanical laboratory issued publications about the failure behaviour of Hungarian rocks already from 1878 (HORVÁTH, 1878).

According to the modern principles of measurement techniques and to the international regulations, the compression strength tests were standardised in 1978, being completed with the analysis of strain behaviour (MSZ 18285/1).

Because of the importance of compression tests and their qualifying results, researchers return to this problem now and again – both concerning measurement techniques and from theoretical – microstructural point of view (GÁLOS – KÜRTI, 1977; BAGI – BOJTÁR – GÁLOS, 1993).

2. Theoretical Background

Uniaxial compression tests are based on the most simple mechanical assumptions. The material is considered as a continuous domain and assuming simple uniaxial stress state in every point in it, failure strength is calculated as the axial force that breaks continuity (F_{ci}) divided by the area of orthogonal cross-section (A):

$$\sigma_{ci} = \frac{F_{ci}}{A}. \tag{1}$$

Compression failure strength is calculated from the total area of the cross-section under compression. However, rocks generally consist of pores as well as solid components (GÁLOS, 1981). Instead of uniform stress field, a better estimation is illustrated in *Fig. 1.*

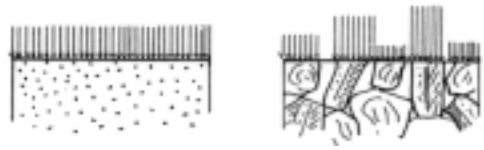

Fig. 1. Strength dispersion on surface

Failure of an elementary cell under uniaxial stress state is mostly due to the destroying of contact bonds between solid particles in those materials we are dealing with. This may be the result of either transversal tension or shear slip (GÁLOS – KÜRTI – VÁSÁRHELYI, 1994). Among them, the shear type failure is usually applied as the fracture condition for rocks (Mohr condition for fracture). The conditions are shown in *Fig. 2.*

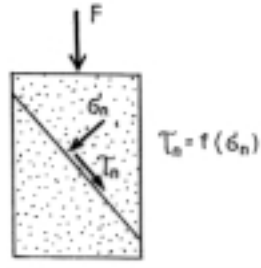

Fig. 2. Mohr crack theory

Failure strength is a result of complex processes going on in the internal structure of the material. The assumption of homogeneous isotropic behaviour is only a theoretical estimation; forming of shear planes is mostly based on the internal built-up of the stone. This structure – solid components, pores also as components, form and strength of bonds between solid particles, etc. – determines the resistance to external loads.

Suitable modelling of the internal structure may help to get a better understanding of fracture process. Complex analysis of theoretical considerations and experimental results can lead us to a deeper understanding of the behaviour of rocks.

3. Laboratory Experiments

Compression failure tests have to be performed according to standardised instructions all over in the world. In Hungary the measurement technical details are prescribed in the Standard MSZ 18285/1. According to it, samples are cylindrical bodies with a diameter of 5 ± 0.5 cm, and with a diameter : height = 1 : 2 ratio. Velocity of loading is also fixed in the standard so that any experiment would be performed under the same conditions. Number of tests on samples from the same material is $n = 5$, and beside the $+ + +$ average of the five results for failure strength, their corrected standard deviation

$$\overline{\sigma}_c = \frac{\sum_{k=1}^{n} \sigma_{ck}}{n}, \tag{2}$$

$$s = \sqrt{\frac{\sum_{k=1}^{n} (\overline{\sigma}_c - \sigma_{ck})^2}{n-1}} \tag{3}$$

also has to be determined (here $++$, $++$, …$++$ are the results of the five tests.

Our experimental results show a definite correlation between the average magnitude and deviation. Compression strength results of samples from a rough limestone can be seen in *Fig. 3.* Similar relations were gained also on other types of rocks.

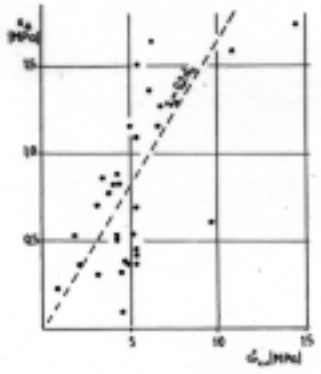

Fig. 3. Unaxial compressive strength and dispersion of laboratory test by miocenic limestone

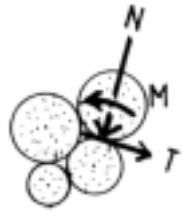

Fig. 4. Internal structure

Remarkably the standard deviation is about 20–25% of the compression strength which does not fit to the theoretical assumption of homogeneity. This large scatter is produced by the variability of internal structure as well as the technical, personal, etc. errors.

4. Numerical Simulations

4.1. Introductory Remarks

Application of numerical simulations is a powerful modern tool for the deeper understanding of microstructural fundaments of macro-level material behaviour. Most microstructural models consider the material as a set of randomly or regularly arranged, rigid *grains*, with elastic – frictional, elastoplastic, etc. *contacts* ('glue') between them. Physical characteristics of real grains are expressed through the material parameters of contacts in the model. The models are able to follow how the individual grains and contacts react to the effect of external loads: grain displacements, contact forces, crack opening, etc. can be followed step-by-step during the loading process.

Two main methods are applied today for following the state-changes. Both of them analyse the behaviour in finite steps, but the theoretical background of determining the displacement increments is different. *Quasi-static* models are based on compiling the stiffness matrix of the whole system or of the individual grains; the displacements are calculated by inverting the stiffness matrix (similarly to the methods applied in structural analysis). *Dynamic* models follow the processes in time. At the beginning of a new time step the model determines the accelerations of the grains, from the forces acting on them and from their inertia; then assuming the accelerations to be constant during the time step, velocities and displacements are calculated.

Grains are perfect circles in most of the models. A few special versions may use elliptical or polygonal particles but because following the geometry and the detection of contacts are very complicated problems in these cases, these models have a huge need of memory and computer time, which makes them rather difficult to apply on most of the commonly available PC-s today.

Recently the *two-dimensional* models are much more widespread in research applications than the 3D versions. Beside their lower price, 2D models have a more acceptable memory and speed requirement than the 3D versions. However, it is important to underline that though 2D models can be very helpful in understanding what is happening in the microstructure, they are definitely *not* suitable for directly predicting the quantitative results of real (three-dimensional) laboratory tests or practical problems.

4.2. The Applied Numerical Model

Our simulations were performed by the software 'PFC-2D', a two-dimensional dynamic model. The grains in it are perfect circles. Their contacts resist to normal forces, shear forces, and bending moments (*Fig. 5*). Contact strength has three components: tensional strength, shear strength, and a strength for bending expressed through the length of the contact ('length of the glue').

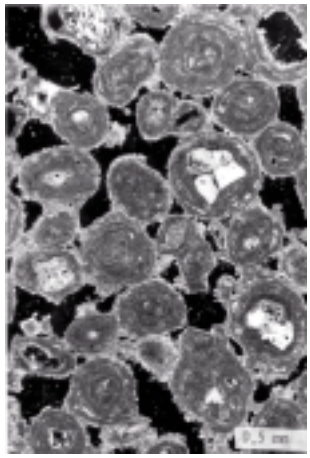

Fig. 5. Microscopic view of limestone texture

Position and size of every grain, contact properties, and characteristics of 'walls' surrounding the system have to be defined as initial data. Loads can be specified as external forces acting on the grains, or as prescribed displacements of any walls or grains.

4.3. The Simulations and their Results

The aim of our simulations was to see whether the randomness of microstructural geometry had any effect on the compression failure strength of the material. Five

tests were carried out on structures having the same size and the same grain-size distribution, and differing from each other only in the geometrical arrangement (this difference was due to the random generation of the sets of particles). The assemblies had the same micro-level (contact) characteristics. *Fig. 6* illustrates an example for initial arrangement. The geometrical and physical data for every assembly:

Domain size: 200 × 200
Grain number: 1000
Grain size: 1.80 – 4.50
(uniform distribution)
Grain–grain contact stiffnesses:
Normal: 1.e8 (Force/disp.)
Shear: 2.e7 (Force/disp.)
Grain–grain contact strengths:
Tension: 1.e5 (Force)
Shear: 1.e5 (Force)
Radius: 15% of the radius of the smaller grain
Wall–grain contact stiffness:
Normal: 1.e12 (Force/disp.);
no strength for tension
Shear: $\cong 0$;
no strength for shear
Friction: $\cong 0$

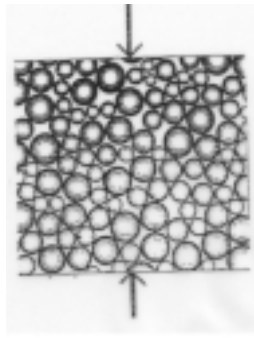

Fig. 6. Simulation of limestone

4.4. Loading Process

The upper wall was very slowly translated down by strain increments of 0.5 e–9 magnitude, until 1 600 000 steps. The failure was detected from following the magnitude of normal force resultant acting on the wall. (Failure, i.e. the drop in

the force started at around 800 000 steps, differently in every test.) The test results were as follows:

TEST 1
Limit force: 1.37 e6
Failure strength: $\sigma_{c1} = 6.85$ e3
Way of failure: shear
A diagonal slip line was formed along which the upper left part of the sample was translated downwards left.

Fig. 7. Failure mode (Test 1)

TEST 2
Limit force: 0.93 e6
Failure strength: $\sigma_{c2} = 4.65$ e3
Way of failure: cleavage fracture
A tensional crack opened up in the middle of the bottom of the sample; left part moved to left and the right part slipped to the right.

TEST 3
Limit force: 1.42 e6
Failure strength: $\sigma_{c3} = 7.1$ e3
Way of failure: partial shear
An initial tensional crack was opened up at the bottom; this crack weakened the sample so that a slip line could be formed along which the upper left corner was moved down left.

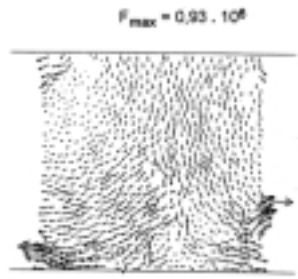

Fig. 8. Failure mode (Test 2)

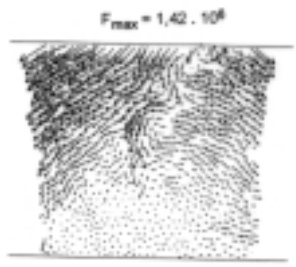

Fig. 9. Failure mode (Test 3)

TEST 4
Limit force: 0.96 e6
Failure strength: $\sigma_{c4} = 4.8$ e3
Way of failure: double local shear
The upper left and right corners were locally sheared down to
the left and to the right.

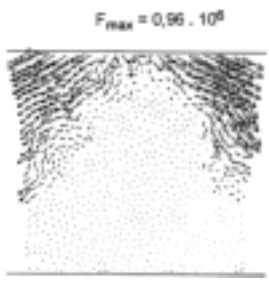

Fig. 10. Failure mode (Test 4)

TEST 5
Limit force: 1.26 e6
Failure strength: $\sigma_{c5} = 6.3$ e3
Way of failure: double local shear
Similarly to the previous test, the lower left and right corners were sheared 'up' as compared to the rigid intact block of the sample that was translated down.

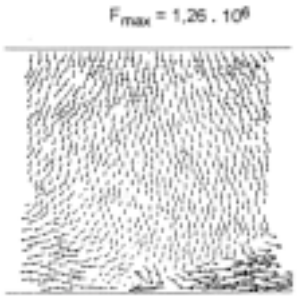

Fig. 11. Failure mode (Test 5)

Figs 7 – 11 illustrate the translations of particles with respect to their original positions. The directions of arrows show the directions of translations. Lengths are proportional to the magnitudes of translations, however, they are increased by a suitably chosen factor to support the visibility of the displacement pictures.

It can clearly be seen that due to the random differences in the initial geometrical arrangements, failure strength results significantly *differed* from each other.

Average failure strength was calculated as

$$\overline{\sigma}_c = \frac{\sum_{k=1}^{n} \sigma_{ck}}{n},$$

where $n = 5$ in our case. The result was $\overline{\sigma}_c = 5.94$ e3.

The scatter was determined as

$$s = \sqrt{\frac{\sum_{k=1}^{n} (\overline{\sigma}_c - \sigma_{ck})^2}{n-1}},$$

and resulted as large as $s = 1.147$ e3 that is *19.3% of the average failure strength.*

5. Conclusion

The simulations clearly underline the importance of internal built-up of the material. Mechanical changes during the loading process – breaks of bonds, displacements

of particles – basically depend on the structure of the analysed stone. Random geometrical arrangement of the particles leads to different failure modes even in the case of the same material. The simulations show that failure is a rather complex internal process: shear planes may be formed at different places; and transversal tensional opening-up may also have a significant role in the behaviour.

Comparison of the computer simulations with the laboratory tests on the modelled rough limestone produced a remarkable result. Though the five different samples in the computer simulations differed from each other only because of the random deviations of geometrical arrangement, failure modes were strongly different and the force causing the failure had a significant standard deviation (nearly the same magnitude like that in the laboratory experiments).

These simulation results inspire us to continue the failure analysis of rough limestones. Calling the attention on the importance of the internal structure to be taken into account, the applied method is able to broaden our knowledge and give a deeper understanding on fracture phenomena.

Acknowledgement

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