periodica polytechnica

Civil Engineering
58/2 (2014) 93–104
doi: 10.3311/PPci.7051
http://periodicapolytechnica.org/ci

RESEARCH ARTICLE

Sounds of Silence: a sampling-based bi-criteria harmony search metaheuristic for the resource constrained project scheduling problem with uncertain activity durations and cash flows

Sándor Danka
Received 2013-09-27, accepted 2014-01-10

Abstract

In this paper, we present a new sampling-based bi-criteria hybrid harmony search metaheuristic for the resource-constrained project-scheduling problem (RCPSP) with uncertain activity durations (UAD) and uncertain cash flows (UCF), with the total project duration (TPD) and the net present value (NPV) as objectives. The proposed problem-specific Sounds of Silence (SoS) metaheuristic is an appropriate hybridization of the robust SoS developed to minimize the project makespan with uncertain activity durations, and the crisp SoS developed for several a primary-secondary (PS) and bi-criteria (BC) project scheduling problems. In the presented hybrid approach, we applied a sampling-based approximation to cope with the uncertain cash flows. In order to illustrate the efficiency and stability of the proposed problem-specific SoS, which is a new member of the SoS family, we present detailed computational results for a larger and challenging project instance. The computational results reveal the fact that the modified and extended SoS is fast, efficient and robust algorithm, which is able to cope successfully with the project-scheduling problems when we replace the traditional crisp parameters with uncertain-but-bounded parameters.

Keywords

Crisp project scheduling · Uncertainty in project scheduling · Heuristic scheduling algorithms · Harmony search · Sample-based solution approximation · Hybrid algorithms

Sándor Danka
University of Pécs, Boszorkány u. 2., H-7624 Pécs, Hungary
e-mail: sandordanka@gmail.com

1 Introduction

Traditionally, project schedule uncertainty has been addressed by considering the uncertainty related to activity duration. In general, there are two approaches to dealing with uncertainty in a scheduling environment (Davenport and Beck [1]; Herroelen and Leus [2], and Van de Vonder et al. [3]): proactive and reactive scheduling. *Proactive scheduling* constructs a predictive schedule that accounts for statistical knowledge of uncertainty. The consideration of uncertainty information is used to make the predictive schedule more *robust*, i.e., insensitive to disruptions. *Reactive scheduling* involves revising or re-optimizing a schedule when an unexpected event occurs. At one extreme, reactive scheduling may not be based on a predictive schedule at all: allocation and scheduling decisions take place dynamically in order to account for disruptions as they occur. A less extreme approach would be to reschedule when schedule breakage occurs, either by completely regenerating a new schedule or by repairing an existing predictive schedule to take into account the current state of the system.

According to the author's opinion, from managerial point of view the "rescheduling of rescheduling" like reactive process, as a problem solving conception, is far from the reality. To avoid the combinatorial explosion of scenario-oriented approaches, we have to go back to the proactive schedule, and have to immunize it against the uncertainty in activity durations.

In this paper, we present a new sampling-based bi-criteria (total project duration (TPD) and net present value (NPV)) hybrid harmony search metaheuristic for the resource-constrained project-scheduling problem (RCPSP) with uncertain activity durations (UAD) and uncertain cash flows (UCF). The proposed problem-specific Sounds of Silence (SoS) metaheuristic is an appropriate hybridization of the robust SoS developed to minimize the project makespan with uncertain activity durations, and the crisp SoS developed for several primary-secondary (PSC) and bi-criteria (BIC) project scheduling problems in a wide range [4], [5], [6]. The first members of the SoS family were developed by [4] for standard single-mode and multiple-mod RCPSP. In the proposed BIC-RCPSP-UAD-UCF approach, the robust (immunized) schedule-searching phase is combined with a sampling-based duration-and-cost-oriented solution approximation phase.

The results are the approximated pareto-optimal schedules, which are immune against the uncertainties in the activity durations and which are the bests for TPD as primary and NPV as secondary (NPV as primary and TPD as secondary) criterion on the set of the uncertain activity duration and cash flow parameters using a sampling-based solution approximation hybridized with linear programming (LP). A problem specific fast and efficient harmony search algorithm for large uncertain problems, namely the RCPSP-UAD-UCF version of Sounds of Silence (SoS), which was previously developed for a wide range of the RCPSP family, will be presented in a forthcoming paper.

2 Problem formulation

The theoretical description of the investigated problem, according to the applied bi-criteria (BIC) approach may be the following: The project consists of N activities $i \in \{1, 2, \dots, N\}$ with nonpreemptable integer duration of D_i periods. In the traditional approach, it is assumed that each activity duration is a crisp value. Naturally, in the project-planning phase this assumption may be far from the reality. Imagine, for example, a new R&D project with several more or less new activities and an extremely long planning horizon.

Furthermore, activity $i = 0$ ($i = N + 1$) is defined to be the unique dummy source (sink) with zero duration. The activities are interrelated by precedence and resource constraints: Precedence constraints force an activity not to be started before all its predecessors are finished. Let

$$NR = \{i \to j | i \neq j, i \in \{q, 1, 2, ..., N\}, j \in \{1, 2, ..., N+1\}\}$$

denote the set of immediate predecessor-successor network relations (NR).

Resource constraints arise as follows: In order to be processed, activity i requires R_{ir} units of resource type $r \in \{1, \dots, R\}$ during every period of its duration. Since resource r, $r \in \{1, \dots, R\}$ is only available with the constant period availability of R_r units for each period, activities might not be scheduled at their earliest (network-feasible) start time but later. Let $\overline{T}$ denote the resource-constrained project's makespan and fix the position of the dummy sink in $\overline{T} + 1$.

Without loss of generality, let α define the crisp discount rate in the planning horizon. Naturally, for a long-time project this assumption may be far from the reality, but from a methodological point of view, it can be replaced by a not necessarily continuous time function: $\alpha = \alpha(t)$, $t \in \{0, \dots, \overline{T}\}$ without difficulty. Let C_i, $i \in \{1, 2, ..., N\}$ denote the cash flow connected to activity i. By definition, the cash flow C_i, $i \in \{1, 2, ..., N\}$ may be negative, zero or positive and it is evaluated at the completion time of activity i. This assumption would be replaced by a more realistic one by introducing dummy activities with zero duration and without resource requirements as cash flow events, which connected to the real activities with predecessor-successor relations. Naturally, the essence of our model is not affected by this event-oriented modification, which methodological point of view similar to the hammock activity handling [7], [8].

The traditional crisp RCPSP-NPV maximization can be defined as a mixed integer linear programming problem (MILP) as follows:

$$\max \left[NPV = \sum_{i=1}^{N} \sum_{T \in T_i} C_{it} * X_{it} \right] = NPV^* \tag{1}$$

$$X_i + D_i \leq X_j\ ,\ i \to j \in NR \tag{2}$$

$$X_{N+1} = \overline{T} + 1 \tag{3}$$

$$\begin{aligned} & X_i = \sum_{t \in T_i} X_{it} * t \\ & T_i = \left\{ \underline{X_i}, \underline{X_i} + 1, ..., \overline{X_i} \right\} \\ & i \in \{1, 2, ..., N\} \end{aligned} \tag{4}$$

$$\begin{aligned} & \sum_{t \in T_i} X_{it} = 1 \\ & X_{it} \in \{0, 1\} \\ & i \in \{1, 2, ..., N\} \end{aligned} \tag{5}$$

$$\begin{aligned} & A_t = \{ i | X_i \leq t < X_i + D_i, i \in \{1, 2, ..., N\}\} \\ & t \in \{1, 2, ..., T\} \end{aligned} \tag{6}$$

$$\begin{aligned} & U_{tr} = \sum_{i \in A_t} R_{ir} \\ & t \in \{1, 2, ..., T\} \\ & r \in \{1, 2, ..., R\} \end{aligned} \tag{7}$$

$$\begin{aligned} & U_{tr} \leq R_r \\ & t \in \{1, 2, ..., T\} \\ & r \in \{1, 2, ..., R\} \end{aligned} \tag{8}$$

$$\begin{aligned} & C_{it} = C_i * e^{-\alpha(t+D_i-1)} \\ & i \in \{1, 2, ..., N\} \\ & t \in T_i \end{aligned} \tag{9}$$

$$\begin{aligned} & X_{it} \in \{0, 1\} \\ & t \in T_i \\ & i \in \{1, 2, ..., N\} \end{aligned} \tag{10}$$

The binary decision variable set Eq. 10 specifies the possible starting times for each activity. By definition, the cash flow

$C_i$connected to activity i, $i \in \{1, 2, ..., N\}$ may be negative, zero or positive and it is evaluated at its completion time. The discount rate is denoted by α. Objective Eq. 1 maximizes the discounted value of all cash flows that occur during the life of the project. Note that the early schedules do not necessarily maximize the NPV of cash flows. Constraints Eq. 2 represent the precedence relations. In constraint Eq. 3 the resource-constrained project's makespan $\overline{T}$ can be replaced by its estimated upper bound. Constraints Eq. 4, Eq. 5 ensure that each activity i, $i \in \{1, 2, ..., N\}$ has exactly one starting time within its time window $T_i = \left\{\underline{X}_i, \underline{X}_i + 1, \ldots, \overline{X}_i\right\}$ where $\underline{X}_i$ ($\overline{X}_i$) is the early (late) starting time for activity i according to the precedence constraints and the latest project completion time $\overline{T}$. Constraints (6 - 8) ensure that resources allocated to activities at any time during the project do not exceed resource availabilities. Constraint set Eq. 9 for each activity describes the change of the cash flow in the function of the completion time.

In the case of the RCPSP-UAD-UCF model, we have to assume, that

- Each activity duration D_i, $i \in \{1, 2, ..., N\}$ is a discrete (positive) uncertain-but-bounded parameter:

$$D_i \in \{A_i, A_i + 1, ..., B_i\} \tag{11}$$

where A_i and B_i are the optimistic and pessimistic estimations of D_i, respectively.

Each activity cash flow C_i, $i \in \{1, 2, ..., N\}$ is a continuous (positive or negative) uncertain-but-bounded parameter:

$$C_i \in \left[\underline{C}_i, \overline{C}_i\right]. \tag{12}$$

After that, we have to generate a schedule with an appropriate resource-conflict repairing relation set $RR^* \subset RR$, which repairs all visible or hidden resource usage conflicts on the feasible

$$\begin{aligned} &D_i \in \{A_i, A_i + 1, ..., B_i\} \\ &i \in \{1, 2, ..., N\} \\ &D = \{D_1, ..., D_N\} \end{aligned} \tag{13}$$

scenario set such a way that on the feasible

$$\begin{aligned} &C_i \in \left[\underline{C}_i, \overline{C}_i\right] \\ &i \in \{1, 2, ..., N\} \\ &C = \{C_1, ..., C_N\} \end{aligned} \tag{14}$$

cash flow set it is "somehow" the best, according to managers values observing completion times and NPV's.

Naturally it is an open and very hard question, that how can we evaluate the quality of the schedules in an uncertain scheduling environment from managerial point of view. In this paper, we introduce a very simple, easy-to-understand measure to characterize such schedule (see Figure 1).

Figure 1 is a good visualization of a dilemma, whether which schedule would be the better from managerial point of view.

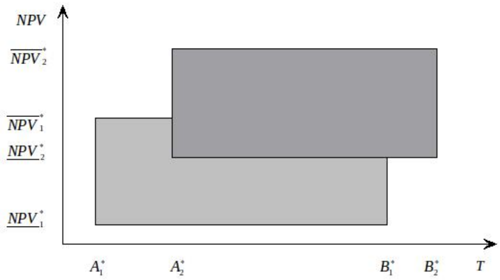

Fig. 1. A simple measure for BC-RCPSP-UAD-UCF

Naturally, the answer depends on the habit of the project manager and the TPD optimal schedule (which visualized by a light gray bar) not necessary will be selected by the project manager as the best, because in this case the reality of the statement of "time is money" proverb is not necessarily true.

In the primary-secondary criteria approach the "best" schedule means a TPD-minimal resource-constrained schedule for which NPV is maximal [Eq. 9, Eq. 10, Eq. 11]. In the investigated bi-criteria approach a project has to be characterized by its pareto-optimal schedules. We have to note, that there is a "natural" conflict between these performance measures because the longer the "playfield" the higher the chance for the NPV minimization and vice-versa.

Theoretically, in the crisp case, the optimal schedule searching process consists of two time-oriented steps: In the crisp primary-secondary criteria approach, in the first step, we solve the traditional time-oriented RCPSP problem to minimize TPD. In the second step we maximize NPVon the set of the feasible activity movements according to the optimal TPD given from the first step.

In the crisp bi-criteria approach to generate the pareto-optimum solutions we have to repeat these steps in reversed order. Naturally, in the first step of reverse case, when we maximize NPV, we have to introduce an additional constraint to define a $TPD \leq \overline{TPD}$ upper bound for safety reason. Without it, in the case of an unprofitable project with negative NPV, namely when the discounted total out-flow is greater than the discounted total inflow, than the problem solving process, to minimize the loss (to solve the problem), move TPD to the positive infinity. Naturally, the crisp bi-criteria approach for TPD and NPVas objectives, a standard multi-objective mixed integer linear programming problem (MOMILP) which can be managed by several different ways.

In the case of the BIC-RCPSP-UAD-UCF we have to introduce a sample-based approximation step, to handle the uncertain activity durations and cash flows. According to our opinion, it is very hard to imagine, that somebody would be able to develop a problem solving process, which consists only of exact methodological approach without sampling-based elements or additional assumptions to simplify the problem.

The core element of the sampling-based solution approximation is a MILP problem, in which we try to maximize the NPV fixing the activity durations and the cash-flow values according to the generated random numbers. Generally, the solution of a MILP problem is a costly operation. When the MILP problem is a core element of a simulation process, the total time requirement of the MILP problem solutions may be a limiting factor of the sample size of the simulation, therefore the smaller sample size may degrade the quality of the sample-based approximated solutions.

Fortunatelly, the implicit resource-constraint handling, which is based on the forbidden set concept, as a good idea, can help to manage this problem. The implicit resource-constraint handling is the core element of our problem-specific SoS algorithm, which will be presented in the next section. The introduced SoS for BIC-RCPSP-UAD-UCF exploits the fact that the heuristic frame around the optimization problem, resolves the resource usage conflicts in an implicit way, therefore eliminates the explicit resource-constraints from the MIP formulation by adding appropriate conflict-repairing relations to the network-relations before we call the MILP solver.

The essence of the implicit resource-conflict management is very simple: Replacing the standard precedence constraint description:

$$\begin{aligned} & S_i + D_i \leq S_j \\ & i \rightarrow j \in NR \cup RR^* \subset RR, \end{aligned} \tag{15}$$

with a totally unimodular (TU) formulation, the resource-constrain-free net present value model can be solved in polynomial time as a LP problem (see Pritsker et al., [12]):

$$\max \left[NPV = \sum_{i=1}^{N} \sum_{t \in T_i} C_{it} * X_{it} \right] = NPV^* \tag{16}$$

$$\begin{aligned} & \sum_{p=T_i}^{\overline{X}_i} X_{ip} + \sum_{q=\underline{X}_p}^{T_i+D_i-1} X_{jq} \leq 1 \\ & T_i \in \left\{ \underline{X}_i, \underline{X}_i + 1, ..., \overline{X}_i \right\} \\ & i \rightarrow j \in PS \cup RR^* \end{aligned} \tag{17}$$

$$X_{N+1} = \overline{T} + 1,. \tag{18}$$

$$\begin{aligned} & \sum_{t \in T_i} X_{it} = 1 \\ & i \in \{1, 2, ..., N\} \end{aligned} \tag{19}$$

$$\begin{aligned} & C_{it} = C_i * e^{-\alpha(t+D_i-1)} \\ & i \in \{1, 2, ..., N\} \\ & t \in T_i \end{aligned} \tag{20}$$

$$\begin{aligned} & X_{it} \in \{0, 1\} \\ & t \in T_i \\ & i \in \{1, 2, ..., N\} \end{aligned} \tag{21}$$

Objective Eq. 16 maximizes the discounted value of all cash flows that occur during the life of the project. Note that early schedules do not necessarily maximize the NPV of cash flows. Constraints Eq. 17 represent the "strong" precedence relations. In constraint Eq. 18 the resource-constrained project's makespan $\overline{T}$ can be replaced by its estimated upper bound. Constraints Eq. 19 ensure that each activity i, $i \in \{1, 2, ..., N\}$ has exactly one starting time within its time window $T_i = \left\{ \underline{X}_i, \underline{X}_i + 1, \ldots, \overline{X}_i \right\}$, where $\underline{X}_i$ ($\overline{X}_i$) is the early (late) starting time for activity$i$ according to the precedence constraints and the latest project completion time $\overline{T}$. Constraint set Eq. 20 describes for each activity the change of the cash flow in the function of its completion time. The binary decision variable set Eq. 21 specifies the possible starting times for each activity. Using a fast interior-point-solver (for example: BPMPD developed by Mészáros, 1966) than the modified MILP as an LP problem can be solved nearly 100 times faster than with a traditional simplex solver.

3 Algorithm

Harmony search (HS) algorithm was recently developed by Lee and Geem [13], in an analogy with music improvisation process, where music players in a jazz band improvise to obtain better harmony. In HS, the optimization problem is specified as follows:

$$\max \left\{ f(X) \mid X = \left\{ X_i \mid \underline{X}_i \leq X_i \leq \overline{X}_i, i \in \{1, 2, ..., N\} \right\} \right\} \tag{22}$$

In the language of music, X is a melody, which aesthetic value is represented by $f(X)$. Namely, the higher the value $f(X)$, the higher the quality of the melody is. In the jazz band, the number of musicians is N, and musician i, $i = \{1, 2, \ldots, N\}$ is responsible for sound X_i. The improvisation process is driven by two parameters:

- According to the repertoire consideration rate (RCR), each musician is choosing a sound from his/here repertoire with probability RCR, or a totally random value with probability $(1 - RCR)$;
- According to the sound adjusting rate (SAR), the sound, selected from his/here repertoire, will be modified with probability SAR. The algorithm starts with a random “repertoire uploading” phase, after that, the jazz band begins to improvise. During the improvisations, when a new melody is better than the worst in the repertoire, the worst will be replaced by the better one. Naturally, the two most important parameters of HS algorithm are the repertoire size and the number of improvisations. The HS algorithm is an “explicit” one, because it operates directly on the sounds.

In the case of RCPSP, we can only define an “implicit” algorithm, and without introducing a “conductor”, we cannot manage the problem efficiently. Naturally, when we introduce a conductor, then, according to the real-life, we have to replace the jazz band with an orchestra. In the world of music, the resource profiles form a “polyphonic melody”. Therefore, assuming that in every phrase only the “high sounds” are audible, the transformed problem will be the following: find the shortest “Sounds of Silence” melody by improvisation! Naturally, the “high sound” in music is analogous to the resource-overload in scheduling.

In the language of music, the RCPSP can be summarized as follows:

- The orchestra consists of N musicians;
- The polyphonic melody consists of R phrases and N polyphonic sounds;
- Each $i \in \{1, 2, ..., N\}$ musician is responsible for exactly one polyphonic sound;
- Each $i \in \{1, 2, ..., N\}$ polyphonic sound is characterized by the set of the following elements: $\{X_i, D_i, \{R_{ir} \mid r \in \{1, 2, ..., R\}\}\}$;
- The polyphonic sounds (musicians) form a partially ordered set according to the precedence (predecessor-successor) relations;
- each $r \in \{1, 2, ..., R\}$ phrase is additive for the simultaneous sounds;
- In each phrase only the high sounds are audible: $\{U_{tr} \mid U_{tr} > R_r,\ t = 1, 2, ..., T\}$;
- In each repertoire uploading (improvisation) step, each $i \in \{1, 2, ..., N\}$ musician has the right to present (modify) an idea $IP_i \in [-1, +1]$ about X_i where a large positive (negative) value means that the musician want to enter into the melody as early (late) as possible;
- In the repertoire uploading phase the musicians improvise absolutely freely, $IP_i \leftarrow RandomGauss(0, 1)$, where, $\eta \leftarrow RandomGauss(\mu, \sigma)$ generates random normal random numbers from a truncated $(-1 \leq \eta \leq 1)$ normal distribution with mean μ and standard deviation σ;
- In the improvisation phase, the “freedom of imaginations” is decreasing step by step, $IP_i \leftarrow RandomGauss(IP_i, \sigma)$, where standard deviation σ is an exponentially decreasing function $\sigma = \sigma(g)$ of the progress from generation to generation (see Figure 2, 3;
- Each of the possible decisions of the harmony searching process (melody selection and idea-driven melody construction) is the conductor’s responsibility;
- The band tries to find the shortest “Sounds of Silence” melody by improvisation.

The conductor solves a linear programming (LP) problem to balance the effect of the "more or less opposite ideas" about a shorter “Sounds of Silence” melody. The LP problem, which

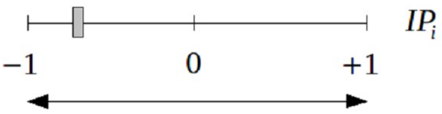

Fig. 2. An idea IP_i about the "best" position

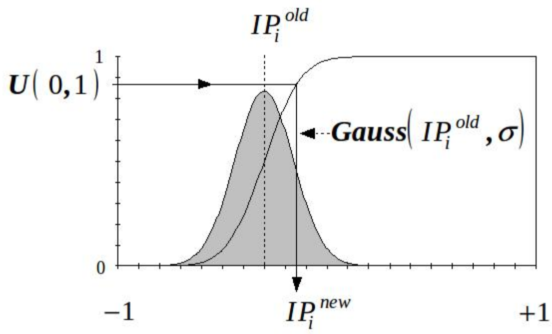

Fig. 3. Perturbation of IP_i

maximizes the satisfaction of the musicians with the sound positions, is the following.

$$\min \left[\sum_{i=1}^{N} IP_i * X_i \right], \tag{23}$$

$$X_i + D_i \leq X_j, \quad i \rightarrow j \in NR, \tag{24}$$

$$\underline{X}_i \leq X_i \leq \overline{X}_i, \quad i \in \{1, 2, \ldots, N\}. \tag{25}$$

The result of the optimization is a schedule (melody) which is used by the conductor to define the final starting (entering) order of the sounds (musicians). The conductor generate a soundless melody by taking the selected sounds one by one in the given order and scheduling them at the earliest (latest) feasible start time. After that, using the well-known forward-backward improvement (FBI) methods (see, for example, Tormos and Lova, [14]) the conductor tries to improve the quality of the generated melody. Naturally, the conductor memorizes the shortest feasible melody found so far.

The conflict-repairing version of SoS is based on the forbidden set concept. In the conflict- repairing version, the primary variables are conflict-repairing relations, and a solution will be a makespan minimal resource-feasible solution set, in which every movable activity can be shifted without affecting the resource feasibility. In the traditional time-oriented model the primary variables are starting times, therefore an activity shift may be able to destroy the resource feasibility.

The makespan minimal solutions of the conflict-repairing model are immune against the activity movements, so we can introduce a not necessarily regular secondary performance measure to select the “best” makespan minimal resource feasible solution from the generated solution sets. In SoS, according to the applied replacement strategy (whenever the algorithm obtains a solution superior to the worst solution of the current population, the worst solution will be replaced by the better one) the quality of the population is increasing step by step. According to the progress of the searching process, the size of the makespan minimal subset of the population is increasing. The larger the makespan minimal subset size, the higher the chance to get a good solution for the secondary criterion.

It is well-known, that the crucial point of the conflict repairing model is the forbidden set computation. In the SoS the conductor using a simple (but fast and effective) “thumb rule” to decrease the time requirement of the forbidden set computation. In the forward-backward list scheduling process, the conductor (without explicit forbidden set computation) inserts a precedence relation $i \rightarrow j$ between an already scheduled activity i and the currently scheduled activity j whenever they are connected without lag ($S_j + D_j = S_j$). The result will be schedule without “visible” conflicts. After that, the conductor (in exactly one step) repairs all of the hidden (invisible) conflicts, inserting always the “best” conflict repairing relation for each forbidden set. In this context “best” means a relation $i \rightarrow j$ between two forbidden set members for which the lag ($S_j - S_i - D_i$) is maximal.

In the language of music, the result of the conflict repairing process will be a robust (flexible) “Sounds of Silence” melody, in which the musicians have some freedom to enter to the performance without affecting the aesthetic value of the composition. Naturally, when we introduce a secondary criterion (in our case, for example, the NPV measure), for which the aesthetic value is a function of the starting times, the freedom of the musicians totally disappears.

In the presented bi-criteria approach:

- The modified selection mechanism alternatively select a "more or less" TPD-minimal and NPV-maximal or a "more or less" NPV-maximal and TPD-minimal melody, where, in this context, "more-or-less" means that the better the current measure value pair of a melody , the higher the chance that it will be selected by the conductor in the improvisation process;

- According to the modified selection mechanism, the conductor memorizes the best pareto-solutions found so far, which consists of the currently best TPD-minimal and NPV-maximal and the currently best NPV-maximal and TPD-minimal schedules;

- According to the uncertain activity durations and cash flows, the TPD-minimal and NPV-maximal (NPV-maximal and TPD-minimal) means a schedule for which, according to the sampling-based approximation, $\underline{A} + \overline{B}$ minimal and $\underline{NPV} + \overline{NPV}$ maximal ($\underline{NPV} + \overline{NPV}$ maximal and $\underline{A} + \overline{B}$ minimal).

Naturally, it is very interesting and challenging question that what would be the best measure to characterize a schedule in an uncertain activity duration and cash flow environment. For example, the presented average-like approach could be replaced by an average-and-range-like approach. The investigation of this question will be presented in a forthcoming paper.

4 Implementation details

The algorithm of the proposed model has been programmed in Compaq Visual Fortran® Version 6.5. The algorithm, as a DLL, was built into the *ProMan* system (Visual Basic® Version 6.0) developed by Ghobadian and Csébfalvi [15]. To solve the MILP problem a state-of-the-art solver, namely the CPLEX 12.0 in AIMMS 3.10 for Windows environment was used. The solver, as an AIMMS COM object, was integrated into *ProMan*. The computational results were obtained by running *ProMan* on a 1.8 GHz Pentium IV IBM PC with 256 MB of memory under Microsoft Windows XP® operation system. At the running of the resource-constrained project borrowed from Golenko-Ginzburg and Gonik [17] we changed the default optimality tolerance parameters (Relative Gap = 0.01 % and Absolute Gap = 5 period) and the Time Limit parameter (10 hours). In the presented example, the $\{WA, \ WB\} = \{1, \ 1\}$ weight set was used.

To solve the LP problems a fast primal-dual interior point solver, namely the DLL version of BPMPD developed by Mészáros [16], was used. The computational results were obtained by running ProMan on a 1.8 GHz Pentium IV IBM PC with 256 MB of memory under Microsoft Windows XP® operation system.

At the running of the resource-constrained project borrowed from Golenko-Ginzburg and Gonik instance, we used the SoS for BC-RCPSP-UAD-UCF algorithm with the following global parameters (see Table 1):

$$\left\{\underline{RCR}, \ \overline{RCR}\right\} = \{0.8, \ 0.9\}$$
$$\left\{\underline{SAR}, \ \overline{SAR}\right\} = \{0.1, \ 0.9\},$$
$$\left\{\underline{\sigma}, \ \overline{\sigma}\right\} = \{0.01, \ 1.0\}.$$

Our algorithm, according to its “robust” nature, is not so sensitive to the “fine-tuning” of the parameters. Probably, the robustness of the algorithm can be explained by the robustness of the developed “unusual” activity list generator. In other words, each of the global parameters can be kept “frozen” in the algorithm, which results in a practically “tuning-free” algorithm.

5 Computational experiments

In this section, as a motivating example, we consider a larger resource constrained project with 36 real activities borrowed from Golenko-Ginzburg and Gonik [17]. In contrast to the instances of the well-known and popular PSPLIB developed by Kolisch and Sprecher [18], this instance already includes information of random activity durations that is, for each activity i the optimistic and pessimistic duration time $[A_i, B_i]$, $i \in \{1, 2, \ldots, 36\}$ is given.

In this problem, there is only one renewable resource type and it is assumed that 50 units are available for each period. In this study, we assumed that each activity-duration is an uncertain-but-bounded parameter without any possibilistic or probabilistic interpretation.

The instance contains $F = 3730$ forbidden sets and the cardinality of the resource-conflict repairing relations (the total number of binary indicators) is $|RR| = 938$, which means that the exact solution of the robust RCPSP-UAD is challenging problem from methodological point of view. The unfeasible early start schedule of the project in activity-on-node representation mode is presented in Figure 4, 5. In these figures, the activities are bars, where the bar lenghts are proportional to the activity duration. The random part of each activity-duration is represented by a lighter grey colour. The resource-usage histogram is presented in theoretical correct form, according to the cumulative resource constraints. The predecessor-successor relations are represented by lines. The unconstrained optimistic (pessimistic) makespan is 173 (265) time units, respectively. The initial data of the project are given in Table 1, 2.

Tab. 1. The initial data of the Golenko-Ginzburg and Gonik project

a	A_a	B_a	R_{ia}	IP_a
0	0	0	0	
1	16	60	16	{0}
2	15	70	15	{0}
3	18	35	18	{0}
4	19	45	19	{0}
5	10	33	10	{0}
6	18	15	18	{1}
7	24	50	24	{1}
8	25	18	25	{6}
9	16	24	16	{6}
10	19	38	19	{2}
11	20	22	20	{2}
12	18	32	18	{3}
13	15	45	15	{4}
14	16	78	16	{5}
15	17	45	17	{14}
16	19	35	19	{14}
17	21	60	21	{10, 13}
18	24	50	24	{10, 13}

For each real activity, according to the usual managerial assumptions, which define an unsymmetrical optimistic (pessimistic) range around the crisp nominal (most probable or most likely) value, the randomly generated uncertain-but-bounded cash flow values are presented in Table 3.

Using the CPLEX 12.0 solver with the mentioned setting to solve RCPSP-UD, the solving process was terminated prematurely, as a result of reaching the extremely large 10 hours (36000 sec) time limit. Therefore, the given final solution $\{A^*, B^*\} = \{340, 500\}$ is only a good one. After inserting the conflict-repairing relations, the possibilistic range of the makespan $\left[\underline{A}, \overline{B}\right] = [395, 465]$ and the net present value $\left[\underline{NPV}, \overline{NPV}\right] = [1579, 2111]$ were estimated by simulation. The total number of the generated random schedules was 1000(S 1000) using a uniform random number generator to generate the integer durations and the cash flow values. We run the BC-RCPSP-UAD-UCF specific robust SoS algorithm 30

Tab. 2. The initial data of the Golenko-Ginzburg and Gonik project

a	A_a	B_a	R_{1a}	IP_a
19	13	42	13	{17}
20	16	30	16	{15, 18}
21	12	21	12	{15, 18}
22	14	20	14	{15, 18}
23	16	42	16	{16}
24	15	40	15	{12}
25	13	28	13	{12}
26	14	35	14	{8, 11}
27	18	24	18	{7, 9}
28	22	22	22	{7, 9}
29	10	18	10	{26, 27}
30	18	38	18	{24, 28}
31	16	55	16	{24, 28}
32	17	30	17	{25}
33	19	37	19	{20, 23}
34	20	38	20	{21, 32}
35	15	55	15	{19, 22}
36	24	22	24	{29, 30, 34}
37	0	0	0	{31, 33, 35, 36}

times independently with the following setting and randomly generated starting seeds and with frozen global parameters: $\{G = 10, P = 500, S = 1000\}$, where G is the number of generations, P is the population size and S is the sample size in the sampling based solution approximation phase (see Table 4, 5).

We have to note again, that in our case the simulation is not a costly operation, because using a fast interior point solver and a totally unimodular predecessor-successor formulation for activities, the NPV optimization problem in this size can be solved within a fraction of a second.

The computational results reveal the fact that the robust SoS for BC-RCPSP-UAD-UCF is a fast, effective and robust algorithm, which is able to produce good quality results with extremely small spreading within a quarter hour.

Naturally, in the sample-based solution approximation phase, the uniform random number generator can be replaced by a more sophisticated tool to figure the future, but, without a crystal ball, such a modification not necessarily would be able to produce better forecasting result.

In Table 6, we show the estimated sample-based approximations of pareto-optimal solutions for $S = 1000$. In the presented bi-criteria approach for$\{TPD, NPV\}$, the "currently-best" pareto-optimal schedules were defined by the set of the first (last) element of the following two ordered sets:

$$\left\{\overset{>}{\mathbf{TPD}}, \overset{<}{\mathbf{NPV}}\right\} \text{ and } \left\{\overset{<}{\mathbf{TPD}}, \overset{>}{\mathbf{NPV}}\right\}$$

which were selected from the current repertoire, where in the presented formalism, symbol < (>) means an ascending (descending) primary (secondary) sorting key in the order of columns. In Table 6, the best estimations for $S = 1000$ are presented with bold characters.

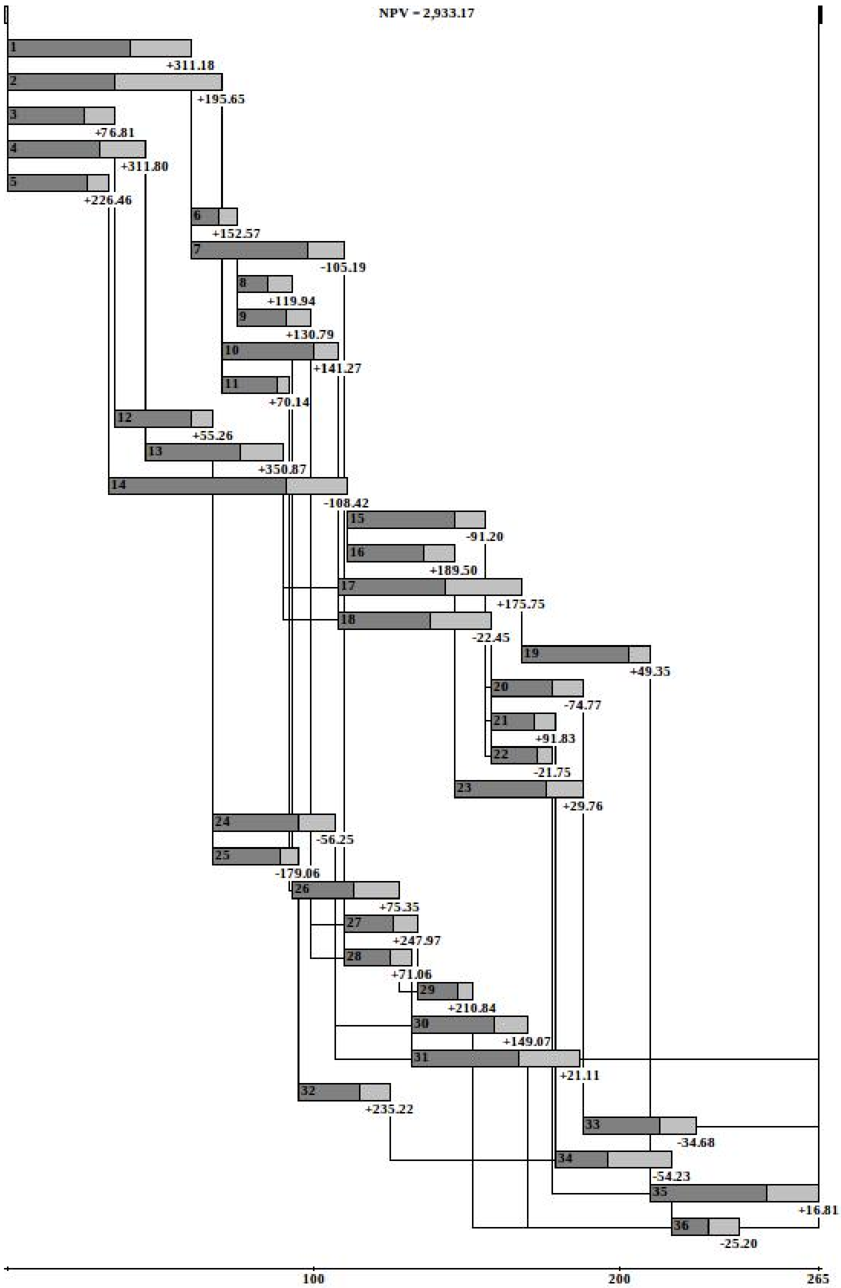

Fig. 4. Cash flow oriented early start project visualization with the nominal cash flow values

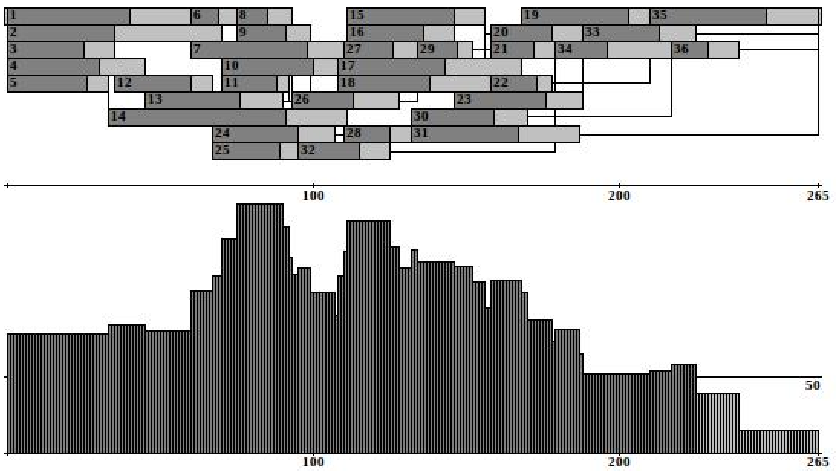

Fig. 5. Resource usage oriented theoretically correct early start project visualization

Tab. 3. The generated uncertain-but-bounded cash flow values

a	CF_A_a	CF_M_a	CF_B_a	a	CF_A_a	CF_M_a	CF_B_a
1	482	567	595	19	343	403	423
2	335	394	414	20	-563	-490	-465
3	93	109	114	21	468	550	578
4	416	489	513	22	-148	-129	-123
5	268	315	331	23	166	195	205
6	275	323	339	24	-189	-164	-156
7	-363	-316	-300	25	-532	-463	-440
8	258	304	319	26	230	271	285
9	299	352	370	27	805	947	994
10	354	416	437	28	226	266	279
11	150	176	185	29	819	964	1012
12	92	108	113	30	694	816	857
13	734	863	906	31	116	137	144
14	-378	-329	-313	32	698	821	862
15	-499	-434	-412	33	-378	-329	-313
16	694	816	857	34	-546	-475	-451
17	802	943	990	35	202	238	250
18	-125	-109	-104	36	-316	-275	-261

6 Conclusions

In this paper, we presented a new hybrid harmony search metaheuristic combined with sampling-based solution approximation for a bi-criteria resource-constrained project-scheduling problem (RCPSP) with uncertain-but-bounded activity durations and cash flows (BIC-RCPSP-UAD-UCF).

The presented Sound of Silence (SoS) algorithm, which is an appropriate combination of mathematical programming, metaheuristic and sampling-based elements, is a straightforward modification of the originally time oriented SoS harmony search metaheuristic developed by [6], [7], [8] and extended for a wide range of different resource-constrained project scheduling models [9], [10], [11] and [19], [20], [21], [22]. The presented algorithm, as a new member of the SoS family produces optimal "robust" proactive schedules, which are immune against the uncertainties in the activity durations. The presented robust schedule searching heuristic is based on a "forbidden set" oriented reformulation of the originally time oriented algorithm. In the presented algorithm, it is assumed that each activity duration and each cash flow value is an uncertain-but-bounded parameter, which characterized by their optimistic and pessimistic estimations. The searching process is driven by the sampling-based approximation of the pareto-optimal solutions. The evaluation of a given robust schedule is based on the investigation of variability of the makespan and the net present value criterion on the set of randomly generated scenarios given by a sampling-on-sampling-like process.

We have to note, that in the simulation phase, the presented uncertain-but-bounded approach can be replaced by a possibilistic (membership function oriented fuzzy) or a probabilistic (density function oriented stochastic) approach, because the best solution searching process is invariant to the "real meaning" of the optimistic and pessimistic estimations. In this paper, in the simulation phase a uniform random number generator was used to generate the uncertain parameters of the scenarios. Naturally, this simple approach can be replaced by more sophisticated parameter estimation process, but according to our experiences and the robust nature of the Central Limit Theorem, the simulation is process not so sensitive to the applied parameter generation method. In highly uncertain and usually very long range, a more sophisticated forecasting method (a possibilistic or probabilistic approach with several estimated parameters and special operators) is not necessarily a crystal ball in the imagination of the future. In order to illustrate the efficiency and stability of the proposed SoS metaheuristic we presented detailed computational results for a larger and challenging project instance borrowed from Golenko-Ginzburg and Gonik [17] and discussed by several authors in the literature.

The presented reproducible results can be used for testing the quality of exact and heuristic solution procedures to be developed in the future in this area. The computational results reveal the fact that the modified and extended SoS is fast, effective and robust algorithm, which is able to cope successfully with bi-criteria project-scheduling problems when we replace the traditional crisp duration and cash flow parameters with uncertain-but-bounded ones. It is an open and very challenging question that what would be the best bi-criteria measure to evaluate a schedule in an uncertain environment. Our primary results about this problem will be presented in a forthcoming paper.

Tab. 4. The Cplex solution and the results of 30 independent SoS runs (G10P500)

Run	$A+B$	A	B	$E(A+B)$	$E(A)$	$E(B)$	*Time*
				%	%	%	*sec*
Cplex	**840**	**340**	**500**				**36000**
1	842	343	499	0.24	0.88	-0.20	14.837
2	842	344	498	0.24	1.18	-0.40	14.052
3	849	336	513	1.07	-1.18	2.60	10.619
4	849	339	510	1.07	-0.29	2.00	11.370
5	849	345	504	1.07	1.47	0.80	11.074
6	849	346	503	1.07	1.76	0.60	10.183
7	849	349	500	1.07	2.65	0.00	12.011
8	850	341	509	1.19	0.29	1.80	14.221
9	850	343	507	1.19	0.88	1.40	15.079
10	850	343	507	1.19	0.88	1.40	12.035
11	850	343	507	1.19	0.88	1.40	12.778
12	850	344	506	1.19	1.18	1.20	14.927
13	850	345	505	1.19	1.47	1.00	14.964
14	850	345	505	1.19	1.47	1.00	14.067
15	850	345	505	1.19	1.47	1.00	12.428
16	850	348	502	1.19	2.35	0.40	11.311
17	851	344	507	1.31	1.18	1.40	15.839
18	852	340	512	1.43	0.00	2.40	10.258
19	852	341	511	1.43	0.29	2.20	11.416
20	852	345	507	1.43	1.47	1.40	12.505
21	852	345	507	1.43	1.47	1.40	13.296
22	853	338	515	1.55	-0.59	3.00	13.455
23	853	345	508	1.55	1.47	1.60	13.138
24	853	347	506	1.55	2.06	1.20	10.358
25	853	347	506	1.55	2.06	1.20	10.288
26	853	351	502	1.55	3.24	0.40	15.189
27	854	346	508	1.67	1.76	1.60	12.607
28	854	346	508	1.67	1.76	1.60	10.841
29	855	351	504	1.79	3.24	0.80	11.694
30	856	341	515	1.90	0.29	3.00	13.606
Mean	851	344	507	1.28	1.24	1.31	12.681
Range	14	15	17	1.67	4.41	3.40	5.656

Tab. 5. The best Cplex solution and ordered result of the approximated solutions for 30 independent SoS runs: (G10P500 + S1000)

	TPD		NPV			TPD		NPV	
i	$\underline{A}$	$\overline{B}$	$\underline{NPV}$	$\overline{NPV}$	i	$\underline{A}$	$\overline{B}$	$\underline{NPV}$	$\overline{NPV}$
Cplex	**395**	**465**	**1579**	**2111**		**395**	**465**	**1579**	**2111**
1	393	475	1624	2222	16	402	481	1473	1958
2	395	471	1425	1932	17	403	472	1489	1972
3	395	477	1547	2145	18	403	475	1419	1865
4	397	472	1374	1828	19	403	482	1451	1973
5	398	468	1423	1923	20	404	472	1488	1964
6	398	476	1586	2166	21	404	472	1585	2088
7	398	478	1501	2002	22	404	473	1571	2072
8	398	486	1796	2438	23	404	475	1200	1591
9	399	471	1576	2095	24	404	481	1497	1963
10	399	476	1370	1884	25	405	469	1574	2079
11	399	476	1411	1895	26	405	489	1385	1829
12	400	475	1612	2147	27	406	475	1319	1777
13	401	474	1494	1983	28	408	476	1384	1861
14	402	469	1357	1760	29	408	485	1448	1958
15	402	472	1730	2267	30	409	470	1489	1979

Tab. 6. The estimated pareto-optimal solutions

i	$\overset{<}{\text{TPD}}$	$\overset{>}{\text{NPV}}$	$\overset{>}{\text{NPV}}$	$\overset{<}{\text{TPD}}$
1	401	1900	1982	437
2	408	2126	2156	432
3	401	1812	1901	439
4	404	1831	1964	448
5	401	1906	1983	430
6	405	1816	1960	439
7	404	1827	1903	448
8	409	2110	2153	445
9	**397**	**2046**	2053	447
10	403	1973	1973	434
11	405	1741	1777	461
12	405	2101	2222	452
13	405	1943	1943	428
14	403	1759	1829	434
15	405	2374	**2438**	**458**
16	397	1933	1946	447
17	412	2051	2104	438
18	414	2069	2084	453
19	406	1871	1979	420
20	407	1828	1900	455
21	404	2004	2045	426
22	400	1896	1923	432
23	410	2034	2034	426
24	403	1799	1865	433
25	402	1715	1771	425
26	403	2202	2255	422
27	402	1902	1958	451
28	402	1518	1564	459
29	404	2048	2163	445
30	414	1778	1850	448

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