A proposed methodology for the improvement of the simplified calculation of thermal bridges for well typified facades

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Abstract

The precise calculation of the multi-dimensional heat losses through the external building fabric is a key issue in the accuracy of building energy calculations. With the help of an extensive thermal bridge database created for a well typifiable group of buildings – 19th century urban apartment houses – a detailed investigation is presented about the accuracy of the currently used simplified thermal bridge calculation method. A Monte Carlo simulation approach is applied to explore the various geometrical and constructional parameters that affect multi-dimensional heat losses and to generate a sufficiently large sample for the investigation. The inaccuracy of the current method is demonstrated and a possible new approach is introduced for its improvement.

Keywords

Thermal bridges · building energy calculation · Monte Carlo simulation

1 Introduction

In the years 2011-2012 the Department of Building Construction - Faculty of Architecture, Budapest University of Technology and Economics - conducted a government grant aided research into the possibilities of the thermal insulation of historical buildings with protected facades. Multi-story apartment buildings from the second half of the 19th century and the first decade of the 20th represent a significant portion of the Hungarian building stock, one which is in need of building energy refurbishment. However the façade of these buildings doesn’t allow for a conventional external thermal insulation since it would destroy their historical quality and architectural value. As this is not a unique predicament in the building industry internal insulation systems, which have previously been considered too dangerous due to building physics problems for decades, are now emerging at a rapid pace all-over Europe [11]. This was made possible by advances in materials and the development of sophisticated hygrothermal simulation tools such as the ones presented in [12] or [19]. The research project aimed at analyzing the use of these advanced materials for the construction of internal insulations for the specific requirements of the Hungarian historical building stock and subject to our climate, and to provide possible assistance for planners and decision makers for the planning process and the evaluation of such measures. Many of the results of these investigations were already published, e.g. in [8] and [10].

Among the many aspects of building energy refurbishments the precise calculation of the energy saving potential is perhaps the most important. Internal insulations however can pose a significant challenge in this respect, as the insulation layer cannot be continuous at the connections of internal constructions to the external walls (.partition walls, slabs, etc.); the treatment of multi-dimensional heat flows and thermal bridges is therefore a key issue. In Hungary such calculations are done according to the method described in the effective Government Regulation (Ministerial Decree 7/2006 TNM [1]). This method is used by both the planners and the authorities e.g. for the application for and the awarding of subsidies for retrofit measures. In this article we present a detailed investigation about the precision of the simplified form of this calculation method (as it is the
most widely used) when applied to the historical building stock and internal thermal insulations and we propose a possible new methodology for its improvement. As shown in [5] other EU member states use similar simplified techniques in their building energy regulations, so the impact of such investigations could go further than just the current Hungarian regulation.

2 Treatment of thermal bridges in building heat loss calculations in Hungary

The definition of a thermal bridge is a part of the external thermal envelope where the heat flux lines are not parallel to each other but instead become more-dimensional. If we observe this definition strictly every part of every building would have to be considered a thermal bridge, because perfectly parallel heat flux lines are only possible in homogeneous, infinitely wide surfaces with a constant thickness. However, to simplify their treatment and to lessen the calculation workload we can differentiate between two distinct groups of the thermal bridges:

1 repeating thermal bridges, under which we understand inhomogeneities in the external constructions demonstrating a recurring pattern within a single planar construction (e.g. wooden studs in lightweight walls or wall-ties in cavity walls), and

2 non-repeating thermal bridges, which occur at the large scale details and junctures of different constructions (e.g. wall corners, slab to wall connections, the connection of partition walls to exterior walls, etc.), where the interior and exterior surface dimensions are not equal or where materials with different thermal conductivities are present.

According to current regulations heat losses resulting from repeating thermal bridges must be incorporated into either the thermal conductivity of the materials (e.g. the thermal conductivity of a masonry must represent the joint characteristics of both brick and mortar), or into the U-value of the individual construction (e.g. the correction for mechanical fixing in ETICS - see MSZ-EN-ISO-6946 [13]). Repeating thermal bridges are not a subject of this article.

The detailed calculation of non-repeating thermal bridges (henceforth just thermal bridges) is described in the standard MSZ-EN-ISO-10211 [14]. For such a calculation one must make a 2 or 3 D thermal model of the detail in question (some numerical solution of the stationary heat equation over the domain) according to the specific thermal boundary conditions described in the standard (temperatures and surface heat transfer coefficients). This model yields the heat flux densities over the surface of the construction from which we can derive the thermal transmittance of the thermal bridge the following way (as illustrated with a simple 2D case seen in Fig. [1]).

By integrating the surface normal component of the heat flux density over 1 we get the total heat flux for unit length through the entire detail:

$$Q_l = \int_{x_0}^{x_1} q(x) \, dx$$  \hspace{1cm} (1)

where \(Q_l\) [W/m] is the total heat flux for a unit length, and \(q_n\) [W/m²] the surface normal component of the heat flux density. If we divide this value with the temperature difference we get the so called 2D thermal coupling coefficient which gives the heat flux through 1 [m] of the detail for a temperature difference of 1 [K]:

$$L_{2D} = \frac{Q_l}{\Delta T}$$  \hspace{1cm} (2)

where \(L_{2D}\) [W/mK] is the 2D thermal coupling coefficient, \(Q_l\) [W/m] the total heat flux for a unit length and \(\Delta T\) [K] the temperature difference. The \(L_{2D}\) value represents the heat losses of the detail with complete accuracy, so an ideal calculation method would be to compute the heat losses of the whole building in a similar manner. However to model an entire building thusly is not yet practical even with today’s computer capacities, therefore we have to calculate individual thermal bridges separately, and then divide the calculated heat fluxes into one-dimensional (U-value) and multi-dimensional (ψ-value) parts that we can later use in the whole building's heat loss calculation:

$$L_{2D} = \psi + \sum_{i=1}^{n} l_i U_i \quad \text{v.} \quad \psi = L_{2D} - \sum_{i=1}^{n} l_i U_i$$  \hspace{1cm} (3)

where \(\psi\) [W/mK] is the linear thermal transmittance, \(L_{2D}\) [W/mK] the 2D thermal coupling coefficient, \(l_i\) [m] the length of surface \(i\), and \(U_i\) [W/m²K] the thermal transmittance of surface \(i\).

So as we can see the total heat transmittance of a detail is the sum of the one-dimensional heat losses (in our example the heat loss through the planar wall given by its U-value and surface area) and the multi-dimensional heat losses given by the so called linear thermal transmittance value. In other words the linear thermal transmittance value is the difference between a strictly one-dimensional heat loss calculation and an exact calculation that accurately models the multi-dimensional heat transfer effects. In a way it is the calculation error of the one dimensional models. Keeping this in mind it is clear that there are no “thermal-bridge free” constructions or designs in this sense, because this calculation error is never exactly zero. A further observation we have to make is that the exact value of the linear transmittance is dependent on the coordinate system we use for our calculations. If one uses the internal surface dimensions of a building (surface areas as measured in the interior) to calculate the heat losses the \(\psi_i\) (interior) value of the thermal transmittance must be used in order reproduce the exact value according to Eq. (3), whereas by using the external dimensions the \(\psi_e\) (exterior) value is needed. For every detail where the internal and external dimensions are not identical we can calculate two values for \(\psi\), and in some extreme cases the external value may even be negative, as demonstrated in Fig. [2]. This does not mean that the detail has heat gains, but simply indicates that by
Fig. 1. Isotherm and heat flux line in a constructional detail and the normal heat flux density on the surface

Fig. 2. An example for the calculation of the linear thermal transmittance of an external wall corner based on external and internal dimensions

\[
u_{\text{fail}} = 1.29 \text{ [W/m}^2\text{K]} \\
L_{2D} = 2.865 \text{ [W/mK]}
\]

\[
\Psi_i = 2.865 - 2 \times 1 \times 1.29 = 0.285 \text{ [W/mK]}
\]

\[
\Psi_e = 2.865 - 2 \times 1.44 \times 1.29 = -0.953 \text{ [W/mK]}
\]
using only the one-dimensional U-value and the external surface area of the building the heat losses were overestimated.

In the Hungarian building energy certificate calculation according to the effective Government Regulation  [1] the multi-dimensional heat loss effects of the thermal bridges must be incorporated into the specific net heating energy demand of the building. This can be done with a detailed or a simplified calculation. According to the detailed calculation:

\[
q = \frac{1}{V} \sum_{i=1}^{n} A_i U_i + \sum_{j=1}^{m} l_j \psi_j - \frac{Q_{sd} + Q_{sid}}{72}
\]

(4)

where \( q \) [W/m²K] is the specific net heating energy demand of the building, \( V \) [m³] is the heated air volume, \( A_i \) [m²] the area of surface \( i \), \( U_i \) [W/m²K] the thermal transmittance of surface \( i \), \( l_j \) [m] the length of the linear thermal bridge \( j \), \( \psi_j \) [W/m³K] the linear thermal transmittance of thermal bridge \( j \), \( Q_{sd} \) [kWh/a] the direct solar heat gains and \( Q_{sid} \) [kWh/a] the indirect solar heat gains.

In Eq. (4) the one- and multi-dimensional heat losses are in separate terms and if sufficient data is available for every thermal bridge we can calculate the exact heat transmittance values for the whole building as shown in Eq. (4). The question is: do we have the \( \psi \) values to substitute into Eq. (4)? Although even the weakest PC or notebook currently on the market has sufficient computational capacity to perform the necessary thermal simulation to get these required values the manual workload necessary to build all the necessary thermal models is still too big. It is simply not practical to use simulations for every building energy calculation done in practice. Therefore the regulation also allows for a simplified version of the calculation which is used by most practitioners, with the following equation:

\[
q = \frac{1}{V} \sum_{i=1}^{n} A_i U_{R,i} + \sum_{j=1}^{m} l_j \psi_j - \frac{Q_{sd}}{72}
\]

(5)

where \( q \) [W/m²K] is the specific net heating energy demand of the building, \( V \) [m³] the heated air volume, \( A_i \) [m²] the (internal!) area of surface \( i \), \( U_{R,i} \) [W/m²K] the effective thermal transmittance value of surface \( i \), \( l_j \) [m] the length of the plinth detail \( j \) (slab-on-grade perimeter) or basement wall, \( \psi_j \) [W/m³K] the linear thermal transmittance value of the planar construction. \( U_{R} \) is calculated as:

\[
U_{R} = (1 + \chi) U
\]

(6)

where \( U_{R} \) [W/m²K] is the effective thermal transmittance value of the thermal bridges as well. \( \psi \) is the effective thermal transmittance factor, \( \chi \) [-] the thermal bridge correction factor, and \( U \) [W/m²K] the thermal transmittance value of the planar construction. Eq. (5) and Eq. (6) are based on the assumption that the following equality is approximately true:

\[
\sum_{i=1}^{n} (1 + \chi_i) U_{R,i} \equiv \sum_{i=1}^{n} U_i A_i + \sum_{j=1}^{m} \psi_j l_j
\]

(7)

The \( \chi \) values to be used on which the accuracy of the simplified method depends are given in the regulation (acc. to II.3.b in [1]) in a tabulated form which is shown here in Table I. To use this method the geometry of the individual surfaces must be calculated with their internal dimensions, than the ratio of the total length of thermal bridges to the wall area (\( \Sigma/A \)) must be determined. For external walls the \( \chi \) value to be used is only dependent on whether the wall has a continuous thermal insulation layer or not and on the value of \( \Sigma/A \).

<table>
<thead>
<tr>
<th>( \Sigma/A ) [1/m]</th>
<th>geometrical limits for choosing the correction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.8</td>
<td>0.8 – 1.0</td>
</tr>
<tr>
<td>&gt;1.0</td>
<td></td>
</tr>
</tbody>
</table>

The values to be used on which the accuracy of the simplified method depends are given in the regulation (acc. to II.3.b in [1]) in a tabulated form which is shown here in Table I. To use this method the geometry of the individual surfaces must be calculated with their internal dimensions, than the ratio of the total length of thermal bridges to the wall area (\( \Sigma/A \)) must be determined. For external walls the \( \chi \) value to be used is only dependent on whether the wall has a continuous thermal insulation layer or not and on the value of \( \Sigma/A \).

3 The thermal bridge atlas created

In the research project mentioned earlier we created a comprehensive thermal bridge atlas, basically a database containing an extensive set of linear thermal transmittance values, and other numerical results like minimum internal surface temperatures according to the standard DIN-4108-2 [9]. The atlas was intended to be used in the energy certification and the planning of building energy retrofits for late 19th century urban apartment buildings. It contains the most important external details (e.g. partition wall to external wall connection, slab to external wall connection, window jamb, sill and head...) with the most typical building constructions of the time (solid brick walls, slabs with segmental masonry brick vaults supported by steel beams, casement windows with two layers of sashes, etc.). Each detail is presented in its original state, with an internal thermal insulation system (with a varying thickness) and with an external thermal insulation system (also with a varying thickness) for reference. The details with the internal insulation are given with both a thermally optimized solution (thermal insulation extended inwards on adjoining constructions and into the window jambs for the reduction of thermal bridging) and a basic solution (without these enhancements). Of course no thermal bridge atlas can claim to be exhaustive given that the number of possible details
and solution is infinite, but if the building to be investigated is well typifiable a collection of the most important cases can be enough for most of the calculations. The atlas currently covers more than 100 different details in several variations on over 220 pages.

4 A Monte Carlo simulation to investigate the effect of the different parameters

With the thermal bridge database we created for this building type we were able to assess the precision of the simplified calculation method by comparing the thermal transmittance results obtained with the detailed calculation method, Eq. (4), and our database with the results of the simplified calculation, Eq. (5) and Eq. (6) using Table 1. As the detailed calculation theoretically reproduces the true multi-dimensional heat losses of a construction this comparison yields the relative and absolute errors of the simplified method with regards to the exact heat-losses (within the limits of the numerical thermal simulations performed to obtain the thermal bridge values in the atlas).

In a previous article [2] we presented a case study done for a sample building where we found large discrepancies between the simplified and the detailed calculations results. Other authors have reached similar conclusions in other studies of both the Hungarian and other states’ building energy calculation methods. In [17] Talamon and Csoknyai described the inadequacy of the simplified calculation method in [1] to accurately account for thermal bridging in prefabricated “panel” buildings. As mentioned earlier very similar simplified methods also exist in other countries calculation methodologies. Theodosius and Papadopoulos in [18] demonstrated similar discrepancies with regards to the Greek building energy code, buildings with brick cavity walls and the Greek version of the simplified calculation. Finally, in their article [3] Berggren and Wall conducted a comprehensive survey among building energy professionals in Norway and found that there was a lack of sufficient knowledge of how to use simplified thermal bridge calculations correctly. They also demonstrated that even if properly used these simplified methods have strong limitations when calculating buildings with and increased thickness of thermal insulation: having only a few discrete values (or even just one) for the thermal bridge correction can’t possibly deliver accurate results for all of the cases.

For the further investigation of the problem we concluded, that one case study is not sufficient to draw far reaching conclusions. There are multiple geometrical and structural parameters on which the results may depend. We can choose from several mathematical methods to address this uncertainty and get a picture of the distribution of the possible results. As shown in [4] many such methods have already been applied to building energy and building physics problems before. For the purpose of this article we have chosen to perform a Monte Carlo simulation.

The error between the two methods depends on the thermal transmittance of the individual thermal bridges (the \( \psi \) values, depending on the wall and insulation thickness as well as the quality of the details) and the building geometry (the composition and lengths of the individual thermal bridges, as well as the difference between internal and external dimensions). The thermal bridge data was already gathered in the atlas, but to obtain a statistically significant sample of possible building geometries by simply measuring individual buildings was not practical. Therefore a method was devised to get the necessary building geometries similar to the one presented in [16], where the authors generated a sample of building geometries artificially by determining the range of possible geometrical parameters and then combining them together randomly to get a sample. As they pointed out, if the sample is large enough and the geometrical parameters are evenly distributed and have a finite variance, according to the central limit theorem the result will be approximately normally distributed. However this criterion will not be met in our case since e.g. the thickness of the thermal insulation on the façade or the thickness of the walls can only take on certain discrete values (the thickness of a brick masonry is not a continuous function and therefore the thermal bridge data was also limited to certain masonry and thermal insulation thicknesses).

To calculate the effect of thermal bridges on the thermal transmittance of external walls we only needed to generate the façade geometry. 19th century urban apartment buildings all show a very similar structure and façade if we only focus on the geometry and neglect the stylistic elements. Furthermore, since they are geometrically very repetitive we can faithfully represent them with just a well-chosen small portion or patch (e.g. a façade element belonging to a single flat).

We investigated both a typical street side façade and an internal courtyard façade with the cantilever stone corridors characteristic for the epoch. The expected limits for the geometrical parameters were determined and in absence of a better guess a uniform random distribution was assumed. For a summary of the investigated façade elements and the geometrical parameters see Fig. 3.

In terms of different constructional types the current simplified calculation method in the government regulation [1] only differentiates between external walls with and without a continuous layer of thermal insulation. We however set up three distinct cases: external wall in the original state (no thermal insulation), external wall with interior (discontinuous) insulation and external wall with external (continuous) insulation. The possible thickness of the masonry is known from the building regulations of the period (1.5 brick walls for top floors, 2 and 2.5 brick walls below acc. to the loads), while the thickness of the thermal insulation was assumed between 2 and 8 cm. A thicker internal insulation is rarely possible due to hygrothermal reasons, while the thickness of an external insulation in this case is limited by current building regulations and simple geometry to 10 cm including plaster. For the internal insulation both standard and thermally optimized details were investigated. For a summary
of the examined constructional parameters and their assumed values see Fig. 4.

With the thermal bridge atlas and the method for generating geometrical and constructional descriptions of the façade we performed a Monte Carlo simulation by performing both the simplified and the detailed calculation for each individual sample to get a distribution of the produced results. We conducted 100,000 calculations for each façade and insulation type shown in Fig. 3 and Fig. 4 to guarantee a sufficient coverage of the space of possible variations.

5 The analysis of the results and development of a new approximation method

By varying all the parameters shown in Fig. 3 and Fig. 4 at the same time we get the results shown in the histograms in Fig. 5. As expected the histograms are not all continuous and not normally distributed since the geometrical and structural parameters shown in Fig. 3 and Fig. 4 were not continuous either. By comparing the computed \( \chi \) values with the standard ones from Table 1 it is immediately obvious how inaccurate the simplified method can be. Only the results for the original state (Fig. 5 left) show comparable values to Table 1, but here the exact values seem to fall shorter than the ones in the regulation, while at the two other cases the opposite is true. But these histograms don’t allow for much further study, as we can’t distinguish between the influences of the individual parameters.

For further investigation it was reasonable to assume, that the \( \chi \) value would be dependent not just on the geometry of the façade, but also on the thickness (or thermal resistance) of the external wall, the thickness (or thermal resistance) of the thermal insulation and the type of details used. Therefore we defined groups for each façade and thermal insulation type where the thickness of the wall, the thickness of the thermal insulation and the quality of the details (for internal insulations) were held constant and only the geometrical parameters were varied, so we could investigate the influence of the different parameters one at a time. The calculated results (\( \chi \) values) were then plotted against the specific length of the thermal bridges on the façade (\( \Sigma \bar{l} / A \)) and a very good linear correlation could be observed (see Fig. 6 for a few examples).

Because of the linear correlation we can represent the data with reasonable accuracy with a single line starting from the origin and described by the equation:

\[
\chi = s \cdot \frac{\sum l}{A}
\]  

(8)

where \( \chi \) [-] is the thermal bridge correction factor, \( s \) [m] a constant and \( \Sigma l / A \) [1/m] the specific length of the thermal bridges on the façade.

What Eq. (8) states is that the thermal bridge correction factor is directly proportional to the total length of the thermal bridges on the investigated façade. The constant of proportionality is dependent on the type of external construction investigated and the overall façade geometry (the types of thermal bridges present). The simplified method also implies this, but the dependence of Table 1 on these factors is far too digital, and some \( \Sigma l / A \) ranges it stipulates don’t even exist on real façades (at least for this type of building). The value of \( s \) was determined with a least squares fit for every group investigated and the resulting regression lines are also shown in Fig. 6.

We could substitute the lengthy detailed thermal bridge calculations or vastly improve on the accuracy of the simplified calculation with just equation Eq. (8) if we could only give the right \( s \) value for every case. But as stated \( s \) is still dependent on the thickness of the masonry, the type of insulation (external, internal, ore none), the thickness off the insulation layer and the thermal quality of the details used, the number of necessary \( s \) values would be impractically high. The question is: can we find a more general equation to obtain \( s \) from these parameters?

In Fig. 7 we can see the dependence of \( s \) on the thermal resistance (thickness) of the masonry construction and on the thermal resistance (thickness) of the thermal insulation. The \( s \) values for the original state of the wall only depend on the thermal resistance of the masonry approximately linearly. The values of \( s \) for the different types of insulated constructions all define distinct curved surfaces over \( R_{wall} \) and \( R_{ins} \). We can find an equation, \( s = f(R_{wall}, R_{insulation}) \), e.g. a 2D polynomial with the help of a non-linear least squares fit for every such surface, and we can increase the accuracy of the fit by increasing the order of the polynomial (as long as we have enough data points to do the calculation). But for different types of constructions the equations (the coefficients of the polynomials) will be different (see Fig. 7). In a way the independent variables capture \( s \)’s dependence on the thermal conductivity of the thermal insulation and the masonry, while the polynomial coefficients capture the type of the construction (internal or external insulation, the quality of the details, etc.), and then substituting \( s \) into Eq. (8) the exact geometry is represented by \( \Sigma l / A \). The polynomial equations for \( s \) and the coefficients determined for the constructions investigated in this article are shown in Eq. (9) and Table 2.

\[
s = p_{00} + p_{10}R_{wall} + p_{01}R_{ins} + p_{20}R_{wall}^{2} + p_{11}R_{wall}R_{ins} + p_{02}R_{ins}^{2} + p_{21}R_{wall}R_{ins} + p_{12}R_{wall}^{2}R_{ins} + p_{03}R_{ins}^{3}
\]  

(9)

where \( s \) [m] is the constant for Eq. (8), \( p \) are the polynomial coefficients, \( R_{wall} \) [m²K/W] the thermal resistance of the masonry and \( R_{ins} \) [m²K/W] the thermal resistance of the thermal insulation layer.

For the time being we failed to find a more general (single equation) formulation to calculate a precise \( \chi \) value and it is not immediately clear that there can be one at all. The proposed method is only capable to fit data previously calculated in a much more compact and easy to use form, and to accurately interpolate between individual data points. It is not however capable of extrapolation to constructional variants and façade types previously not investigated.
Fig. 3. The typical façade patches and the geometrical parameters investigated

<table>
<thead>
<tr>
<th>Geometrical parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{\text{win}} )</td>
<td>2-4 [-] number of window axis</td>
</tr>
<tr>
<td>( n_{\text{part}} )</td>
<td>0.5-1 [-] number of partitions per axis</td>
</tr>
<tr>
<td>( l_{\text{win}} )</td>
<td>2.5 - 3 [m] length of window axis</td>
</tr>
<tr>
<td>( b_{\text{win}} )</td>
<td>1-1.5 [m] window width</td>
</tr>
<tr>
<td>( h_{\text{win}} )</td>
<td>2-2.5 [m] window height</td>
</tr>
<tr>
<td>( h_{\text{headroom}} )</td>
<td>3.5-4 [m] headroom</td>
</tr>
<tr>
<td>( l_{\text{part}} )</td>
<td>0.18 [m] partition wall</td>
</tr>
</tbody>
</table>

\[ A_{\text{wall}} = n_{\text{win}} \cdot l_{\text{win}} \cdot \left( \frac{l_{\text{part}}}{n_{\text{part}}} \right) \cdot l_{\text{part}} \]

Fig. 4. The structural parameters investigated

| Tab. 2. Calculated polynomial coefficients for Eq. (9) |
|-----------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| type            | case          | \( P_{00} \) | \( P_{10} \) | \( P_{01} \) | \( P_{20} \) | \( P_{11} \) | \( P_{21} \) | \( P_{12} \) | \( P_{03} \) |
| original        | -             | 0.000         | 0.000         | 0.000         | 0.000         | 0.000         | 0.000         | 0.000         | 0.000         |
| int. ins.       | optimized     | 0.095         | -0.262        | 0.189         | 0.259         | -0.149        | 0.0098        | 0.031         | 0.118         | 0.035         |
| standard        | 0.088         | -0.074        | 0.084         | 0.148         | -0.066        | 0.153         | 0.164         | 0.039         | -0.063        |
| ext. ins.       | street        | 0.105         | -0.116        | 0.147         | 0.172         | -0.392        | 0.125         | 0.182         | 0.019         | -0.034        |
|                 | courty        | 0.109         | -0.124        | 0.286         | 0.177         | -0.491        | 0.136         | 0.205         | 0.028         | -0.039        |
**Fig. 5.** Histogram plot of the calculated correct $\chi$ values for an internal courtyard façade from the detailed calculation when all the parameters are varied at the same time: original state (left), internal insulation (middle) and external insulation (right).

**Fig. 6.** 3 examples of the calculated $\chi$ values and regression lines plotted against $\Sigma l/A$ and compared to the standard $\chi$ values of the simplified method, for wall thickness: 59cm, thermal insulation: 6 cm; original state (left), internal insulation with optimized details (middle) and internal insulation with standard details (right).

**Fig. 7.** The calculated values of $s$ plotted against $R_{\text{wall}}$ and $R_{\text{insulation}}$. 
6 Comparing the calculation errors
For all the constructional groups investigated in part 5 the calculation error of both the existing and the proposed simplified calculation methods were investigated with regards to the detailed calculation depending on the geometrical parameters. Some of the results are shown with the histograms in Fig. 8. As previously discussed the existing method was found to be very inaccurate. The expected percentage error in $\chi$ is very large, and the sign that this error takes is also very unfavorable. For existing solid brick masonry walls it significantly overpredicts heat losses while for internal insulation it mostly underpredicts them. This can be a very dangerous combination of errors when calculating the possible energy saving potential of retrofit measures.

The percentage error obtained with the proposed method was centered at zero with a standard deviation of less than 5% for the vast majority of cases.

7 Conclusions
The simplified thermal bridge calculation in the Hungarian regulation today was demonstrated to be very inaccurate (at least for historical buildings), therefore a new methodology is proposed to improve on it. The algorithm for the method is:

1. describe a typical façade element for which to make the calculation, determine the geometrical parameters and their expected range
2. determine the constructional variants to be investigated (e.g. wall thickness, type, position and thickness of the thermal insulation) and define their range
3. make a list of the typical (non-repeating) thermal bridges on the façade and prepare their thermal simulation to obtain their linear thermal transmittance value
4. perform a Monte Carlo simulation on the variables from points 1 and 2 and the thermal bridge database from point 3 to calculate the correct $\chi$ values
5. perform a least squares regression to obtain the constants $s$ for Eq. (8)
6. perform a non-linear least squares fit on the data in step 5 to obtain a more general equation for $s$

Points 3 to 6 could even be performed automatically with customized software. Once derived, Eq. (7) with Table 2 and Eq. (8) can be used as a simplified calculation method to get an approximately correct $\chi$ value to use in Eq. (5) for the simplified building energy calculation.

The obtained method was shown to be much more accurate than the use of the generic $\chi$ values in Table 1. Basically this proposed method can approximate all the data gathered in a thermal bridge atlas without the designer having to look through several hundred pages in search of $\psi$ values. But there are limitations: the equations for the $s$ values can only be determined for a construction from an already existing thermal bridge database, and Eq. (7) is only valid if the façade being investigated is monotonous (it can be described by a characteristic portion) like the ones investigated in this article. Furthermore the obtained data however should always be accompanied by a detailed account of the assumed façade type, geometrical parameters and constructional parameters on the basis of which it was determined (like Fig. 3 and Fig. 4 in this article). Similarly to the details given in the German standard [7], or in another proposed methodology for the improvement of thermal bridge calculations described in [15], the designer could then always compare the building he investigates with these assumptions and determine whether the published data is valid for the concrete case or not.

Because of these limitations and because different types of constructions and details seem to behave differently (as demonstrated in Fig. 7) it is questionable that such a method could be perfected for new buildings, where the architectural and constructional freedom is extremely large. The best usage of such a method would be the building energy calculation of all existing buildings the façade of which is well-typifiable and repetitive and where the number of geometrical and constructional parameters are limited. Besides 19th century urban apartment buildings investigated in this article this could include post war brick buildings in the Socialist Realism style, large format block buildings, panel buildings and public buildings built according to type plans. Most of these buildings are in need of a thermal retrofit for which there are numerous government subsidies. To improve the well-roundedness of these applications it could be possible to provide new mandatory thermal bridge correction factors for the specific building and thermal insulation types. An example for a similar method for prefabricated “panel” buildings was already demonstrated in [17]. The methodology described in this article could be used to provide such calculation aids for all of the mentioned building types.

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Fig. 8. Calculation errors for the range of geometrical parameters: original state (left), internal insulation with optimized details (middle) and internal insulation with standard details (right).

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