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RESEARCH ARTICLE

Localization of bearing errors using spline method

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Abstract

This paper describes a method to locate bearing errors in transistion curve geometry. The method is based on the function of special splines fitted on the setup points of transition curves. When errors occur in coordinates of transition curve, the method is able to localize and value them.

Keywords

transition curve · spline

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1 Kinematic characteristics of railway tracks

In railway transportation it is a fundamental requirement to have track geometry that corresponds to the desired motion since in this track vehicle system the track defines the motion primarily. Tracks constructed with appropriate kinematic precision reduce the magnitude of internal forces. High speed railways require appropriate design of the motion geometry of the track.

For this purpose a unified motion geometry theory can be applied, which enables the determination of correct track geometry on differential geometric basis [1, 7–10].

The curvature function of the railway track plays a primary role in kinematically induced internal forces. Therefore the track is designed such that the site plan of the transition curve is determined based on the shape of the curvature function.

The exact relationship between the function on the site plan and the curvature function is given as

$$G = \frac{d^2y}{dx^2} \Bigg/ \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} \tag{1}$$

It is complicated to deduce the function of the track curve from this formula. The Cartesian coordinates of the setout are obtained from the integral equations of the arc length parameter below using two terms in the power series.

$$\begin{aligned} x &= \int_0^x dx = \int_0^l \cos\tau dl = \int_0^l \cos \int_0^l Gdldl \\ y &= \int_0^y dy = \int_0^l \sin\tau dl = \int_0^l \sin \int_0^l Gdldl \end{aligned} \tag{2}$$

This calculation of the setout coordinates are used for the physical layout of the track geometry.

In practice the Cartesian coordinates are computed for points on the arc at equal 5 meter distances.

The track to be built by the setout can be modelled by a new mathematical procedure that is based on older principles.

All this can be done using the splines. Splines are a family of polynomials in which the polynomials are connected to one another in a certain order resulting in a curve that closely approximates the shape of the railway track fitted to the setout points.

Thus the entire function of the segment of track in question is obtained.

2 The spline approximation

Based on the setout points we intend to determine a track function where where the total change of curvature is minimal is minimal. It yields a curve that closely approximates the shape of the railway track fitted to setout points.

Using these two viewpoints of the analysis we applied the following mathematical model:

The track function to be determined is approximated by a spline to give the best approximation using the setout points.

Given are the setout points (x_i, y_i), $i = 0, \ldots, N$ in a Cartesian coordinate system with its origin and point (X, Y) at the start and at the end of the transition curve, respectively. The radius R of the circular arc is known. The spline is

$$s(x) = \begin{cases} s_1(x), & \text{where } 0 = \tilde{x}_0 \le x \le \tilde{x}_1 \\ s_2(x), & \tilde{x}_1 \le x \le \tilde{x}_2 \\ \vdots & \vdots \\ s_n(x), & \tilde{x}_{n-1} \le x \le \tilde{x}_n = X \end{cases} \tag{3}$$

where $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_{n-1}$ are suitably chosen subpoints in the interval $[0, X]$ and $s_1(x)$, $s_2(x)$, $\ldots$, $s_n(x)$ are the individual polynomials in the spline.

The solution is sought by the minimization of the functional

$$F(s(x)) = \int_0^X \left(s'''(x)\right)^2 dx + \lambda \sum_{i=1}^{N-1} p_i \left(s(x_i) - y_i\right)^2 \tag{4}$$

where P_i are the approximation weights and λ is the Lagrange multiplier.

The minimization of the functional (using Sard [2]) yields that all polynomials are fifth order functions, that is for all $(i = 1 \ldots \mathrm{n})$:

$$s_i(x) = \sum_{k=0}^{5} a_i \left(x - \tilde{x}_{i-1}\right)^k . \tag{5}$$

The solution is a differentiable function to the fourth order.

The minimum is obtained as the solution of a conditional extremum problem of multivariable functions, where it is a functional [3]:

$$\begin{aligned} &F\left(a_{1,0}; \ldots; a_{n,5}; \Lambda_{1,0}; \ldots; \Lambda_{n-1,4}\right) = \\ &= \sum_{j=1}^{N-1} p_j \left(s\left(x_j\right) - y_j\right)^2 - 2 \sum_{k=0}^{4} \sum_{i=1}^{n-1} \Lambda_{ik} \left(s_i^{(k)}(\tilde{x}_i) - s_{i+1}^{(k)}(\tilde{x}_i)\right) \end{aligned} \tag{6}$$

It leads to an iterative procedure where the weights p_j are recomputed again and again until the required precision of the track curve is obtained.

3 Localization and determination of bearing errors

There are several kinds of bearing errors. The track may deviate from the line in a straight segment, from the perfect circle in an arc, and in a transition curve from the exact direction.

There are several reasons for errors, such as rail abrasion, deformations caused by centrifugal forces due to lateral acceleration of vehicles, faulty rail fastening, rail lifting.

Bearing errors cause changes in curvature, which in turn may lead to the increase of kinematic internal forces with respect to the design values.

Various correctional methods exist to tackle this phenomenon.

4 Localization of errors using the spline method

The track curve is approximated by a spline function to give the best approximation of the setout points thus the polynomials do not exactly fit the points but pass by them by a certain small distance resulting in a minimal total change of curvature.

The polynomials are fitted to the points in a way that the distance is supposed to be small and at the same time the change of curvature is minimal.

From the fact that the polynomial behaves as a bent stick, one can conclude that there may exist points outside the domain of a particular polynomial, thus they are preferably ignored.

The spline method is able to filter out such points because during the fitting of the function to the points the approximation weight of that point is set at the value of the inverse of the distance measured from the curve of the function with minimal total change of curvature.

Transition curves are segments with curvatures to have minimal total change of curvature, thus can be ideally modelled by splines. In other words, the setout points will be aligned with the shape of a bent stick.

In the case of a point lying off the spline further than the adjustable limit, the method considers it to a tiny extent (the inverse of the distance) and incorporates it in the calculation with a tiny weight as a measurement error.

Thus it is possible to localize the bearing errors and to give the value of the error in an (x, y) coordinate system.

The approximative nature of this method enables us to make a good model for the curve of the railway track in spite of the measurement and computational errors commonly present in engineering practice.

The measurement errors can be localized and corrected when known.

The practical importance of this conclusion is utilized in the filtering out of certain bearing errors on tracks.

A cosine transition curve ($L = 160$ m) between straight and circle ($R = 1500$ m) has been examined using the spline method to demonstrate its behaviour in the case of bearing errors.

First step we layed a spline on the cosine transition curve with the parameters determined above and we computed the distance between setout points and the spline and plotted it on a coordinate system.

Tab. 1. Spline layed on cosine transition curve coordinates [5]

x	y	Approximation	Spline-point
(m)	(m)	weight	distance (m)
0.000	0.000	1000	0.0
5.000	0.000	1000	0.000002074609010
10.000	0.000	1000	0.00004577085877
15.000	0.000	1000	0.0002510361562
20.000	0.001	1000	0.0001805796342
25.000	0.002	1000	0.000029847594
30.000	0.004	1000	0.000234387972
35.000	0.008	1000	0.000145970560
40.000	0.013	1000	0.00037444954
45.000	0.021	1000	0.00034178717
50.000	0.032	1000	0.00035841440
55.000	0.047	1000	0.00007867612
60.000	0.066	1000	0.00020379302
65.000	0.091	1000	0.00052535860
70.000	0.121	1000	0.0003363033
75.000	0.158	1000	0.0004330560
79.999	0.202	1000	0.0000139033
84.999	0.255	1000	0.0002424721
89.999	0.317	1000	0.0003088859
94.998	0.389	1000	0.0004194377
99.998	0.471	1000	0.000246710
104.997	0.565	1000	0.0004233720
109.996	0.672	1000	0.0001396333
114.994	0.791	1000	0.0002287938
119.992	0.924	1000	0.0002015321
124.99	1.071	1000	0.000383357
129.988	1.233	1000	0.000325981
134.984	1.41	1000	0.000449027
139.981	1.603	1000	0.000284567
144.976	1.812	1000	0.000074160
149.971	2.037	1000	0.000180802
154.965	2.279	1000	0.000234448
159.959	2.537	1000	0.0

Tab. 2. An error of 10 mm intaked at coordinate N^o 17.

N^o	x	y	Error	Approximation	Spline-point
	(m)	(m)	(mm)	weight	distance
1	0.000	0.000		1000	0.0
2	5.000	0.000		1000	0.000003271768696
3	10.000	0.000		1000	0.00005457960179
4	15.000	0.000		1000	0.0002782875810
5	20.000	0.001		1000	0.0001215833373
6	25.000	0.002		1000	0.000134673619
7	30.000	0.004		1000	0.000398479981
8	35.000	0.008		1000	0.000089000978
9	40.000	0.013		1000	0.00068917448
10	45.000	0.021		1000	0.00074173994
11	50.000	0.032		1000	0.00084526733
12	55.000	0.047		1000	0.00065015279
13	60.000	0.066		1000	0.00085374820
14	65.000	0.091		1000	0.00019329261
15	70.000	0.121		1000	0.0004379774
16	75.000	0.158		1000	0.0003809779
17	79.999	**0.212**	10	108	0.0091782084
18	84.999	0.255		1000	0.0005953015
19	89.999	0.317		1000	0.0005107173
20	94.998	0.389		1000	0.0003620524
21	99.998	0.471		1000	0.000971504
22	104.997	0.565		930	0.0010754798
23	109.996	0.672		1000	0.0004275551
24	114.994	0.791		1000	0.0007035190
25	119.992	0.924		1000	0.0005814126
26	124.99	1.071		1000	0.000671280
27	129.988	1.233		1000	0.000529793
28	134.984	1.410		1000	0.000580848
29	139.981	1.603		1000	0.000359581
30	144.976	1.812		1000	0.000109165
31	149.971	2.037		1000	0.000192222
32	154.965	2.279		1000	0.000232882
33	159.959	2.537		1000	0.0

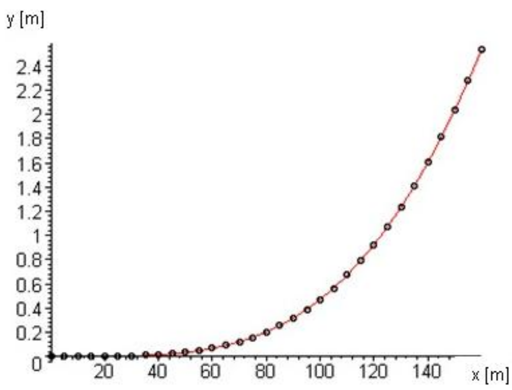

Fig. 1. Transition curve modeling with spline method

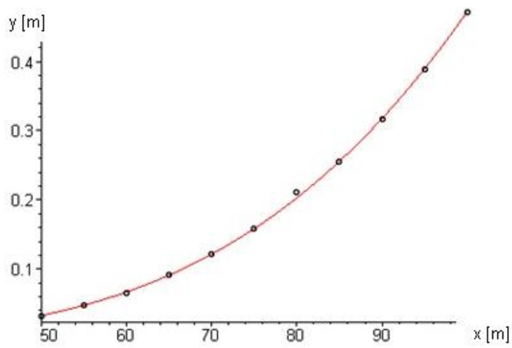

Fig. 2. Detail of transition curve at intaked error at 17. setout point

Tab. 3. Many errors intaked at several coordinates

N^o	x (m)	y (m)	Error (mm)	Approximation weight	Spline-point distance
1	0.000	0.000		1000	0.0
2	5.000	0.000		1000	0.000003397015374
3	10.000	0.000		1000	0.00005544463941
4	15.000	0.000		1000	0.0002807777332
5	20.000	0.001		1000	0.0001166207037
6	25.000	0.002		1000	0.000142682316
7	30.000	0.004		1000	0.000409671619
8	35.000	0.008		1000	0.000102985715
9	40.000	0.013		1000	0.00070501865
10	45.000	0.021		1000	0.00075802187
11	50.000	0.032		1000	0.00086020602
12	55.000	0.047		1000	0.00066180855
13	60.000	0.066		1000	0.00086026727
14	65.000	0.091		1000	0.00019312649
15	70.000	0.121		1000	0.0004300482
16	75.000	0.158		1000	0.0003647900
17	79.999	**0.212**	10	109	0.0092025113
18	84.999	0.255		1000	0.0005636663
19	89.999	0.317		1000	0.0004731221
20	94.998	0.389		1000	0.0003203529
21	99.998	0.471		1000	0.000927875
22	104.997	0.565		969	0.0010322055
23	109.996	0.672		1000	0.0003867796
24	114.994	0.791		1000	0.0006670445
25	119.992	0.924		1000	0.0005505452
26	124.990	1.071		1000	0.000646744
27	129.988	1.233		1000	0.000511699
28	134.984	**1.400**	-10	95	0.010568721
29	139.981	**1.608**	5	215	0.004647538
30	144.976	**1.832**	20	50	0.019894247
31	149.971	2.037		1000	0.000191080
32	154.965	2.279		1000	0.000233041
33	159.959	2.537		1000	0.0

This method may constitute a significant advance in track adjusting because maintenance machine adjust tracks not only in the setout points at every 4 or 5 metres but at all ties placed at about 60 centimetres usually. The correct position of the ties is calculated by interpolation. Coordinates of the track using spline curves would give much more precise approximation avoiding unnecessary deformations in the rails.

Using the spline method during the construction likewise would be preferably by the same reasoning.

The function of the spline is a family of third order polynomials [4, 6]. The spline has been fitted to the setout points just as well the track bends to fit the setout points. The setout data are determined by approximation using the first two terms of the power series of the given curvature function.

The theoretical and the constructed curvature function coincides only at the setout points. Between the points the bent shape of the rail track determines the curvature function the same way as the spline bends to the setout points.

The theoretical curvature function and the curvature function

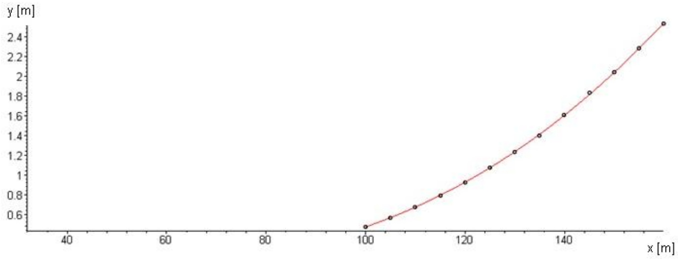

Fig. 3. Detail of transition curve at intaked errors at 28., 29. and 30. setout points

of the track fitted to setout points spaced at 5 metre distance based on the theoretical curvature function are different.

Leaving 5-metre distances between consecutive setout points, the track or in our model the spline is enabled to accommodate elastically and find the minimal total curvature.

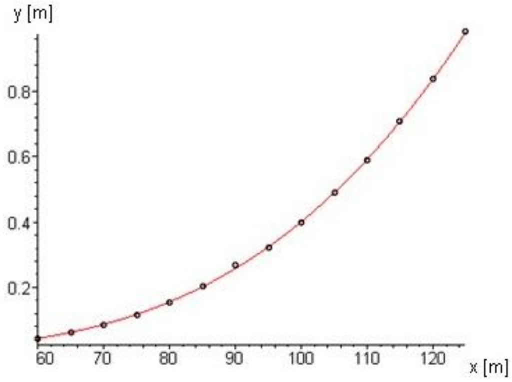

Fig. 4. Details of transition curve at intaked errors at 19. setout point

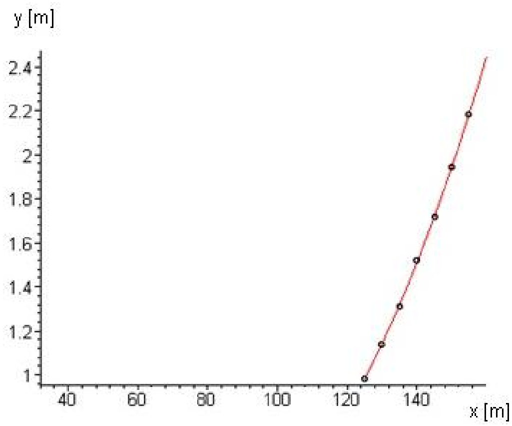

Fig. 5. Details of transition curve at intaked errors at 28. and 29. setout points

5 Conclusions

The bearing errors can be localized with adequate precision using the spline method.

During the procedure the coordinates of the track need to be determined using the set of input survey data, then the boundary conditions are created from the design values, and the method points out the regions subjected to possible errors in a way shown above.

Tab. 4. Many errors intaked at several coordinates (19., 28., 29.)

N^o	x	y	x_{spline}	y_{spline}	Error	Approximation	Spline-point
	(m)	(m)	(m)	(m)	(mm)	weight	distance
1	0.000	0.000	0.000	0.000		1000	0.0
2	5.000	0.000	5.000	0.000		1000	0.00001098638630
3	10.000	0.000	10.000	0.000		1000	0.00007855929441
4	15.000	0.001	15.000	0.001		1000	0.0007512402237
5	20.000	0.001	20.000	0.001		1000	0.0007133359331
6	25.000	0.002	25.000	0.002		1000	0.000790023152
7	30.000	0.003	30.000	0.003		1000	0.000677737696
8	35.000	0.005	35.000	0.005		1000	0.000763242288
9	40.000	0.008	40.000	0.008		1000	0.000623718833
10	45.000	0.012	45.000	0.012		1000	0.00026372184
11	50.000	0.019	50.000	0.019		1000	0.00051216802
12	55.000	0.029	55.000	0.029		1000	0.00081503520
13	60.000	0.043	60.000	0.043		1000	0.0009361940
14	65.000	0.062	65.000	0.062		1000	0.00070007752
15	70.000	0.086	70.000	0.086		1000	0.00098124174
16	75.000	0.117	75.000	0.117		1000	0.0006925398
17	80.000	0.155	80.000	0.155		1000	0.0007727507
18	84.999	0.202	84.999	0.202		1000	0.0001640092
19	89.999	0.257	89.999	**0.267**	10	109	0.0091618326
20	94.998	0.323	94.998	0.323		1000	0.0007147477
21	99.998	0.400	99.998	0.400		1000	0.0007385115
22	104.997	0.489	104.997	0.489		1000	0.0007607938
23	109.996	0.591	109.996	0.591		1000	0.0006294605
24	114.995	0.706	114.995	0.706		895	0.0011173462
25	119.993	0.836	119.993	0.836		1000	0.0008966939
26	124.991	0.981	124.991	0.981		1000	0.000603013
27	129.989	1.141	129.989	1.141		1000	0.000752476
28	134.985	1.317	134.985	**1.312**	-5	176	0.005683256
29	139.982	1.509	139.982	**1.519**	10	109	0.009189462
30	144.977	1.718	144.977	1.718		1000	0.000243506
31	149.972	1.943	149.972	1.943		1000	0.000213059
32	154.966	2.185	154.966	2.185		1000	0.000232947
33	159.960	2.443	159.960	2.443		1000	0

Tab. 5. Many errors intaked at several coordinates (13., 15., 27., 28.)

N^o	x	y	x_{spline}	y_{spline}	Error	Approximation	Spline-point
	(m)	(m)	(m)	(m)	(mm)	weight	distance
1	0.000	0.000	0.000	0.000		1000	0.0
2	5.000	0.000	5.000	0.000		1000	$0.6360920387\ 10^{-4}$
3	10.000	0.000	10.000	0.000		1000	0.00003255018176
4	15.000	0.000	15.000	0.000		1000	0.0002321852278
5	20.000	0.001	20.000	0.001		1000	0.0001708192988
6	25.000	0.002	25.000	0.002		1000	0.000142282429
7	30.000	0.005	30.000	0.005		1000	0.000432380663
8	35.000	0.009	35.000	0.009		1000	0.000431747631
9	40.000	0.015	40.000	0.015		1000	0.00033627658
10	45.000	0.024	45.000	0.024		1000	0.00058032812
11	50.000	0.036	50.000	0.036		1000	0.00056079031
12	55.000	0.051	55.000	0.051		1000	0.00035876011
13	60.000	0.072	60.000	**0.066**	-6	171	0.00584565514
14	65.000	0.097	65.000	0.097		1000	0.00059580030
15	70.000	0.129	70.000	**0.137**	8	130	0.0076686604
16	74.999	0.167	74.999	0.167		1000	0.0007899037
17	79.999	0.213	79.999	0.213		1000	0.0007543101
18	84.998	0.268	84.998	0.268		1000	0.89051 10
19	89.998	0.331	89.998	0.331		1000	0.0003173741
20	94.997	0.404	94.997	0.404		1000	0.0005044755
21	99.995	0.488	99.995	0.488		1000	0.0003353349
22	104.993	0.584	104.993	0.584		1000	0.0004040986
23	109.991	0.691	109.991	0.691		1000	0.0000333264
24	114.987	0.811	114.987	0.811		1000	0.0003098984
25	119.983	0.945	119.983	0.945		1000	0.000135522
26	124.977	1.093	124.977	1.093		1000	0.000065786
27	129.970	1.255	129.970	**1.265**	10	107	0.009324626
28	134.960	1.433	134.960	**1.423**	-10	96	0.010375425
29	139.949	1.626	139.949	1.626		1000	0.000646520
30	144.935	1.835	144.935	1.835		1000	0.000750910
31	149.917	2.060	149.917	2.060		1000	0.000899721
32	154.896	2.302	154.896	2.302		1000	0.000316371
33	159.870	2.560	159.870	2.560		1000	0

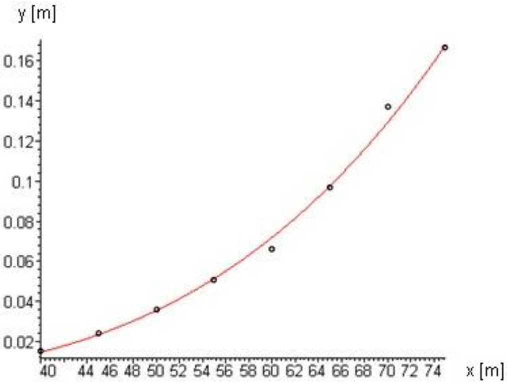

Fig. 6. Details of transition curve at intaked errors at 13. and 15. setout points

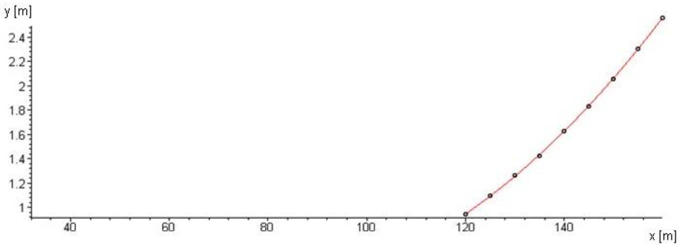

Fig. 7. Details of transition curve at intaked errors at 27. and 28. setout points

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