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RESEARCH ARTICLE

Optimal Design of Multiple Tuned Liquid Column Dampers for Seismic Vibration Control of MDOF Structures

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Abstract

This paper proposes a systematic optimization method to design optimal multiple tuned liquid column dampers ($MTLCDs$) for improving the seismic behavior of structures. A constrained optimization problem is formulated and solved using Genetic algorithm (GA) to generate the optimum parameters of $TLCDs$ that minimizes an objective function defined in terms of minimization of either (a) the maximum displacement or (b) the maximum acceleration of the structure. To illustrate the design procedure, a ten-storey shear frame subjected to a filtered white noise excitation has been considered and for different values of $MTLCD$ mass ratios and $TLCD$ numbers, optimal $MTLCDs$ have been designed for both objective functions and tested under real earthquakes. The results of numerical simulations show the simplicity and effectiveness of the method. Also it has been found that the performance of $MTLCDs$ has been affected by its mass ratio and earthquake characteristics while in this case study, increasing the number of $TLCDs$ has had no significant effect on its performance. Finally, comparison has been made between the performance of $MTLCDs$ and multiple tuned mass dampers (MTMDs), which show no significant difference in performance of these control systems in most of the simulated cases especially under the design record

Keywords

Passive Control · Multiple Tuned Liquid Column Dampers ($MTLCDs$) · Optimization · Genetic Algorithms (GAs) · Objective Function

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Notation

ρ	Liquid density
μ	Total mass ratio
ξ	Head loss coefficient
L_f	Length of liquid in the container
A_f	Cross-sectional area of tube
α	Length ratio
B_f	Horizontal portion of liquid in the container
N_{TLCD}	Number of $TLCD$s
g	Acceleration of gravity
$\ddot{X}_g$	Earthquake ground acceleration

1 Introduction

Over the past decades, the idea of using structural control systems in design and construction of tall buildings and other vulnerable structures has been recognized as an alternative approach to protect structures against the damaging effects of dynamic forces such as winds and earthquake excitations. This has led to investigation and development of numerous control techniques and mechanisms in this field which can be broadly classified into four main categories: passive, active, semi-active or hybrid control [1]. The most mechanically and technologically simple set of control schemes belongs to the passive control categorization, which has thus far been the most accepted for practical civil engineering applications. Different passive control mechanisms have been studied theoretically and experimentally and in some cases applied in real full-scale buildings [2]. Some examples are: mass dampers, tuned liquid and tuned liquid column dampers, visco-elastic dampers and base isolation systems [2].

Tuned liquid column dampers ($TLCD$s) [3] in particular, have been proposed for reducing the response of structures subjected to wind and earthquake excitations. This device includes a U-tube container with an orifice opening in the middle. The $TLCD$ dissipates the structural vibration energy by the combined action involving the motion of the liquid mass in the tube-like container, the restoring force due to the gravity acting on the liquid and the damping effect due to the orifices inside the tube. Lower costs, fewer maintenance requirements as well as simplicity and versatility are some advantages of this device. In addition, when the damper is incorporated into the design as a required water tank for water supply or firefighting it in fact imposes no weight penalty on the structure.

Different experimental and numerical studies have been carried out on designing $TLCD$s and evaluating their effectiveness in suppressing the structural response. Gao et al. [4] optimized the parameters of $TLCD$ with different cross-sectional area in its vertical and horizontal sections for controlling structural vibrations. The performance of liquid column vibration absorber (LCVA) which allows the column cross-section to be non-uniform was studied by Hitchcock et al. [5]. Sadek et al. [6] tried to determine the optimum tuning ratio, tube width to liquid length ratio and head loss coefficient for a given mass ratio of $TLCD$ using a deterministic analysis under 72 earthquake records. Chang et al. [7] studied optimal designing of $TLCD$s and proposed a method for determination of optimal frequency and damping ratio of $TLCD$s. The effectiveness of $TLCD$ in reducing the along-wind response of tall buildings with different mass-stiffness distributions as well as using $TLCD$ for vibration control of various structural systems such as frame, shear wall and frame-shear wall under random wind loading were investigated by Balendra et al. [8]. Yallah and Kareem [9] studied determination of the optimum parameters of $TLCD$ under wind and earthquake loads and suggested a method for head loss coefficient calculations. Xue et al. [10] examined the capability of $TLCD$ in suppressing pitching motion of structures where through theoretical and experimental simulations they found that $TLCD$ can efficiently reduce structural pitching motion. Also different researches such as using $TLCD$ for seismic vibration control of short period structures [11], dynamics of vibrating systems with tuned liquid column dampers and limited power supply [12], optimum design of $TLCD$s under stochastic earthquake load considering uncertain bounded system parameters [13] and modified tuned liquid column damper [14] have previously been carried out on $TLCD$s.

Though application of a single $TLCD$ could be helpful in suppressing the structural response under external excitations, it does come with some limitations such as sensitivity problem to detuning the $TLCD$ frequency or its damping ratio and uncertainty in dynamic properties of the main structure. These shortcomings are quite similar to that of tuned mass damper (TMD) and could decrease the effectiveness of both devices significantly. To overcome these drawbacks and improve the performance of a $TLCD$, different systems such as using active tuned liquid damper (A$TLCD$) [15] or multiple tuned liquid column dampers ($MTLCD$s) [16] could be used.

$MTLCD$ includes a number of $TLCD$s each having different dynamic characteristics which can be located at one floor or distributed over the floors of a building. This control mechanism is very similar to multiple tuned mass dampers (MTMDs) [17] in concept and design procedure.

Many researches confirm the certain advantages that using $MTLCD$ could have over single $TLCD$ considering detuning issues. By studying the performance of $MTLCD$s in controlling the vibration of structures under earthquake excitations, Samali et al. [16] concluded that the sensitivity of $MTLCD$s to uncertainty of structural dynamic parameters is less than a single $TLCD$. A similar result has been found by Hitchcock et al. [18] who conducted some experiments on $MTLCD$s system using different mass and frequency ratios. Sadek et al. [6] studied the optimal design and performance of single and multiple $TLCD$s under 72 earthquake excitations and determined the optimum central tuning ratio, tuning bandwidth and number of $TLCD$s for $MTLCD$s system for a given mass ratio. Chang et al. [19] studied designing of single and multiple $TLCD$s for buildings and presented some design formulas and design procedure for $MTLCD$s and showed that the sensitivity of single $TLCD$ to loading intensity could be reduced by using $MTLCD$s. In another research conducted by Goa et al. [20] the performance of $MTLCD$s in mitigating the response of structures has been investigated, where the effect of different parameters of $TLCD$s such as frequency domain, central frequency, head loss coefficient and the number of $TLCD$s on $MTLCD$s effectiveness has been evaluated and it has been found that the frequency range and head loss coefficients affect the performance of $MTLCD$s. The capability of $MTLCD$s system in reducing coupled lateral and torsional vibration of structures has also been shown by Shum and Xu [21]. In addition, Kim et al. [22] have proposed using tunable $TLCD$s with multiple-cell for controlling the response of tall buildings subjected to wind excitations to shift the frequency of $TLCD$ to a new frequency. In most of the previous researches conducted on designing $MTLCD$s, the design procedure has been based on some simplifying assumptions for $TLCD$s' parameters such as identical masses, uniform distribution for $TLCD$ frequency or damping. These constraints on distribution of $TLCD$ parameters for simplifying the design procedure, along with tuning the frequency of $TLCD$s to a specified frequency of structural mode or the need for extensive numerical analysis have been some of the drawbacks of previous design procedures used for designing $MTLCD$s for multi degree of freedom (MDOF) structures.

Recently Mohebbi et al. [23–26] have combined structural control strategies and optimization techniques to come up with efficient approaches to design different control systems. These approaches are simple and computationally efficient, and are previously used to design MTMDs [23], MR dampers [24] for linear structures as well as MTMDs [25], and active mass damper (AMD) [26] for nonlinear structures. In this paper, a new approach based on using intelligent optimization techniques is suggested for optimal design of $MTLCD$s to overcome limitations of the previous $MTLCD$ design methods. In the proposed method which is an adaptation of a recently proposed method by Mohebbi et al. [23] to design optimal MTMDs for MDOF linear structures, an optimization problem is formulated and solved to find the optimal parameters of $TLCD$s. The optimal parameters of each $TLCD$ unit are determined and the $MTLCD$s control system is designed to minimize certain response of the structure and pursue different design objectives while satisfying some practical considerations regarding $TLCD$ parameters. Genetic algorithm (GA) [27] is used to solve the optimization problem. The proposed method is employed to design optimal $MTLCD$s to control the response of a ten–storey linear shear building frame subjected to earthquake excitation. Two cases of design including: a) minimization of maximum displacement and b) minimization of maximum acceleration of the structure has been studied. Also the effect of different parameters such as total mass of $TLCD$s, number of $TLCD$s, input earthquake characteristics as well as design criterion on performance of the $MTLCD$s are discussed.

2 Structure-$MTLCD$s equation of motion

Consider an n-degree-of-freedom linear shear building frame equipped with N_{TLCD} $TLCD$s located on its top floor in parallel configuration ,as shown in Fig. 1, and subjected to an earthquake ground motion, $\ddot{X}_g$. The dynamic equation of motion of the entire structure-$MTLCD$s system can be written in the following form:

$$[M]\left\{\ddot{X}\right\} + [C]\left\{\dot{X}\right\} + [K]\{X\} = \{F\} \tag{1}$$

$$\{F\} = [M']\, e\ddot{X}_g \tag{2}$$

where X, $\dot{X}$ and $\ddot{X}$ are $(n + N_{TLCD})$- dimensional vectors which include n components of horizontal displacement, velocity and acceleration of the structure with respect to the ground and N_{TLCD} components of vertical displacement, velocity and acceleration of liquid in vertical part of the tube, $e^T = [-1, -1, \ldots, -1]_{1\times(n+N_{TLCD})}$ is ground acceleration-mass transformation vector. [M], [C] and [K] are, respectively, the $(n + N_{TLCD}) \times (n + N_{TLCD})$ mass, damping and stiffness matrices which for the case of installing $TLCD$s at the top floor in parallel configuration, can be determined from Eqs (3) - (5).

Also [M′] is a matrix that includes the mass of floors and $TLCD$s, as defined in Eq. (6), and is used for determination of the external force {**F**}.

$$M = \begin{pmatrix} m_1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & m_2 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & m_{n-1} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & m_n + \sum_{i=1}^{N_{TLCD}} m_{di} & (\alpha m_d)_1 & (\alpha m_d)_2 & \cdots & (\alpha m_d)_{N_{TLCD}-1} & (\alpha m_d)_{N_{TLCD}} \\ 0 & 0 & \cdots & 0 & (\alpha m_d)_1 & m_{d1} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & (\alpha m_d)_2 & 0 & m_{d2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & (\alpha m_d)_{N_{TLCD}-1} & 0 & 0 & \cdots & m_{dN_{TLCD}-1} & 0 \\ 0 & 0 & \cdots & 0 & (\alpha m_d)_{N_{TLCD}} & 0 & 0 & \cdots & 0 & m_{dN_{TLCD}} \end{pmatrix} \tag{3}$$

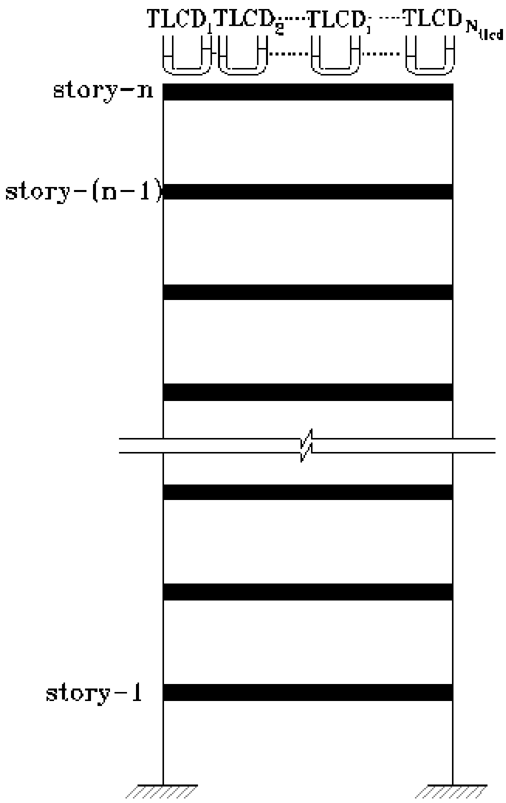

Fig. 1. Multiple tuned liquid column dampers ($MTLCD$s) - Multi degree of freedom (MDOF) structure model

$$K = \begin{pmatrix} k_1 + k_2 & -k_2 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & k_{n-1} + k_n & -k_n & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & -k_n & k_n & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & k_{d_1} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & k_{d_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & k_{d_{N_{TLCD-1}}} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & k_{d_{N_{TLCD}}} \end{pmatrix} \tag{4}$$

$$C = \begin{pmatrix} c_1 + c_2 & -c_2 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -c_2 & c_3 + c_4 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & c_{n-1} + c_n & -c_n & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & -c_n & c_n & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & c_{d_1} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & c_{d_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & c_{d_{N_{TLCD-1}}} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & c_{d_{N_{TLCD}}} \end{pmatrix} \tag{5}$$

$$
M' = \begin{pmatrix}
m_1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & m_2 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & m_{n-1} & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & m_n + \sum_{i=1}^{N_{TLCD}} m_{d_i} & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & (\alpha m_d)_1 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & (\alpha m_d)_2 & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & (\alpha m_d)_{N_{TLCD}-1} & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & (\alpha m_d)_{N_{TLCD}}
\end{pmatrix} \tag{6}
$$

where m_{di}, k_{di} and c_{di} = the mass, stiffness and damping of the i^{th} $TLCD$ that are defined as:

$$m_{d_i} = \rho L_{f_i} A_{f_i} \tag{7}$$

$$c_{d_i} = \frac{1}{2} \rho A_{f_i} \xi_{f_i} \left| \dot{X}_{f_i}(t) \right| \tag{8}$$

$$k_{d_i} = 2 \rho A_{f_i} g \tag{9}$$

where $\dot{X}_{f_i}$ = velocity of liquid in i^{th} column in vertical direction, ξ_f = head loss coefficient, w_d = natural frequency of the i^{th} $TLCD$, ρ = liquid density, L_{fi} and A_{fi} = total length of liquid in the container and cross-sectional area of i^{th} tube, respectively. Also α_i = length ratio = B_{fi} / L_{fi} where B_{fi} is the horizontal portion of liquid in the i^{th} $TLCD$ container. To guarantee that the liquid will retain in horizontal portion of U-tube, the following equation should be satisfied as a constraint for each $TLCD$ [6]:

$$2 \dot{\mathbf{X}}_{f_i} + B_{f_i} \leq L_{f_i} \tag{10}$$

The equation of motion of the structure can be solved using any numerical methods, where in this paper, Wilson's-θ numerical procedure has been used.

3 Optimal design of $MTLCD$s

In this paper, following the method proposed by Mohebbi et al. [23] for optimal design of multiple tuned mass dampers (MTMDs), an effective method has been proposed to design optimal $MTLCD$s for multi-degree-of-freedom linear structures subjected to any desired excitation. The method is based on defining an optimization problem that takes the parameters of $TLCD$s as design variables and determines them to meet certain design objectives. Taking $v_i(i = 1, 2, \ldots, N_{TLCD})$ as the vector representing the parameters of i^{th} $TLCD$, the optimization problem to design optimal $MTLCD$s can be defined as follows:

$$
\begin{aligned}
&\text{Find: } v_1, v_2, \ldots\ldots, v_{N_TLCD} \\
&\text{Minimize: } F(R) = f(r_1, r_2, \ldots, r_p) \\
&\text{Subject to: } g_i(r_1, r_2, \ldots, r_p) \leq 0.0 \;\; i = 1,2,\ldots,q \\
&h_j(r_1, r_2, \ldots, r_p) = 0.0 \;\; j = 1, 2, \ldots, r
\end{aligned} \tag{11}
$$

where R is a vector used to define the objective function and constraints of the optimization problem; g_i and h_i are inequality and equality constraints, respectively, where q and r show the number of inequality and equality constraints. The objective function in the optimization problem can be chosen to design optimal $MTLCD$s to pursue different design objectives, that is, different response of structure including maximum displacement, acceleration or internal force can be considered in the objective function for minimization. The parameters of $TLCD$s will then be determined in a way that they can optimally meet these design objectives and satisfy the required practical limitations. The design objective for $MTLCD$s can be varied depending on their application purposes. In this research, in order to maintain safety and serviceability of the structure and ensure occupant's comfort criteria, designing $MTLCD$s has been studied for the two cases of: a) minimizing the maximum displacement and b) minimizing the maximum acceleration of structure, as explained below:

3.1 Case (a): Optimal $MTLCD$s based on minimizing the maximum displacement

In most of the previous researches on designing control systems, the amount of reduction in the maximum displacement of the structure has been used to assess the effectiveness of the structural control system. In this paper, too, minimization of the maximum displacement of structure has been considered as the objective function in case (a). Assuming a constant value for length ratio, α, the mass, length and head loss of each $TLCD$ have been considered as the variables of the optimization problem. Using the proposed method, these variables will be optimally chosen to minimize the maximum displacement, X_{max}, of the structure as the objective function. At the same time, some limitations on $TLCD$'s parameters will be applied as constraints of optimization problem. For this case the optimization problem defined in Eq. (11) takes the form of:

$$Find: \;\; m_{d_1}, L_{d_1}, \xi_{d_1}, \ldots, m_{d_{N_{TLCD}}}, L_{d_{N_{TLCD}}}, \xi_{d_{N_{TLCD}}} \tag{12}$$

$$
\begin{aligned}
\text{Minimize: } & X_{max} = max(|x_k(i)|), \\
& k = 1, 2, \ldots, k_{max}, \;\; i = 1, 2, \ldots, n
\end{aligned} \tag{13}
$$

$$\text{Subject to: } 0 < m_{d_i} < m_{d_{\max}} \;\; i = 1, 2, \ldots, N_{TLCD} \tag{14}$$

$$0 < L_{d_i} < L_{d_{\max}} \;\; i = 1, 2, \ldots, N_{TLCD} \tag{15}$$

$$0 < \xi_{d_i} < \xi_{d_{\max}} \;\; i = 1, 2, \ldots, N_{TLCD} \tag{16}$$

where m_{dmax}, L_{dmax} and ξ_{dmax} are the upper limit of mass, length and head loss of $TLCD$s and k_{max} is the total number of time steps. By assuming a specified value for the total mass ratio, μ, and uniform distribution for $TLCD$s mass, the mass of each $TLCD$ can be considered as:

$$m_{d_1} = m_{d_2} = \ldots = m_{d_{N_{TLCD}}} = \mu.\frac{m_{tot}}{N_{TLCD}} \quad (17)$$

where m_{tot} is the total mass of the structure. Hence in this case the variables of optimization problem have been the length and head loss of $TLCD$s.

3.2 Case (b): Optimal MTLCDs based on minimizing the maximum acceleration

For this case, to improve serviceability of the structure for occupant's comfort criterion, minimization of maximum acceleration of structure, $\ddot{\mathbf{X}}_{\max}$, has been considered as the objective function in designing optimal $MTLCD$s. The optimization problem in this case is defined as follows:

$$\text{Find: } m_{d_1}, L_{d_1}, \xi_{d_1}, \ldots, m_{d_{N_{TLCD}}}, L_{d_{N_{TLCD}}}, \xi_{d_{N_{TLCD}}} \quad (18)$$

$$\text{Minimize: } \ddot{\mathbf{X}}_{\max} = max\left(\left|\ddot{\mathrm{X}}_{\mathrm{k}}(\mathrm{i})\right|\right), \mathrm{k} = 1,2,\ldots,k_{\max}, \mathrm{i} = 1,2,\ldots,n \quad (19)$$

$$\text{Subject to: } 0 < m_{d_i} < m_{d_{\max}} \quad i = 1, 2, \ldots, N_{TLCD} \quad (20)$$

$$0 < L_{d_i} < L_{d_{\max}} \quad i = 1, 2, \ldots, N_{TLCD} \quad (21)$$

$$0 < \xi_{d_i} < \xi_{d_{\max}} \quad i = 1, 2, \ldots, N_{TLCD} \quad (22)$$

The optimization problem for designing optimal $MTLCD$s for both cases has a large number of variables, consequently, using traditional optimization techniques such as gradient based methods would be extremely complicated and a powerful algorithm is required to solve the problem. In this paper, Genetic Algorithm (GA) [27], which has been found to be an effective optimization technique especially for problems with large number of variables, has been used for solving the optimization problem. Fig. 2 demonstrates the flowchart of the GA-based design procedure used for optimal design of $MTLCD$s. More details on solving the optimization problem and the design procedure are given in section 5. Also a brief explanation of GA has been presented in the following section.

4 Genetic algorithms (GAs)

An optimization problem is defined as finding the best solutions for design variables that make the value of an objective function maximum or minimum. To solve an optimization problem using traditional optimization method, the domain is searched using the gradient of the objective function. The limitation of this method arises in problems such as designing optimal $MTLCD$s, the case studied in this paper, where the parameters of the objective function and the constraints of the optimization problem are not continuous and it is not possible to calculate the gradient of the functions. Genetic algorithm (GA) is an effective computational method for solving linear and nonlinear optimization problems with large number of variables. In GAs, the variables are represented in binary or real value format. In this paper the real-valued coding method has been used for representing the variables which has some advantages such as simpler programming, less memory required, no need to convert chromosomes and greater freedom to use different genetic operators over binary versions [28].

There are three genetic algorithm operators including selection, cross over and mutation. In GA, in each generation, using selection operator a set of chromosomes is selected for mating based on their relative fitness. In this paper stochastic universal sampling (SUS) method [29] is used for selecting the individuals for reproduction according to their fitness in the current population as:

$$P(\mathbf{x}_i) = \frac{F(\mathbf{x}_i)}{\sum_{i=1}^{N_{ind}} F(\mathbf{x}_i)}, i = 1,2,...,N_{ind} \quad (23)$$

where $\mathrm{F}(\mathbf{x}_i)$ = fitness of chromosome $\mathbf{x}_i$, $\mathrm{P}(\mathbf{x}_i)$ = probability of selection of x_i and N_{ind} = number of individuals.

The basic operator for producing new individuals in the GA is cross over. Cross over produces new individuals that have some parts of both parents' genetic materials. For real-coded GA, several types of recombination such as intermediate, line and uniform recombination have been proposed [28]. In this paper, intermediate recombination method [30] has been used for crossover, where values of newborns genes are determined as:

$$O = P_1 + \beta(P_2 - P_1) \quad (24)$$

where O = the value of newborn gene, P_1 and P_2 are the parent chromosomes genes and β is a scaling factor chosen randomly over [-0.25, 1.25] interval typically .This method uses a new β for each pair of parent genes. The main purpose of using mutation operator in GAs is providing a guarantee that the probability of searching any given string will never be zero. In this paper, the method proposed by Mühlenbein and Schlierkamp-Voosen [30] has been used for mutation.

To maintain the size of the original population, the new chromosomes have to be reinserted into the old population. An insertion rate,η, determines the number of newly produced chromosomes inserted in the old population according to:

$$N_{ins} = N_{new} \times \eta \quad (25)$$

where N_{ins} =number of inserted newborn and N_{new} = number of newborns. Therefore, N_{elites} of the best chromosomes in the current population advance to the next generation without modification as:

$$N_{elites} = N_{ind} - N_{ins} \quad (26)$$

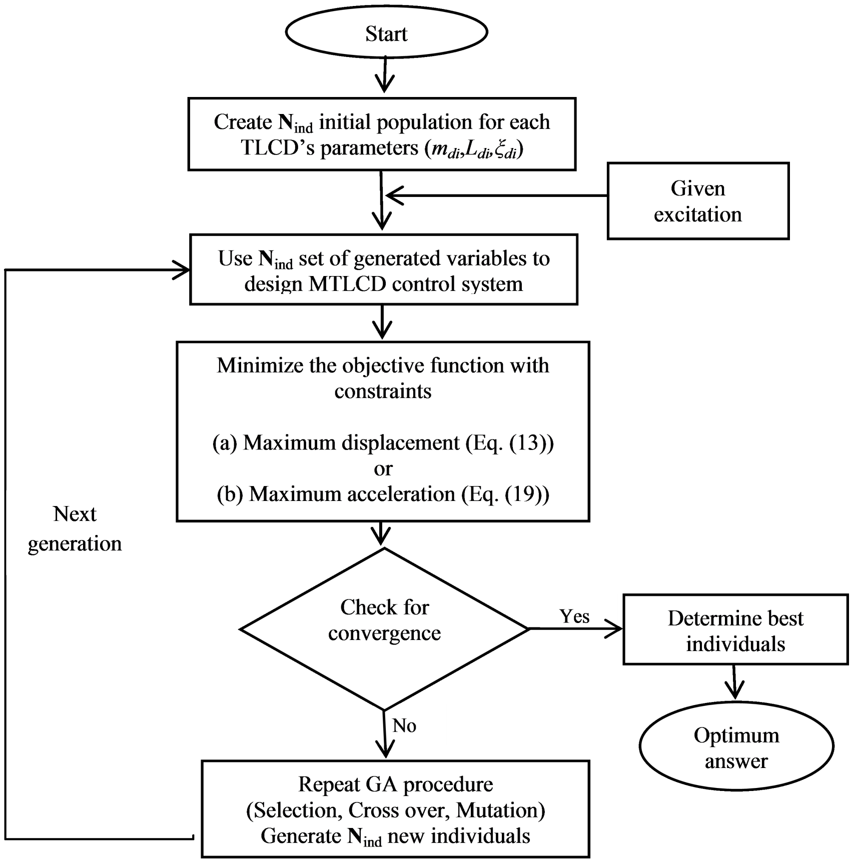

Fig. 2. Flowchart of the GA-based design procedure for optimal design of $MTLCD$s

where N_{ind} = number of individuals in each generation. The rest of the chromosomes in the population are replaced by N_{ins} inserted newborns.

5 Numerical example

To illustrate the procedure of the proposed method for designing optimal $MTLCD$s as well as assessing the effect of different factors such as mass ratio, $TLCD$ numbers, design criteria and input excitation on $MTLCD$s performance, a ten-storey shear frame with uniform properties for all storeys and linear material behaviour for the structure and $TLCD$s has been considered. The properties of each storey including mass, stiffness and damping coefficient has been assumed to be $m = 360$ tons, $k = 650$ MN/m and $c = 6.2$ MN.s/m, respectively.

For different values of $MTLCD$'s mass ratio and $TLCD$ numbers, the structure has been subjected to a filtered white noise excitation, W(t), as shown in Fig. 3 and optimal $MTLCD$s have been designed to minimize the maximum displacement and acceleration of the structure. The input excitation with a peak ground acceleration (PGA) = 0.475 g used to design $MTLCD$s has been simulated in the optimization procedure by passing a Gaussian White Noise process through Kanai-Tajimi filter (Kanai, 1961; Tajimi, 1960) [23]. While $MTLCD$s has been designed for the white noise excitation their performance has also been tested under a number of real earthquakes including both near and far-field earthquakes. The maximum response of the uncontrolled structure under $W(t)$ excitation has been reported in Table 1.

5.1 Designing optimal $MTLCD$s for $N_{TLCD} = 5$ and $\mu = 2\%$.

Here as an example to illustrate the proposed method, the GA-based design procedure for designing optimal $MTLCD$s is presented using five $TLCD$s ($N_{TLCD} = 5$) located in parallel configuration on the top floor of the structure where the total mass ratio has been assumed as $\mu = 2\%$ and uniform distribution for $TLCD$ masses has been considered. Also based on suggestion of Sadek et al. [6], the length ratio, $\alpha = 0.8$ has been selected. In this case, there are 10 variables which are the length and head loss of $TLCD$s and should be determined through solving the optimization problem using GA. The details and parameters used in the GA analysis have been given in appendix A.

Here to better illustrate the details of the method, the design procedure for minimization of the maximum displacement of structure according to case (a) has been explained. For determining the optimum values of variables using GA, following the GA-based design algorithm shown in Fig. 2, first an initial population consisting 25 randomly generated vectors (chromosomes) of $MTLCD$s parameters (length and head loss) with each chromosome having 10 genes has been generated. The $MTLCD$s have been designed using each set of generated parameters and the maximum controlled displacement of the frame has been obtained and saved. Then, the value of the objective function has been calculated for each set of generated $MTLCD$ parameters. By monitoring the response, the fittest individuals in each gen-

Tab. 1. Maximum response of uncontrolled structure under W(t) excitation

Storey No.	Disp.(cm)	Acc.(cm/s^2)	Drift(cm)	Disp.RMS(cm)	Acc. RMS(cm/s^2)
1	2.17	345.35	2.17	0.64	101.66
2	4.3	571.6	2.13	1.27	157.16
3	6.28	708.65	1.99	1.86	184.12
4	8.01	708.89	1.76	2.41	192.85
5	9.5	749.88	1.64	2.91	194.08
6	10.75	719.11	1.57	3.34	195.23
7	11.86	713.77	1.38	3.70	198.43
8	12.88	713.73	1.12	3.98	205.41
9	13.63	730.84	0.79	4.17	217.73
10	14.03	802.68	0.41	4.26	229.00

RMS = root-mean-square, Disp. = displacement, Acc. = acceleration

eration have been identified and N_{elites} of them have been kept to move on to the next generation. Iteratively, the populations have been modified by GA and new generations have been created until convergence has finally been attained. 4 different runs of GA with four different initial populations have been performed for the considered example to ensure the accuracy of the optimization procedure. In each generation, for all individuals, the maximum displacement of the controlled structure has been divided to maximum uncontrolled displacement and the best value of the objective function (normalized maximum displacement) has been shown in Fig. 4(a) during 400 generations of GA for four different runs. It is clear that all runs have ended to the same result while their convergence speeds have been different.

Also in Fig. 4(b) the normalized maximum displacement of the structure for each individual of GA at first and final generations has been reported. Based on the results it can be said that at final generation most of the individuals have the same value of objective function, which shows the simplicity and desirable convergence of the optimization procedure used for optimal design of $MTLCD$s.

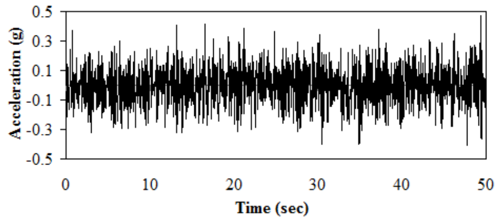

Fig. 3. Filtered White noise excitation, W(t), with PGA = 0.475 g

The optimum answer obtained at the final stage has been $X_{max} = 7.46$ cm while the maximum displacement of uncontrolled structure has been 14.03 cm (see Table 1), hence about 46.7% reduction in the maximum displacement of the frame has been achieved. The final results for the optimum values of length and head loss of 5 $TLCD$s have been reported in Table 2 which shows at the optimum point, the parameters of $TLCD$s are different.

Following the same procedure, for $N_{TLCD} = 5$ and $\mu = 2\%$, optimal $MTLCD$s have also been designed for minimizing the maximum acceleration of structure according to case (b). Figs. 5(a) and 5(b) show the maximum displacement and acceleration of the uncontrolled and controlled structures when using both cases (a) and (b) as objective functions. From the results it has been found that for case (a), i.e. minimization of the maximum displacement as the objective function, the maximum displacement and acceleration of the frame have been decreased by about 46.7% and 5.5%, respectively, while the corresponding reductions have been 36.8% and 19.6% for case (b) in which the maximum acceleration has been considered as the objective function. Hence, it can be concluded that when it is desired to reduce a specific response of structure, minimization of that response should be considered as the objective function. Also it has been found that for both cases the maximum displacement has been reduced more, therefore, $MTLCD$s has been more effective in reducing the maximum displacement.

5.2 Designing optimal $MTLCD$s for different mass ratio

Following the same procedure explained for $N_{TLCD} = 5$ and $\mu = 2\%$, optimal $MTLCD$s have been designed for different values of mass ratio for both cases of (a) and (b) under W(t) , excitation. In Fig. 6 the reductions in maximum displacement and acceleration of the structure for both objective functions have been reported. Results show improvement in the effectiveness of $MTLCD$s by increasing the mass ratio of $MTLCD$s for both cases. Also it is clear that for different values of mass ratio the reduction in maximum displacement has been more than reduction in maximum acceleration even for the case (b) where the maximum acceleration has been used as the objective function. These conclusions in designing optimal $MTLCD$s are similar to that of obtained for MTMDs in previous researches [23, 31].

5.3 Performance of optimal $MTLCD$s under real excitations

In the design procedure applied in this paper, the optimal $TLCD$ parameters have been determined based on minimization of the maximum displacement or acceleration of the struc-

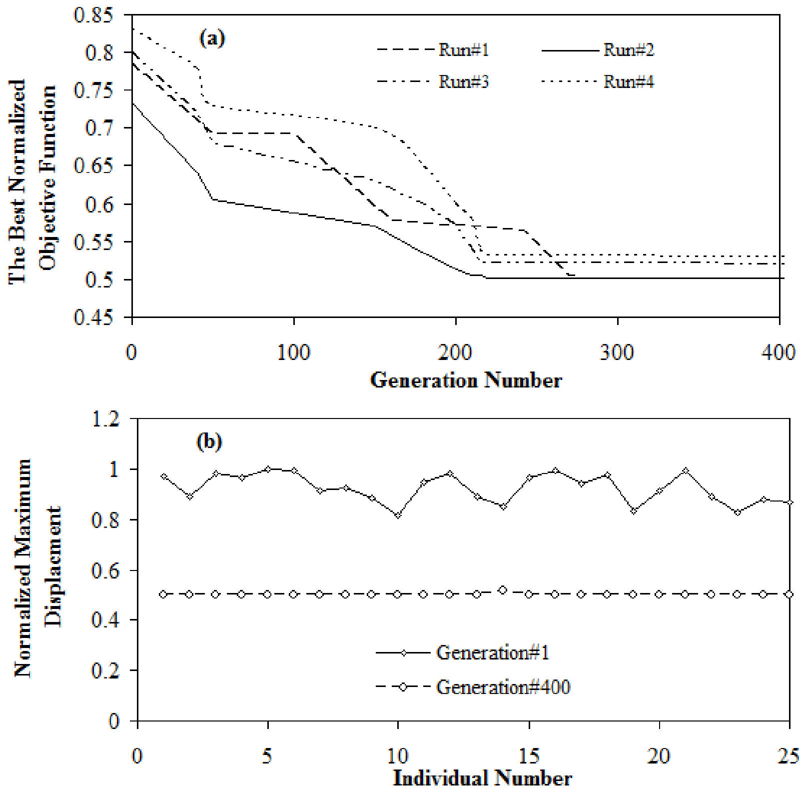

Fig. 4. (a) The best value of objective function (Normalized maximum displacement) for different runs in GA; (b) Normalized objective function value at 1st and final generation for each individual.

Tab. 2. Optimum length and head loss of $TLCD$s for $N_{TLCD} = 5$ and $\mu = 2\%$.

$TLCD$	m_d (tons)	L_{opt} (m)	ξ_{opt}
$TLCD_1$	14.4	0.707	0.053
$TLCD_2$	14.4	0.59	0.267
$TLCD_3$	14.4	0. 437	0.055
$TLCD_4$	14.4	0.527	0.051
$TLCD_5$	14.4	0.441	0.056

ture under a filtered white noise excitation. To assess the effectiveness of the designed optimal $MTLCD$s in reducing the response of structure under other excitations, for different values of $MTLCD$'s mass ratio as well as both objective functions, the uncontrolled and controlled structures have been subjected to El-Centro (1940, PGA = 0.34 g), and Hachinohe (1968, PGA = 0.23 g) records as far-field earthquakes as well as Northridge (1994, PGA = 0.84 g) and Kobe (1995, PGA = 0.83 g) records as near-field earthquakes, respectively. While the maximum response of uncontrolled structure under these real earthquakes has been reported in Table 3, Figs. 7 and 8 show the reductions achieved in the maximum displacements and acceleration of the structure using the designed $MTLCD$ control mechanisms for both objective functions under the test earthquakes. Results show that the effectiveness of $MTLCD$s depends on the characteristics of earthquake which for this case of study the best performance has been achieved under the El-Centro (1940) excitation as a far-field earthquake while the worst performance has been under the Northridge (1994) excitation which is a near-field and much sever earthquake. For example, the $MTLCD$ system with $\mu = 2\%$, and designed based on maximum displacement minimization objective function has decreased the maximum displacement and acceleration of the structure by about 23% and 17%, respectively, when subjected to the El-Centro excitation. While these reductions in the response have been different for four considered earthquake records, in all cases increasing the mass ratio of $MTLCD$s has led to more reductions in the response of the structure under testing excitations. The performance of the $MTLCD$s under the test earthquakes has also been influenced by their design criterion. $MTLCD$s designed based on maximum acceleration minimization objective function have been more effective in reducing the peak acceleration of the structure under different excitations, where for $\mu = 2\%$ and under El-Centro excitation, 25% reduction in the peak acceleration of the structure has been achieved.

According to the results obtained from testing the controlled structure under real earthquakes it can be said, choosing an appropriate design record and a proper design criterion can result in designing optimal $MTLCD$s which can effectively reduce the response of the structure using proposed method. Hence, in order to improve the effectiveness of $MTLCD$s in a special area, it is suggested that the design earthquake of that area be used as design record of $MTLCD$s. However, to generalize this conclusion and suggest a useful procedure for selecting design record, an extensive research and numerical analysis should be carried out.

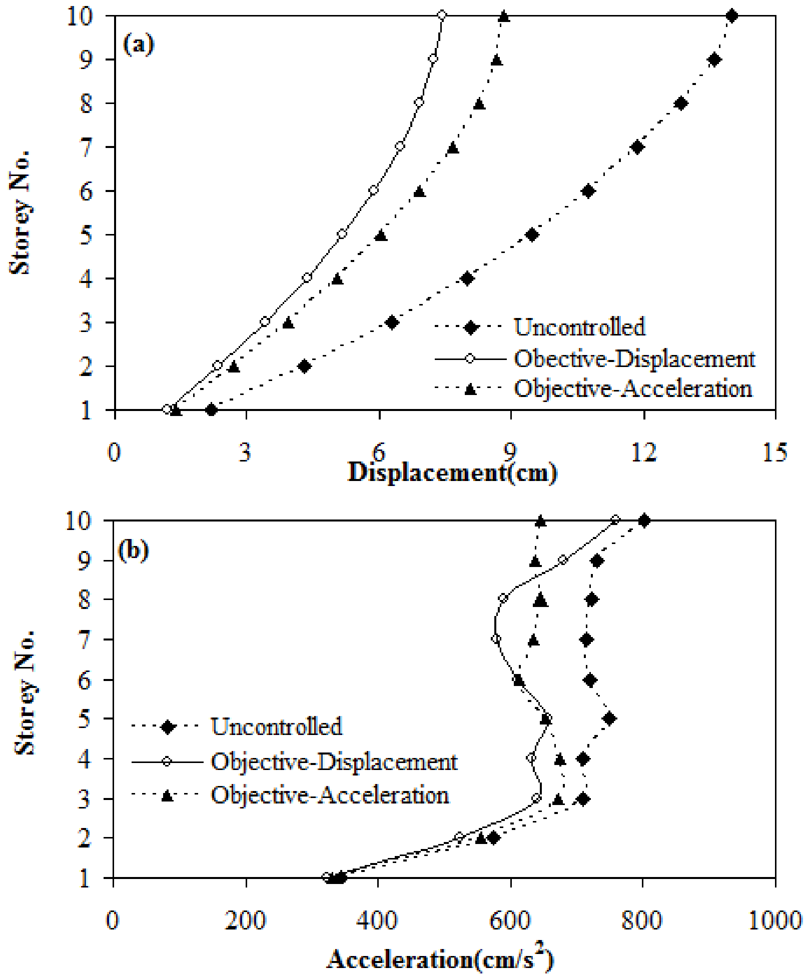

Fig. 5. Maximum (a) displacement and (b) acceleration of uncontrolled and controlled structures when minimizing the maximum displacement and acceleration as objective function for $\mu = 2\%$ and $N_{TLCD} = 5$

5.4 Designing optimal $MTLCD$s for different number of $TLCD$s

In this part, the effect of $TLCD$'s number on performance of $MTLCD$s in mitigating the response of the structure has been assessed. To this end, for different number of $TLCD$s, optimal $MTLCD$s have been designed to minimize the maximum displacement of the structure under the W(t) excitation. The maximum displacement, drift, acceleration and root-mean-square (RMS) of displacement and acceleration of controlled frame have been divided to the corresponding maximum uncontrolled responses and has been shown in Fig. 9 for different numbers of $TLCD$s. As can be seen, using 5 $TLCD$s instead of one $TLCD$ has caused about 5% more reductions in the response of the structure, while for other numbers of $TLCD$s the reductions have almost been the same. According to the results, it can be said that for this case of study the performance of $MTLCD$s has not been affected significantly by increasing the number of $TLCD$s. Though there is no significant difference between the effectiveness of $MTLCD$s and single $TLCD$, it is clear that increasing the number of $TLCD$s has some advantages such as need to smaller mass and required space for installation [6].

5.5 Comparing the performance of MTMDs and $MTLCD$s:

To overcome the shortcomings of single tuned mass damper (TMD), multiple tuned mass dampers (MTMDs) has been proposed and studied extensively in researches [17, 23, 31]. It has been shown that the sensitivity of MTMDs to uncertainty of structural dynamic parameters is less than a single TMD, also the performance of MTMDs depends on the total number of dampers, damping ratio, frequency range selected for designing optimal MTMDs, the distribution of TMDs on the floors and the stroke length of mass dampers [23]. While MTMDs and $MTLCD$s control systems are similar in main concept, they have different construction details which lead to difference in application regarding the simplicity and cost. In this paper, it has been decided to compare the performance of $MTLCD$s and MTMDs in mitigating the response of structures under seismic loads. For this purpose, for the ten-storey shear frame studied in this paper, the maximum response of controlled structure using $MTLCD$s and MTMDs [23] under design and testing records for different values of mass ratio and $N_{TLCD} = 5$ has been compared in Figs. 10 and 11 for both objective function cases. According to the results it can be concluded that (i) under the design record, $W(t)$, $MTLCD$s and MTMDs have had approximately the same reduction in maximum displacement when the case of minimiz-

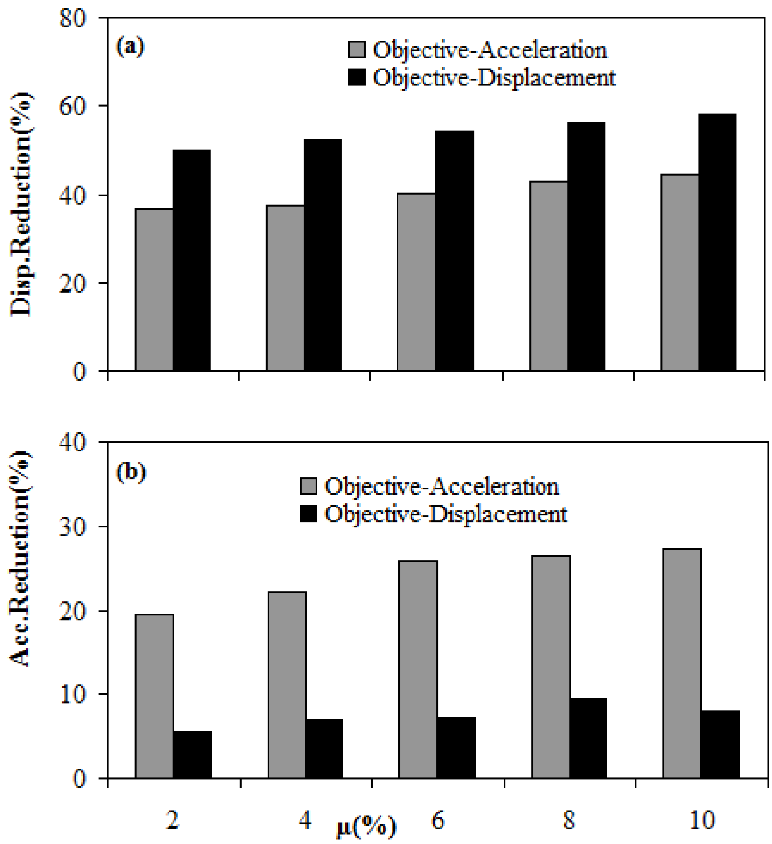

Fig. 6. Reductions in the maximum (a) displacement; and (b) acceleration of structure subjected to $W(t)$ excitation versus different values of $MTLCD$'s mass ratio for both objective functions when $N_{TLCD} = 5$.

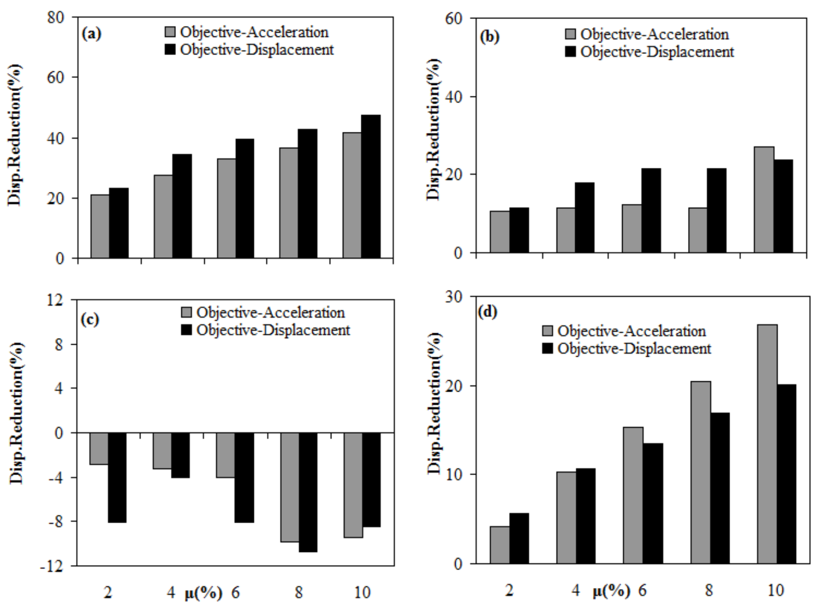

Fig. 7. Reductions in maximum displacements for both objective functions under (a) El-Centro (1940); (b) Hachinohe(1968); (c) Northridge(1994); and (d) Kobe (1995) testing excitations versus different values of $MTLCD$s mass ration when $N_{TLCD} = 5$.

Tab. 3. Maximum response of uncontrolled structure under real earthquakes

Earthquake	Disp.(cm)	Acc.(cm/s^2)	Drift(cm)	Disp.RMS(cm)	Acc. RMS(cm/s^2)
Elcentro	18.81	974.82	3.05	3.4	157.37
Hachinohe	12.69	716.2	1.93	4.98	208.54
Northridge	27.09	1756.87	5.53	4.14	194.37
Kobe	52.58	2718.17	7.31	6.06	270.54

RMS = root-mean-square, Disp. = displacement, Acc. = acceleration

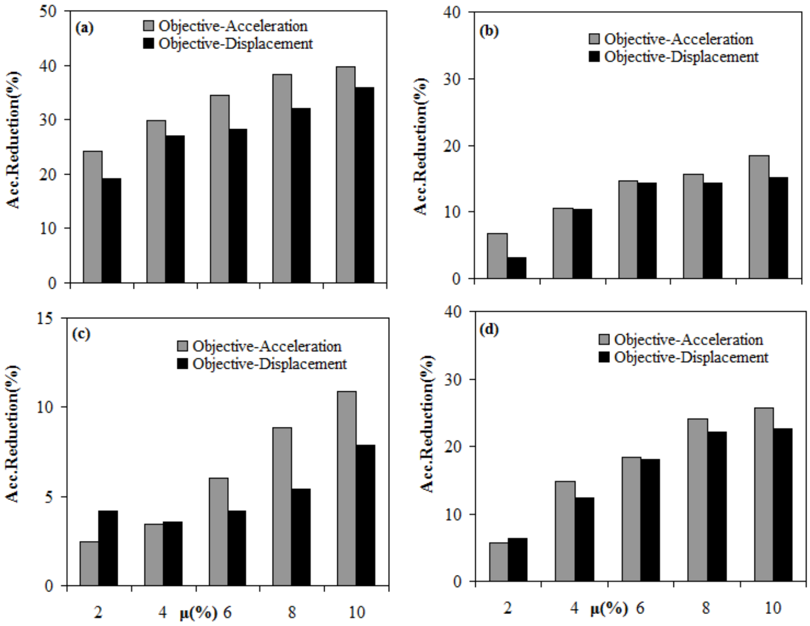

Fig. 8. Reductions in maximum acceleration for both objective functions under (a) El-Centro (1940); (b) Hachinohe(1968); (c) Northridge(1994); and (d) Kobe (1995) testing excitations versus different values of $MTLCD$s mass ration when $N_{TLCD} = 5$.

ing the maximum displacement has been considered as the objective function. A similar result has been obtained for the maximum acceleration for the case of selecting minimization of the maximum acceleration in the design procedure. For some mass ratios, there is a slight difference in maximum acceleration reduction in case (a) and maximum displacement reduction in case (b) between $MTLCD$s and MTMDs under W(t). However overall, it can be concluded that for the same objective function under the design record, $MTLCD$s and MTMDs have worked similarly in mitigating the response of structure, especially, when minimization of that response has been chosen as the objective function. This conclusion is similar to results of previous researches [6]. (ii) under testing earthquakes, though for most values of mass ratio $MTLCD$s and MTMDs have had approximately similar performance in reducing the maximum response such as maximum acceleration under all earthquakes, but for some values of mass ratio, MTMDs has been slightly better (at maximum value about 20%) and has reduced the maximum response especially maximum displacement more. For example, for $\mu = 8\%$ under Kobe and Hachinohe excitations, about 20% more reduction has been obtained by using MTMDs. Though the performance of the $MTLCD$s has been similar to MTMDs for most values of mass ratios, but $MTLCD$s have some advantages over MTMDs such as no need to large stroke lengths and easy tuning of the frequency by adjusting liquid column length. Also $TLCD$s are easily capable of dissipating energy in two directions simultaneously by using bi-directional U-tube [6]. This balanced energy dissipation capability of this device can facilitate controlling the response of a structure in two directions against strong seismic excitations which may cause damaging deformations or even nonlinear behavior in structural components. Note that the design methodology proposed in this paper can also be extended for nonlinear structures where other important indices such as minimizing the maximum accumulated hysteretic energy [32] can be applied as the objective function in the design process of $TLCD$s which has been planned for subsequent studies by the authors.

6 Conclusions

In this paper an effective method has been proposed for optimal design of multiple tuned liquid column dampers ($MTLCD$s) where the parameters of $TLCD$s including the length and head loss of each $TLCD$ have been considered as design variables and the optimum values have been determined through solving an optimization problem. For both cases of (a) minimizing the maximum displacement and (b) minimizing the maximum ac-

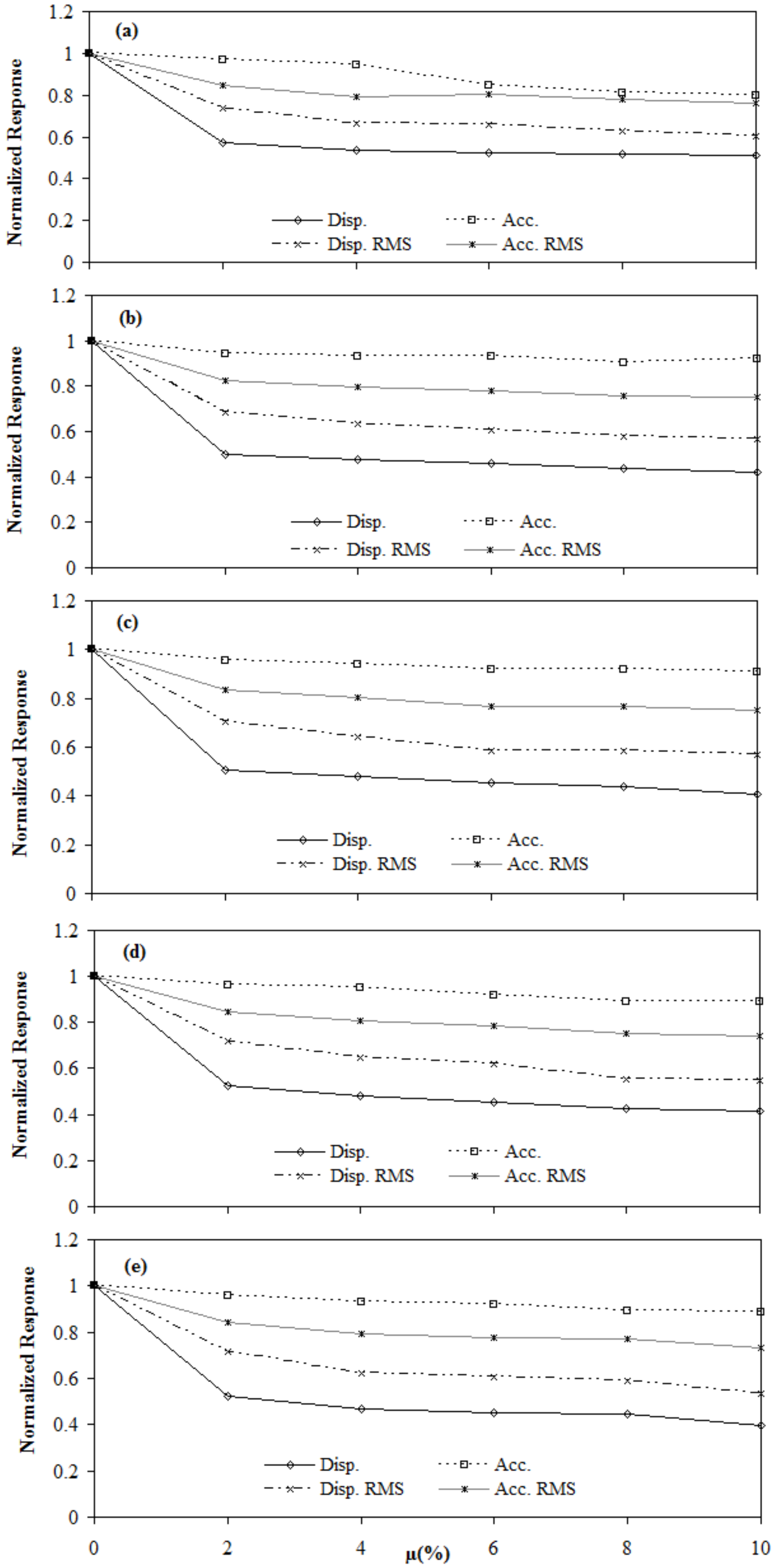

Fig. 9. Normalized response of controlled structure when minimizing the maximum displacement as objective functions for (a) $N_{TLCD} = 1$; (b) $N_{TLCD} = 5$; (c) $N_{TLCD} = 10$; (d) $N_{TLCD} = 15$; and (e) $N_{TLCD} = 20$ number of $TLCD$s versus different values of mass ratio under W(t) excitation.

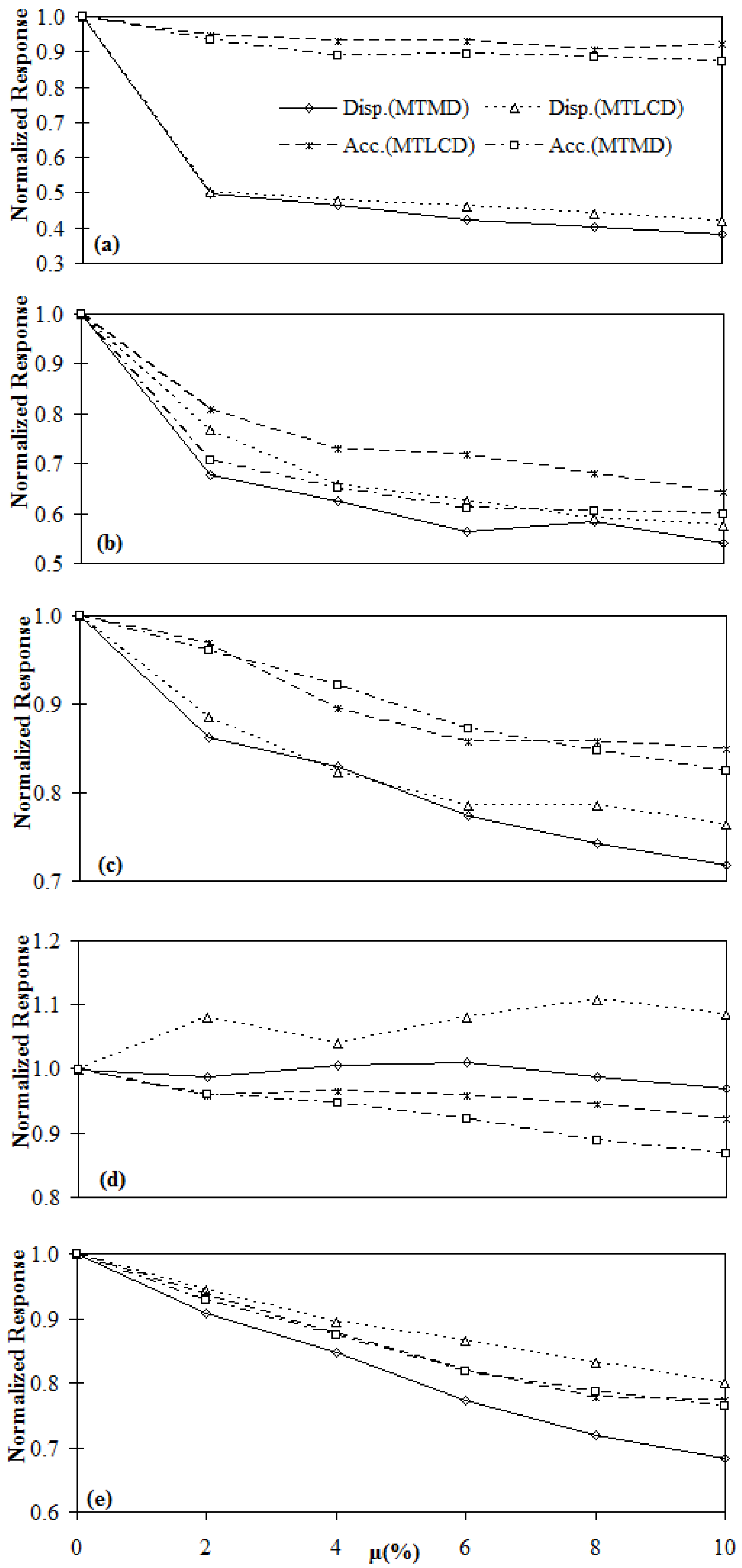

Fig. 10. Normalized response of controlled structure using *MTLCD*s and MTMDs when minimizing the maximum displacement as objective functions under (a) W(t); (b) El-Centro (1940); (c) Hachinohe(1968); (d) Northridge(1994); and (e) Kobe (1995) excitations versus different values of mass ratios for $N_{TLCD} = 5$.

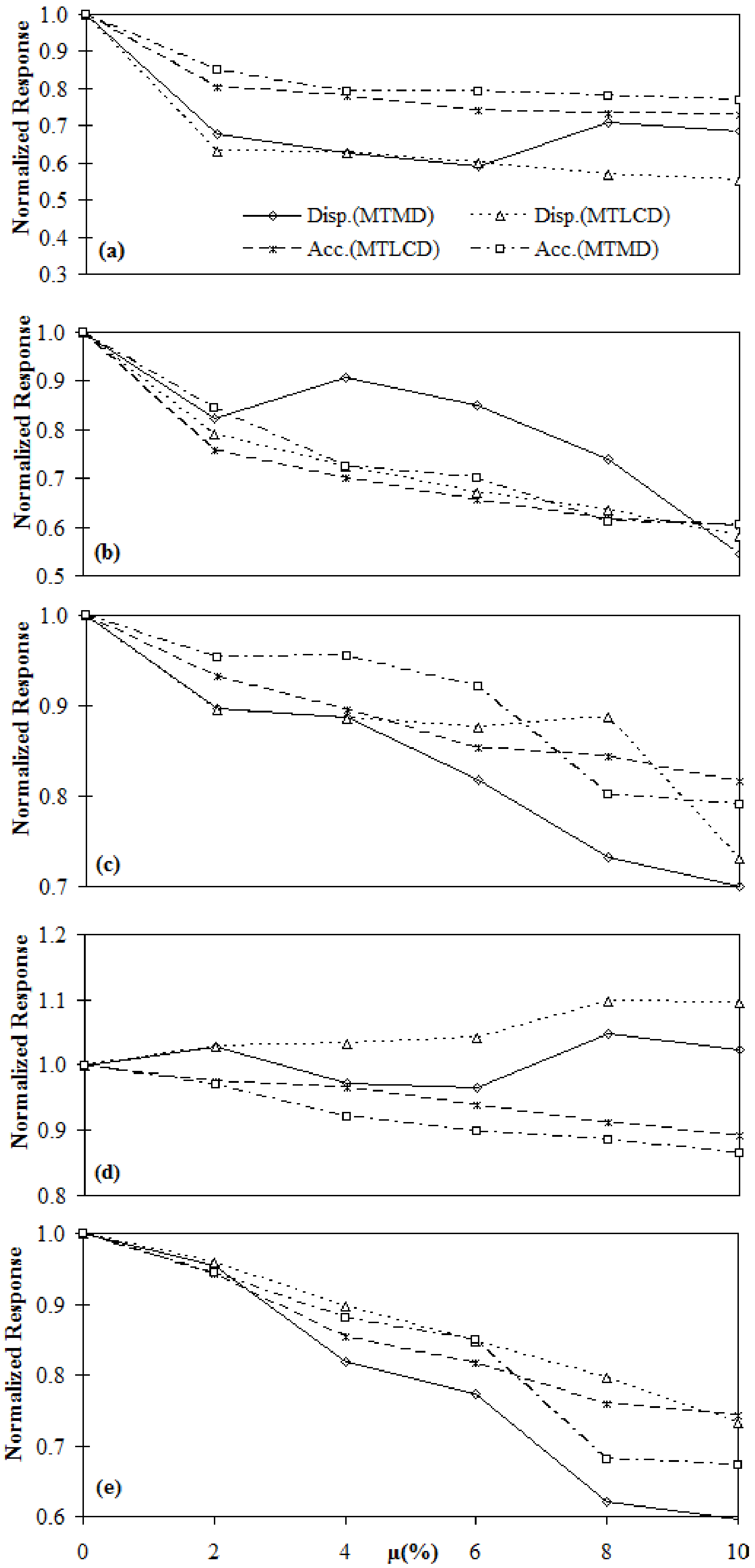

Fig. 11. Fig. 11. Normalized response of controlled structure using *MTLCD*s and MTMDs when minimizing the maximum acceleration as objective functions under (a) W(t); (b) El-Centro (1940); (c) Hachinohe(1968); (d) Northridge(1994); and (e) Kobe (1995) excitations versus different values of mass ratios for $N_{TLCD} = 5$.

celeration of structure, optimal $MTLCD$s have been designed. Since the optimization problem especially for larger number of $TLCD$s has a large number of variables, Genetic algorithm (GA) has been used for solving the optimization problem. To illustrate the procedure of design method and assess the effectiveness of $MTLCD$s in reducing the response of structures under earthquake excitations, a ten-storey linear shear frame subjected to a filtered white noise excitation has been considered and for different values of $MTLCD$'s mass ratio and $TLCD$s number optimal $MTLCD$s have been designed for both cases of objective function. According to the results it can be said that the proposed method for designing optimal $MTLCD$s has been effective regarding the simplicity and convergence behavior of the method. Also results show that $MTLCD$s has been more effective in reducing the maximum displacement than maximum acceleration even for the case of selecting minimization of maximum acceleration as the objective function. The results of designing optimal $MTLCD$s for different mass ratio and $TLCD$ numbers show that increasing the mass ratio leads to improvement in the performance of $MTLCD$s while in this paper, for the domain studied for $TLCD$ numbers, effectiveness of $MTLCD$s has not been affected significantly by $TLCD$ numbers. To evaluate the performance of $MTLCD$s under real earthquakes, optimal $MTLCD$s has been subjected to both Far-field and near-field real earthquakes which are different in frequency content and peak ground acceleration (PGA) from the design white noise excitation and it has been found that the efficiency of $MTLCD$s depends on the characteristics of excitations. Also comparing the performance of $MTLCD$s and multiple tuned mass dampers (MTMDs) for both cases (a) and (b) under design record and testing excitations has shown that under design record there is no significant difference in performance of these control systems, especially in reducing the response that has been included in objective function while under testing records for some values of mass ratio, MTMDs has slightly worked better than $MTLCD$s.

Appendix A: GA parameters used to design $MTLCD$s

The following parameters have been considered in the GA analysis used for designing $MTLCD$s:

Tab. 4. GA parameters used to solve the optimization problem

Parameter	Definition	Selected value
N_{gen}	number of generations	400
N_{ind}	number of individuals in each generation	25
N_{elites}	number of elites in each generation	2
N_{new}	number of the newborns	25
m_r	mutation rate	0.04
η	insertion rate	0.9
Sp	Selective Pressure [29]	2

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