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RESEARCH ARTICLE

Damage Diagnosis of Structures Using Modal Data and Static Response

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Abstract

This paper is aimed at presenting three methods to detect and estimate damage using modal data and static response of a damaged structure. The proposed methods use modal data with and without noise or static displacement to formulate objective functions. Damage location and severity in structural elements are determined using optimization of the objective functions by the simulated annealing algorithm. These methods have been applied to three examples, namely a three-story plane frame, cantilever plate and benchmark problem provided by the IASC-ASCE Task Group on Structural Health Monitoring. Also, the effect of the discrepancy in mass and stiffness between the finite element model and the actual tested system has been investigated. The obtained results indicate that the proposed methods can be viewed as a powerful and reliable method for structural damage detection and estimation.

Keywords

damage detection, modal analysis, static response, optimization, simulated annealing algorithm

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1 Introduction

Damage detection and estimation in engineering structures such as offshore structures, bridges and tall buildings is a vital part of structural health monitoring during their service life. The objective of damage detection is to determine the damage in a structural system from the measured responses of the structures. Structural damage progressively impacts on the physical properties of structures such as stiffness and damping at damage location.

Many researchers studied modal parameters of structure including natural frequencies and mode shapes that are so sensitive to structural properties like stiffness [1-4]. On the other hand, static responses are more locally sensitive to damage than natural frequencies [5] and the equipments of static testing, and precise static displacements of structures could be obtained rapidly and economically [6]. However, there are two main drawbacks in the static damage identification methods: (1) Static testing provides less information as compared to dynamic testing; (2) The effect of damages on static responses for damage detection may be cryptic due to limited load paths [6].

Hajela and Soeiro [7] presented a damage detection algorithm based on static displacements, mode shapes and frequencies. To solve an unconstrained optimization problem, an iterative non-linear programming method was developed. Hwu and Liang [8] used static strain measurement from multiple loading models for identification of the hole and cracks in linear anisotropy elastic materials with nonlinear optimization. Paola and Bilello [9] proposed a damage identification procedure based on a least-square constrained nonlinear minimization problem for Euler-Bernoulli beams under static loads. Yam et al. [10] proposed sensitivities analysis in static and dynamic parameters damage indices quantification for their identification capabilities over plate-like structures. Hua et al. [11] proposed a new damage detection procedure for cable-stayed bridges by changes in cable forces. Also, Lee et al. [12] developed a method used continuous strain data from fiber optic sensor and neural network model.

Recently, Cao et al. [13] presented the sensitivity of fundamental mode shape and static deflection for damage identification in cantilever beams, wherein these features are extremely similar in configurations. Radzieński et al. [14] proposed a new method for structural damage detection based on experimentally obtained modal parameters. The new method is suitable for detection of fatigue damage occurring in an aluminum cantilever beam. Lakshmanana et al. [15] proposes a methodology for damage identification from measured natural frequencies of a contiguously damaged reinforced concrete axial rod and beam, idealized with distributed damage model. Zhua et al. [16] proposed an effective method for damage detection in shear buildings using the change (s) in the first mode shape slopes. Also, Kourehli et al. [17] presented a damage detection and estimation method in structures using incomplete static responses and pattern search algorithm. The performance of the proposed method is evaluated using three numerical examples.

In this study, three methods are introduced to detect and estimate damage in structures using modal data and static displacements of a damaged structure. The damage detection is carried out through applying simulated annealing method to minimize objective function. Simulated annealing is a probabilistic method proposed by Kirkpatrick et al. [18] and Cerny [19] for finding the global minimum of a cost function that may possess several local minima. It works by emulating the physical process whereby a solid is slowly cooled so that when eventually its structure is “frozen” this happens at a minimum energy configuration [20]. The simulated annealing method has been widely used in different fields of engineering as a robust and promising method [21-23]. The presented method for damage identification has been applied to two numerical examples, namely a three-story plane frame and cantilever plate. In addition, the benchmark problem provided by the IASC-ASCE Task Group on Structural Health Monitoring [24] is used for demonstration of efficiency of the presented approaches. Moreover, the performance of the proposed method using modal data has been evaluated through comparison the obtained results with energy index method presented by Sharifi and Banan [25].

2 Problem formulation

In this section, three methods are proposed for structural damage detection and estimation. The first method uses the mode shapes and frequencies to formulate a dynamic residue vector as an objective function. The second method uses the difference between the measured displacements and the corresponding computed displacements. Then, in the third method, objective function is formulated using static residue force vector. Finally, the simulated annealing optimization algorithm for minimizing the objective functions is presented.

2.1 First objective function

The modal characteristics of a structure without damage are described by the equations:

$$\left[\mathbf{K}-\omega_i^2\mathbf{M}\right]\mathbf{\Phi}_i=0 \quad i=1,2,...,m \tag{1}$$

where, $\mathbf{K}$ and $\mathbf{M}$ are stiffness and mass matrices, respectively; ω_i is the natural frequency corresponding to mode shape $\mathbf{\Phi}_i$; and m is the number of structural mode shapes.

The damage can be quantified through a scalar variable d whose values are between 0 for the undamaged member of structure and 1 for a rupture member. The damage can be described by a decrease in the stiffness of the element:

$$\mathbf{K}_e^d=\left(1-d_e\right)\mathbf{K}_e \tag{2}$$

where, K_e^d and $\mathbf{K}_e$ are the damaged and undamaged local stiffness matrices of the eth element in the finite element method, respectively; and d_e is the damage of the eth element.

In the finite element method, for a damaged structure the damaged global stiffness matrix is obtained by transformation and assemblage. The damaged local stiffness matrix K_e^d is first transformed into $\overline{\mathrm{K}}_e^d$, which is the damaged element stiffness matrix in the global coordinate system, using the transformation matrix $\mathbf{T}$, by:

$$\overline{\mathbf{K}}_e^d=\mathbf{T}^T\mathbf{K}_e^d\mathbf{T} \tag{3}$$

The damaged element stiffness matrix in the global coordinate system $\overline{\mathrm{K}}_e^d$ is then expanded into ($p \times p$) matrix denoted by $\tilde{\mathrm{K}}_e^d$, K where, p is the number of degrees of freedom (DOFs) of the structure. The damaged global stiffness matrix $\mathbf{K}^d$ is obtained by summation of $\tilde{\mathrm{K}}_e^d$ for all elements:

$$\mathbf{K}^d=\sum_{e=1}^{n}\tilde{\mathbf{K}}_e^d \tag{4}$$

where, n is the total number of elements in structure.

Thus, Eq. (1) for a damaged structure can be written as follows:

$$\left[\mathbf{K}^d-\left(\omega_i^d\right)^2\mathbf{M}\right]\mathbf{\Phi}_i^d=0 \quad i=1,2,\ldots,m \tag{5}$$

where, ω_i^d is the natural frequency corresponding to mode shape $\mathbf{\Phi}_i^d$ for damaged structure.

Applying the mode shapes and natural frequencies of damaged structure to Eq. (5) leads to form the inverse problem of determining the damage severity parameter. The definition of a local damage severity parameter d in the finite element model allows estimating damage quantity and location together since damage identification is then carried out at the element level. The problem can be formulated as optimization problem of objective function while using some transforms as a direct inversion to obtain solution is impossible most of the time.

Localizing and quantifying damage is often considered as a difficult and complex problem, requiring a sophisticated optimization procedure. In typical optimization problem, there may be lots of locally optimal layout; therefore, a downhill-proceeding algorithm in which steady declining value of objective function is created in iterations, may be stuck into a locally optimal point instead of providing global optimal solution. For this reason, global search algorithms like the simulated annealing method are adopted by the authors in order to characterize damage. Simulated annealing is a popular technique due to its ability to 'escape' from local minima; it is able to move to areas of less desirable solutions than that which it currently explores, so that it can eventually locate the global optimum [26]. A particular attraction of simulated annealing is the existence of the proof of Geman et al. [27] that guarantees convergence to the global minimum provided that the annealing rate is sufficiently slow.

The simulated annealing method attempts to find the best solution to a given problem by minimizing an objective function. In any optimization process, existence of objective function is indispensable part of problem.

The general statement for the objective function is:

$$F = f\left(d_1, d_2, \ldots, d_n\right) \tag{6}$$

In the event of substituting the measured modal parameters of the damaged structure in Eq. (5), a dynamic residue vector can be defined over each measured mode as follows:

$$\mathbf{E}_i = \left[\mathbf{K}^d - \left(\omega_i^{d,m}\right)^2 \mathbf{M}\right] \boldsymbol{\Phi}_i^{d,m} \quad i = 1,2,...,k \tag{7}$$

where, $\omega_i^{d,m}$ and $\boldsymbol{\Phi}_i^{d,m}$ are the ith natural frequency and the ith mode shape from measurements, respectively; and k is total number of mode shapes for damage detection.

Therefore, the problem of damage detection can be formulated as an optimization problem. So, the first objective function can be formulated as follows:

$$f_1(d) = \sum_{i=1}^{k}\left(\sum_{j=1}^{p}\left(E_i\right)^2\right) \quad i = 1,2,...,,k \tag{8}$$

where, p is the number of DOFs of the structure.

2.2 Two last objective functions

The static equilibrium equation of a structure in a displacement based finite element frame work can be expressed as follows:

$$\left[\mathbf{K}\right]\left\{\mathbf{x}\right\} = \left\{\mathbf{F}\right\} \tag{9}$$

where, F and x are the force and displacement vectors; respectively.

From Eq. (9), the static equilibrium equation of damaged structure can be obtained as:

$$\left[\mathbf{K}^d\right]\left\{\mathbf{x}^d\right\} = \left\{\mathbf{F}\right\} \tag{10}$$

where, superscript d is noted the damage state.

Based on Eq. (10), the measured displacement of damaged structure can be obtained as:

$$\left\{\mathbf{x}^d\right\} = \left[\mathbf{K}^d\right]^{-1}\left\{\mathbf{F}\right\} \tag{11}$$

The second objective function is defined in terms of output errors between computed and measured displacements as follow:

$$f_2(d) = \sum_{i=1}^{p}\left(\beta\left(\mathbf{x}_{m,i}^d - \mathbf{x}_{t,i}^d\right)^2\right) \tag{12}$$

where, $\mathbf{x}^d_{m,i}$ and $\mathbf{x}^d_{t,i}$ are the measured and theoretically computed displacement of the ith node of damaged structure; respectively, and β is a weighting factor.

Also, the third objective function f_3 formulates as a static residue force vector as follow:

$$f_3(d) = \left\|\gamma\left(\mathbf{F} - \mathbf{K}^d \mathbf{x}_m^d\right)\right\|^2 \tag{13}$$

where, $\| \; \|$ represents the Euclidean length and γ is a weighting factor. The weighting factors are introduced to produce a more appropriate value of the objective functions.

2.3 Optimization using simulated annealing algorithm

Simulated annealing is the simulation of annealing of a physical many particle system for finding the global optimum solutions of a large combinatorial optimization problem [28]. It uses a temperature parameter that controls the search. At each step the temperature is slowly „cooled" or lowered and a new point is generated using annealing function. At each step, new points distance from the current point is proportional to the temperature. If the energy (cost) of this new point is lower than that of the old point, the new point is accepted. If the new energy is higher, the point is accepted probabilistically, with probability dependent on a "temperature" parameter. This unintuitive step sometime helps identify a new search region in hope of finding a better minimum. In this paper, fast annealing function that takes random steps with size proportional to temperature and exponential temperature update are used.

The proposed algorithm uses the exponential temperature update for temperature, T in annealing-time k:

$$T = 0.95^k T_0 \tag{14}$$

where T_0 is initial temperature.

Also, Reannealing is performed after a certain number of points (Reanneal Interval) are accepted. Reannealing raises the temperature in each dimension, depending on sensitivity information and the search is resumed with the new temperature values [29].

- The Simulated annealing optimization algorithm stops when any of following situations occurs [29]:
- The number of iteration or evaluation of objective function reaches the max value.
- Alteration in the improvement of objective function is less than the function tolerance.
- The time algorithm runs reaches the max value.

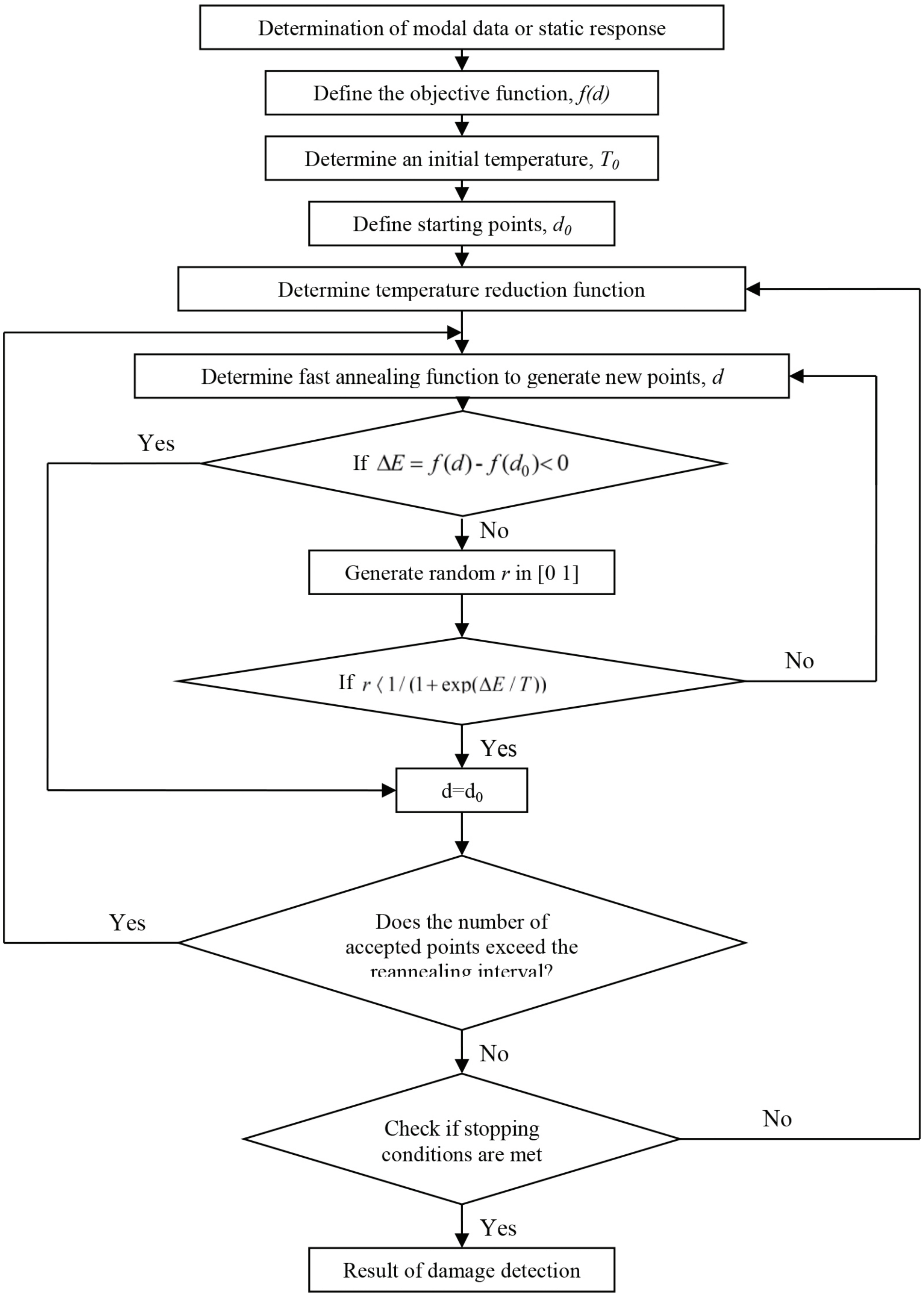

Fig. 1 Flowchart of the damage detection method using the simulated annealing method

Figure 1 shows the flowchart of the proposed method for estimation and localization of the damage via simulated annealing method.

3 Validation of the proposed methods

To evaluate the feasibility of the proposed methods, they were applied to the three numerical examples such as a three-story plane frame, cantilever plate and finally the first phase of IASC-ASCE benchmark structure.

3.1 Three story plane frame

A three-story plane steel frame as illustrated in Fig. 2 with finite element model consists of nine elements (six columns and three beams) and six free nodes are considered. The numerical studies are carried out within the MATLAB environment, which is used for the solution of finite element problems.

For the considered steel frame, the material properties of the steel include Young's modulus of E=200 GPa, mass density of ρ=7850 kg/m^3. The mass per unit length, moment of inertia, and cross-sectional area of the columns are: m=117.75 kg/m, I=3.3 × 10^{-4} m^4 and A=1.5 × 10^{-2} m^2, respectively; for the beams are: m=119.71 kg/m, I=3.69 × 10^{-4} m^4 and A=1.52 × 10−2 m^2.

In the finite-element model, the damage is represented as the elements with reduction in stiffness. In order to quantify damage detection methods, three different patterns from the aspect of number and damage severity were employed. In this numerical example, the three following damage patterns have been considered as:

1. The damage severity of element 3 is 0.25.
2. The damage severity of elements 2 and 3 are 0.2 and 0.25, respectively.
3. The damage severity of elements 2, 3 and 8 are 0.2, 0.25 and 0.3, respectively.

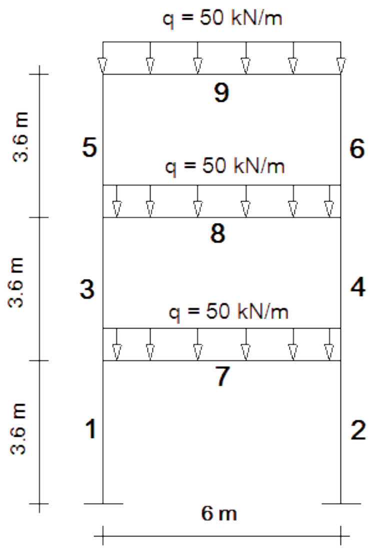

Fig. 2 The three-story plane frame with the finite element model

Considering the low values of the applied loads on the structure, so the formed objective functions will be small. Therefore, it has been tried to prevent diverging of the proposed algorithm by defining two magnifier weighting factors β and γ that are large enough and magnifying the objective function. So, the values of the weighing factors β and γ are defined 1030 and 1020, respectively. Also, the simulated annealing method input parameters adopted for the following analyses are summarized in Table 1.

Table 1 Input parameters for the simulated annealing algorithm

Maximum function evaluations	1000-50000
Annealing function	Fast annealing
Temperature update function	Exponential temperature update
Reannealing interval	100
Initial temperature	100

To be more suited with the real dynamic cases, another examination has been performed in which the natural frequencies with 2 % noise are utilized to damage identification considering the same patterns mentioned before. To perform this, some random noise has been added to the theoretically calculated natural frequencies. The frequency contaminated with noise can be obtained from the frequency without noise using the following equation:

$$\bar{\omega}_i = \omega_i \left(1 + \eta \; rand()\right) \tag{15}$$

Where $\bar{\omega}_i$ and ω_i are the frequencies of ith mode contaminated with noise and without noise, respectively. η is the noise level (e.g., 0.02 relates to a 2 % noise level) and *rand*( ) is a random number in the range [-1 1].

Damage in the three-story plane frame can be determined using the proposed methods. Using modal parameters include two and four mode shapes and natural frequencies of damaged frame with and without noise, the proposed method was applied to detect and quantify the damage in the considered frame. Figure 3 illustrates that the proposed method is robust and promising in detecting and quantifying various damage patterns with 2% noise. In addition, the obtained results based on the last two objective functions are shown in Fig. 4. It can be seen that the damage severity and locations can be obtained for three different scenarios considered.

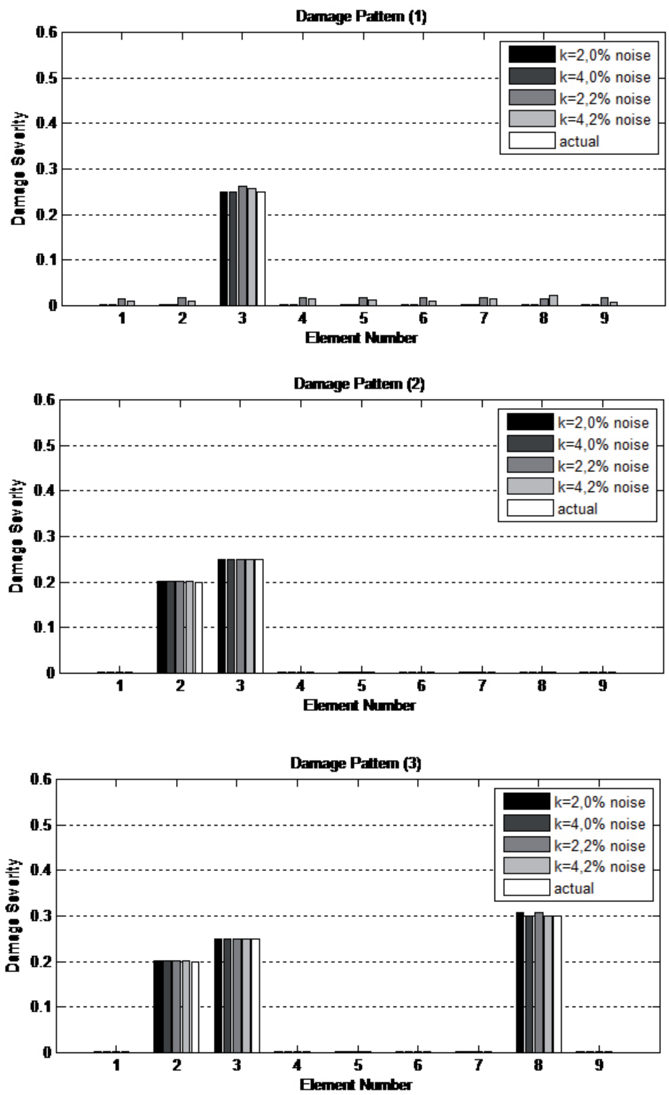

Fig. 3 The obtained results of the three-story plane frame based on the first objective function

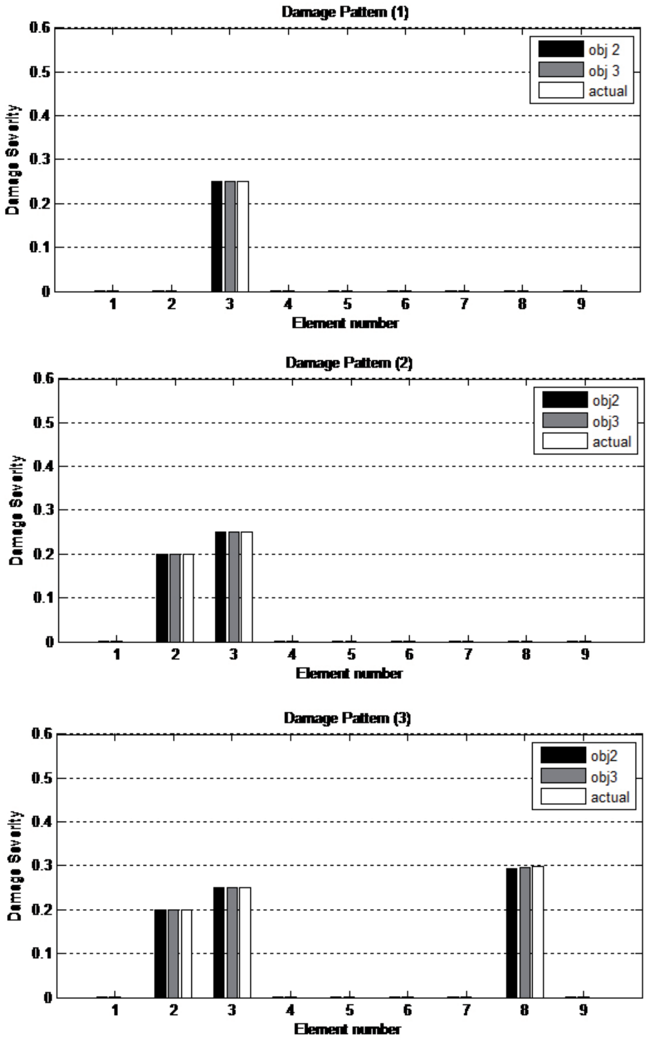

Fig. 4 The obtained results of the three-story plane frame based on the last two objective functions

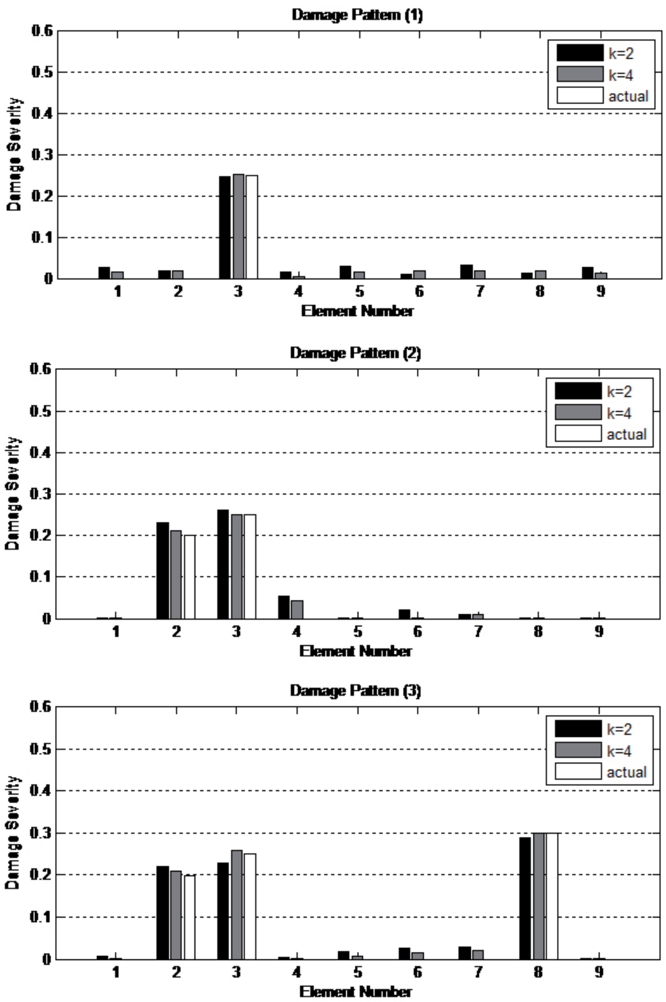

Fig. 5 The obtained results of the three-story plane frame based on the first objective function with modeling errors

Finally, the modeling errors in the analytical model have been studied. It is assumed that the actual tested frame has perturbations of stiffness of 3 % and 4 % at elements 3 and 7; respectively, and perturbations of mass of 5 % and 4 % at elements 2 and 9; respectively. Figures 5 and 6 illustrate the efficiency and effectiveness of the proposed method in detecting and quantifying of various damage patterns considering the modeling errors.

3.2 Cantilever plate

A cantilever plate as illustrated in Fig. 7 with finite element model consists of 18 elements and 42 nodes are considered. The thickness of considered plate is t=0.15m and the material properties include Young's modulus of E=20 GPa, mass density of ρ=2400 kg/m^3 and Poisson's ratio of μ=0.2. Also, in the static base methods, concentrated load P=100 N at the end of cantilever plate has been used.

The two damage patterns for this case are:

1. The damage severity of elements 3, 11 and 13 are 0.1, 0.15 and 0.2, respectively.
2. The damage severity of elements 2, 9 and 17 are 0.1, 0.1 and 0.15, respectively.

Figure 8 shows the results of damage identification in the cantilever plate using only one and three mode shapes and frequencies for two damage patterns using the proposed method based on the simulated annealing method, which was described before. Also, Fig. 9 shows the obtained results using the last two objective functions. The obtained results indicate that the proposed methods are robust and effective methods in detecting and quantifying various damage patterns in plate structures.

Also, the modeling errors in the analytical model have been investigated. It is assumed that the actual tested frame has perturbations of stiffness of 2 % and 4 % at elements 1 and 8; respectively, and perturbations of mass of 5 % and 3 % at elements 6 and 7; respectively. Figures 10 and 11 show the efficiency and effectiveness of the proposed method considering the modeling errors.

3.3 IASC-ASCE benchmark structure

The benchmark structure is a four-story steel frame, two-bay by two-bay and quarter-scale model structure constructed in the Earthquake Engineering Research Laboratory at the University of British Colombia. Geometry of the benchmark structure is shown in Fig. 12. Details of the first phase of IASC-ASCE benchmark problem was presented by Johnson et al. [24].

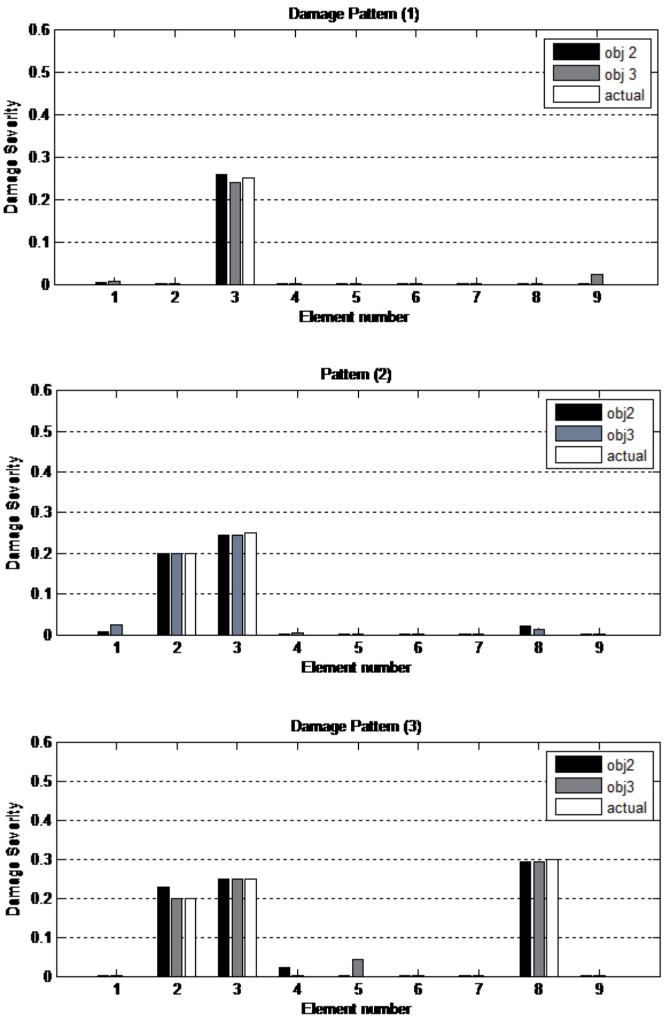

Fig. 6 The obtained results of the three-story plane frame based on the last two objective function with modeling errors

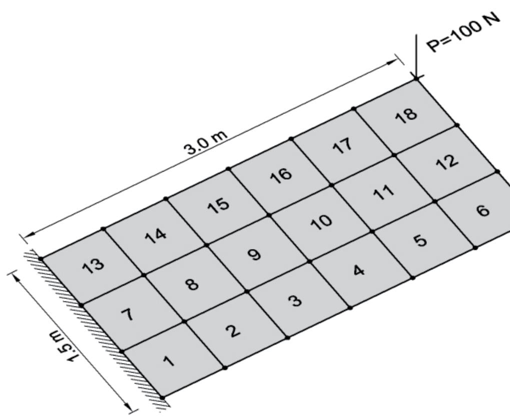

Fig. 7 The cantilever plate with finite element model

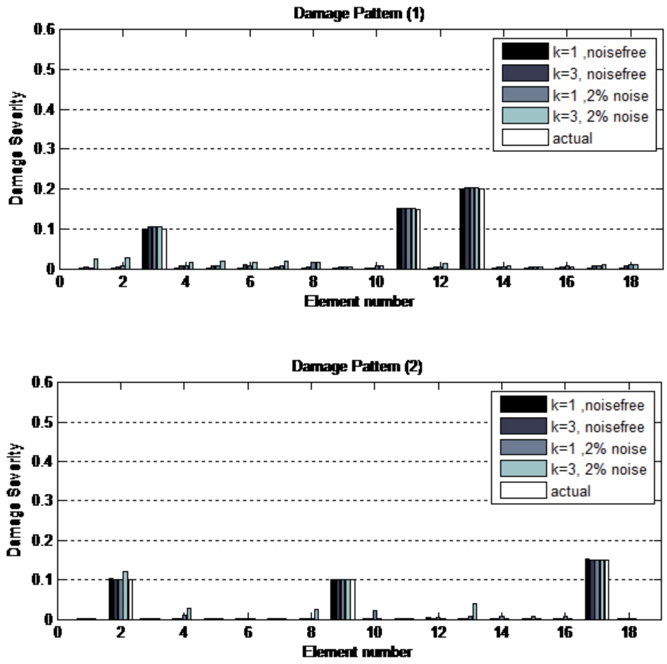

Fig. 8 The obtained results of the cantilever plate for the first objective function

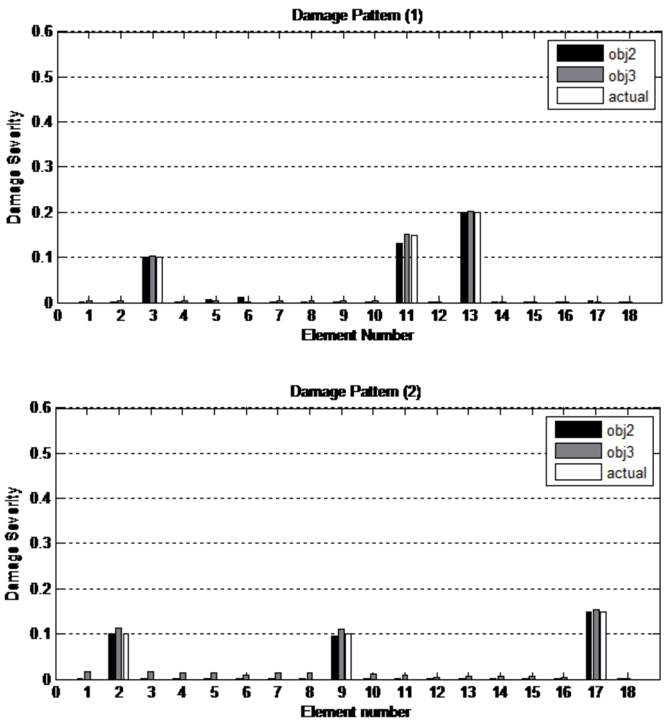

Fig. 9 The obtained results of cantilever plate for the last two objective functions

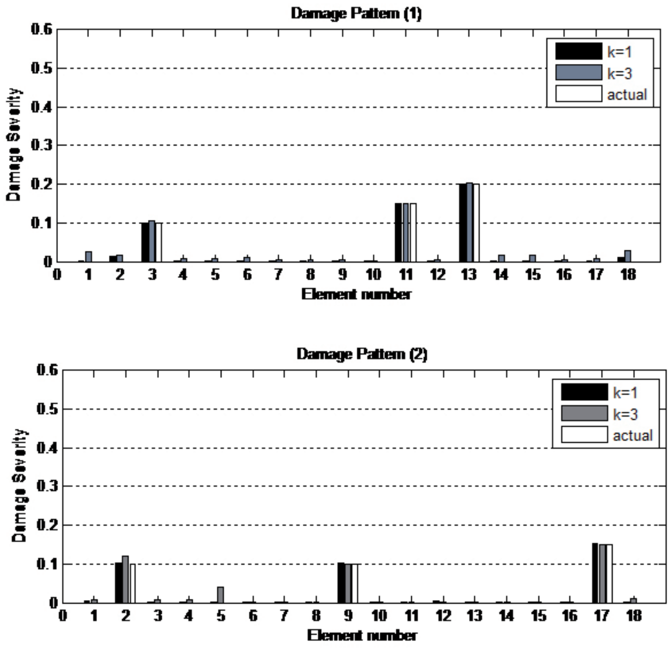

Fig. 10 The obtained results of the cantilever plate for the first objective function with modeling errors

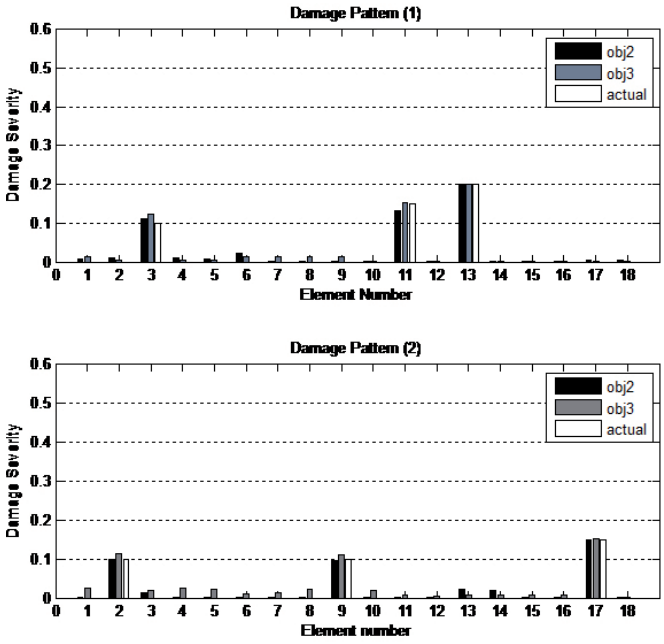

Fig. 11 The obtained results of cantilever plate for the last two objective functions with modeling errors

The proposed method is applied to case 1 of this phase benchmark problem, and the finite element model of the 12 DOF shear building model is used. In this case, the five following damage patterns have been considered:

1. All of the braces of the first story are broken
2. All of the braces of the first and third stories are broken
3. One brace of the first story is broken
4. One brace at the first and third stories is broken
5. 1/3 of area of one brace at the first story is cut

For each damage pattern, the mass and horizontal story stiffnesses are illustrated in Table 2.

Damage in the structures can be determined using the proposed method based on modal data which employ the simulated annealing algorithm. The number of modal data used for damage detection is three for the first and second damage pattern and is twelve for other patterns. The obtained results of the application of the proposed method using modal data to the benchmark structure are cited in Table 3. The structural damage was detected and estimated within different level of damage. The results revealed that there is a good agreement between the actual and estimated damage of the benchmark structure.

Table 2 Mass and horizontal story stiffness (MN/m) of undamaged and damaged 12 DOF model

Story	DOF	Mass (kg)	Undamaged	Pattern (1)	Pattern (2)	Pattern (3)	Pattern (4)	Pattern (5)
			stiffness	Stiffness	Stiffness	Stiffness	Stiffness	Stiffness
1	x	3452.4	106.60	58.37	58.37	106.6	106.6	106.6
1	y	3452.4	67.90	19.67	19.67	55.84	55.84	63.88
1	θ_z	3819.4	232.00	81.32	81.32	213.12	213.12	225.71
2	x	2652.4	106.60	106.60	106.6	106.60	106.60	106.60
2	y	2652.4	67.90	67.90	67.9	67.90	67.90	67.90
2	θ_z	2986.1	232.00	232.00	232.00	232.00	232.00	232.00
3	x	2652.4	106.60	106.60	58.37	106.60	94.54	106.60
3	y	2652.4	67.90	67.90	19.67	67.90	67.90	67.90
3	θ_z	2986.1	232.00	232.00	81.32	232.00	213.12	232.00
4	x	1809.9	106.60	106.60	106.6	106.60	106.60	106.60
4	y	1809.9	67.90	67.90	67.9	67.90	67.90	67.90
4	θ_z	2056.9	232.00	232.00	232.00	232.00	232.00	232.00

Table 3 The obtained results of damage detection for the benchmark structure (%)

Story	DOF	Pattern (2)		Pattern (3)		Pattern (4)		Pattern (5)	
1	x	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual
1	y	45.24	45.24	0	0	0	0	0	0
1	θ_z	71.02	71.03	17.76	17.76	17.76	17.76	5.91	5.92
2	x	64.95	64.95	8.14	8.14	8.14	8.14	2.71	2.71
2	y	0	0	0	0	0	0	0	0
2	θ_z	0	0	0	0	0	0	0	0
3	x	0	0	0	0	0	0	0	0
3	y	45.24	45.24	0	0	11.31	11.31	0	0
3	θ_z	71.02	71.03	0	0	0	0	0	0
4	x	64.95	64.95	0	0	8.14	8.14	0	0
4	y	0	0	0	0	0	0	0	0
4	θ_z	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Table 4 Comparison of the results of the methods for damage detection of the benchmark structure (%)

Story	DOF	Damage pattern (1)			Damage pattern (2)		
		Presentd method	Energy index Method [25]	Actual	Presented method	Energy index Method [25]	Actual
1	x	45.24	41.81	45.24	45.24	41.98	45.24
1	y	71.1	66.26	71.03	71.02	69.48	71.03
2	x	0	0	0	0	0	0
2	y	0	0	0	0	0	0
3	x	0	0	0	45.24	45.13	45.24
3	y	0	0	0	71.02	69.5	71.03
4	x	0	0	0	0	0	0
4	y	0	0	0	0	0	0

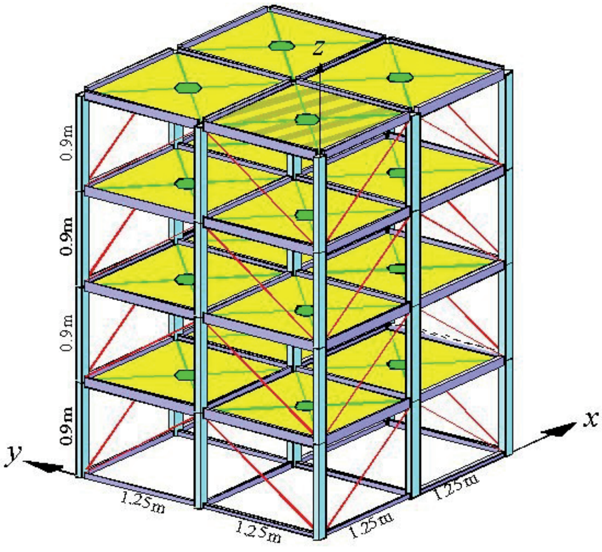

Fig. 12 Geometry of the benchmark structure [24]

Furthermore, Table 4 provides comparison between the obtained results from the presented method in this paper and energy index method proposed by Sharifi and Banan [25]. As it is shown, the damages obtained by the proposed model are closer to actual damages compared to the energy index method.

4 Conclusions

This study presented three methods for detection and identification of structural damage based on the modal data and static responses of damaged structure using simulated annealing algorithm. In the first method, objective function is formulated via mode shapes and frequencies, while the last two methods use static responses. The proposed methods employ simulated annealing to determine the damage by optimizing different objective functions.

To validate the efficiency and effectiveness of the proposed method a study on the damage detection was conducted with noisy modal data. Also, the effect of the modeling errors in the finite element model and the actual tested system has been investigated. The obtained results indicated that the proposed method is a strong and viable method to the problem of detection and estimation of damage in the structures.

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