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RESEARCH ARTICLE

Numerical Investigation on the Reference Crushing Stress of Granular Materials in Triaxial Compression Test

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Abstract

Particle crushing dominates the deformation behaviour of granular materials under significantly high compressive and shear stress. A proposed constitutive model has been verified to predict crushing behaviour of granular materials with different crushability and adopted one kind of reference crushing stress. It is noted that no positive dilatancy of granular material in triaxial test occurs once the confining pressure exceeds a certain stress level. That stress is defined as the reference crushing stress. This study presents a parametric study on the reference crushing stress in the constitutive model and examines its variation for different distributed ranges of grain size gradation and relative densities. Predicted results demonstrate that the peak stress ratio increases and contractive behaviour becomes less obvious with a larger reference crushing stress. Reference crushing stress increases with a wider grain size gradation and larger relative density for the same granular material. A linear relationship between the reference crushing stress and single particle strength has been obtained from the numerical and experimental results. The reference crushing stress can be recognized as one effective index to evaluate the strength of granular material in triaxial tests.

Keywords

Constitutive model · particle crushing · single particle strength · particle diameter · relative density

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1 Introduction

Estimation of crushing stress is vital to comprehend the crushing mechanism of granular material. In geotechnical practice, it is significantly important to consider and decide the crushing stress of granular material when we estimate the bearing capacity of pile penetrating into crushable sand and analyze the stability of soil at bottom of the dam. For an individual particle, tensile strength is a very useful index because that it expresses the average stress or force on a single grain. It can be measured from a single particle crushing test. The volume of a grain is assumed to be spherical for simplification. Dexter and Kroesbergen (1985) [1] had compared a numbered of methods to calculate the tensile stress of granular materials. Many researchers (Jaeger (1967), McDowell *et al.* (1996), McDowell and Bolton (1998), Nakata *et al.* (2001b), McDowell and Debono (2013)) [2–6] had rewritten the equation by revising the expression of grain diameter. Essentially, the tensile stress expresses the strength of a grain from a micro viewpoint. In most cases, the crushing strength for a specimen of assembled grains but not an individual particle is in urgent demand for laboratorial tests and field practice. However, very limited studies have been conducted on the crushing stress of granular material in triaxial compression tests.

The influence of such a kind of strength index on the mechanical behaviour of granular materials needs further examination. Harbin (1985) [7] had defined a relative breakage index containing the breakage reference stress. However, this breakage reference stress did not focus on its relevant form of granular material in triaxial tests from a macro viewpoint but was directly linked to the mechanical behaviour under specific loading. Nakata *et al.* (1999) [8] had only correlated the maximum value of mean normal stress with breakage factor. The constitutive model proposed by Yao *et al.* (2008) which adopts a reference crushing stress provides an option to solve this difficulty [9]. It is noted that no positive dilatancy of granular material in triaxial test occurs once the confining pressure exceeds a certain stress level. That stress is defined as the reference crushing stress. This constitutive model has been employed to evaluate the crushing stress of granulated coal ash at critical state by Wu *et al.* (2014) [10].

This study examines the validity of a constitutive model adopting a reference crushing stress to predict the mechanical behaviour of granular materials with different crushability. This study also presents a parametric study on the reference crushing stress in the model and examines its variation for different distributed ranges of grain size gradation and relative densities. Predicted results demonstrate that the peak stress ratio increases and contractive behaviour becomes less obvious with a larger reference crushing stress. It is noted that the reference crushing stress is greatly dependent on the type, grain size gradation and compactness of granular material and it also affects the prediction accuracy of the constitutive model. The wider the distributed range of grain size of the granular material and higher the relative density are, the larger the reference crushing stress becomes.

To specify the mechanical meaning of the reference crushing stress, a linear relation between the reference crushing stress and the single particle strength is displayed for five kinds of granular materials. It is concluded that the reference crushing stress could be regarded as an effective index to evaluate the strength of granular material in triaxial compression tests.

2 Constitutive model for granular material with crushing and its reference crushing stress

In the last decade, a series of elasto-plastic constitutive models have been proposed by researchers (Daouadji *et al.* (2001), Daouadji and Hicher (2010), Kikumoto *et al.* (2010), Hu *et al.* (2011), Wei (2012)) [11–15] to describe crushing behaviour for granular materials. Although most of them are capable of representing the variation in mechanical behaviour before and after particle crushing occurrence, less of them directly adopt the crushing stress in the specific expression of constitutive relation. One of them, a simple constitutive model for sand with particle crushing proposed by Yao *et al.* (2008) [9], is shortly reviewed here. The reference crushing stress affects the evolution of characteristic state curves controlling the volumetric variation and the failure judgment in the entire loading process. To furtherly understand this strength index, the determination of reference crushing stress is explained in detail using an example of Toyoura sand. In addition, the prediction capacity of the constitutive model is examined by other granular materials with different crushability.

2.1 Constitutive model for granular material with crushing

The constitutive model can predict the dilatancy behaviour of granular material from negative to positive at low confining pressures but can only predict negative dilatancy at high confining pressures. It also demonstrates the peak strength reduction with increasing confining pressure.

The theory for dilatancy prediction in the constitutive model for granular material with particle crushing is explained as follows. The newly revised hardening parameter H in Eq. (1), which represents both positive and negative dilatancy, consists of the characteristic state curve M_c, the failure state curve M_f, the stress ratio $\eta = q / p$ and the plastic volumetric strain increment $d\varepsilon_v^p$. The characteristic state curve M_c in Eq. (2) represents the boundary curve for variation of volumetric strain, whereas the failure state curve M_f in Eq. (3) provides the failure boundary for sand.

$$H = \int dH = \int \frac{M_c^4}{M_f^4} \frac{M_f^4 - \eta^4}{M_c^4 - \eta^4} d\varepsilon_v^p \tag{1}$$

$$M_c = M \left(\frac{p}{p_c} \right)^n \tag{2}$$

$$M_f = M \left(\frac{p}{p_c} \right)^{-n} \tag{3}$$

where M and p_c are the stress ratio at critical state and the reference crushing stress, respectively, and n is a material parameter.

Fig. 1 demonstrates the characteristic state and failure curves on the mean stress p and deviatoric stress q plane. *AB*, *CD* and *EF* denote different stress paths at low, medium and high initial confining pressures, respectively. Along path *AB*, the volume initially contracts from *A* to *K* and expands in phase *KB*. For path *CD* at medium confining pressure, M_c and M_f intersect at point *D* where no volumetric variation appears and failure occurs simultaneously. As the ratio of failure state curve M_f decreases as the mean stress p increases, the stress path *EF* reaches the failure state curve M_f prior to the characteristic state curve M_c. Only the volumetric contraction is predicted by the constitutive model at medium and high confining pressures. The determination methods for three parameters M, p_c and n will be specified in a later section.

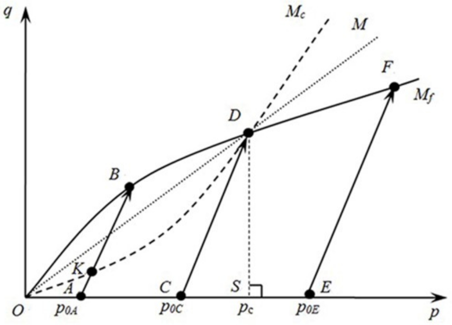

Fig. 1. The M_c and M_f curves on p and q plane

$$\varepsilon_v^e = C_e \left[\left(\frac{p_x}{p_a} \right)^m - \left(\frac{p_o}{p_a} \right)^m \right] \tag{4}$$

$$\varepsilon_v^p = (C_t - C_e) \left[\left(\frac{p_x}{p_a} \right)^m - \left(\frac{p_o}{p_a} \right)^m \right] \tag{5}$$

where p_x is the isotropic consolidation stress on yield surface of the constitutive model.

It has been revealed by Nakai (1989) [16] that the linear relation between the elastic volumetric stain increment $d\varepsilon_v^e$ or plastic volumetric stain increment $d\varepsilon_v^p$ and $(p / p_a)^m$ for granular material could be obtained based on the experimental results of Toyoura sand using Eq. (4) and Eq. (5). Herein, p_a is the atmosphere pressure. C_e and C_t represent the swelling and virgin compression index and m is a coefficient for sand. These three parameters can be obtained from drawing the results for isotropic loading compression and unloading test.

The stress-dilatancy equation in this constitutive model is written in Eq. (6).

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = \frac{M_c^2 - \eta^2}{2\eta} \tag{6}$$

where $d\varepsilon_d^p$ is the plastic deviatoric strain increment. The orthogonality condition is

$$dp \cdot d\varepsilon_v^p + dq \cdot d\varepsilon_d^p = 0 \tag{7}$$

The expression of the yield surface of the constitutive model can be obtained by the combination of Eq. (6) and Eq. (7). From the relationship between the isotropic consolidation stress p_x and the plastic volumetric stain increment $d\varepsilon_v^p$ in Eq. (5), the yield function of the constitutive model for sand with particle crushing is given in Eq. (8).

$$\begin{aligned} f = {} & \frac{C_t - C_e}{p_a^m} \cdot \\ & \cdot \left\{ \left[\frac{(2n+1)\, p_c^{2n}}{M^2} \cdot \frac{q^2}{p} + p^{2n+1} \right]^{\frac{m}{2n+1}} - p_o^m \right\} - H = 0 \end{aligned} \tag{8}$$

where p_o is the initial mean stress. The crushing model takes the associated flow rule, so that plastic potential function g is identical to yield function f.

The constitutive model incorporates seven parameters. A Poisson's ratio ν of 0.3 is assumed. The predicted results by this constitutive model show good agreement with the results of triaxial compression tests for Toyoura sand. This model has also been employed to simulate the mechanical behaviour of sand in the significantly high stress concentration area such as in the region surrounding pile tips (Wu *et al.* (2013a), Wu and Yamamoto (2013b), Wu and Yamamoto (2014)) [17–19].

2.2 M_c and M_f curves with different reference crushing stresses and determination of the reference crushing stress

Fig. 1 shows that the characteristic state curve M_c and failure state curve M_f intersecting at two points on the critical state line M including the zero point. The reference crushing stress p_c corresponds to the point by drawing the straight line from the non-zero point perpendicular to the p -axis. The variation tendency of M_c and M_f is largely determined by the reference crushing stress p_c as well. The reference crushing stress is determined as 5.85 MPa for Toyoura sand which is believed to be harder than some other kinds of granite soils. Therefore, the M_c and M_f curves for the constitutive model adopting p_c as 0.5 MPa, 2 MPa and 5.85 MPa are shown in Fig. 2. It is observed that the gradient of the failure state curve M_f decreases as the reference crushing stress becomes small. Oppositely, the gradient for characteristic state curve M_c increases as the reference crushing stress decreases. In this model, dilatancy behaviour is permitted to be predicted when the mean stress varies from 0 to p_c. The predicted positive dilatancy region shrinks as the reference crushing stress p_c becomes small. The constitutive model predicts significantly volumetric contraction and the stress path is liable to reach the failure state curve with decreasing reference crushing stress. Additionally, the influences of reference crushing stress on the mechanical behaviour of granular materials in triaxial tests will be discussed later.

The determination method of the reference crushing stress p_c is explained with an example of Toyoura sand. It was found that both of the stress ratio at failure q_f / p and the strain increment ratio $-(d\varepsilon_v / d\varepsilon_a)$ became constant values under different confining pressures. Also, connecting the points according to failure states on the q_f / p and $-(d\varepsilon_v / d\varepsilon_a)$ plane provides a straight line as shown in Fig. 3. The stress ratio at failure takes the peak stress ratio or the value when the axial strain is 15%. On the linear relation between these two ratio values, the elastic deformation part is ignored. The peak stress ratio is assumed to be equal to M when the strain increment ratio is zero ($d\varepsilon_v / d\varepsilon_a = 0$). Utilizing the above linear relationship, we can determine M and then make a rearrangement of Eq. (3), obtaining Eq. (9).

$$\ln M_f = -n \ln p + n \ln p_c + \ln M \tag{9}$$

According to the relationship between the failure state curve M_f and the mean stress p in the test, we can draw the line on Fig. 4 to express the relationship between $\ln\left(M_f\right)$ and $\ln(p)$. n is the gradient of the line, and then we can obtain the exact value of p_c.

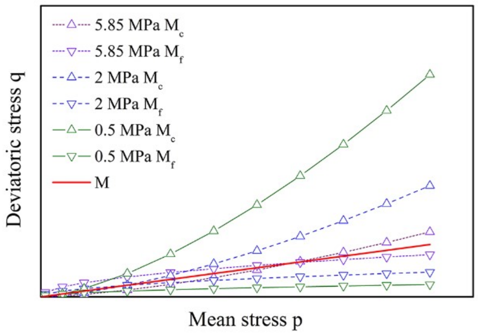

Fig. 2. Variation of M_c and M_f curves with different reference crushing stress p_c

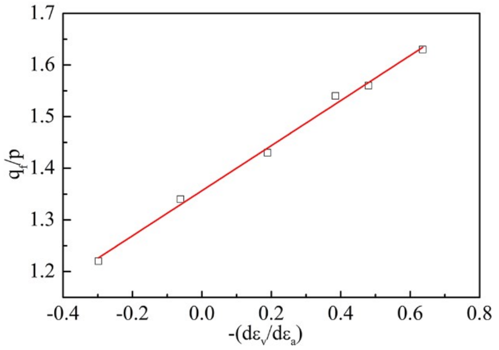

Fig. 3. The relationship between q_f / p and $-(d\varepsilon_v / d\varepsilon_a)$ at failure state under different confining pressures

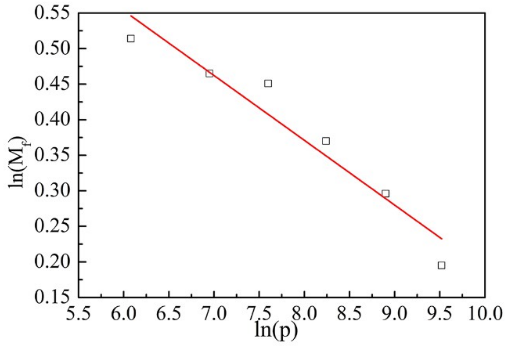

Fig. 4. The relationship between $ln\left(M_f\right)$ and $ln\,(p)$

2.3 Validation of constitutive model for crushable granular materials

Although the constitutive model is verified to predict the mechanical behaviour of Toyoura sand with particle crushing, the prediction capacity of the constitutive model for more crushable soil needs further examination. Crushing failure of relative crushable granular material initially occurs at relatively low stress level and displays significantly contractive behaviour with increasing external force.

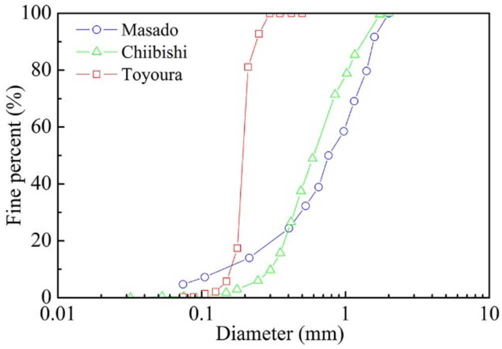

Fig. 5. Grain size distribution curves of three granular materials

Another two kinds of relatively weak materials, Masado and Chiibishi sand, are employed in this study to examine the validity of the constitutive model with particle crushing. Masado is a kind of decomposed granite soil, distributed in large areas of land reclamation in coastal regions, and has been employed by many researchers (Toyota *et al.* (2004), Tsuchida *et al.* (2008), Kumruzzaman and Yin (2012)) [20–22] in laboratory test. Chiibishi sand is a skeletal carbonate beach sand from Okinawa, Japan. The physical properties for the Masado, Chiibishi and Toyoura sand are shown in Table 1. Fig. 5 shows the grain size distribution curves of these three granular materials. To compare the reference crushing stress in the same condition, the selected two kinds of specimens composed of relatively crushable granular materials have the same relative density as 90% to that of Toyoura sand. The positive dilatancy of sand specimens in triaxial compression test disappears at confining pressures as 200 kPa, 1000 kPa and 4000 kPa for the above three kinds of granular materials (Murata *et al.* (1988), Shinoda (2002), Sun *et al.* (2007)) [23–25]. The criterion for evaluating the crushability of granular material is simply employed by comparing those critical confining pressures in triaxial tests. The parameters of constitutive model for three kinds of granular materials are shown in Table 2.

Fig. 6 represents the experimental and predicted results of the relationship between stress ratio and axial strain in triaxial compression tests for Masado sand. Predictions agree well with the measured results except when the confining pressure is at low level. It is believed that the failure state curve corresponding to low confining pressure underestimates the actual strength of the material. It also can be seen that peak strength reduction is also represented by the constitutive model with increasing confining pressure. The volumetric strain plotted against the axial strain for Masado sand is shown in Fig. 7. The predicted values can predict the dilatancy from negative to positive at confining pressures as 60 kPa and 100 kPa, showing agreement with test results, although only the negative dilatancy when confining pressure is 200 kPa and 400 kPa. The predicted values overestimate the positive dilatancy at low confining pressure as 60 kPa and the negative dilatancy at high confining pressure as 400 kPa. It could be explained that the crushing of some large particles for Masado sand causes the larger volumetric strain. The validity of the constitutive model for relative crushable soil is confirmed at a wide range of confining pressures.

The predicted values of the triaxial compression tests for the Chiibishi sand specimen at different confining pressures are compared with the experimental results; the results show good agreement. Triaxial compression tests on saturated dense Chiibishi sand were performed by Shinoda (2002) [24] at confining pressures as 0.2 MPa, 0.5 MPa, 1 MPa, 2 MPa and 5 MPa. Fig. 8 shows the predicted and experimental relationships between the stress ratio and the axial stain. The peak stress ratio tends to reduce as the confining pressure increases. Fig. 9 represents the predicted and experimental relationships between the volumetric strain and the axial strain. The constitutive model displays negative to positive dilatancy at confining pressures as 0.2 MPa and negative dilatancy at confining pressures as 0.5 MPa, 1 MPa, 2 MPa and 5 MPa. Chiibishi sand displays intensively contractive behaviour under high confining pressure and the predicted volumetric strain attains to 16% in compression side and the

Tab. 1. Property for three kinds of granular material

	Specific gravity	e_{max}	e_{min}	d_{50} (*mm*)	Relative density (%)
Masado	2.62	0.967	0.491	0.760	90
Chiibishi	2.65	1.574	0.983	0.613	90
Toyoura	2.66	1.646	1.332	0.200	90

Tab. 2. Parameters of constitutive model for three kinds of granular material

	C_e	C_t	m	p_c (MPa)	M	n	ν
Masado	0.00637	0.01494	0.8	0.412	1.80	0.1782	0.3
Chiibishi	0.00285	0.02664	0.4	0.961	1.73	0.1209	0.3
Toyoura	0.00160	0.00440	0.5	5.850	1.5	0.0850	0.3

constitutive model is capable of describing such extremely high volumetric strain.

From the numerical results for these three representative granular materials, the constitutive model is verified to predict the crushing behaviour of granular materials with different crushability. However, this constitutive model has no capacity of predicting the strain softening phenomena. The constitutive model is capable of describing the strength and deformation behaviour of granular material in triaxial compression until the peak stress ratio appears.

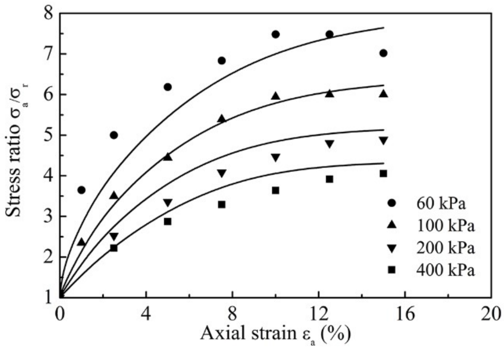

Fig. 6. Stress ratio σ_a / σ_r plotted against axial strain ε_a (Masado sand)

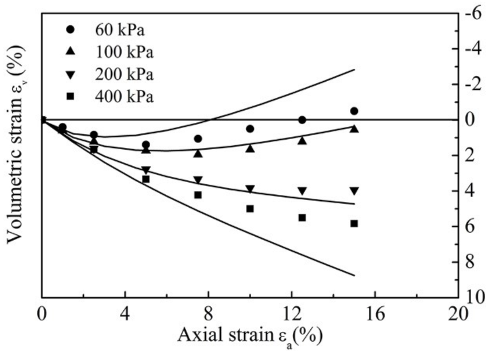

Fig. 7. Volumetric strain ε_v plotted against axial strain ε_a (Masado sand)

3 Parametric study on the reference crushing stress

It is recognized that the reference crushing stress p_c is dependent on the kind of granular material from the predicted results for Masado and Chiibishi sand. "p_c", determined from the results of triaxial compression tests, takes variable values for different kinds of granular material. The value of reference crushing stress p_c plays a significant role in predicting the dilatancy behaviour of granular materials in the constitutive model. Therefore, parametric study on the reference crushing stress p_c for Toyoura sand is conducted to investigate its influence on the predicted mechanical behaviour. In parametric analysis, the reference crushing stress takes the value as 0.5 MPa, 2.0 MPa and 4.0 MPa, respectively. The other six parameters keep constant. The predicted mechanical relationship is expressed as the confining pressure varying from 0.2 MPa to 8 MPa corresponding to the loading condition in the experiment.

The numerical results adopting different reference crushing stresses p_c are represented from Fig. 10 to Fig. 15. It can be concluded that the peak stress ratio increases as the reference crushing stress is increased. The peak stress ratio is around 3.5

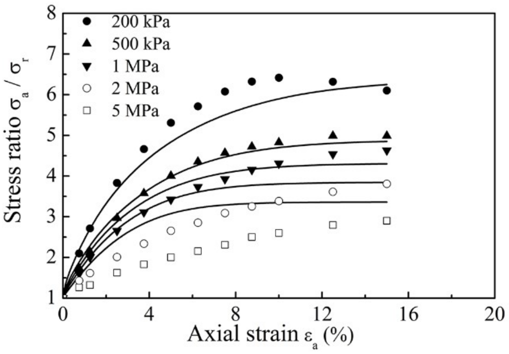

Fig. 8. Stress ratio σ_a / σ_r plotted against axial strain ε_a (Chiibishi sand)

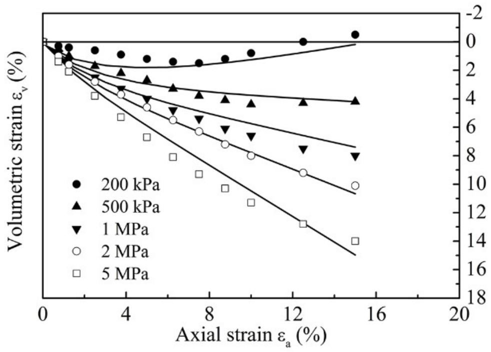

Fig. 9. Volumetric strain ε_v plotted against axial strain ε_a (Chiibishi sand)

at a confining pressure of 0.2 MPa with p_c at 0.5 MPa in Fig. 10, while it reaches 4.5 at a confining pressure of 0.2 MPa with p_c at 4 MPa in Fig. 14. The stress ratio reaches the maximum value at lower axial strain level as the reference crushing stress decreases in Fig. 10, Fig. 12 and Fig. 14.

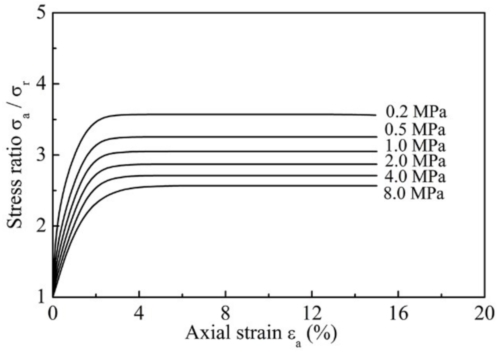

Fig. 10. Stress ratio σ_a / σ_r plotted against axial strain ε_a when $p_c = 0.5$ MPa (Toyoura sand)

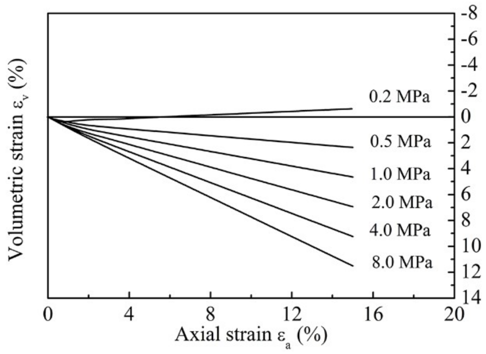

Fig. 11. Volumetric strain ε_v plotted against axial strain ε_a when $p_c = 0.5$ MPa (Toyoura sand)

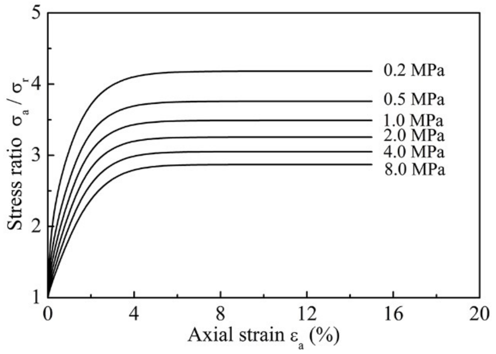

Fig. 12. Stress ratio σ_a / σ_r plotted against axial strain ε_a when $p_c = 2.0$ MPa (Toyoura sand)

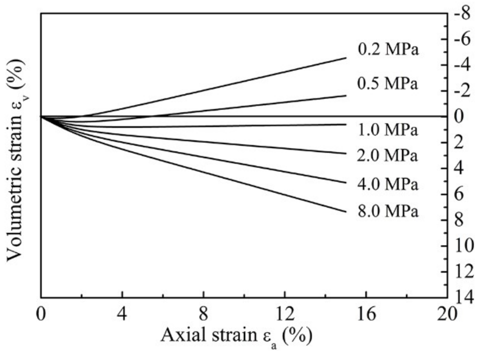

Fig. 13. Volumetric strain ε_v plotted against axial strain ε_a when $p_c = 2.0$ MPa (Toyoura sand)

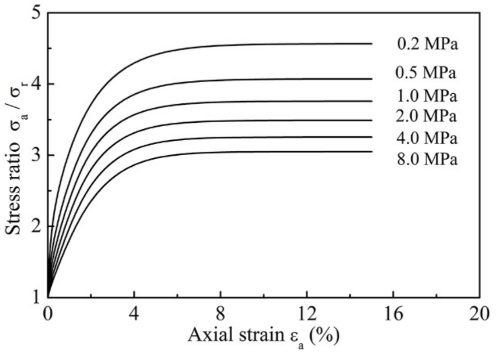

Fig. 14. Stress ratio σ_a / σ_r plotted against axial strain ε_a when $p_c = 4.0$ MPa (Toyoura sand)

The numerical results between the volumetric strain and axial stain adopting different reference crushing stresses p_c are shown in Fig. 11, Fig. 13 and Fig. 15. The predicted results represent remarkable positive dilatancy as reference crushing stress p_c increases. Simultaneously, the predicted negative dilatancy becomes weak. The maximum contractive volumetric strain is 12% when the reference crushing stress p_c is at 0.5 MPa in Fig. 11. The maximum contractive volumetric strain reduces to half when the reference crushing stress p_c is raised to 4 MPa in Fig. 15. Also, the positive dilatancy appears only at a confining pressure of 0.2 MPa when the reference crushing stress p_c is at 0.5 MPa. The constitutive model adopting p_c as 4 MPa can predict positive dilatancy even when confining pressure is increased to 2.0 MPa. As the reference crushing stress p_c increases, the constitutive model displays the obvious tendency of predicting positive dilatancy. It is noted that the scope between the maximum predicted positive dilatancy and maximum negative dilatancy is not affected by the reference crushing stress level.

Based on the above parametric analysis, it is concluded that the constitutive model predicts much larger peak stress ratio and positive dilatancy as the reference crushing stress p_c increases.

4 Influences of distributed range of grain size and initial relative density on the reference crushing stress

Particle crushing is a failure process dominated by its inherent physical and mechanical properties. However, this constitutive model has no capacity of directly describing the effect of physical property on the crushing behaviour of granular material. The reference crushing stress is a sensitive parameter to the kind, grain size gradation and compactness of granular materials. The reference crushing stress varying with different kinds of granular material is testified in the previous section. It is quite meaningful to discuss the reference crushing stress in the constitutive model for the same granular material with different physical conditions.

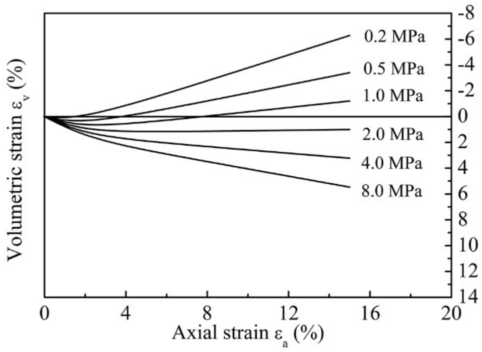

Fig. 15. Volumetric strain ε_v plotted against axial strain ε_a when $p_c = 4.0$ MPa (Toyoura sand)

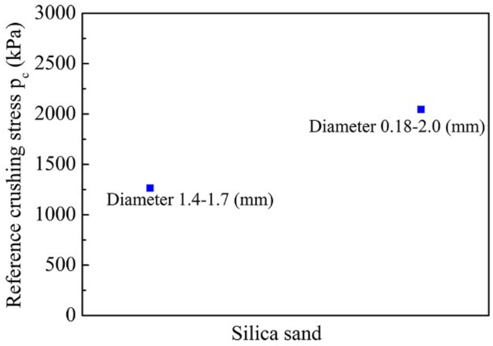

Fig. 16. Effect of diameter size on the reference crushing stress p_c

4.1 Influence of distributed range of grain size

It was pointed out by Miura and Ohara (1979) [26] that grain size distribution greatly affected the compressive characteristics of granular material. Zhang *et al.* (2013) [27] discussed the relative breakage factor for cemented aggregates considering the effects of particle grain size distribution. This important physical parameter is investigated in this section.

Previous investigation has revealed that the grain size gradation influenced particle crushing in one dimensional tests as well as triaxial compression test. The Silica sand with two different kinds of distributed range of grain size was tested by triaxial compression tests and one-dimensional compression tests. One was uniformly graded sand with particle size concentrated between 1.4 mm and 1.7 mm. The other kind was a well distributed material containing particle sizes varying from 0.18 mm to 2.0 mm. The main component of the silica sand was quartz. The reference crushing stress is obtained based on the results of triaxial compression tests for two groups of silica sand. Numerical results show that p_c for silica sand with uniformly and well-distributed grain sizes are around 1250 kPa and 2000 kPa respectively in Fig. 16. It is demonstrated that the reference crushing stress is dependent on the distributed range of grain size. This phenomenon had been proved using a series of one-dimensional compression tests by Nakata *et al.* (2001a) [28].

For the well-distributed sand, the number of the small particles surrounding the large particles is quite high, while the opposite is right. Although the reference crushing stress for large particles is relative low, but the reference crushing stress for the large volume of small particles is quite high. On the whole, it is understandable that the high reference crushing stress is obtained for well-distributed graded sand.

4.2 Influence of initial relative density

Lade and Bopp (2005), Bopp and Lade (2005) [29, 30] performed a series of high-level triaxial compression tests on Cambira sand at different initial relative densities. They pointed out that the initial relative density had a pronounced effect on the Mohr-Coulomb secant friction angle of sand for triaxial compression tests in the low pressure region. The secant friction angle is generally regarded as the strength index for granular material. To clarify the effect of initial relative density on the reference crushing stress in triaxial compression test, numerical examination based on the experimental results of Cambira sand is implemented.

The reference crushing stress is calculated for the Cambria sand at loose, medium and dense states respectively. The reference crushing stress is plotted against the relative density D_r in Fig. 17. It can be seen that the reference crushing stress increases linearly with the relative density. The reference crushing stress is 582.8 kPa at lose state ($D_r = 30\%$), while it is raised to 1486.4 kPa at high state ($D_r = 90\%$) as shown in Table 3. The predicted results of the reference crushing stress are in accordance with the experimental results on the secant friction angle. Also, the incremental degree of secant friction angle shows a linear relationship with increasing relative density in the low pressure region as well. It is believed that the contact surface of the particle becomes large for granular material at dense state. There is limited space for particle movement or rotate unless crushing failure occurs.

Herein, a new reference crushing stress considering relative density is defined as a ratio value of the reference crushing stress divided by the relative density D_r. Fig. 18 represents that this new ratio value slightly decreases with the increasing relative density. This is due to higher occurrence of particle crushing for specimen in the dense state.

Tab. 3. Reference crushing stress p_c for Cambria sand at different relative densities

Relative density (%)	Reference crushing stress (kPa)
30	582.8
60	1086.5
90	1486.4

5 The reference crushing stress related to the single particle strength

The single particle strength is the average characteristic tensile stress σ_{sp} acting on a particle using a simplified theoretical expression in Eq. (10). Various granular materials were tested in single particle crushing experiments. The single particle strength was also discussed with reference to the yield stress

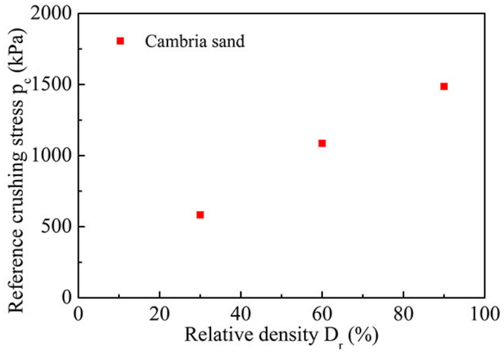

Fig. 17. Effect of relative density D_r on the reference crushing stress p_c

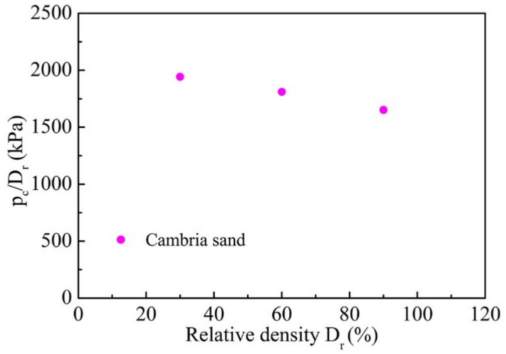

Fig. 18. Relationship between the p_c / D_r and the relative density D_r

obtained in one-dimensional compression which was commonly adopted to understand crushing of granular materials in significantly high stress region. Both the single particle strength and the reference crushing stress are dependent on the kind and composition of granular material but the strength indexes in the different forms in micro and macro viewpoint. The relationships between these two strength indexes are investigated here.

$$\sigma_{sp} = F_{sp}/d^2 \tag{10}$$

where F_{sp} means the force acted on the particle and d is the mean particle diameter.

The single particle strength expresses a linear relationship with the reference crushing stress on a log-log scale plot for five kinds of granular material as shown Fig. 19. The results of the single particle strength and the reference crushing stress are detailed given in Table 4. The difference of the reference crushing stress is basically dependent on the mineral composition of material. Fig. 19 demonstrates that the single particle strength is greater than the reference crushing stress in the same physical conditions. The major reason for the difference is that reference crushing stress does not represent the actual failure stress in triaxial compression tests but a strength index affecting the failure state curves during the entire loading process. In addition, the crushing failure of a grain particle is greatly affected by the shear stress in triaxial compression tests. However, the reference crushing stress still can describe the strength characteristics of granular materials as the single particle strength. Therefore, there are some relations between single particle strength and the reference crushing stress in triaxial compression tests. It is noted that the inclination of the plot between the reference crushing stress and single strength is influenced by the initial packing relative density. It is because that the reference crushing stresses for five granular materials will all decrease once the specimens of granular material are prepared in medium or loose states.

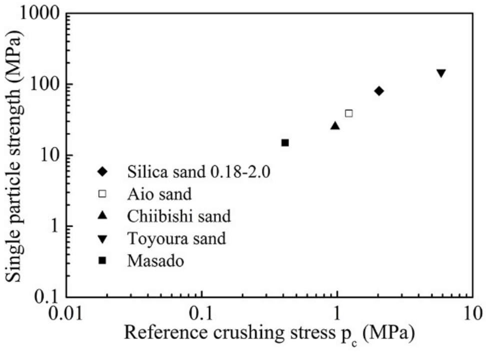

Fig. 19. Relationship between the single particle strength and the reference crushing stress p_c

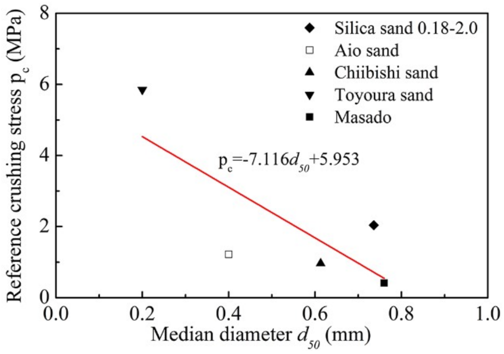

Fig. 20. Relationship between the reference crushing stress p_c and the particle median diameter d_{50}

The reference crushing stress is plotted against the median diameter d_{50} in Fig. 20. It can be seen that the median diameter has a marked influence on the determination of reference crushing stress. The results by Nakata *et al.* (2001b) [5] also showed the relationship between the single particle strength and the median diameter. Both the reference crushing stress and the single particle strength display a decreasing tendency with the increasing median diameter size. This is due to the reasons explained by Fukumoto and Hara (1998) [31] that smaller grains have a higher mean crushing strength because they are formed by the breakage at the boundary between different minerals until they are composed of a single mineral, which has a strong, homogeneous internal structure.

6 Conclusions

The parametric study on the influence of reference crushing stress on the mechanical behaviour and the variation in the characteristic as well as failure state curves have been carried out in this study. The effects of the distributed range of grain size and relative density on the reference crushing stress have been examined. It is essential that the reference crushing stress is entitled with the mechanical meaning in triaxial compression tests with

Tab. 4. Single particle strength and reference crushing stress p_c for granular materials

Name of granular materials	Single particle strength (MPa)	Reference crushing stress p_c (MPa)
Silica sand 0.18-2.0	2.040	11.20
Aio sand	1.220	5.26
Chiibishi sand	0.965	3.26
Toyoura sand	5.850	17.46
Masado sand	0.412	2.23

reference to the single particle strength. It is understandable that the reference crushing stress p_c in the constitutive model is determined by the composition of micro-structure. It is dependent on not only the kind but also the grain size gradation and compactness for the same granular material. p_c is not the specific ultimate strength of the particle when crushing occurs in triaxial tests but is dependent on the ultimate strength of the particle to some degree. Based on the results, some conclusions can be made as followed.

1. The constitutive model with a reference crushing stress is verified to be applicable to granular materials with different crushability. The constitutive model is capable of describing the mechanical behaviour of relative crushable materials under triaxial compression until the peak stress ratio appears. However, the strain softening phenomena cannot be predicted by this simple constitutive model. It also has no capacity of directly considering the effects of the grain size gradation, relative density and stress path on the shear strength and deformation caused by particle crushing.
2. The gradient of the failure state curve M_f decreases as the reference crushing stress becomes small. Conversely, the gradient for the characteristic state curve M_c increases as the reference crushing stress decreases. The predicted dilatancy region shrinks as the reference crushing stress becomes smaller.
3. The constitutive model predicts much larger peak stress ratio and positive dilatancy as the reference crushing stress p_c is increased. It is noted that the scope between the maximum predicted positive dilatancy and negative dilatancy is not affected by the reference crushing stress level.
4. The reference crushing stress is sensitive to the grain size gradation and compactness of the granular material. The reference crushing stress of well-distributed granular material is higher than that of uniformly distributed granular material.
5. It can be seen that the reference crushing stress increases linearly with the increasing relative density for Cambria sand.
6. The single particle strength and reference crushing stress on a log-log scale plot expresses a linear relationship for five kinds of granular materials. It is noted that the inclination of the plot between the reference crushing stress and single strength is influenced by the initial packing relative density. It is because that the reference crushing stresses for five granular materials will all decrease once the specimens of granular material are prepared in medium or loose states. p_c is proved to be an effective index to evaluate the strength of granular materials in triaxial compression tests. It is also found that the reference crushing stress displays a decreasing tendency with increasing median diameter size.

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