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RESEARCH ARTICLE

On Design Method of Lateral-torsional Buckling of Beams: State of the Art and a New Proposal for a General Type Design Method

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Abstract

After introducing the Eurocode standards several theses have been published on the now much-discussed phenomenon of lateral-torsional buckling of steel structural elements under pure bending. According that, researchers are working on the development of such new design methods which can solve the problems of the design formulae given by the EN 1993-1-1. This paper gives a detailed review on the proposals for novel hand calculation procedures for the prediction of LT buckling resistance of beams. Nowadays, the application of structural design softwares in practical engineering becomes more common and widespread. Recognizing this growing interest, the main objective of our research work is the development of a novel, computer-aided design method. In this paper the details of a general type stability design procedure for the determination of the LT buckling resistance of members under pure bending are introduced. Here, the theoretical basis of the proposed method is clarified, the calculation procedure is detailed and some results for the evaluation of the appropriateness of the method is also presented. Based on the evaluations it can be stated that the new, general type design method is properly accurate and has several advantages on the stability check of beams under bending.

Keywords

stability resistance · lateral-torsional buckling · Ayrton-Perry formula · design method

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1 Introduction

During the analysis of steel structures the determination of the stability resistance is one of the most significant verification since usually the loss of stability is the governing problem. For these complex mechanical behaviors the Eurocode standards endeavor to give simplified methods to make the design process easier. However, after the introduction of the EN version of Eurocodes the design of steel members against lateral-torsional buckling became one of the most controversial topic. According to the given standard design procedures many theses have been published on the problems and open questions. Recognizing the need for an appropriate stability design method several researches are dealing with the problem of lateral-torsional buckling of beams. Naumes et al. proposed a "general method" for assessing the out-of-plane stability of members based on the determination of the critical cross-section in [8]. However, the presentation of the widespread validation of the method is still needed. For the clarification of the theoretical background of the lateral-torsional buckling behavior Szalai and Papp determined the Ayrton-Perry type resistance formulae for simple beams under pure bending and also for beam-columns in [4]. Afterwards, Taras et al. published a similar, Ayrton-Perry type solution for the case of simple beams and proposed a calculation procedure for the prediction of lateral-torsional buckling resistance of members subjected to bending in [11]. According to the modern intentions we started a new research work for the development of a computer-aided and general stability design method, which is based on the generalized Ayrton-Perry formulae published in [4]. In this paper a novel method is proposed which reduces the stability problem to evaluation of cross-sectional problem using in structural design software. The method is appropriate to predict the behavior of the beams under bending with arbitrary moment distributions and boundary conditions.

2 Eurocode methods for LT buckling of beams

In the 1960's an extensive test program was carried out with both laboratory and numerical deterministic tests as well as Monte-Carlo evaluation on steel members subjected to bending. As the results of these investigations the lateral-torsional (LT) buckling curves belonging to the different cross-sections were determined. Then, the numerical confirmation and theoretical verification of these curves began. Similarly as in the case of the flexural buckling of columns the Ayrton-Perry type solution [1] was chosen for the description. The determination of these formulae for columns resulted in simple equations which can be properly applied through design methods, see in [2] and [3]. However, due to the complexity of the LT buckling type problems the researchers got much more intricate solution for the case of beams. Namely, the derived formulae were too complicated for calculation procedures and they were unfit for standard applications [4]. In lack of the proper determination of the mechanical background of this behavior the researchers started to calibrate the original, flexural buckling based Ayrton-Perry formula for LT buckling of beams. Finally, the design parameters of the column design procedure were fitted to the resistance curves of the members in bending. This is the reason why EN1993 Part 1-1 (EC3-1-1) standard uses the same multiple buckling curves for the determination of the stability resistance of columns and beams as well [5].

At present, the Eurocode standards give two alternative methods for the stability design of beams under bending. The designer can choose the applied procedure with regard to the specifications of the National Annexes. These alternative methods define the $M_{b,Rd}$ standardized LT buckling resistance in the same way. The calculation for the case of beams with compact sections of class 1 or 2 is:

$$M_{b,Rd} = \chi_{LT} \cdot W_{pl,y} \cdot f_y / \gamma_{M1} \tag{1}$$

where χ_{LT} is the reduction factor for LT buckling, $W_{pl,y}$ is the strong axis sectional modulus, f_y is the yield strength of the material and γ_{M1} is the partial factor for stability checks. The two given alternative methods differ in the calculation procedure of the χ_{LT} reduction factor. One of them is the ‘General Case’ procedure, which was already included in the ENV version of the standards. This method applies analogue formulae for the LT buckling of beams as the formulae given for the flexural buckling of columns. Only the parameters are derived for the behavior of members subjected to bending. According to the EC3-1-1 Part 6.3.2.2 ‘General Case’ procedure the form of the LT buckling curves:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1,0 \tag{2}$$

where the ϕ_{LT} factor and the $\bar{\lambda}_{LT}$ slenderness for LT buckling can be determined through the following equations:

$$\begin{aligned} \phi_{LT} &= 0,5 \cdot \left[1 + \eta_{LT} + \bar{\lambda}_{LT}^2\right] = \\ &= 0,5 \cdot \left[1 + \alpha_{LT} \cdot \left(\bar{\lambda}_{LT} - 0,2\right) + \bar{\lambda}_{LT}^2\right] \end{aligned} \tag{3}$$

$$\bar{\lambda}_{LT} = \sqrt{W_{pl,y} \cdot f_y / M_{cr}} \tag{4}$$

In Eq. (3) the η_{LT} is the imperfection factor for LT buckling whose calibrated form is $\eta_{LT} = \alpha_{LT} \cdot \left(\bar{\lambda}_{LT} - 0,2\right)$ with α_{LT} imperfection constant. In Eq. (4) M_{cr} is the elastic critical bending moment. The most important remarks on the ‘General Case’ method:

- this standard procedure takes into account the type of the bending moment distribution (thereby the load distribution and boundary conditions) of the beam only in the determination of the slenderness;
- regarding the calibrated form of η_{LT} imperfection factor it can be stated that the standard specifies reduction for LT buckling only for beams with $\bar{\lambda}_{LT} > 0{,}2$ slenderness, i.e. the LT buckling curves have a plateau length under $\bar{\lambda}_{LT} = 0{,}2$;
- according to the given tables for the standardized values of α_{LT} constant it can be seen that the EC3-1-1 applies the same LT buckling curve for a group of different profiles and it does not make a distinction between them regarding their behavior and resistance.

The other procedure for the design of beams for LT buckling is the ‘Special Case’ method. The possibility of choosing this alternative option is one of the most significant changes of Eurocode 3 during the conversion from ENV to EN status [6]. This new method can be applied only for beams with hot-rolled and equivalent welded I profiles. Compared to the original procedure the standardized LT buckling curves are considerably changed. The formula for the determination of the χ_{LT} reduction factor in the EC3-1-1 Part 6.3.2.3 ‘Special Case’ method is:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \cdot \bar{\lambda}_{LT}^2}} \leq \min\left(1,0\,;\ \frac{1}{\bar{\lambda}_{LT}^2}\right) \tag{5}$$

where the definition of the $\bar{\lambda}_{LT}$ slenderness is the same as it is written by Eq. (4), and for the calculation of the ϕ_{LT} factor the following expression can be used:

$$\phi_{LT} = 0,5 \cdot \left[1 + \alpha_{LT} \cdot \left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta \cdot \bar{\lambda}_{LT}^2\right] \tag{6}$$

Considering Eq. (5) and Eq. (6) it can be established that the original, ‘General Case’ shape of the LT buckling curves is modified through the application of the β and $\bar{\lambda}_{LT,0}$ parameters. Furthermore, the given values are also changed for the α_{LT} imperfection constant belonging to the separated groups of profiles. This also causes differences in the calculated values of the LT buckling resistance. The EC3-1-1 allows to calculate with *0,4* as the maximum value of the $\bar{\lambda}_{LT,0}$ plateau. Therefore, designers do not have to count with reduction for LT buckling in the $\bar{\lambda}_{LT} < 0,4$ slenderness range. The standards recommend for choosing the β with minimum value of *0,75*. The values of β and $\bar{\lambda}_{LT,0}$ parameters are established in the National Annexes.

Beside the modified shape of the LT buckling curves the new, ‘Special Case’ method contains another important change regarding the original, ‘General Case’ procedure. This difference is that the type of the bending moment distribution of the examined beam is taken into account not only through the determination of the slenderness but in the calculation of the reduction factor also. To this, the EC3-1-1 defines a modifier f factor which carries the effect of the load distribution. Finally, using the f factor the LT buckling resistance of the examined member can be calculated with the following equations:

$$M_{b,Rd} = \chi_{LT,\,\text{mod}} \cdot W_{pl,y} \cdot f_y/\gamma_{M1} = \frac{\chi_{LT}}{f} \cdot W_{pl,y} \cdot f_y/\gamma_{M1} \qquad (7)$$

where the f factor:

$$f = 1 - 0,5 \cdot (1 - k_c) \cdot \left[1 - 2 \cdot \left(\bar{\lambda}_{LT} - 0,8\right)^2\right] \leq 1,0 \qquad (8)$$

In Eq. (8) k_c is a correction factor whose value depends on the bending moment distribution. The recommended formulae for the calculation of this factor are given in tables.

3 Revision of the Eurocode methods

After the introduction of the EN version of Eurocodes the scientific community had the opportunity to get to know and revise the given methods through the translation works and determination of the National Annexes. In a short time during this implementation phase the design of steel members against LT buckling became one of the most controversial parts of the standards. Up to now, several theses have been published which draw attention to the problems and open questions regarding the given design procedures for this structural behavior. Some of these observations:

- compared the numerical LT buckling curves as the results of geometrically and materially nonlinear imperfect (GMNI) analyses with the curves given in EC3-1-1 more or less large differences can be found between them; occasionally, the standard curves are on the unsafe side [6];
- using the 'General Case' method, it seems to be disadvantageous solution to take into consideration the type of the bending moment distribution only through the determination of the slenderness; this means that the effect of the plastic zone reduction belonging to different configurations is quite neglected which causes relevant underestimation of the LT buckling resistance;
- based on detailed examinations the appropriateness of the present grouping of profiles is questionable; namely, the separation of the cross-sections purely based on the height/width (h / b) ratio which does not properly represent the different behaviors [7];
- some theses focus on the requirement of the harmonization of design rules; however, the theoretical basis of the design methods for LT buckling does not fit properly to the principles of other stability problems (e. g. the flexural buckling of columns) [4], [8].

3.1 Evaluation of the Eurocode resistance model for LT buckling

The appropriateness and accuracy of the resistance model for LT buckling is evaluated in the [6] paper of Simões da Silva et al. For the examinations numerical LT buckling curves belonging to beams with 3 chosen profiles, different load distributions and boundary conditions were determined. These resistances were the results of GMNI calculations and they were compared with standard resistances from the two alternative methods. The main points of the summary of the evaluation [6]:

- The resistances calculated with the 'General Case' method are clearly on the safe side but they are generally over-conservative for non-uniform bending moment diagrams. It is important to emphasize that quality of a design method relies essentially on the low variance of its results. Based on the examinations it can be stated that the differences between the resistances of the 'General Case' procedure and numerical calculations show a high scatter. With this significant variance the uniform safety level cannot be guaranteed on the whole practical range despite the correction of the mean value with γ_{Rd} safety factor.
- Large number of the resistances calculated with the 'Special Case' method are on the unsafe side. However, the differences of the standard and numerical results show much lower variance than in the case of the 'General Case' procedure. Based on the findings of the examination the 'Special Case' procedure cannot be considered safe enough.

To solve the above problems the authors give a recommendation with the "union" of the two alternative design methods. According that, their proposal is to apply the 'Special Case' procedure with the f factor for taking into account the effect of the bending moment distribution of the beam. But, with $\bar{\lambda}_{LT,0} = 0,2$ and $\beta = 1$ values and the same buckling curves as for the 'General Case'. (Basically, this means the use of the 'General Case' method with f factor.) It has to be noted, that this methodology is already adopted by the Portuguese National Annex [6].

3.2 The theoretical background of the LT buckling check

Most of the publications dealing with the standard design methods for LT buckling of beams point on a theoretical contradiction as the most important problem. This contradiction arises from that the standard procedures use the flexural buckling curves for the calculation of the LT buckling resistances instead of the clarification of the theoretical background of this behavior. This means that the determination procedure of the LT buckling resistance uses the column buckling curves whose theoretical, Ayrton-Perry type formulae are derived for the flexural buckling behavior.

The essence of the Ayrton-Perry formula is that it defines the load intensity which belongs to the first yield of the member at the most compressed fiber. The starting condition of the determination of this formula is that the elastic member has geometric imperfection. In this way, the Ayrton-Perry formula does not take into account the possibility of plastic behavior and it neglects the effect of the residual stresses. Notwithstanding, this formula is a very popular model for the standard definition of the buckling resistances of steel members. These standards benefit from the simplicity and flexibility, but most of all from the clear mechanical background of this model. For the case of simple columns the formulae and their theoretical bases are properly determined. And, with a simple calibration of their chosen design parameters these equations are proved to be appropriate for the description of the flexural buckling curves based on test results.

In contrast with the compressed columns the mechanical background of the beams under bending was not described, the determination of the formulae for the LT buckling behavior was not solved. Recognizing this need Szalai and Papp give a possible solution of the Ayrton-Perry type description in [4]. The derived formulae are valid for the LT buckling of simple beams with I profiles. In [4] the authors introduce the Ayrton-Perry formula in a form appropriate for the basic equations of a new standard design method. As it was stated above, the main problem of the determination arose from the complexity of the LT buckling behavior. Namely, that it contains two kinds of deformation components: lateral deflection (v) and rotation (φ). According to Szalai and Papp, the key component of the determination is the choice of the proper condition for the initial geometry. In [4] it is proved that if the geometric imperfection of the examined member is chosen to be identical to the first eigenshape of the examined member the determination of the Ayrton-Perry formula becomes possible. This means that the condition for the ratio of the amplitudes of initial lateral deflection (v_0) and initial rotation (φ_0) is:

$$\frac{v_0}{\varphi_0} = \frac{M_{cr}}{N_{cr,z}} \tag{9}$$

where $N_{cr,z}$ is the elastic critical normal force belonging to the weak axis flexural buckling. After using this condition for the initial geometry of the simple beams the next step is the creation of the first yield criteria for the most compressed fiber at the middle of the beam.

According to Fig. 1, this criteria for prismatic beams with I profiles, end-fork boundary conditions and loaded by uniform major axis bending moment can be written in the following form:

$$\frac{M_y}{W_{el,y}} + \frac{M_z^{II}}{W_{el,z}} + \frac{B^{II}}{W_{el,\omega}} = f_y \tag{10}$$

In Eq. (10) $W_{el,y}$, $W_{el,z}$ and $W_{el,\omega}$ are the elastic major axis, minor axis and warping sectional modules of the cross-section respectively, M_y is the loading major axis bending moment, M_z^{II} and B^{II} are the second order minor axis bending moment and bimoment from the deformations of the beam.

The second order internal forces can be written as the function of the loading bending moment and the displacement components. With the introduction of the $\overline{\lambda}_{LT}$ slenderness and the χ_{LT} reduction factor for LT buckling:

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}} \quad \text{and} \quad \chi_{LT} = \frac{M_y}{W_y \cdot f_y} \tag{11}$$

after the transformations of the initial equation the second order form of the generalized Ayrton-Perry formula for LT buckling can be determined:

$$\chi_{LT} + \chi_{LT} \cdot \eta_{LT} \cdot \frac{1}{1 - \chi_{LT} \cdot \overline{\lambda}_{LT}^2} = 1 \tag{12}$$

where the η_{LT} imperfection factor:

$$\eta_{LT} = v_0 \cdot \frac{W_{el,y}}{W_{el,\omega}} + \varphi_0 \cdot \frac{W_{el,y}}{W_{el,z}} - \varphi_0 \cdot \frac{G \cdot I_t}{M_{cr}} \cdot \frac{W_{el,y}}{W_{el,\omega}} \tag{13}$$

In the equations G is the shear modulus of the material and I_t is the inertia for St. Venant torsional stiffness. The Ayrton-Perry formula written by Eq. (12) has the form similar to the solution of the column buckling stability problem. This proves the accuracy of the new formula for the description of the LT buckling behavior taking into account the new meaning of the imperfection factor. Solving the determined equations the LT buckling curves can be written in the well-known form of EC3-1-1:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \overline{\lambda}_{LT}^2}} \tag{14}$$

where

$$\phi_{LT} = 0,5 \cdot \left[1 + \eta_{LT} + \overline{\lambda}_{LT}^2\right] \tag{15}$$

This is the fundamental solution of the Ayrton-Perry formula based LT buckling curves which belongs to the first yield criteria and specifically chosen initial geometric imperfection. Nevertheless, these formulae detailed in [4] are not appropriate for design purposes in practice because they do not take into account the plastic behavior of the material and the effect of the residual stresses from manufacturing procedures. These formulae give appropriate basis for the development of a new standard procedure for LT buckling but a comprehensive probabilistic calibration is still needed. The main results of the research in [4] are the mathematical determination of the generalized Ayrton-Perry formula and the description of the mechanical background of the LT buckling behavior.

3.3 New proposal based on equivalent geometric imperfection

According to Naumes et al. the common definition of equivalent geometric imperfections could be the solution to harmonizing the standard procedures for the stability design of steel

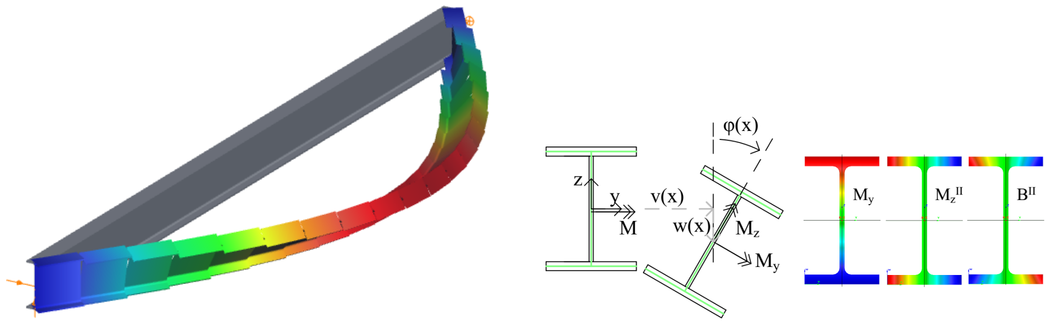

Fig. 1. Basic model for the determination of the Ayrton-Perry formula for LT buckling [12]

structural members [8]. The equivalent out-of-plane geometric imperfection which can represent all the effects of 2any geometric or material imperfections itself has to be defined with its shape, direction and amplitude as well.

The shape of the equivalent geometric imperfection has to be chosen identical to the critical flexural buckling or LT buckling mode which belongs to the lowest positive value of elastic critical load multiplication factors of the examined member. This equivalent initial geometry for the case of simple, prismatic columns under uniform compression is given in EC3-1-1 Part 5.3.2 (11). In this case the equivalent geometric imperfection (η_{init}) which can be described by the critical buckling mode belonging to the flexural buckling behavior:

$$\eta_{init} = \left[e_0 \cdot \frac{\alpha_{cr} \cdot N_E(x)}{E \cdot I(x) \cdot \eta''_{crit}(x)} \right]_{x=x_d} \cdot \eta_{crit}(x) \qquad (16)$$

where η_{crit} is the first eigenshape belonging to the flexural buckling, α_{cr} is the appropriate critical load amplifier, $I(x)$ and $N_E(x)$ are the function of the cross-sectional inertia and the diagram of the loading normal force along the member length respectively, x is the coordinate along the length, x_d is that cross-sectional coordinate where the in-plane loads and out-of-plane imperfections produce the maximum effect together, and e_0 is the amplitude of the equivalent geometric imperfection which is given in standards. The x_d location is usually called critical cross-section or design location which belongs to the maximum displacement of the eigenshape in the case of simple columns. Taking into account the amplitude of the equivalent geometric imperfection (e_0), the normal force (N_E) and the first-order bending moment ($N_E \cdot e_0$) the second order bending moment can be calculated. With these the utilization of the critical cross-section can be determined which is appropriate for the evaluation of the resistance of the whole examined member. The resistance of the column meets with the requirements when:

$$1 \geq \frac{N_E}{N_R} + \frac{N_E \cdot e_0}{M_R} \cdot \frac{1}{1 - \frac{N_E}{N_{cr}}} \qquad (17)$$

In Eq. (17) N_R and M_R are the cross-sectional normal force and bending moment resistances respectively.

The authors propose a solution also for the determination of the LT buckling resistance of beams, similarly as in the case of columns. The proposed equivalent geometric imperfection is based on a derivation procedure analogous to the case of simple columns. The LT buckling type first eigenshape of simple beams and therefore the equivalent initial shape as well can be characterized by the lateral deflection (η_{crit}) and the rotation (φ_{crit}) components, see Fig. 2.

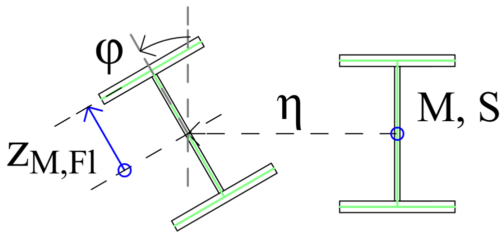

Fig. 2. The deformation components of LT buckling

This solution detailed in [8] handles the LT buckling behavior as the flexural buckling of the upper flange. This approach has a great benefit. Namely, the behavior of the members in bending can be converted into the flexural buckling of columns. Therefore, through the determined "column-like" formulae the standard parameters given for the case of simple columns can be applied for beams also. With these initial conditions the equivalent geometric imperfection for the LT buckling behavior can be written in the following form for the most utilized upper flange:

$$\eta_{init,Fl} = \left[e_0^* \cdot \frac{\alpha_{cr} \cdot N_{E,Fl}(x)}{E \cdot I_{Fl} \cdot \left(\eta''_{crit} + z_{M,Fl} \cdot \varphi''_{crit}\right)} \right]_{x=x_d} \cdot \left(\eta_{crit} + z_{M,Fl} \cdot \varphi_{crit}\right) \qquad (18)$$

where $\eta_{crit,Fl} = \eta_{crit} + z_{M,Fl} \cdot \varphi_{crit}$ is the total lateral deflection of the examined upper flange due to the LT buckling. According to the initial conditions, in this case the amplitude of the equivalent geometric imperfection (e_0^*) can be written in a form similar to the simple column case:

$$e_0^* = \frac{M_{R,Fl}}{N_{R,Fl}} \cdot \alpha^* \cdot \left(\bar{\lambda} - 0,2\right) \qquad (19)$$

In Eq. (19) $M_{R,Fl}$ and $N_{R,Fl}$ are the characteristic values of the resistances of the critical compression flange to weak axis bending moment and normal force respectively, $\bar{\lambda}$ is the slenderness of the member, and α^* can be calculated from the α imperfection constant whose values are given in standards based on column buckling tests:

$$\alpha^* = \frac{\alpha^*_{crit}}{\alpha_{crit}} \cdot \alpha \tag{20}$$

This reduction of the α imperfection constant makes it possible that the standard values of α belonging to the column buckling case can be applied for the LT buckling behavior also. The modification means nothing more than making the LT buckling behavior "column-like" through the neglect of the St. Venant torsional stiffness of the structural member. In Eq. (20) α_{crit} and α^*_{crit} are the critical load amplifiers with and without taking into account the It torsional stiffness. The authors provide a diagram in [9] to help the calculation of this reduced α^* constant.

The above formulae can be applied through practical design only when the x_d location of the critical (or design) cross-section is known. Namely, the resistance of the structural member has to be evaluated at the $x = x_d$ design point. However, this critical cross-section is generally unknown. Therefore, the authors propose equations for the determination of the x_d location which are based on the results of numerical simulations and are given in a tabular form. Creating the condition for the resistance evaluation of the critical compressed flange at $x = x_d$ which is known by now:

$$1 \geq \frac{N_{E,Fl}}{N_{R,Fl}} + \frac{M_{E,Fl}}{M_{R,Fl}} \tag{21}$$

After modifying the initial equations the formulae for the definition of the stability curves can be determined in the well-known form of the standards. Then, the χ reduction factor is:

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}} \leq 1,0 \tag{22}$$

where

$$\phi = 0,5 \cdot \left[1 + \alpha^* \cdot \left(\overline{\lambda} - 0,2\right) + \overline{\lambda}^2\right] \tag{23}$$

In the formulae α^* marks the imperfection constant for the "column-like" LT buckling, and $\overline{\lambda}$ describes the slenderness belonging to the design location:

$$\overline{\lambda}(x_d) = \sqrt{\frac{\alpha_{ult,k}(x_d)}{\alpha_{crit}}} \tag{24}$$

So, after the determination of the χ reduction factor the stability resistance of the examined structural member can be evaluated. In this case the requirement:

$$\frac{\gamma_{M1}}{\chi \cdot \alpha_{ult,k}(x_d)} \leq 1,0 \tag{25}$$

where $\alpha_{ult,k}(x_d)$ is the multiplication factor for the compression force in the relevant flange to reach the characteristic value of the resistance.

Using the above formulae we have a design method for the case of beams under pure bending. It has to be noted that the theoretical basis of the introduced method is identical with the Ayrton-Perry type solution detailed in *Section 3.2*. Namely, the transformation of the formulae applied here leads to the same equations for the case of simple beams. To determine a design method from this theoretical basis the authors chose the solution to handle the LT buckling behavior as the flexural buckling of the compressed upper flange. Thus, the equations of EC3-1-1 as the results of previous calibration process can be applied for the calculation of the imperfection factor. So, a design method for the case of simple beams is given. For the extension of this method for the case of beams with various boundary conditions and load distributions the chosen solution is the methodology based on the design cross-section. For the determination of this location and then for the evaluation according to this design point the authors propose the above formulae.

3.4 New LT buckling curves for beams under pure bending

To solve the problems of the standard design methods for LT buckling Greiner and Taras found it necessary to develop a new procedure which is appropriate for code amendments. Through the evaluation of the current rules they established that the applied mechanical background does not describe properly the LT buckling behavior of beams. Therefore, the authors determined the correct description of this stability problem which is able to avoid the previous contradictions in the EC3-1-1. Similarly to Szalai and Papp, they also chose the Ayrton-Perry type solution as a proper basis for the new design method. Actually, Greiner and Taras in [7] give a determination procedure similar to the solution in [4] and as the final result they introduce the same Ayrton-Perry formula shown in *Eq. 12*. The difference in this new solution can be found in the description of certain parameters. With these definitions and simplifications the authors give the imperfection factor belonging to the LT buckling behavior in the following form [7]:

$$\eta_{LT} = \frac{A \cdot e_0}{W_{el,z}} \cdot \frac{\lambda^2_{LT}}{\lambda^2_z} \tag{26}$$

where $e_0 = v_0 + \varphi_0 \cdot h/2$ is the total lateral deflection of the most utilized fiber, i.e. the maximal displacement of the member. Nevertheless, the η_{LT} factor in this theoretical form is inappropriate for design procedures because of the neglect of several effects (e.g. the plastic behavior and residual stresses). Therefore, the authors carried out a comprehensive calibration procedure based on the results of numerous GMNI analysis. Finally, the proposed, calibrated expression for the calculation of the imperfection factor is:

$$\eta_{LT} = \alpha_{LT} \cdot \left(\overline{\lambda}_z - 0,2\right) \cdot \frac{\overline{\lambda}^2_{LT}}{\overline{\lambda}^2_z} \tag{27}$$

For this calculation the numerically determined values of α_{LT} imperfection constant for hot-rolled and welded I profiles are given in [11].

According to Taras' examinations it can be stated that the present standard procedures with this new definition of the imperfection factor can properly follow the behavior of beams with uniform bending moment distribution. So, the calculated resistances are in good agreement with the numerical results [10]. However, when the load distribution of the beam is changed (e.g. to linear moment diagram or distributed loading) the given formulae cannot describe appropriately the numerically determined behavior. For these cases, Taras proposes the application of a new, additional design parameter which is marked with φ and its value is called "over-strength" factor in [10]. Essentially, this new factor helps to characterize the bending moment distribution and to take into account the differences between the LT buckling curves belonging to the different load distributions. For the calculation of this new parameter Taras proposes such equations which are fitted to numerical results. These equations are given in tabular form for the different configurations in [11].

As the result of an extensive research work the proposal of a novel design method reached completion. This proposed procedure was introduced on a TC8 session in 2012 by Taras and Unterweger. The report about the presentation can be found in [11]. This proposed new method aims the replacement of the present standard procedures for the check of LT buckling. Similarly to the current methodology, it starts with the definition of the $\overline{\lambda}_{LT}$ slenderness, see in Eq. (4). After determining this slenderness, based on the above parameter definitions and formulae the χ_{LT} reduction factor for LT buckling can be calculated:

$$\chi_{LT} = \frac{\varphi}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \varphi \cdot \overline{\lambda}_{LT}^2}} \leq 1,0 \tag{28}$$

where

$$\varphi_{LT} = 0,5 \cdot \left[1 + \varphi \cdot \left(\frac{\overline{\lambda}_{LT}^2}{\overline{\lambda}_z^2} \alpha_{LT} \cdot \left(\overline{\lambda}_z - 0,2\right) + \overline{\lambda}_{LT}^2\right)\right] \tag{29}$$

In the above equations the α_{LT} imperfection constant and the φ "over-strength" factor can be determined according to [11]. Finally, knowing the value of the reduction factor the LT buckling resistance of the examined structural member can be calculated:

$$M_{b,Rd} = \chi_{LT} \cdot W_{pl,y} \cdot f_y / \gamma_{M1} \tag{30}$$

In summary, the code amendment of Taras et al. is based on the Ayrton-Perry formula determined for the LT buckling behavior. To have a design procedure for the case of simple beams the authors give a calibrated expression for the calculation of the imperfection factor. Using this proposal with the formulae from the Ayrton-Perry type solution we have a simple calculation procedure to determine the LT buckling resistances. To extend this method for a wide range of beam configurations the authors give additional factors based on the results of numerical simulations. It has to be stated that this proposed method follow properly the stability behavior of the beams subjected to pure bending thanks to the appropriate mechanical background and the comprehensive calibration procedure.

4 New design method for LT buckling of beams

As it was mentioned above, through the revision of the Eurocode methods for LT buckling the scientific community drew attention to several problems and open questions. Maybe the most important statement is that the theoretical basis of these given procedures is not acceptable because the mechanical background of the stability behavior of members subjected to bending is not clarified. Perhaps, this is the reason why the LT buckling curves defined by the standard methods cannot properly follow the experimental and numerical results. Therefore, the given procedures in EC3-1-1 do not describe appropriately the LT buckling behavior of beams. Realizing these problems, some researches are dealing with the development of a proper design methodology for the determination of the LT buckling resistance.

In this paper, two new proposals are introduced for the determination of the LT buckling resistance of members. Both of them are based on an Ayrton-Perry type theoretical solution but they use different adaptations. The above detailed proposal of Naumes et al. applies a basic method for simple beams with the calibration of the imperfection factor. Here, the LT buckling behavior is handled as the flexural buckling of the compressed upper flange. The extension of this procedure is based on the knowledge of the x_d design location where the authors give numerically confirmed equations for its determination. However, the presentation of the widespread validation for this method is still needed. The secondly detailed proposal from Taras et al. also uses the Ayrton-Perry formulae with a calibrated equation for the imperfection factor. But, for the extension of this basic method it requires additional factors fitted to numerical results to be able to follow the behavior of different beam configurations. Nevertheless, the physical explanation of these factors is not complete or perfectly clarified. At the same time, the modern intentions focus on the development of such methods which are suitable to be applied in computer aided design procedures. However, this kind of utilization of the above mentioned procedures has some difficulties through the tabular definition of the parameters.

Following the modern intentions, we started a new research on the development of a general type stability design methodology for beams which is appropriate for computer-aided design work also. For the theoretical basis of this new procedure we chose the generalized Ayrton-Perry formula detailed in *Section 3.2* which is determined for the LT buckling behavior of beams under bending. To determine an appropriate design method for the case of simple beams a calibration procedure was carried out. As the result, contrary to the above detailed proposals we give a calibrated expression for the measure of the imperfection instead of the imperfection factor. This makes possible the more accurate description of the LT buckling behavior. Taking into account the calibrated equation and using the above detailed Ayrton-Perry formulae an appropriate design method is given for the simple beams. The calibration process and this new proposal for design procedure is detailed in *Section 4.1*. The numerical model used for the determination of LT buckling resistances of beams needed to the evaluations is introduced in *Section 4.2*. To extend the proposed basic procedure for different beam configurations our idea was to use such methodology where the stability check of the structural members is reduced to the evaluation of specific cross-sections. This makes possible the general application of the method. The appropriateness and applicability of this new proposal has been verified for prismatic beams under pure bending. This new segmental methodology as well as the details of the determination and application are introduced in *Section 4.3*.

4.1 New design method for simple beams

As it is mentioned above, the theoretical basis of the new design method is the generalized Ayrton-Perry formula determined for simple beams. However, these equations in the form introduced in [4] are not appropriate for the determination of the resistances of beams. The reason is that the formulae define the load-carrying capacity equal to the first yield of the member and do not take into account several effects, e.g. the plastic behavior of the beam and the effect of residual stresses. To make these theoretical equations applicable for the design of simple beams we started the development of the new method with widespread deterministic calibration of the Ayrton-Perry type formulae. Henceforward, the prismatic beams with I profiles, end-fork boundary conditions, loaded by uniform bending moment distribution (which is the basic model of the theoretical determination) are called reference models.

The database needed for the deterministic calibration is created by the results of GMNI analyses carried out in ANSYS software using the numerical model detailed in *Section 4.2*. For the numerical test program 20 hot-rolled I profiles were chosen where each of them belongs to the compact sections of class 1 or 2. With the chosen profiles 7 different long beams were modeled where the L member lengths were determined belonging to discrete values of $\bar{\lambda}_z$ relative slenderness for weak axis flexural buckling. In the test program, to take into account an appropriately wide variation of the behavior the values of $\bar{\lambda}_z$ were defined in the range of *0,3 - 3,0*. The material grade of the members was S235, with yield strength *235 N/mm*^{2}. As the result of the numerical tests the LT buckling resistances of 140 reference model beams were determined.

The final results of the GMNI analyses were the $M_{b,Rd}$ LT buckling resistances of the tested beams. From these values, using Eq. (11), (14) and (15) the η_{LT} imperfection factors were calculated. Then, based on Eq. (9) and (13) the v_0 amplitudes of the lateral deflections were determined. It is important to note that the cross-sectional properties in the above formulae were

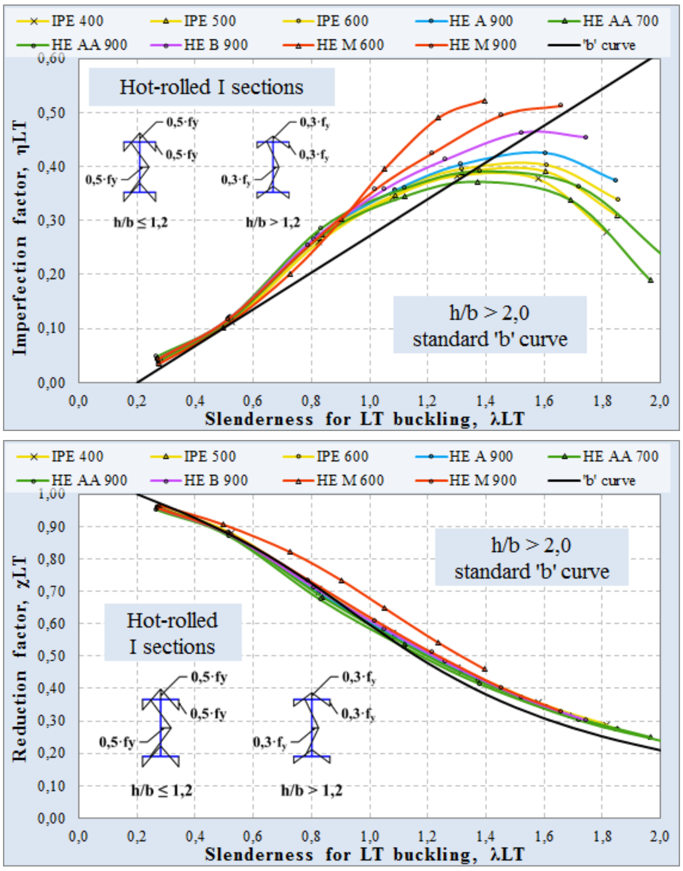

Fig. 3. Comparison of the numerical results to standard a - imperfection factors and b - LT buckling curves

calculated belonging to the plastic behavior allowed for cross-sectional class 1 and 2. At first, it was investigated how the application of a linear function of the $\bar{\lambda}_{LT}$ slenderness for η_{LT} imperfection factor affects the final results. This is how it happens in EC3-1-1. For this investigation the calculated η_{LT} values were grouped based on the cross-sectional geometry according to the Eurocode rules, as it is shown in Fig. 3. In diagrams of the hot-rolled profiles with $h/b > 2$ it can be seen that the numerical results show various behavior. So, substituting the got curves with one linear relation would cause relevant neglect. This can be observed by the comparison of the numerical results to the calculated standard LT buckling curves also. The resistance curve given by EC3-1-1 does not fit properly to the behavior of the different profiles.

Evaluating the possible ways of the calibration it was stated that the determined form of the imperfection factor in Eq. (13) carries such a real physical meaning which gives the opportunity to follow uniquely the behavior of each cross-sections. So, this determined formula has a very important benefit regarding the novel method which has to be taken into account. Therefore, through the calibration procedure the definition of the imperfection factor was left in the original form and the imperfection amplitudes in it were chosen as the basic of the calibration. According to the results of examination the $L\,/\,v_{FI}$ (member length / total lateral deflection of the midpoint of upper flange) ratios proved to be most appropriate for the determination of the calibration equation. The $v_{Fl} = v_0 + h / 2 \cdot \varphi_0$ values which characterize the measure of the imperfection were calculated from the numerical resistances. These results were plotted in diagrams over the slenderness for LT buckling, shown in Fig. 4.

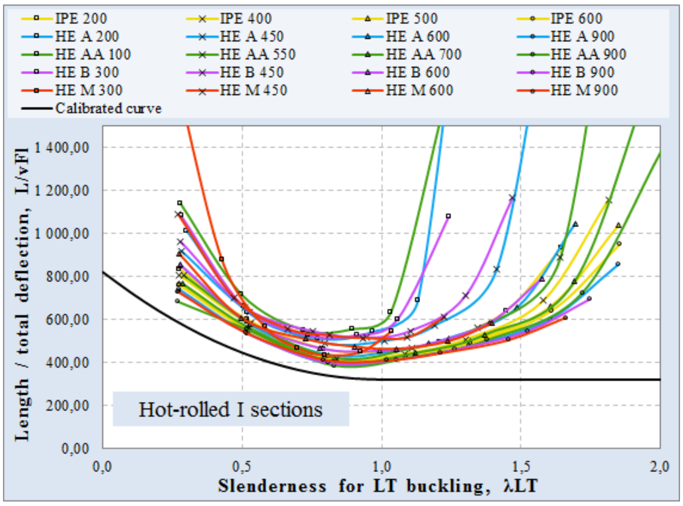

Fig. 4. L / v_{Fl} values and the calibrated curve for hot rolled profiles

For the results collected in the above diagram a bottom covering curve was fitted (shown with black solid line). Through the determination of this curve the main aspect was to ensure the accurate enough fit on the practical slenderness range. To this, the choice of a nonlinear function became reasonable for the members with $\bar{\lambda}_{LT} < 1,0$. In the range of higher slenderness a constant value is proposed for L / v_{Fl} ratios. It has to be noted for this constant value, that the difference between the numerical values and the calibrated curve does not cause significant errors. Finally, the proposed, calibrated formula for the calculation of the L / v_{Fl} ratios of hot-rolled profiles:

$$\frac{L}{v_{Fl}} = \begin{cases} 500 \cdot (\bar{\lambda}_{LT} - 1,0)^2 + 320 & \text{if} \quad \bar{\lambda}_{LT} < 1,0 \\ 320 & \text{if} \quad \bar{\lambda}_{LT} \geq 1,0 \end{cases} \tag{31}$$

Using the above introduced, calibrated equation and applying the Ayrton-Perry based formulae written by Eq. (9) and (13), (14), (15) we have a new method for the determination of the LT buckling resistance of beams belonging to the reference model. The method has benefits from the simple, clear theoretical background which is determined for the LT buckling behavior. Furthermore, a very important advantage arises from the application of the theoretical form of imperfection factor through taking into account its physical meaning. Therefore, it makes possible to properly follow the different behaviors of the profiles through unique LT buckling curves depending on the cross-sectional properties. For the results of the new method Fig. 5 shows an example. In this diagram the GMNIA results (solid lines) are also drawn to be able to evaluate of the appropriateness and accuracy of the curves calculated with the proposed procedure (dashed lines).

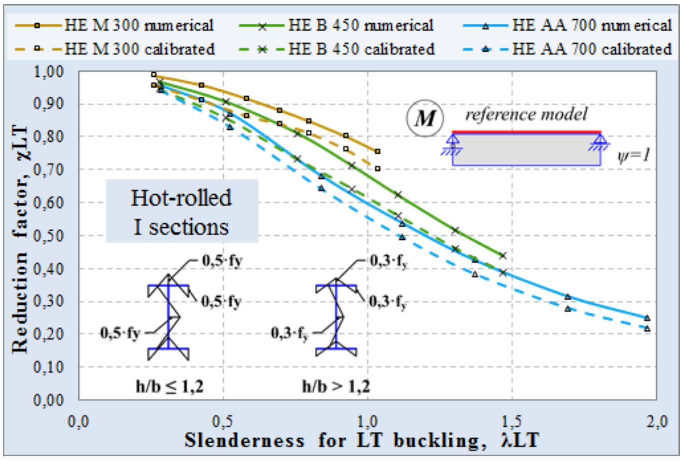

Fig. 5. Numerical and calculated LT buckling curves for hot rolled profiles

4.2 The numerical model

For the development of the new design method for beams numerical LT buckling resistance values were taken into consideration. These load carrying capacities of steel members subjected to bending with different load distributions and boundary conditions were determined by GMNI analyses of shell finite element models carried out in ANSYS software. The model of members were constructed with 4-node, SHELL181 type finite strain shell elements, which can model the nonlinear behavior. The material behavior of the beams was modeled with linear elastic-ideally plastic material model with $E = 210$ GPa Young-modulus and yield criterion belonging to the standard yield strength of the material grade.

In the following of this paper the case of beams with hot-rolled, I-shaped profiles is examined and detailed. These kinds of sections were modeled with simplified cross-section where the web-to-flange zone received specific treatment. Through the shell finite element modeling of the profiles this region includes an overlap of material and disregards the so-called "flange radius" areas. In order to get closer to the real characteristics of such steel cross-sections the finite elements of the web at the web-to-flange zones are modified, see in Fig. 6. To determine the two parameters: height and width of these elements two special conditions were used. First, the cross-sectional area of the special element has to be equal to that of the radius zones plus the substituted web section (substituted section) area. The other condition is that the center of gravity of this element within the web height has to be at the exact vertical position of the centroid of the radius zone. Using these conditions the height and the width of the special finite element can be determined. With this construction the profiles whose "radius zones minus the overlapped" area becomes negative (typically the HEM profiles) also can be handled, through the definition of an element whose thickness is lower than the webs.

The steel members were modeled with special cross-sections at the ends for the application of the boundary conditions. In this construction every node of the two end cross-sections were connected to a master node, see in Fig. 6. This made possible

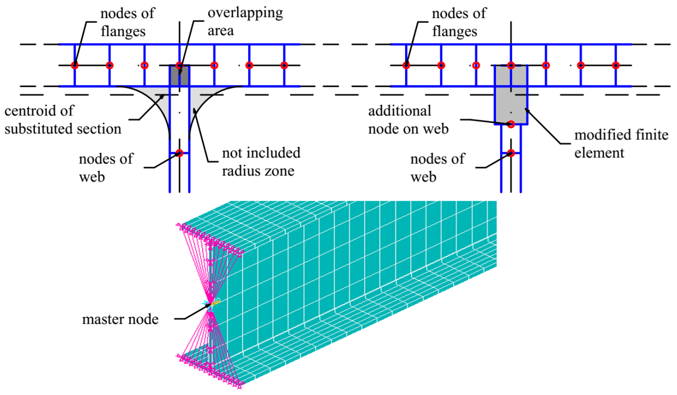

Fig. 6. FE model of beams

the specification of the boundary conditions of warp and different types of supports on one node, and at the same time the avoidance of the numerical errors arising from the concentrated conditions. The bending moment type loads of beams were defined in form of stresses on the lines of end-sections.

The model of the members was defined with geometric and material imperfections. The imperfect geometry of the beams was affine to the first eigenshape of the beams under uniform bending. The amplitude of the geometric imperfection was defined with $L/1000$ value for the maximum lateral displacement of the midpoint of upper flange. The material imperfection was modeled by triangular residual stress distribution, see in Fig. 7. The amplitude of the stresses was defined by the specification of the maximum compression stress at the top of the flanges.

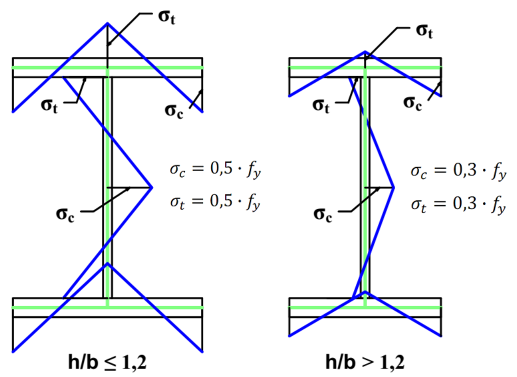

Fig. 7. Residual stress model

4.3 New design method for LT buckling of various beam configurations

In *Section 4.1* a new design method was introduced for the determination of the LT buckling resistance of beams belonging to the reference model. Using its theory and calibrated expressions we started to develop a general type procedure. The generality here means that the same methodology can be applied for the determination of the resistances of beams with arbitrary boundary conditions and load distributions. Before the development of this new method preliminary examinations were carried out to compare and evaluate the behavior of different beam configurations. To this, the results of GMNI analyses of the above detailed ANSYS model were taken into account.

For the examinations a widespread numerical test program was carried out on beams under bending with different boundary conditions and load distributions. The 20 hot-rolled profiles used for the reference model were chosen here also. The examined configurations were built with:

- end-fork (without or with prevented end-warp) or clamped boundary conditions and
- linear bending moment distribution with the end-moment ratios from $\psi = 1$ to $\psi = -1$ ($\psi = 1$; 0,75; 0,5; 0,25; 0; -0,25; -0,5; -0,75 and -1) or uniform load distribution.

Combining these alternatives the LT buckling resistances of beams with different lengths were determined. The member lengths were defined belonging to specific values of $\bar{\lambda}_z$ relative slenderness which were chosen from the range of: $\bar{\lambda}_z = 0{,}3 - 3{,}6$. The material grade of the tested members was S235, with yield strength $235\ N/mm^2$. From the given LT buckling resistances the reduction factors for LT buckling were calculated using the definition in Eq. (11). For these results some examples can be seen in Fig. 8. In the diagrams the numerical LT buckling curves are shown for beams with different configurations.

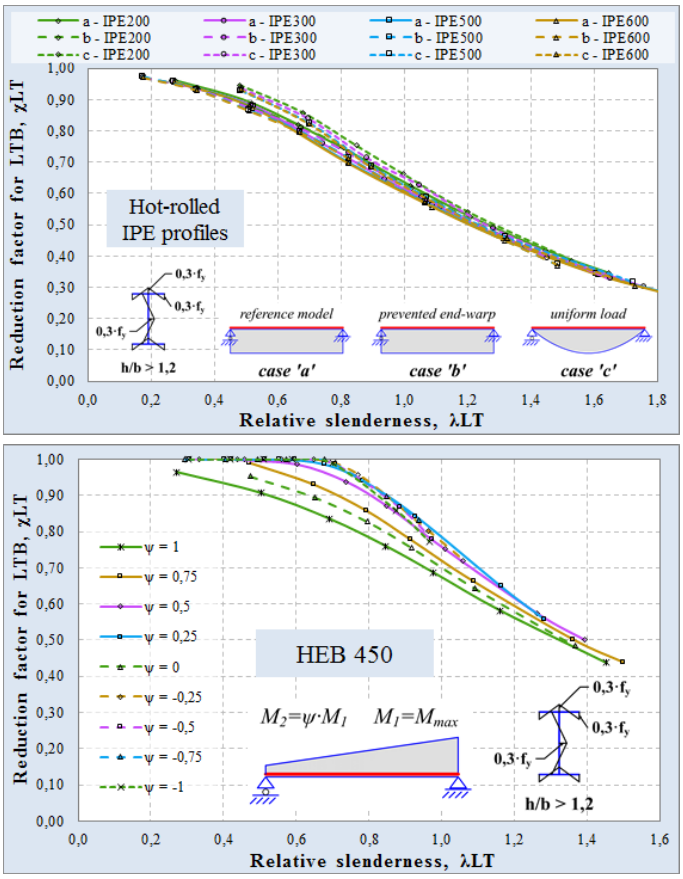

Fig. 8. Numerical LT buckling curves for beams with different configurations

In Fig. 8a the LT buckling curves of symmetric distributions are drawn. According to the diagram it can be stated that the indicated behaviors are quite similar. Namely, these LT buckling curves are very close to each other. The differences between them are caused by the variance of the plastic zones of the examined beams which arises from the bending moment diagrams. However, if the load distribution or the boundary conditions are changed asymmetrically we get significantly different behaviors. In Fig. 8b some examples can be seen for this variance of the LT buckling curves belonging to different beam configurations.

Comparing our examinational results to the statements of Naumes et al. we came to the conclusion that the main reason of the differences between the behaviors arise from the variance of the critical (design) location of x_d. In the case of symmetric configurations definitely the cross-section belonging to the middle of the member length ($x_d = L/2$) is the critical. Because, taking into account the bending moment distribution and the buckled shape (the eigenshape) their total effect has its maximum at this location, see Fig. 9a. (Except that for very short beams with clamped ends the $x_d = 0$ end-sections can be the critical through the very large bending moments.) However, in the case of asymmetric configurations this critical cross-section shifts from the middle. E.g. for a beam with end-fork boundary conditions and loaded at one of its ends (i.e. it has a triangular moment distribution) this design location does not belong neither to the maximum bending moment nor the amplitude of the buckled shape. It is somewhere between these two specific cross-section, see in Fig. 9b.

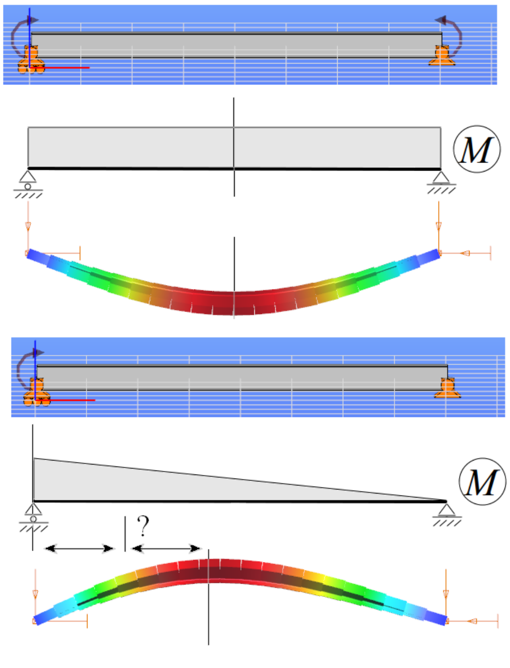

Fig. 9. The demonstration of the x_d design location for a - symmetric and b - asymmetric beam [12]

To develop an optimal design procedure, the resistance of the beam has to be evaluated at the design cross-section. For the determination of this location the proposed procedure of Naumes et al. detailed in [8] can be applied. Nevertheless, that method gives solution only for a limited number of specific configuration and its parameters are defined by numerical results in a tabular form. Therefore, it cannot be applied generally for arbitrary conditions and it is difficult to use in computer-aided design work through the tabular definitions. Using the basic idea of Naumes et al. we developed a practical application of his method. This means, that our new, general type procedure is able to find the design location and it determines the LT buckling resistance of the examined beam belonging to this critical cross-section.

The basic idea here for the determination of the design cross-section is the use of a segmental type methodology. According that, at the first step of the procedure the examined member is divided into an appropriate number of segments of equal length and each cross-section specified by the division is evaluated. Then, as the result of the complete evaluation the critical location can be chosen and according to this cross-section the resistance of the member can be calculated. The basis of the cross-sectional evaluation is the following definition of the slenderness:

$$\bar{\lambda}_{LT,i} = \sqrt{\frac{\alpha_{ult,i}}{\alpha_{cr}}} = \sqrt{\frac{W_{pl,y} \cdot f_y / M_{y,Ed,i}}{\alpha_{cr}}} \qquad (32)$$

where α_{cr} is the minimum load amplifier to reach the elastic critical bending moment resistance of the member, $M_{y,Ed,i}$ is the bending moment in the i-th cross-section, therefore, $\alpha_{ult,i}$ is the minimum load amplifier to reach the characteristic resistance at the i-th cross-section. With the above definition the meaning of the slenderness can be generalized. Therefore, in this form it does not belong merely to the geometrical characteristic of the whole member. With the Eq. (32) definition the slenderness is interpretable for the cross-sections based on their utilization.

After dividing the member and defining the cross-sectional slenderness values the final aim of the calculation methodology is to find the critical cross-section. As it was mentioned before, this cross-section belongs to that location where the bending moment distribution and the buckled shape have together the maximum cross-sectional utilization. To find this cross-section we utilized the Ayrton-Perry formula based method which was detailed in the previous section for the case of the reference model. If the cross-sectional slenderness defined by Eq. (32) is interpreted as the slenderness of an equivalent reference model member for this virtual beam the χ_{LT} reduction factor can be calculated by the Ayrton-Perry formula based method. (i.e. the equivalent members are simple beams whose slenderness value is identical to the given cross-sectional slenderness, see *Part 1* in Fig. 10.)

In that case, when the Ayrton-Perry formula based method is carried out separately for each evaluated cross-sections it is not taken into account that these cross-sections belong to a “global” behavior, to the whole examined member. Therefore, through the procedure they have to be “connected”. To this, the $\eta_{LT,eq,i}$ values determined for the chosen cross-sections through the calculation methodology are normalized with v_i / v_{max} ratios where

v_i is the lateral deflection of the i-th cross-section and v_{max} is the amplitude of the buckled shape.

This weighting is based on the typical LT buckling shape, i.e. the eigenshape of the member, see *Part 2* in Fig. 10. For these modified values of the $\eta_{LT,i}$ imperfection factor the $\chi_{LT,i}$ reduction factors can be calculated by the further steps of the Ayrton-Perry formula based methodology. According to the determined values of the reduction factor a LT buckling resistance can be calculated for each chosen cross-sections in the following way:

$$M_{b,Rd,i} = \alpha_{ult,i} \cdot \frac{\chi_{LT,i}}{\gamma_{M1}} \cdot M_{y,Ed} = \frac{W_{pl,y} \cdot f_y}{M_{y,Ed,i}} \cdot \frac{\chi_{LT,i}}{\gamma_{M1}} \cdot M_{y,Ed} = = W_{pl,y} \cdot f_y \cdot \frac{\chi_{LT,i}}{\gamma_{M1}} \cdot \frac{M_{y,Ed}}{M_{y,Ed,i}} \tag{33}$$

Choosing the minimum value of these load carrying capacities we get the LT buckling resistance of the examined member, see *Part 3* in Fig. 10. And finally, the cross-section which the minimal resistance belongs to that is the critical cross-section (design location).

4.4 The calculation methodology of the new design procedure

As it was written previously and according to the Fig. 10 the main steps of the cross-sectional evaluation according to the new design method:

1 The geometrical properties, boundary conditions and load distribution of the examined beam are known.

2 The α_{cr} multiplication factor has to be determined dividing the M_{cr} elastic critical bending moment of the examined member by the $M_{y,Ed,max}$ maximal value of the loading bending moment. This α_{cr} is valid (one value) for the whole member.

3 The examined member has to be divided into the appropriate number of segments. In our examinations the beams were divided into 20 segments. This meant the evaluation of 21 specific cross-sections.

4 The $\alpha_{ult,i}$ multiplication factors have to be determined for each specific cross-section dividing the $M_{c,Rk}$ characteristic cross-sectional bending resistance by the $M_{y,Ed,i}$ cross-sectional bending moment load. So, the load distribution of the beam is taken into account.

5 Based on the α_{cr} and $\alpha_{ult,i}$ factors the $\bar{\lambda}_{LT,i}$

6 slenderness values can be determined for each chosen cross-section. According to these slenderness values an equivalent, virtual reference member has to be defined for each examined cross-sections which has the same slenderness value.

7 Using the calibrated Eq. (31) with Eq. (9) and Eq. (13) the $\eta_{LT,eq,i}$ generalized imperfection factors have to be calculated for the virtual members belonging to the specific cross-sections.

8 Based on the lateral-torsional buckling type eigenshape of the member the v_{max} maximal and v_i cross-sectional lateral displacements can be calculated. Using the v_i/v_{max} weights the effect of the eigenshape is taken into account.

9 Multiplied the $\eta_{LT,eq,i}$ cross-sectional values with the v_i / v_{max} weights, through Eq. (15) and Eq. (14) the $\chi_{LT,i}$ cross-sectional reduction factors can be calculated.

10 Based on $\alpha_{ult,i}$ and $\chi_{LT,i}$ values an $M_{b,Rd,i}$ *LT* buckling resistance can be determined for each chosen cross-section using Eq. (33).

11 Choosing the $M_{b,Rd,min}$ minimum value of the LT buckling resistances belonging to the cross-sections the resistance of the examined member is got: $M_{b,Rd} = M_{b,Rd,min}$. Nevertheless, the cross-section belonging to the minimum value of the resistances means the critical cross-section.

4.5 Evaluation of the results of the new method

For the evaluation of the new method the LT buckling resistances determined by the cross-section based calculation were examined in different aspects. To this, the load-carrying capacities of numerous beams with various configurations were calculated which results were compared to the results of GMNI analyses in ANSYS software as well as to the resistances determined by the given standard procedures. For the examinations these different kinds of values for the given structural members were plotted on diagrams. In the following, some of these figures are introduced and evaluated.

In Fig. 11 two diagrams are shown for the evaluation of the accuracy of the new method. To this, the LT buckling resistances determined by GMNI analyses (with solid lines) and calculated by the new method (with dashed lines) are compared. Here, the results belonging to beams with end-fork boundary conditions are introduced for 3 different load cases:

- case 'a': uniformly distributed loading
- case 'b': linear bending moment distribution with the end-moment ratios of $\psi = 0$ and
- case 'c': linear bending moment distribution with the end-moment ratios of $\psi = -1$.

For the representation of the examined results the LT buckling curves belonging to two profiles: HEB 900 and IPE 500 were chosen. In the diagrams in Fig. 11 it can be seen, that the curves calculated by the new method follow properly the various behaviors from the different load distribution. The two diagrams show an example for how the new method can benefit from the generalized form of the imperfection factor. Using the above detailed relation unique LT buckling curves can be determined for the different cross-sections which makes possible to follow more properly the behavior of the various configurations.

The LT buckling curves determined by the new method were compared to the curves belonging to the EC3-1-1 'General

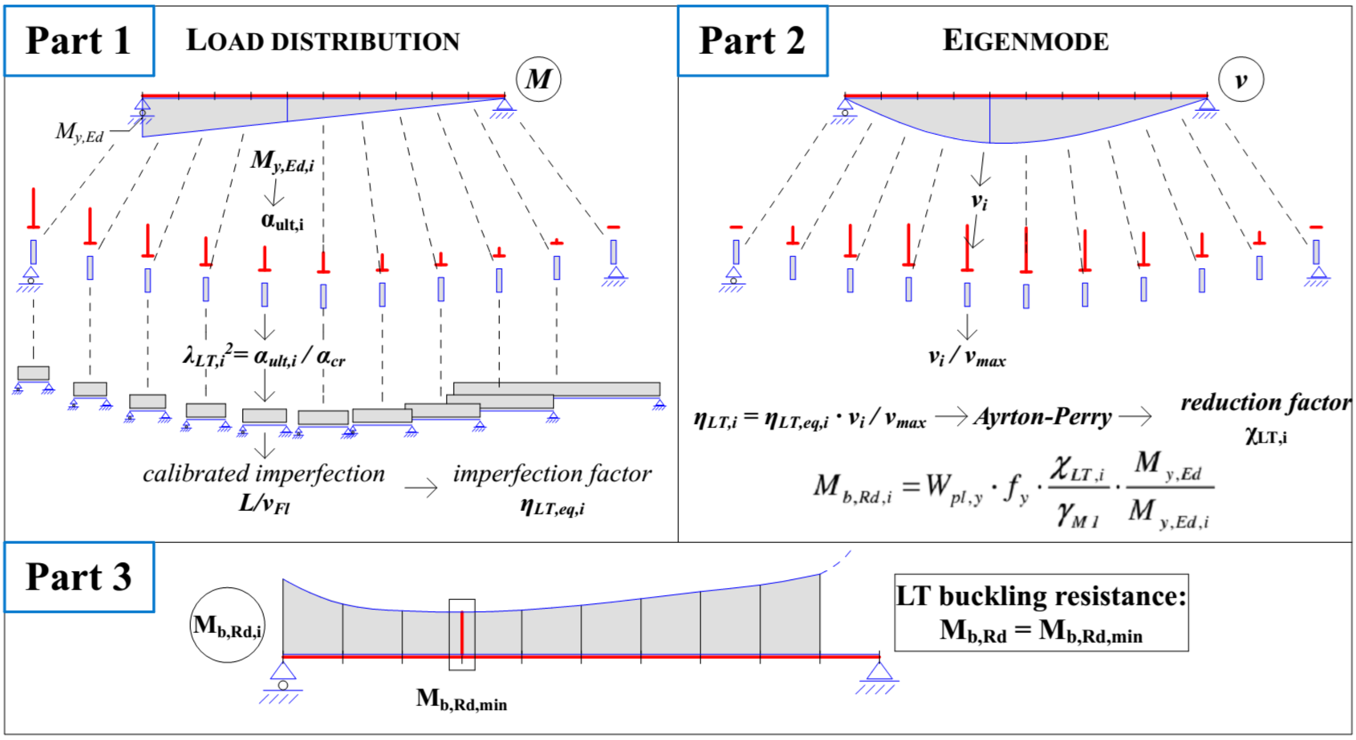

Fig. 10. The main steps of the new, general type segmental methodology

Case' and 'Special Case' procedures. For this evaluation two diagrams are given in Fig. 12 where these different curves are plotted. The results are shown for two different beam configurations:

- case 'a': beam with one hinged and one clamped end, with HEA 450 profile, under uniformly distributed loading
- case 'b': beam with end-fork boundary conditions, with HEA 900 profile, under linear bending moment distribution with the end-moment ratios of $\psi = 0$.

In the diagrams it can be seen that the standard LT buckling curves do not appropriately follow the behavior of the examined beams, i.e. very large differences can be found between the numerical and the calculated resistances. However, the new segmental methodology produced curves fit much more accurately to the real behavior. Therefore, our proposal gives the opportunity for a more optimal structural design work.

5 Summary and conclusions

In this paper a new design method was described for the determination of the lateral-torsional buckling resistance of simple beams. For the basic of the new method the formulae from the Ayrton-Perry type solution were chosen, similar to other new proposals. The very important advantage of this new design procedure is that it takes into account the determined physical meaning of the imperfection factor. As the result of a calibration process an expression was proposed for the calculation of the geometric imperfection amplitude. Using this proposed equation and the Ayrton-Perry type formulae an appropriate method is given for the determination of the LT buckling resistance of simple beams. The accuracy and applicability of this calculation procedure was demonstrated.

For the extension of this basic method for the case of various beam configurations a segmentation based methodology was proposed where the stability problem is evaluated at the cross-sections specified by the division. This means that the determination of the load-carrying capacity of the members is reduced to cross-sectional evaluation. And this gives the advantage of the new methodology. Namely, this cross-sectional calculation makes possible to find the design cross-section and evaluate the resistance of the beams in that location, similar to the proposal of Naumes et al. Through the determination of the critical cross-section the effect of the load distribution of the beam and also the effect of the lateral-torsional buckling type eigenshape (for the boundary conditions) is taken into account. This made possible to create such a methodology which can accurately predict the behavior of the diversely loaded and supported beams. Evaluating the results calculated with the method and comparing them with numerical results it can be stated that the Ayrton-Perry formula based method is properly accurate and has several advantages.

It has to be emphasized that the segmental division and cross-sectional evaluation based method requires long, tabular calculations. And, also requires the knowledge of the eigenvalue and eigenshape of the examined member. Therefore, our intention with this method is not the recommendation of a new manual calculation process. Its advantages can be utilized through computer-aided design procedures. Nevertheless, the cross-sectional evaluation method with the definition of the cross-sectional slenderness makes possible the examination of more

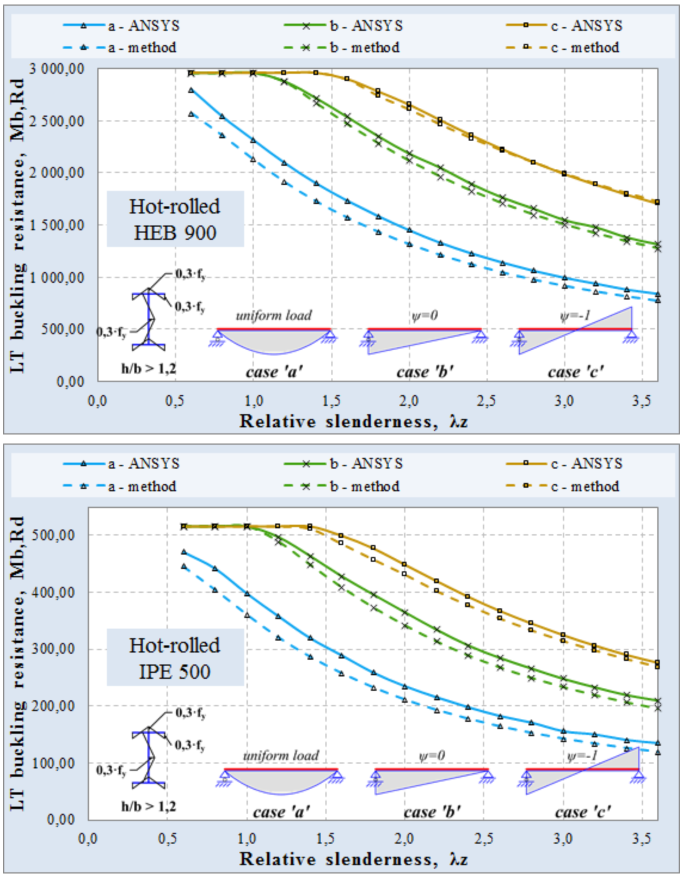

Fig. 11. LT buckling curves from numerical tests and segmental calculations for HEB 900 and IPE 500 profiles

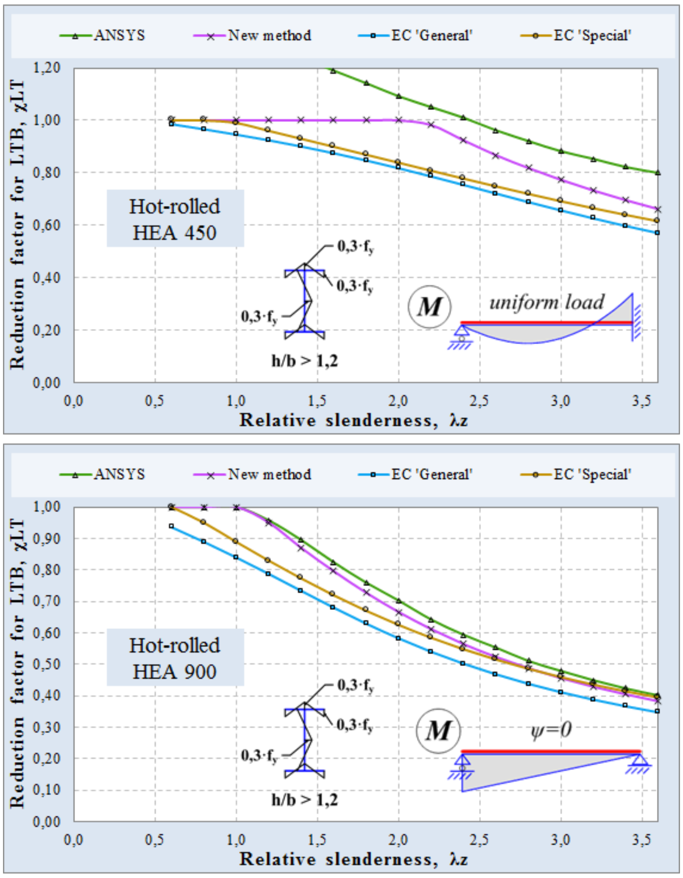

Fig. 12. Comparison of the numerical, proposed and standard LT buckling curves for beam configuration ‘a’ and ‘b’

complex and difficult problems (e.g. tapered beams; complex loading with normal force, biaxial bending etc.). So, we have the possibility for the development of a modern, general stability design method. This generalization is the object of further evaluation and validation research.

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