
Masoud Mohammad Sharifi, Morteza Bagherpour

Received 03-01-2015, revised 17-11-2015, accepted 17-12-2015

Abstract

Project managers normally are facing with difficulties behind management of project cash flow, which requires distinguished methods and appropriate tools to manage negative cash flows. Cash-Flow-at-Risk (CFaR) model is an efficient approach to predict cash flow trend. In this study, all risk factors affecting project management environment have incorporated to predict an accurate project cash flow. Then, a response surface method (RSM) is applied to determine optimal level of risk. The results have successfully implemented through a case study to demonstrate the applicability of the proposed method.

Keywords

Cash-Flow-at-Risk · Gaussian Mixture Model · Project Management · Response Surface Method

1 Introduction

The heart of a project is cash flow especially when the project is going to hand over or at the middle of execution phase. Basically, top level managers, based on cash flow forecasting, make managerial decisions. Therefore, inaccurate cash flow prediction and inadequate cash management lead to financial distresses and negative cash flows through project life cycle analysis [1]. Organizations with different sizes face with such problems which require distinguished approaches and suitable tools according to the nature and complexity of the projects undertaken [2]. The cash flow of the project normally consists of a complete history of all payments, cost, and all revenues. Furthermore, cash flow prediction needs to be effective and fast enough, according to the inadequate time and costs especially at the tendering phase. Contractors scarcely prepare a detailed construction plan at this phase, and usually waiting until the contract being awarded. So, a fast and effective approach for prediction of cash flow is highly desirable [2]. To cope with existing risk factors, for first time, we propose cash flow at risk (CFaR) model in project management.

On the other hand, value at risk (VaR) is a measure of losses because of “normal” market movements. VaR is proposed as a tool to determine the total market risks of a financial institution. VaR accumulates different exposed risks into a single number, and it is easy to communicate, calculate and interpret as well as it is a basis for decision making process [3]. In a lost case, the VaR can be interpreted in the following statement: “with $\alpha$ percent certainty, the loss is not more than $V$ US dollars at the next $N$ days”. We can use conditional probability function (CPF) for VaR calculation [4].

$$\Pr [\Delta P (N) < VaR] = F [\Delta P (-VaR)] = \int_{-VaR}^{VaR} f (\Delta P (x))dx = 1 - \alpha$$

The difference between the CFaR and the well-known value at risk (VaR) measure is that CFaR focuses on operating cash flow, whereas VaR concentrates on the asset values. VaR is inappropriate for non-financial firms which are not concerned with the
The literature survey indicated that there is no a well-organized approach to present an overview for considering risks in project management. Moreover, CFaR represents at-risk framework to set cash flow as target variable [5]. The CFaR is used for longer time horizon (which it is common in project environment) than VaR measure. Schematic VaR is shown in Fig. 1, which $\alpha$ is the confidence level.

![VaR presentation](image)

**2 Literature review & research gap**

Stein et al. (2001) applied the top-down approach which focuses on the cash flow fluctuations. They developed the comparable-based approach to predict cash flow [6]. Because of the insufficiencies of the approach, Andrén et al. (2005) proposed a different method, called exposure-based CFaR. They estimated a company’s cash flow fluctuation by considering corporate macroeconomic exposure and the several factors may affect firm’s cash flow [5].

Chen et al. (2005) presented pattern matching logic and factorial analysis which provide an ability to assess the accuracy of cash flow models. Then, they made a critique of the ability of existing cost-schedule integration (CSI) models to accurately predict cash flows [7]. Al-Joburi et al. (2012) then investigated negative cash flow patterns and their effects on project performance. They reviewed scheduling and financial data for almost 40 projects. The results of data analysis represented negative cash flow for 30 to 70% of projects, at each of the selected projects [8]. Maravas and Pantouvakis (2012) developed a cash flow calculation methodology for projects including activities with fuzzy costs and durations. They indicated project cash flow by an S-surface ensuing by connecting S-curves at different risk possibility levels [9]. Fink and Homberger (2013) considered a multi-agent extension of the non-preemptive single-mode resource-constrained project scheduling problem with discounted cash flow objectives. Also, they proposed a general decentralized negotiation approach which incorporated ideas from ant colony optimization [10]. Zayed and Liu (2014) proposed a model to examine the impact of various factors on cash flow by integrating analytic hierarchy process and simulation [11].

The literature survey indicated there is no a well-organized approach to present an overview for considering risks in project environment and most of researches focused on only some specific risks which it can’t be enough useful. Here, we propose a holistic framework for enabling managers to determine profit according to level of risk aversion. For achieving to this goal, a “bottom up” method is selected for estimating CFaR which begins by enumerating each of risk exposures. Then, risk exposures (rework, change order, exchange rate and raw material movements) are quantified. Afterward, these risks are aggregated to the project’s cash flow and finally cash flow distribution can be further determined (the distribution is used to determine CFaR). Using CFaR, managers enable to estimate how cash flow can be managed with considering volatility in market prices (such as commodity prices, interest rate, and exchange rate). Therefore, CFaR is applied to estimate how much expected future distribution of cash flow can change over time. Although, CFaR is used in manufacturing companies however it has not been applied in project management environment. Also, there is no comprehensive research in project management area considering uncertainty sources for predicting final project cash flow.

**3 Research methodology**

In this section, research methodology is explained through a step by step approach. First, prediction horizon of the project should be initially determined. This step is important because in case of short time prediction, many parameters are deterministically known and some of them would be un-deterministically known. At the next step, Work Breakdown Structure (WBS) is developed and then cost elements related to an individual activity is then identified. Because of the impact of each risk on project cash flow, it is important to identify all influencing factors as well. After identifying such factors, they would be divided into two categories: deterministic and probabilistic items. Following steps are developed for probabilistic analysis.

Risk identification is the first stage in risk management process and it is a basis for further steps. Appropriate risk identification ensures efficiency of risk management. If risk managers fail in identifying all possible affecting risk factors, these non-identified risks will become non-manageable and the organization cannot overcome such issues and further corrective action would not be possible to do.

This phase is basically includes two steps. The first step is to identify the risks and the next stage is the classification of the identified risks. There are many forms of uncertainty which may affect the cash flow. Risks inherently exist in any project. Obviously interaction among expert judgments, experiences and individuals creativity play an important role in identification of risks.

The next step thorough risk management process is risk analysis. The purpose of risk analysis is measuring the effect of the identified risks on a project cash flow. Depends on the available data, risk analysis can be carried out qualitatively or quantitatively or semi quantitatively. Many studies have been applied...
risk analysis techniques to a project. Furthermore, sophisticated simulation techniques can be adapted to multidimensional risk [12].

Risk analysis involves analyzing risks may normally occur in projects. No doubt, there is a strong relationship between usefulness of the results and reliability of input data Thus appropriate analysis of input data is crucial. Two types of data or information can be considered as input including objective and subjective data, however different terminology may be utilized.

The risks such as raw material price fluctuation, inflation rate and exchange are estimated by time series models and for these types of data; variance-covariance matrix should be developed due to interaction between factors. In this study, we propose a novel approach for prediction of delay by Gaussian mixture models (GMM). More information is presented in section 4.1.

Subjective data is extracted from expert’s judgments or expert’s opinions. Perhaps the best-known and most commonly used approach for estimating risk is subjective estimation. Probability distribution can be used for this type of data. Then, the most appropriate distribution is fitted for the identified risks. As mentioned above, a better fitted distribution for each risk may result in a more accurate prediction.

Then, by considering all previous steps, cash flow is simulated for considered periods. In this step, CFaR can be specified in determined confidence level but sometimes this CFaR is not acceptable for project manager, in this situation they can use risk management tools to reduce risk exposure and achieve a better condition to optimize profitability index and trend analysis.

In this step, response surface method (RSM) is applied:

1 Establish a relationship between several explanatory variables (raw materials price and currency rates) and response variable (cash flow) which it can be used for predicting response values.
2 Determine optimal level of the factors affecting the cash flow.

By considering the effect of explanatory variables on cash flow, managers can decide whether variable has an un-desirable effect on cash flow and prevent subsequent results. By using financial derivatives (such as future and forward), some probabilistic data turn into deterministic data. When an acceptable condition is reached, CFaR can be specified. More explanations are presented in section 8.
4 Risk identification
4.1 Delay analysis
In general, claims are common in construction projects and may happen due to several reasons. This may lead to delay in a project and construction disputes. Delay means the time overrun or more time than the planned completion date [13]. Because of the importance of claims in projects and also there isn’t an applicable model for measuring claim effect on cost overrun, we here have applied a methodology for predicting impact of claim through cost overrun as described in the following.

The data related to claim occurrences according to the similar projects or previous projects have been collected and they represented a historical data inspired from all different types of projects. We need to subdivide the data into subsamples, where each subsample is composed of projects with similar characteristics. We thus have an idea of how to predict claim in a project efficiently. Four characteristics have settled which these characteristics seem to be most strongly associated with patterns in claim prediction.

The first one is the relevant experience of previous projects conducted by contractors. The second key characteristic is relevant experience of previous projects being completed by the owner. The third one is similar projects from the perspective of project scope and type. Eventually, last factor is the projects with the same prices. Then, a distribution for each mentioned factors is fitted. Due to heavy intensive data and central limit theorem, we suppose the distribution of all four characteristic follow Gaussian distribution.

Now four Gaussian distribution is fitted and there is a need for a method to merge them altogether to achieve a single distribution for predicting claim. Here, a GMM is used which is a weighted sum of M component Gaussian densities [14]. Here, maximum-likelihood estimation is one of the most widely used methods for estimation of the parameters of GMM model. As well as, Expectation-Maximization (EM) algorithm is one of popular and common algorithm among existing maximum-likelihood estimation methods [15].

\[
p(x|\lambda) = \sum_{i=1}^{M} w_i g(x|\mu_i, \Sigma_i) \tag{2}
\]

\(X\) is a D-dimensional continuous-valued data vector (i.e. measurement or features), \(w_i, i = 1 \ldots M\), are the mixture weights, and \(g(x|\mu_i, \Sigma_i), i = 1 \ldots M\), are the component Gaussian densities. Every component density is a Gaussian function as given using Eq. \(2\):

\[
g(x|\mu_i, \Sigma_i) = \frac{1}{(2\pi)^{D/2} |\Sigma_i|^{1/2}} \exp \left\{ -\frac{1}{2} (x - \mu_i)' \Sigma_i^{-1} (x - \mu_i) \right\} \tag{3}
\]

With covariance matrix \(\Sigma_i\) and mean vector \(\mu_i\), the mixture weights satisfy \(\sum_{i=1}^{M} w_i = 1\). These parameters are represented by the notation as used in Fatma & Çetin 1999, \(\lambda = \{w_i, \mu_i, \Sigma_i\}, i = 1, \ldots, M\).

Same weight factors are considered for analyzing four characteristics. In other word, we considered four characteristics with the same impact on project claims. It is possible for project manager to consider different weights for factors affecting on claim or add (or subtract) other characteristics based on his/her judgment. The resultant distribution for claim factors is shown in Fig. [3]. This distribution is used in CFaR model to predict impact of claims on cash flow.

4.1.1 Raw material cost & exchange rate
The problem of cost overrun, especially in the construction industry is a worldwide incident and its consequences are normally a source of disagreement between client and contractor. If this disagreement is not appropriately handled, this may lead to project failure.

In construction projects, normally a significant portion of the total costs are consumed for purchasing of the required procurements. Commodity prices and exchange rates often have a significant impact on financial statement analysis according to the fluctuations may happen. These markets are characterized by sharp changes. As well as, the level of volatility itself fluctuates over time [16]. We examine the short-run dynamics of commodity prices, with a particular concentration on the role of volatility. The intention is to determine how changes in cash-flow are affected by futures prices.

In this paper using time series analysis, price fluctuations incorporated in our model to study effect of raw material fluctuations on project cash flow.

5 Time series
Time series analysis is a mathematical method using parameter estimation and curve fitting for the observed data and forecast the future trend. Time series analysis is the low cost and an accurate method. We here applied time series analysis for model fitting of commodity prices, exchange rate and inflation rate trend [17]. Applications of the generalized autoregressive conditional heteroskedasticity (GARCH) and autoregressive integrated moving average (ARIMA) approaches are widely applied in situations where the price volatility is a main issue. These models, particularly in financial applications, are important tools in the analysis of time series. They are especially useful when the goal is to analyze and forecast volatility.

GARCH \((p, q)\) model is expressed as following:

\[
\epsilon_t \sim N(0, \delta_t) \tag{4}
\]

\[
\epsilon_t = \sqrt{\delta_t} \epsilon_{t-1}, u_t \sim N(0, 1) \tag{5}
\]

\[
\delta_t = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \delta_{t-j} \tag{6}
\]

Where \(p \geq 0, q \geq 0, \omega > 0, \alpha_i \geq 0 (i = 1, 2, \ldots, p), \beta_j \geq 0 (j = 1, 2, \ldots, q), p\) is the or-
der of GARCH terms $\delta$ and $q$ is the order of the terms $\varepsilon^2$

ARIMA model is used widely in the area of nonstationary time
series forecasting, which is expressed as [17]:

$$
\theta(B)(1 - B)^d X_t = \theta(B)\varepsilon_t
$$

(7)

Where $X_t$ is a nonstationary time series at time $t$, $\varepsilon_t$ is a white
noise (zero mean and constant variance), $d$ is the order of differ-
encercing, $B$ is a backward shift operator defined by $BX_t = X_{t-1}$ $\theta(B) = 1 - \theta_1B - \theta_2B^2 - \ldots - \theta_pB^p$ and $\theta(B)$ is the moving
average operator defined as: $\theta(B) = 1 - \theta_1B - \theta_2B^2 - \ldots - \theta_qB^q$

[17].

Price time series of steel, aluminum, copper and zinc and cur-
rency rates of EUR/USD, GBP/USD and inflation rate have been
collected as inputs and time series analysis is then used for pre-
diction of price trend in future. Akaike information criterion
(AIC) measure is then applied to evaluate the adequacy of the
model by choosing an appropriate model among possible alter-
natives.

6 Variance-covariance matrix

Covariance matrix summarizes the volatilities and correlations
of the returns on a set of assets or risk factors or interest
rates. It contains all the necessary information for simulation of the corre-
lated values, for estimation of the volatility of a port-
folio for its risk factors. Covariance matrix is applied because it
tries to continue the existence correlation among various risk
factors (such as commodity prices and exchange rates) in future
throughout simulation. For instance, in a risk management sys-
tem related to a large organization, all main foreign exchange rates and commodity prices will encompass in one large dimen-
sional covariance matrix [18]. However, generating the covariance matrix depends on the number of exchange rates and com-
modity prices.

variance-covariance matrix =

$$
= \begin{bmatrix}
\text{var}(\beta_1) & \text{cov}(\beta_1,\beta_2) & \ldots & \text{cov}(\beta_1,\beta_k) \\
\text{cov}(\beta_2,\beta_1) & \text{var}(\beta_2) & \ldots & \text{cov}(\beta_2,\beta_k) \\
\vdots & \vdots & \ddots & \vdots \\
\text{cov}(\beta_k,\beta_1) & \text{cov}(\beta_k,\beta_2) & \ldots & \text{var}(\beta_k)
\end{bmatrix}
$$

(8)

Here, covariance matrix is used for raw material prices and
currency rates to establish a relation among the potential risk
factors.

7 Case study

In this section, a case study is examined to demonstrate the
ability and potential applications of the newly proposed model.
A WBS is developed for a construction project including 40 ac-
tivities. For each activity, cost elements (such as manpower, raw
materials and currencies) are assigned to the activity. We sup-
pose different metals such as steel, copper, zinc and aluminum
are used during the construction phase as well as the currencies
such as EUR/USD and GBP/USD are used to supply equipment
abroad. In this stage, the exposure risks for each cost element
must be determined. For example, price fluctuation is an ef-
ective parameter on raw material price and exchange rate. Also,
inflation rate also can effect manpower. In this stage, predictable
risk factors such as price fluctuations are determined using time
series analysis; the most appropriate model is fitted for each
factor to predict the future trend. Price fluctuations for the last
twelve months related to commodity price, exchange rate have
been collected. Here, covariance matrix is a 6*6; because in
this case study 4 commodity prices (steel, copper, zinc and alu-
mimum) and 2 exchange rates (EUR/USD and GBP/USD) have
been considered.
Tab. 1. Result of RSM method

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF (Degree of Freedom)</th>
<th>Mean Square</th>
<th>F Value</th>
<th>p-value</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>1.43E9</td>
<td>6</td>
<td>2.39E8</td>
<td>22.95</td>
<td>&lt; 0.0001</td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>8.44E8</td>
<td>81</td>
<td>1.04E7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lack of fit</td>
<td>2.98E8</td>
<td>70</td>
<td>4.26E6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pure error</td>
<td>5.46E8</td>
<td>11</td>
<td>4.26E6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor total</td>
<td>2.35E9</td>
<td>95</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>3229.02</td>
<td>R-Squared</td>
<td>0.6296</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>99577.91</td>
<td>adjusted R-Squared</td>
<td>0.6022</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.V%</td>
<td>3.24</td>
<td>predicted R-Squared</td>
<td>0.5367</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PRESS</td>
<td>1.05E9</td>
<td>Adequate precision</td>
<td>72.15.602</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4. Results of project cash flow simulation

For each activity, the probability distribution of reworks and change orders are assigned to each individual activity. These risks factors are subjective which they are predicted by expert judgments. Claim is assigned to the whole of the project according to what presented in section 4.1. The simulated data is used for prediction of claim, change orders & reworks. After running simulation, cash flow distribution is presented in Fig. 4.

Project cash flow is shown in Fig. 4. It shows cash flow range falls between 82362 (minimum) and 116641 (maximum) and mean is equal to 99588. The CFaR is equal to 92118 this means given selected confidence level (95%), firm’s cash flow in only 5% of situations is less than 92118 and the firm’s cash flow with 95% certain will not less than 92118.

In Fig. 4 tornado chart is shown for the factors affecting on cash flow. The longer bars in tornado chart indicate the greatest effects on cash flow fluctuations. As seen, the fluctuation in raw material price has the greatest effect on cash flow and it can change cash flow between 94794 and 104468 units.

8 Simulation based Optimization

In this section, we need a methodology assisting top level managers to evaluate the impact of different hedging strategies on cash flow variability and also to determine optimum price of each parameter for hedging procedure. In order to measure cash flow and determine coefficients of risky factors, optimization model should be developed which it contains a lot of necessary information for deciding the size of the hedge position. A response surface method (RSM) is applied to determine the best known scenario of the affecting risk factors. This method has ability to determine an approximate function with a smaller number of runs. The RSM is a set of statistical and mathematical methods, which enables researchers to determine optimum condition. A central composite design (CCD) is selected in order to study the effective factors. Six major factors are determined to study by the RSM-CCD and the values are studied in five levels (−α, −1, 0, +1, +α) through 96 experiments.

Some risk sources in project environment is introduced in section 3 which may have a significant impact on cash flow however only raw material prices and currency rates can be controlled and other risk sources are normally uncontrollable. Therefore, these factors include raw material prices (copper, steel, zinc and aluminum) and currency rates (EUR/USD and GBP/USD). Response function is chosen in order to increase cash flow [19]. Model fitting is executed by RSM and it proposed which a linear model suited the best fit.

The results of Analysis of variance (ANOVA) are given in Table 1. The adequacy (statistical significance) of the linear model is tested through F- and p-values statistics. A large F-
value statistic indicates that the regression equation can explain most of the variation whereas low p-value (< 0.05) indicates that the model is considered to be statistically significant [19]. Here F-value is 22.95 and p < 0.0001 indicates linear model is enough significant. Both values are fairly high and indicate correlation between the observed and the predicted values. According to evaluation of ANOVA results, the statistical significance of linear model for the response is confirmed and the model can be used for more analysis in order to discover the effect of variables (risk factors) on project cash flow. The obtained equation is shown below:

\[
CFaR = 3.93E5 - 63417.3 \times Y_{\text{EUR/USD}} - 57207.5 \times Y_{\text{GBP/USD}} - 9.36 \times X_{\text{Copper}} - 10.11 \times X_{\text{Aluminium}} - 9.7 \times X_{\text{Zinc}} - 36.1 \times X_{\text{Steel}}
\]

(9)

Where \(X\) is price per ton and \(Y\) is the exchange rate on cash flow (after determining the price of raw materials and currencies, CFaR can be specified by put them down in Eq. (9)). Also, it leads to a better hedging decision. The 3D response surface curve is plotted to observe the interaction of factors and the optimum settings of each factor is required for the optimum cash flow configuration. The 3D surface graph for the response is shown in Fig. 6.

After determining optimum situation for all parameters, cash flow is optimally simulated to determine cash flow distribution under the suggested condition. The result is shown in Fig. 7. Cash flow is between 93354 and 122879 and the mean is 109603. In Fig. 7, the CFaR has improved and reached to 102986, this means CFaR is improved 11.8%. In other word, this means the exposed risk is decrease. Moreover, the cash flow mean is improved %10.1.

9 Conclusion remark and further recommendations

In project environment, project managers normally encounter with many risk factors which is required to deal with them efficiently. The most important conclusion of this study is to provide a systematic framework to determine cash flow distribution through uncertain environment. In this research, for the first time, we proposed financial concepts such as VaR, time series models and covariance matrix through project management context. Also, a novel methodology for predicting claims and then quantifying impact of claim on cash flow presented.

Moreover, a new index is applied called risk-aversion parameter (\(\alpha\)) which by adjusting this parameter a project manager can decide about the desirable risk level. The RSM is applied to establish a relationship between several risk factors and cash flow which the resulting model is used to determine the optimal level of risk and making managerial decisions relevant to financing strategies.

Eventually, the presented approach provides excellent insights for project managers by combining internal and external factors altogether. This combination helps to present an accu-
rate estimation of profit in future and uses financial derivatives to increase profitability index. In this paper, the obtained results successfully confirmed CFaR model is a powerful approach in project environments and this model can be safely applied for prediction of project cash flow under risky conditions.

Further research can be conducted on applying multi-objective simulation based optimization for project cash flow management.

References

Fig. 7. Cash flow simulation after optimization