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RESEARCH ARTICLE

Laboratory Study of the Shear Behaviour of Natural Rough Rock Joints Infilled by Different Soils

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Abstract

Natural rock joints infilled with soil materials may show a reduced shear strength, which influences rock mass stability. The aim of this paper is to experimentally investigate the shear behaviour of infilled rock joints, taking into account joint surface characteristics and the properties of the joint and infill materials. A new model for predicting the shear strength of infilled joints is presented, on the basis of a series of tests carried out on natural rock joints with same surface roughness, with clay, sand and sandy-clay used as infill materials. All tests were carried out in a shear box apparatus under constant normal load (CNL) conditions. The empirical model was finally validated based on the experimental data from the literature. The results showed an acceptable confidence level for the model and reported that the new model successfully describes the observed shear behaviour of natural infilled rock joints.

Keywords

Natural rock joint · Shear behaviour · Experimental investigation · Infill soil material · Empirical model

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1 Introduction

Over many years, fine sediments resulting from weathering and other surface processes could subsequently ingress to rock joints, reducing the overall shear strength of the joint surface [1–3]. Rock joints that are naturally filled with fine materials (see 2) are likely to be the weakest elements in a rock mass and can have a dominant influence on its shear behaviour due to of the low frictional properties of the infill [4, 5].

The most effect of filling material is to separate the discontinuity walls and thereby reduce intact rock contact, but shear strength will also be influenced by the nature of the filling material itself and the characteristics of the wall-fill interfaces. Because of the lack of reliable and realistic theoretical or empirical relations and the difficulties in obtaining and testing representative samples, engineers generally rely on judgment, often considering the shear strength of the infill itself to be conservative. In critical cases, in situ tests may be carried out to provide site specific design criteria, but invariably amount of testing that can be undertaken precludes the establishment of fundamental relations. During the past 30 years much more information has become available on the shear behavior of joints infilled with soil material. Several models have been proposed to predict the shear strength of infilled joints under both constant normal load (CNL) and constant normal stiffness (CNS) boundary conditions, considering the ratio of infill thickness (t) to the height of the joint wall asperity (a), i.e., t/a ratio [4–22]. The experimental researches to date have tended to focus on modelled joints or replicas rather than natural rock joints. In other words, although some works have been done on the effect of infill material by using natural rock joints, they have not proposed prediction models and it can be noted that most of the previous models in the literature have been developed based on the laboratory tests over the simulated artificial joints and not on the real rock joints (for simplicity and reproducibility reasons). In addition, there have been no shear behaviour models which consider differences in infill type within the rock joint. However, this research will give an account of application of a statistical analysis on a series of data obtained from a complete testing program on natural infilled rock joints, taking into account three different material filling the rock joint. The output of this analysis has consequently been conducted to propose an empirical model. The tests in this research have been done in constant normal load (CNL) conditions. In general, the CNL condition is more realistic for shearing planar interfaces where normal stress applied to the shear plane remains relatively constant, and the problem of surface (shallow) slope stability. The development of shear resistance is a function of constant normal stiffness (CNS), and based on the opinion of some researchers (e.g. [13, 19]), the use of CNL test results leads to underestimated shear strengths because the surrounding rock freely allows the joint to shear without restricting the dilation or there is no dilation during the shearing process, thereby keeping normal stress constant during shearing process. To evaluate the behaviour of rock joints under conditions more commonly encountered in underground excavations, it is necessary to simulate the stiffness of the rock mass normal to the direction of shearing [23, 24]. However, this condition is only existent in very deep situations, and the consideration of CNL conditions would, therefore, be adequate for an available model and the tests and resulted model proposed in this paper is therefore suitable to be utilized in shallow rock structures and slope stability applications.

Fig. 1. A natural infilled rock joint in shallow depth

2 Laboratory investigation

The natural sandstone joints were sampled along 20 km of rock slopes of the Khosh-Yeylagh Main Road located in northeast of Iran. The specimens (not sheared naturally before sampling, i.e. healthy specimens) were cut into about 70×70 mm surface profiles in order that they can be placed within the shear box. Fig. 2 shows a set of the specimens prepared in this research.

he joint roughness coefficient (JRC) value was calculated for all the specimens using the tilt test, the Schmidt Hammer, and the methodology presented by [25, 26]. Then the specimens with same JRC values (the specimens with JRC almost around 7) were selected as the target specimens for the shear tests. Three types of materials were considered for the infill of the rock joints regarding their graining including sand, clay and sandy-clay. The infill was then spread over the joints using a spatula to give the desired thickness to asperity height ratio, i.e. t/a (Fig. 3).

Fig. 2. A set of specimens prepared for the laboratory investigation

The tests on the soil-infilled rock joints were performed using the Constant Normal Load Direct Shear Test apparatus (Shear Box) (Fig. 4a). The specimens were fixed within the apparatus by molding with a high-strength gypsum plaster (Fig. 4b). Fig. 4c also shows the molded specimens before shear test. The gypsum plaster used for molding the specimens does have the uniaxial compressive strength equal to 25-30 MPa and Young's Modulus equal to 4.8-5.5 GPa (The molded gypsum condition was checked after each test and it made sure that was quite healthy and undamaged.). The shear box is capable of measuring the peak and residual shear strength of rock specimens in CNL conditions. It has two loading systems: the hydraulic loading is used for normal and shear loads, and the pneumatic loading is utilized to maintain (fix) the normal load within the adjusted desired range while shearing the specimen. Nine different t/a ratios were tested: 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6. The tests were also performed in four normal load levels: 0.25, 0.50, 0.75 and 1.0 MPa.

3 Results and discussion

The experimental results for all t/a ratios tested are presented in Fig. 5 to Fig. 7. Stresses have been calculated using the corrected cross-sectional area of the specimen at each displacement. Figs. 8 to 10 show the shear stress - shear displacement plots for five t/a ratios in different conditions of normal load for the rock joints filled with clayey, sandy, and sandy-clayey infill materials, respectively.

As expected, for a thin layer of infill material from any type, there is a considerable decrease in the peak shear stress compared to a clean joint. With increase in infill material relative thickness (t/a), the shear stress would be gradually decrease and would approach the strength of infill soil itself at about t/a =1.4. Actually, at ratios more than 1.4 the shear behavior would be under control of the infill material and the joint surface roughness no longer plays an important role at this point. This ratio can, therefore, be considered as the critical t/a ratio for all tested rock joints and this means that increasing the infill thickness more than it would not be effective to shear behavior. In addition, it is seen that the drop from peak shear to residual shear stress is negligible at higher t/a ratios. This takes place in any type of

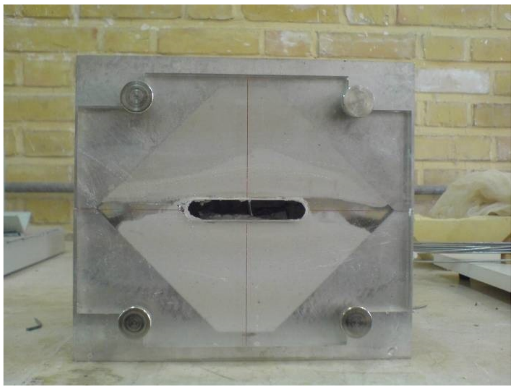
(a)

(b)

(c)

(d)

Fig. 3. Addition of the infill material over the rock joints; (a): Sandy-clayey infill; (b): Sandy infill; (c): Clayey infill; (d): The specimen ready for the shear test



(a)

(b)

(c)

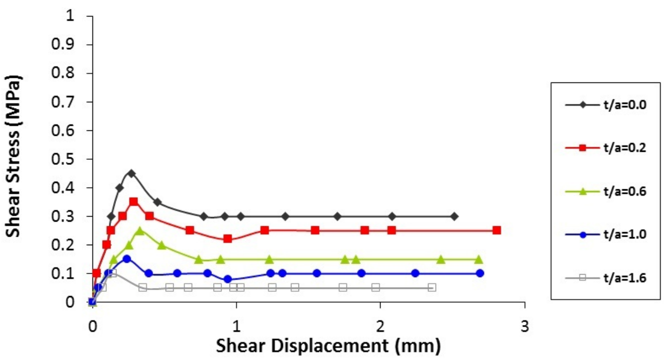
Fig. 4. (a): Gypsum plaster mold; (b): Direct Shear Test Apparatus; (c): The molded specimens prior to shear tests

infill material with any normal load value.

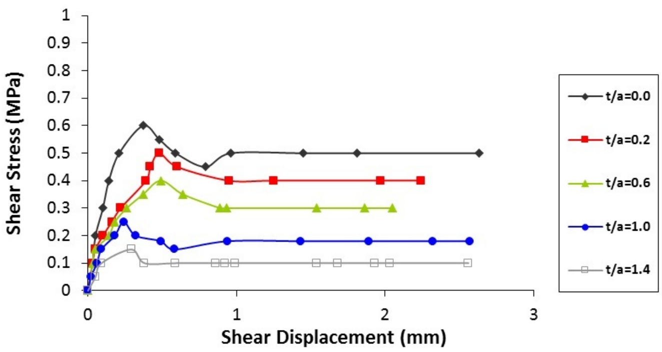
The shear behavior changes with change in the infill type. For instance, at first stage of clayey infill addition to the joints interface (i.e., $t/a = 0.2$), they show peak shear strength of 0.45, 0.50, 0.65 and 0.80 MPa for 0.25, 0.50, 0.75 and 1.0 MPa of normal stress, respectively. In the same stage, addition of sandy infill would result in peak shear strength values of 0.30, 0.45, 0.60 and 0.85 MPa. The sandy-clayey infill material would also show the values equal to 0.40, 0.55, 0.70 and 0.85 MPa.

The experimental results show that the rock joints infilled with sandy material have less shear stress compared to those infilled with clayey material. This can be considered to be resulted from the higher cohesion of clay compared to the sandy materials used in this research.

The behavior that is seen in almost all the plots is that even ad-



(a)



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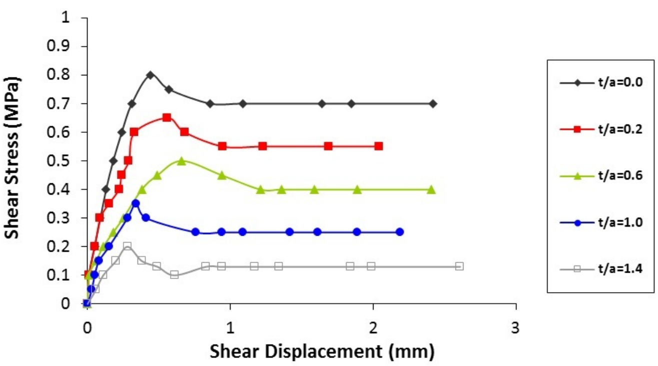

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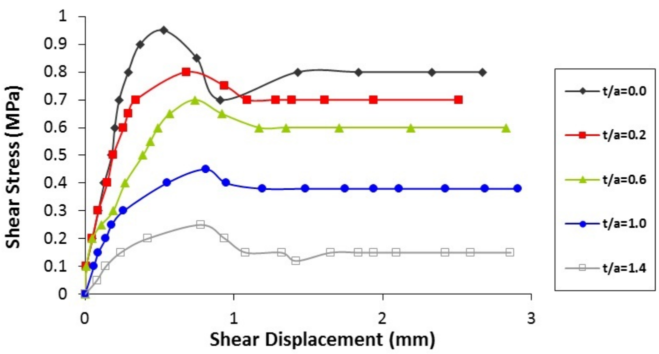
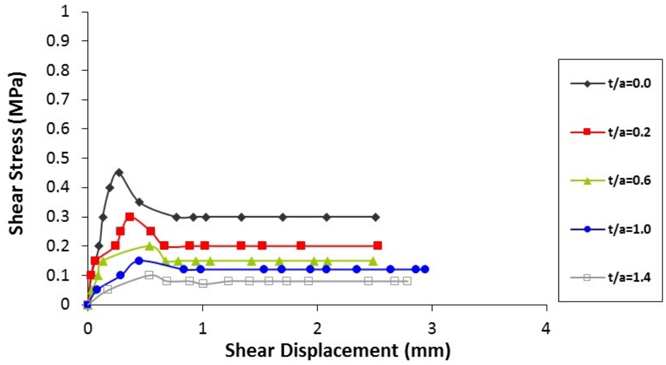
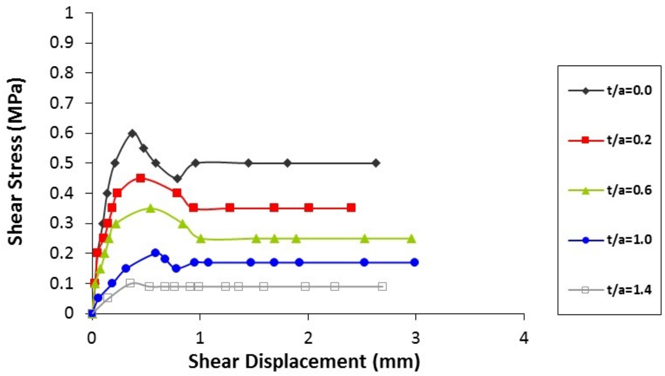


(d)

Fig. 5. Shear stress – shear displacement plots for rock joints infilled with clayey material; (a): Normal load = 0.25 MPa; (b): Normal load = 0.50 MPa; (c): Normal load = 0.75 MPa; (d): Normal load = 1.00 MPa



(a)



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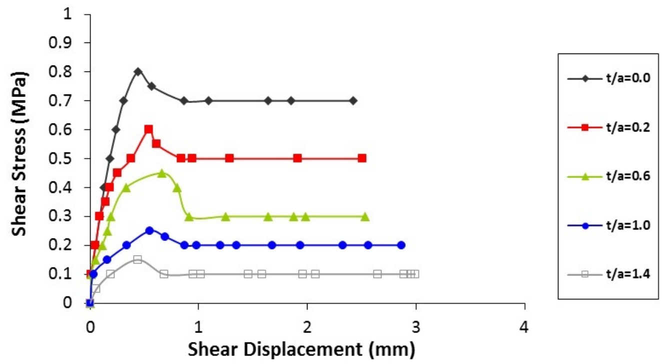

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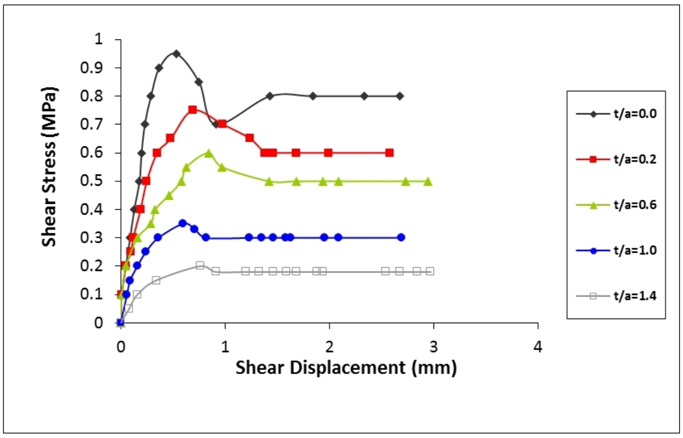


(d)

Fig. 6. Shear stress – shear displacement plots for rock joints infilled with sandy material; (a): Normal load = 0.25 MPa; (b): Normal load = 0.50 MPa; (c): Normal load = 0.75 MPa; (d): Normal load = 1.00 MPa

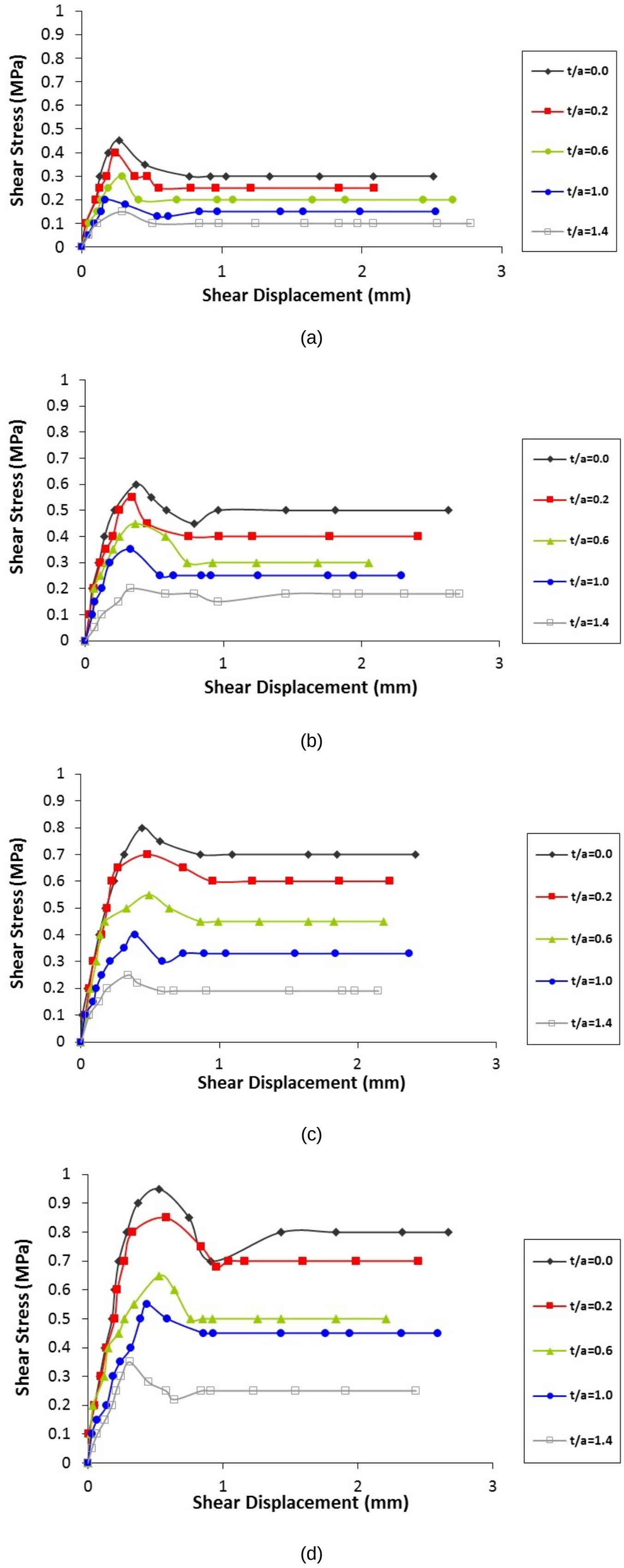


Fig. 7. Shear stress – shear displacement plots for rock joints infilled with sandy-clayey material; (a): Normal load = 0.25 MPa; (b): Normal load = 0.50 MPa; (c): Normal load = 0.75 MPa; (d): Normal load = 1.00 MPa

dition of a thin layer of infill material would drastically decrease the shear strength of the rock joints. While increasing the thickness of the infill, the peak shear stress is gradually decreased and reaching a specific infill thickness, decrease in the stress would be negligible. That is more obvious at lower normal load values.

Fig. 8 shows the shear strength envelops (peak shear stress – normal stress plots) for three types of infill materials at nine t/a ratios. The parameters such as joint friction angle and cohesion can be extracted from these plots. As seen from mentioned plots, the rock joints infilled with sandy-clayey material show higher shear strength compared to those infilled with other types of materials, because of greater strength of that infill itself compared to the other infills.

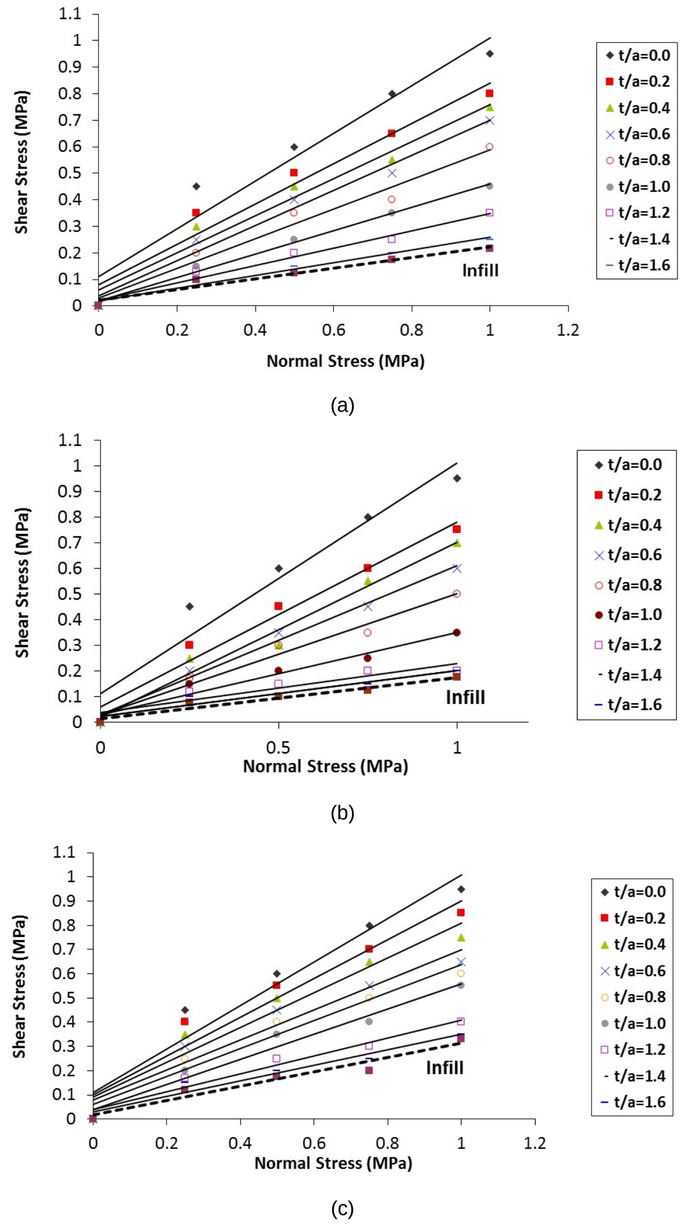
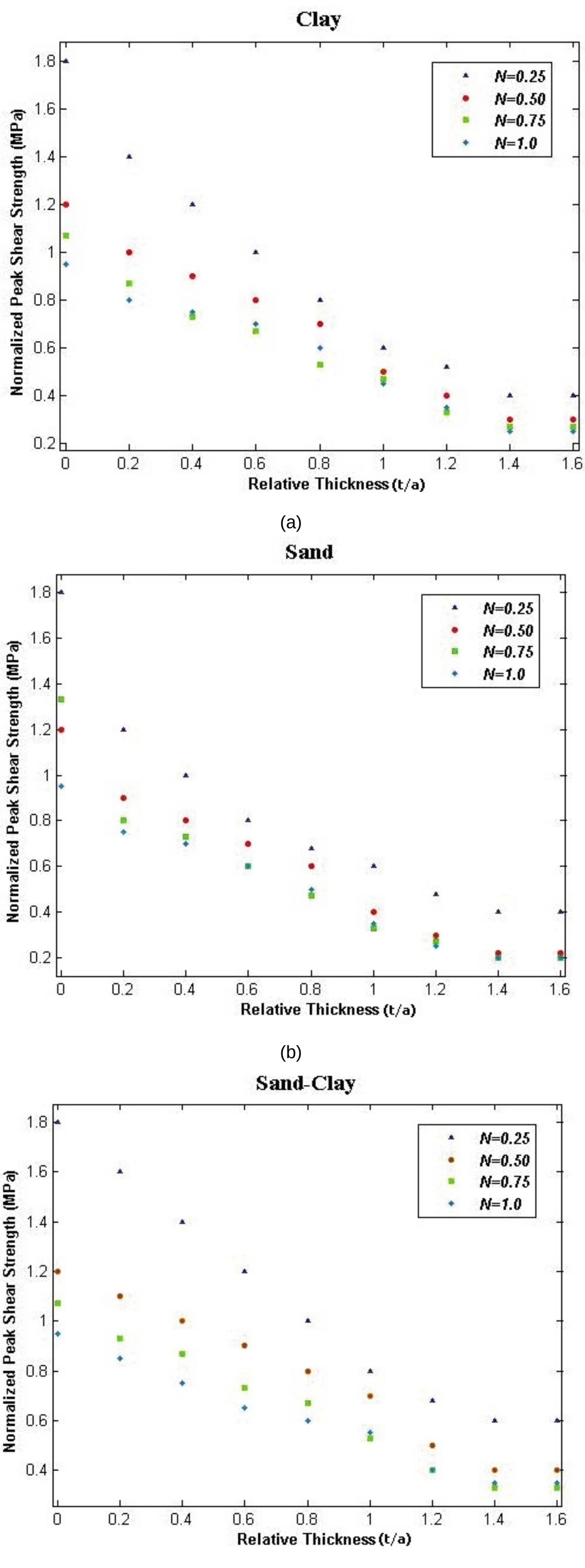


Fig. 8. Shear strength envelops for three types of infill materials; (a): Clayey infill; (b): Sandy infill; (c): Sandy-clayey infill

The greater friction angles in low normal loads show the dilation in the joint while being sheared. In other words, in low



(a)

(b)

(c)

Fig. 9. The experimental normalized peak shear strength – relative thickness data in different normal load values for three types of infill materials; (a): Clayey infill; (b): Sandy infill; (c): Sandy-clayey infill

normal load, the tangential angle measured from shear strength envelop is resulted from basic friction angle of joint surface asperities plus the angle come from dilation. In this case, while shear stress increases, the joint surfaces only slide on each other without any shear in joint surface nor a great damage in asperities.

Therefore, with increase in normal stress, the slope of the shear stress – normal stress plot decreases and reaches its least value. In other words, the increase in normal stress causes the asperities on joint surface to be cut and as a result the dilation angle decreases and reach zero value. As well, the rock joint friction angle reaches the residual friction angle.

The experimental normalized peak shear strength – relative thickness data in different normal load values for three types of infill materials were plotted as shown in Fig. 9. Some criteria such as simplicity, suitability and general type were taken into account to select a series of preliminary mathematical functions to be fitted to the experimental data for generation of the conceptual model. These functions are as follows.

1 Exponential function including exponential and bi-exponential;

2 Fourier function;

3 Gaussian function;

4 Polynomial function including linear, first order and second order;

5 Rational function comprising all possible states in numerator and denominator of the fraction.

The selected mathematical functions were separately fitted to the experimental data and statistical parameters were extracted for each fitting. Three parameters were taken into account to examine the suitability of the functions including *R-Square*, "Sum of Squares due to Error" (*SSE*) and "Root Mean Squared Error" (*RMSE*) [27, 28]. These parameters were calculated for each fit. The main aim in this process was to find out a function which had maximum R-Square and minimum SSE and RMSE. For this purpose, all the calculations and statistical outputs were recorded and compared to the others. Part of the outputs of SSE for clayey infill has been shown in Table 1.

With consideration of three statistical parameters, it was found out that the rational function with constant numerator and second order denominator would be the best function compared to the others. Thus that was selected as the starting state for utilization in development of the conceptual model. General type of the selected rational function is as follows.

$$f(x) = \frac{c_1}{x^2 + k_1 x + k_2} \tag{1}$$

where k_1, k_2 and c_1 are constant values. Fitted rational function to the experimental data of clay infilled rock joints has been shown in Fig. 10 as an instance.

Tab. 1. Outputs of SSE for clayey infill

Function type		Normal load (MPa)			
		0.25	0.5	0.75	1.0
Exponential	I	0.0102	0.0095	0.0073	0.0081
	II	0.0064	0.0079	0.0091	0.0086
Furrier		0.0111	0.0134	0.0134	0.0141
Gaussian		0.0148	0.0152	0.0128	0.0128
Polynomial	I	0.0090	0.0096	0.0083	0.0094
	II	0.0178	0.0199	0.0180	0.0211
	III	0.0844	0.0821	0.0705	0.0605
Rational	I	0.0051	0.0079	0.0086	0.0090
	II	0.0037	0.0069	0.0041	0.0046
	III	0.0092	0.0075	0.0084	0.0067
	VI	0.0049	0.0074	0.0061	0.0051
	V	0.0139	0.0152	0.0175	0.0136
	VI	0.0056	0.0084	0.0074	0.0051

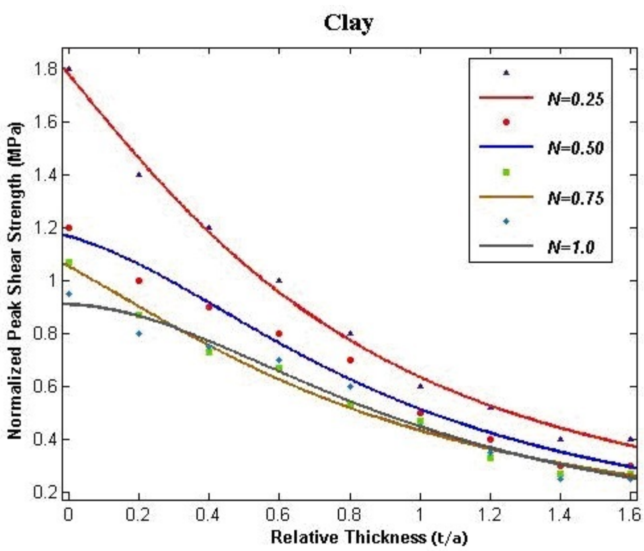


Fig. 10. Fitted rational function to the experimental data of clay infilled rock joints

A normalized conceptual model would be in t form that the shear strength of an infilled joint is described as the effect of its two components combined, i.e. infill material and rock surface. As can also be seen in fitted best function to experimental data (Fig. 10), all the curves start from the point associated with the shear strength value of clean joint and approach an almost horizontal state after passing from the relative thickness of 1.4 (the critical infill relative thickness). For this reason, the derived rational function considered to be "the amount of normalized shear strength loss caused by presence of infill material in joint interface" (Eq. (2)).

$$\left(\frac{\tau_p}{\sigma_n}\right) = \left(\frac{\tau_p}{\sigma_n}\right)_{clean} - \Delta\tau_p \qquad (2)$$

where:

$\left(\frac{\tau_p}{\sigma_n}\right)$ = normalized shear strength of infilled rock joint

$\left(\frac{\tau_p}{\sigma_n}\right)_{clean}$ = normalized shear strength of unfilled (clean) rock joint

$\Delta\tau_p$ = shear strength loss

As a matter of fact, it can be mentioned that $\Delta\tau_p$ is the shear strength loss resulted from the presence of infill material within rock joint interface. It is worthy to note that the main aim of fittings was to achieve this parameter.

After applying the parameters such as relative thickness of infill and maximum normalized shear strength to the general form of rational function and also considering the effect of infill material on strength reduction, the final equation for prediction of shear strength of infilled rock joint would be as follows.

$$\frac{\tau_p}{\sigma_n} = \frac{\tau_0}{\sigma_{n0}} - \frac{c\,(t/a)}{\sigma_n[(t/a)^2 + k_1\,(t/a) + k_2]} \qquad (3)$$

where:

τ_p = shear strength of infilled joint;

τ_0 = shear strength of clean joint under normal load of σ_{no};

σ_n = applied normal load to joint;

t = infill thickness;

a = average roughness of joint surface;

k_1, k_2 and c_1 are empirical constants associated with the applied normal load and type of infill material.

Equation (Eq.(3)) will be reliable to the extent that $\tau_p \geq \tau_{Soil}$ where τ_{Soil} is the shear strength of infill material.

To obtain the constant values of the model, the equation was solved for various quantities of t/a ratio rather than using the values of fittings. This resulted to achieve the values with a high accuracy. The proposed constant values for three different infill materials are shown in Table 2 to Table 4.

Tab. 2. Proposed constant values for clayey infill

Normal load (MPa)	0.25	0.50	0.75	1.0
c	-0.8	-0.4	-2.16	-0.4345
k_1	-2.08	-1.8	-3.4	-1.6897
k_2	-1.08	-0.48	-2.24	-0.2814

Tab. 3. Proposed constant values for sandy infill

Normal load (MPa)	0.25	0.50	0.75	1.0
c	-1.8	-0.5133	-0.7636	-0.56
k_1	-5.6	-2.1689	-2.1636	-1.8
k_2	-1.32	-0.2907	-0.3709	-0.24

Tab. 4. Proposed constant values for sandy-clayey infill

Normal load (MPa)	0.25	0.50	0.75	1.0
c	0.54	-0.8615	-0.75	3.3
k_1	-1.6	-0.7692	-1.7	-0.8
k_2	2.76	-3.4708	-1.2	6.72

It is worthy to note that the proposed new model would be capable of calculation and prediction of the shear strength of infilled rock joints under similar conditions. In addition, that would obviously give more reliable outputs in CNL conditions and surface situations such as road slopes, rock cuts, open pit mines and ditches.

A comprehensive validation with the help of published data has finally been done on the model for determination of the degree of accuracy and reliability. For this purpose, the experimental data of some researches found in the literature were utilized. These data were separately substituted within the model and compared to their original outputs. Then, *RMSE* values between original and predicted amounts for the published date series were calculated. These values are summarized in Table 5. The overall *RMSE* average was 0.2501.

Tab. 5. The RMSE values of comparison between published data and new model's predictions

Published data	RMSE
Goodman (1970) [29]	0.1571
Lama (1978) [7]	0.2120
Phien-Wej (1990) [9]	0.2676
De Toledo (1993) [5]	0.3636
Average	0.2501

The confidence level of the model would accordingly be 0.7499, which can be considered as an acceptable value, hence confirming the relative validity of the proposed model.

4 Conclusion

The main purpose of current study was to develop an empirical CNL model with the help of a wide series of experimental data in order to predict the shear strength of natural infilled rock joints. For this aim, laboratory investigation was conducted to study the shear behavior of sampled sandstone joints infilled with three different materials in various thickness conditions. Then statistical analyses were utilized to achieve the best mathematical function that presents the behavior of infilled joints. The new proposed model represents a variation in the normalized shear stress (τ_p/σ_n) as a function of t/a ratio. This model is capable of explaining the decrease of shear strength with increasing t/a ratio where the critical t/a ratio plays an important role in ultimate shear strength. This criterion has been developed in CNL conditions where the normal load (σ_n) is expected not to change during shearing process. Experimental validation of the model showed an acceptable confidence level and thus it can be used in many of similar positions. However, further experimental investigations are still needed to be undertaken to modify the proposed model and increase the field applicability. In addition, although some steps have been taken by the author to replicate and conduct tests on natural joint profiles, there are still limitations caused by the narrow range of JRC examined in this study. Therefore, further testing of different irregular joint profiles is required to validate the proposed model more comprehensively. Scale effects (the effects of changes in joint surface wave length and asperity height) were not studied which is recommended to be done as the next research.

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