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RESEARCH ARTICLE

A Response Surface Modelling Approach for Resonance Driven Reliability Based Optimization of Composite Shells

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Abstract

The composite materials are extensively used in the structures of civil, aerospace, marine, and automobile engineering due to their tailorable capability. The objective of this article is to address the issue of resonance-free lightweight design of such composite structures coupled with the notion of reliability. Laminated composite spherical shell is considered in this study to optimize width and thickness of the structure corresponding to different level of reliability of the system to avoid resonance. The present study utilizes genetic algorithm in conjunction to surrogate modelling with D-optimal design for this reliability based optimization problem.

Keywords

Reliability based optimization · genetic algorithm · natural frequency · response surface modelling · spherical shells

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1 Introduction

The development of reliable composite structures in production process is always subjected to large variability due to manufacturing imperfection and uncertain operational factors. In practice, an additional factor of safety is assumed by designers due to difficulty in assessing reliability to avoid resonance in conjunction to uncertainties of stochastic natural frequencies. This existing practice of designer results in either an ultraconservative (overestimation of material cost) or an unsafe design. Hence, it is needed to overcome this current limitation wherein the design of composites are restricted to a deterministic regime despite of rapidly increasing demands of technological, economical and safety needs. Many literatures are available dealing with uncertainty quantification of composite structures [1–3]. Moreover, the reliability in conjunction to cost component involved in weight optimization of such composite structures are always a challenge for the designers. The common cause of employing composite structures in many applications (such as aircraft, civil structures) is weight sensitiveness wherein the objective of design optimization [4] is to lower the weight for achieving the better performance. For example, in structural design problem, the need of computation of the natural frequency is required to avoid the resonance which can vary with the uncertain geometric and material properties of the structure. In such engineering applications with complex systems, the consequences of uncertain system behaviour become severe in terms of cost and effort. The assessment of probability of failure and the need to improve the reliability of the systems have become essentially important for structural safety. Such necessities in turn raise the need for reliability based design optimization (RBDO) analysis [5]. The uncertain variation of system parameters can be mathematically coupled with optimization tools such as genetic algorithm (GA) to achieve safety as well as cost-effectiveness.

Many studies are carried out by applying RBDO methods for optimal design of shallow composite structures. The random loading and material properties including manufacturing uncertainties are considered for example in [6–11]. Miki [12] and Fukunaga and Chou [13] proposed a graphical optimization method using lamination parameters for stiffened composite structures. Composites structure with degradation model is investigated by Antonio et al. [14] while buckling instabilities is studied by Su et al. [15]. Many researchers studied on the optimization coupled with uncertainty [16–18]. In contrast, Todoroki and Terada [19, 20] introduced the deterministic optimization method for the stacking sequences of the composite laminates wherein buckling load is maximized by employing fractal branch-and bound (FBB) method. Reliability based design attempts to ensure a minimal probability of failure by controlling of stochastic variables. Hence such method is more flexible and consistent than deterministic analysis as it provides more rational safety levels over various types of structures and takes into account more information than deterministic analysis. Thompson et al. [21] studied the weight minimization problem with a deterministic strength constraint and two probabilistic constraints for fiber-reinforced polymer composite bridge deck panels while Yang et al. [22] explored the use of stochastic approach to the design of stiffened composite panels in composite ship structures under in-plane load.

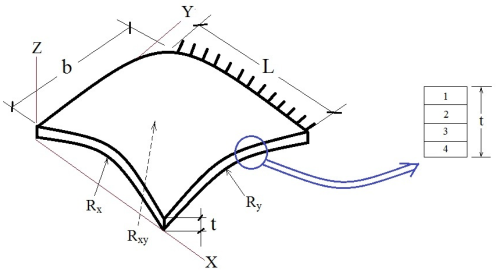

Fig. 1. Composite shallow cantilever shell

In the present study, genetic algorithm (GA) is employed coupled with a local multivariate search function for weight optimization of composite spherical shells to obtain resonance-free design. Most of the previous related studies are limited to deterministic conditions, without considering the effects of uncertainties in the natural frequency of composite shell structures. In this study uncertainties due to material and geometrical properties of composite are accounted to optimize the structure in a computationally efficient way. Novelty of this article includes application of GA in conjunction with surrogate modelling approach for reliability based optimization of composite shells. Moreover, the utilization of the resonance criterion as an optimization constraint in the reliability based optimization of composites is first attempted in this study.

2 Theoretical formulation

A composite cantilever shallow doubly curved shells with length 'L', width 'b', thickness 't', principal radius of curvature R_x and R_y along x- and y-direction, respectively and radius of curvature in xy-plane 'R_{xy}' is considered as furnished in Fig. 1. Based on the first-order shear deformation theory, the displacement field of the shells can be expressed as

$$\begin{aligned} u(x,y,z) &= u^0(x,y) - z\theta_x(x,y) \\ v(x,y,z) &= v^0(x,y) - z\theta_y(x,y) \\ w(x,y,z) &= w^0(x,y) = w(x,y), \end{aligned} \tag{1}$$

Assuming u, v and w are the displacement components in x-, y- and z-directions, respectively and u^0, v^0 and w^0 are the mid-plane displacements, and $\theta_\mathbf{x}$ and $\theta_\mathbf{y}$ are rotations of cross-sections along the x- and y-axes. The strain-displacement relationships for small deformations can be expressed as

$$\begin{aligned} \varepsilon_{xx} &= \varepsilon_x^0 + zk_x \\ \varepsilon_{yy} &= \varepsilon_y^0 + zk_y \\ \gamma_{xy} &= \gamma_{xy}^0 + zk_{yy} \\ \gamma_{xz} &= w_{,x}^0 - \theta_x \\ \gamma_{yz} &= w_{,y}^0 - \theta_y, \end{aligned} \tag{2}$$

where mid-plane components are given by

$$\varepsilon_x^0 = u_{,x}^0\ , \quad \varepsilon_y^0 = u_{,y}^0\ , \quad \gamma_{xy}^0 = u_{,y}^0 + v_{,x}^0$$

and the curvatures are expressed as

$$\begin{aligned} k_x &= -\theta_{x,x} = -w_{,xx} + \gamma_{xz,x} \\ k_y &= -\theta_{y,y} = -w_{,yy} + \gamma_{yz,y} \\ k_{xy} &= -(\theta_{x,y} + \theta_{y,x}) = -2w_{,xy} + \gamma_{xz,y} + \gamma_{yz,x}. \end{aligned}$$

Therefore the strains in the k-th lamina can be expressed in matrix form

$$\begin{aligned} \{\varepsilon\}^k &= \left\{ \begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array} \right\} + z \left\{ \begin{array}{c} k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{array} \right\} = \{\varepsilon^0\} + z\{k\} \\ \text{and } \{\gamma\}^k &= \left\{ \begin{array}{c} \gamma_{yz} \\ \gamma_{xz} \end{array} \right\} = \{\gamma\} \end{aligned} \tag{3}$$

In general, the force and moment resultants of a single lamina are obtained from stresses as [23]

$$\begin{aligned} \{F\} &= \{N_x N_y N_{xy} M_x M_y M_{xy} Q_x Q_y\}^T \\ &= \int_{-t/2}^{t/2} \{\sigma_x \sigma_y \tau_{xy} \sigma_x z \sigma_y z \tau_{xy} z \tau_{xz} \tau_{yz}\}^T dz \end{aligned} \tag{4}$$

In matrix form, the in-plane stress resultant $\{N\}$, the moment resultant $\{M\}$, and the transverse shear resultants $\{Q\}$ can be expressed as

$$\begin{aligned} &\{N\} = [A]\{\varepsilon^0\} + [B]\{k\}\,\{M\} = \\ &= [B]\{\varepsilon^0\} + [D]\{k\}\,\{Q\} = [A*]\{\gamma\} \end{aligned} \tag{5}$$

Here $\varepsilon_{yy} = \varepsilon_y^0 + zk_y$ and $\left[A_{ij}^*\right] = \int_{-t/2}^{t/2} \bar{Q}_{ij} dz$ for $i,\ j = 4{,}5$

$$\left[\bar{Q}_{ij}(\bar{\omega})\right] = \begin{bmatrix} m^4 n^4 2m^2 n^2 4m^2 n^2 \\ n^4 m^4 2m^2 n^2 4m^2 n^2 \\ m^2 n^2 m^2 n^2 (m^4 + n^4) - 4m^2 n^2 \\ m^2 n^2 m^2 n^2 - 2m^2 n^2 (m^2 - n^2)^2 \\ m^3 nmn^3 (mn^3 - m^3 n) 2(mn^3 - m^3 n) \\ mn^3 m^3 n (m^3 n - mn^3) 2(m^3 n - mn^3) \end{bmatrix} \left[Q_{ij}\right]$$

Here $m = Sin\theta(\bar{\omega})$ and $n = Cos\theta(\bar{\omega})$, wherein $\theta(\bar{\omega})$ is the random fibre orientation angle. However, laminate consists of a number of laminae wherein $[Q_{\mathbf{ij}}]$ and $[\bar{Q}_{ij}(\bar{\omega})]$ denotes the on-axis elastic constant matrix and the off-axis elastic constant matrix, respectively. The elasticity matrix of the laminated composite shell can be expressed as,

$$[D'(\bar{\omega})] = \begin{bmatrix} A_{ij}(\bar{\omega}) & B_{ij}(\bar{\omega}) & 0 \\ B_{ij}(\bar{\omega}) & D_{ij}(\bar{\omega}) & 0 \\ q & q & S_{ij}(\bar{\omega}) \end{bmatrix} \tag{6}$$

where

$$[A_{ij}(\bar{\omega}),\ B_{ij}(\bar{\omega}),\ D_{ij}(\bar{\omega})] = \\ = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} [\bar{Q}_{ij}(\bar{\omega})]_k \, [1, z, z^2] dz \quad i, j = 1, 2, 6$$

and

$$[S_{ij}(\bar{\omega})] = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} \alpha_s [Q_{ij}(\bar{\omega})]_k dz \quad i, j = 4, 5$$

where α_s is the shear correction factor and is assumed as 5/6. The mass matrix is expressed as

$$[M(\bar{\omega})] = \int_{Vol} [N][P(\bar{\omega})][N] d(vol) \tag{7}$$

The stiffness matrix is given by

$$[K(\bar{\omega})] = \int_{-1}^{1} \int_{-1}^{1} [B(\bar{\omega})]^T [D(\bar{\omega})][B(\bar{\omega})] d\xi d\eta \tag{8}$$

The strain-displacement relation is expressed as

$$\{\varepsilon\} = [B]\{\delta_e\} \tag{9}$$

where

$$\{\delta_e\} = \{u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \ldots u_8, v_8, w_8, \theta_{x8}, \theta_{y8}\}^T$$

$$[B] = \begin{bmatrix} N_{i,x} & 0 & -\frac{N_i}{R_x} & 0 & 0 \\ 0 & N_{i,y} & -\frac{N_i}{R_y} & 0 & 0 \\ N_{i,y} & N_{i,x} & -\frac{2N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & N_{i,y} & N_{i,x} \\ 0 & 0 & N_{i,x} & N_i & 0 \\ 0 & 0 & N_{i,y} & 0 & N_i \end{bmatrix}$$

The energy functional for Hamilton's principle using Lagrange's equation, the dynamic equilibrium equation for free vibration of graphite-epoxy composite shell can be expressed as [24]

$$[M(\bar{\omega})]\{\ddot{\Delta}\} + [K(\bar{\omega})]\{\Delta\} = 0 \tag{10}$$

The governing equations are derived based on Mindlin's theory [25] incorporating rotary inertia, transverse shear deformation. For free vibration, the stochastic natural frequencies $[\omega_n(\bar{\omega})]$ are determined from the standard eigenvalue problem and is solved by the QR iteration algorithm.

3 Reliability based optimization

Traditional design optimization does not consider the uncertainties present in the actual modelling, imperfection during random manufacturing processes and other external influencing factors for composite structures. In other words, these uncertainties can be occurred due to manufacturing variability like uncertainties in material properties and variability in external conditions like loading, error in modelling or simulation. These uncertainties might cause large variations in certain performance characteristics. Reliable designs are designs at which the chance of failure of structure is low [26]. In reliability based optimization (RBO) problems [27], there is a trade-off between obtaining greater reliability and minimum cost, since greater reliability implies greater cost, but smaller reliability also implies greater cost due to failure costs. Hence there is an optimum reliability that can be achieved specific to design requirement. In the subsequent sections the surrogate modelling approach using D-optimal design, genetic algorithm and finally the reliability based optimization scheme for the present study are discussed.

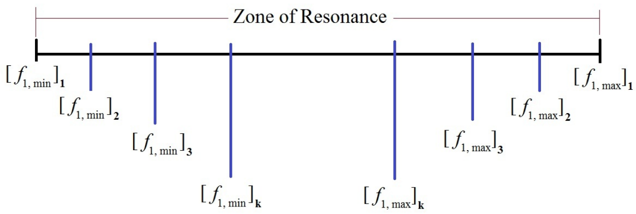

Fig. 2. Reliability based weight optimization for avoiding resonance

3.1 D-optimal design

D-optimal design is a statistical approach with a specific sampling technique which is employed in mapping of the input and output for construction of surrogate model using polynomial regression method. Considering the problem of estimating the coefficients of a linear approximation is modelled by least squares regression analysis

$$Y = X\beta + \varepsilon \tag{11}$$

where 'Y' is a vector of observations of sample size, 'ε' is the vector of errors having normal distribution with zero mean, 'X' is the design matrix and 'β' is a vector of unknown model coefficients and can be estimated by using the least squares method as

$$\beta = (X^T X)^{-1} X^T Y \tag{12}$$

A measure of accuracy of the column of estimators, β is the variance-covariance matrix which is defined as

$$V(\beta) = \sigma^2 (X^T X)^{-1} \tag{13}$$

where $\sigma^2$is the variance of the error. The $V(\beta)$ matrix is a statistical measure of the goodness of the fit. $V(\beta)$ is a function of $(X^T X)^{-1}$and therefore, one would want to minimize $(X^T X)^{-1}$to improve the quality of the fit. If X denotes the design matrix as a set of value combinations of coded parameters and X^T is the transpose of X, then D-optimality is achieved if the determinant of $(X^T X)^{-1}$ is minimal. The letter "D' stands for the determinant of the $(X^T X)$ matrix associated with the model. In the present study, the constructed meta-models provide an approximate meta-model equation which relates the input random parameters 'x_i' (say ply orientation angle, elastic modulus etc. of each layer of laminate) and output 'Y' (say natural frequency) for a particular system [28].

The meta-model is employed to fit approximately for a set of points in the design space using a multiple regression fitting scheme. The position of design points is chosen algorithmically according to the selected number of input variables and their range of variability. Hence the design points are not considered at any specific positions; instead, they are selected in such a fashion so that it meets the optimality criteria. In D-optimal design, the total sample size (n) is the summation of the minimum number of design points [$n_d = 0.5[(k+1)(k+2)]$, additional model points ($n_a = k$) and lack-of-fit points (n_l). (i.e., $n = n_d + n_a + n_l$) where k is the number of stochastic input parameter. For model construction in the present study, an over-determined D-optimal design [29, 30] (number of additional samples n_a, along with the minimum point design and $n_l = 10$ samples to estimate the lack of fit) has been used. The insignificant input features are screened out and not considered in the model formation using analysis of variance (ANOVA) method according to its F-test value. The prediction quality of meta-model is checked by three basic criteria such as coefficient of determination or R^2 (measure of the amount of variation around the mean explained by the model), R^2_{adj} (measure of the amount of variation with respect to mean value explained by the model, adjusted for the number of terms in the model) and R^2_{pred} (measure of the prediction capability of the response surface model) [30].

3.2 Genetic algorithm for composite shells

The concept of Genetic Algorithm (GA) (originated by Charles Darwin) is a computational search tool based on concepts of natural selection and survival of the fittest individual. The prime importance in GAs exists in the way by which the solutions are tracked. Despite of using derivatives or gradients of deterministic approach, GAs work with the objective function based on simple values of individuals. Such feature makes it suitable for solving the problems with discontinuous functions, and non-defined derivatives. GAs work with the population of individuals in each generation similar to deterministic optimization methods wherein the search is performed with focus on a single solution at a time. As several search points are maintained, the convergence or stagnation to local minima, if the starting point is poorly chosen, is prevented. All these aspects result in more chances of finding the optimal solution, even on problems having hard search spaces with multiple local minimum [31]. The design of the optimal sequence of layers in laminated composite materials is a problem of global minimum. Due to the stochastic characteristics of GAs, they are more suitable to optimize than deterministic methods of optimization, which often converge to solutions representing a local minimum. Moreover, in commercial designs, fiber orientation angles and the amount and thickness of layers are discrete variables, a fact which confirms the suitability of GAs for these kinds of problems. Many studies [32, 33] are subsequently carried out by using the method of design optimization for composite structures.

The initial population of individuals is generated randomly for the design parameters of composite shells. It is then encouraged to evolve over generations to produce new better or fitter generations using genetic operators until the problem is satisfactorily solved. An elitist selection scheme is used to obtain the new generation taking organisms from the current population and from the children population just created. This process is repeated until the convergence criterion is met. The three fundamental genetic operators are selection (according to the fitness of individual solutions so that the number of times an individual is selected is dependent on its relative performance in the population) crossover (to form new individuals by exchanging chromosome between two selected individuals segments) and mutation (this prevents premature convergence by randomly changing part of one selected individual's chromosome). Many applications related to GA can be found in the area of structural engineering can be found [34, 35].

In the present study, a multivariable minimization function is coupled with genetic algorithm in order to improve the value of the fitness function. Genetic algorithm searches the results globally first and after the GA terminates a local search is employed with the end results of GA. The output of GA is considered as the initial point for next step of the local optimization. From these initial points, the local minimum point is searched us-

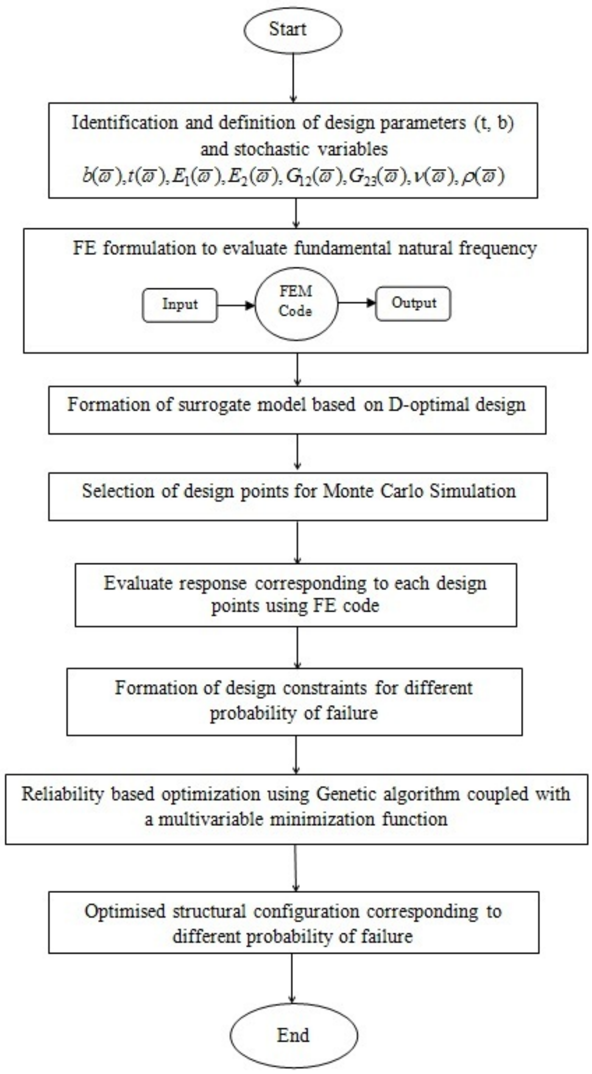

Fig. 3. Flowchart of RBDO using surrogate model for composite shells

ing a multivariable minimization function *fmincon* (MATLAB) [36] which attempts to find the constrained minimum of a scalar function of several variables starting at an initial estimate.

3.3 Detail optimization scheme

There are two types of variables considered in the present analysis, namely stochastic variables (material properties, fibre parameters, laminate dimensional parameters) and design variables (width and thickness) for the composite spherical shell. The upper and lower bounds of design variables and stochastic variables are furnished in Table 1 and Table 2 respectively showing respective upper control limit (UCL) and lower control limit (LCL). The reliability based optimization problem is studied with an objective of weight [i.e., volume(V)×density (ρ)] minimization and to avoid resonance [37–39] as defined below:

Minimize $V(b,t)$, subjected to

$$\begin{aligned} & f_1(b,t) < (f_{1,\min})_i \\ & f_1(b,t) > (f_{1,\max})_i \\ & b_{lcl} \le b \le b_{ucl} \\ & t_{lcl} \le t \le t_{ucl} \end{aligned} \tag{14}$$

where $i = 1,2,\ldots\ldots k$ represent different zone of resonances (ZOR) representing the corresponding level of confidence in the design (refer to Fig. 2). The fitness function can be expressed as

$$F(x) = \{V\} = \pi t\left[\frac{b^2}{4} + R^2\left\{1 - \frac{b^2}{4R^2}\right\}\right] \tag{15}$$

where, for spherical shell, $R_x = R_y = R$is the radius of curvature.

For each aforementioned ZORs, the probability of failure (P_F)can be estimated by performing Monte Carlo simulation on the first- or second-order approximation $\tilde{g}(x^i)$ of the original implicit limit performance function $\tilde{g}(x^i)$and can be expressed as

$$P_F = \frac{1}{N_{samp}}\sum_{i=1}^{N_{samp}} \Pi\left[\tilde{g}(x^i) < 0\right] \tag{16}$$

where x^i is i-th realization of X, N_{samp} is the sampling size, Π is a deciding function of the fail or the safe state such that $\Pi = 1$, if $\tilde{g}(x^i) < 0$otherwise zero. In the present study, if the fundamental frequency for a particular design point falls outside the ZOR, then for that sample$\Pi = 1$, otherwise zero. The reliability index corresponding to the failure probability (P_F) can be obtained by

$$\beta = -\Phi^{-1}(P_F) \tag{17}$$

where $\varphi(.)$ is the cumulative distribution function of a standard Gaussian random variable. In the present analysis, the failure criterion is defined as the occurrence of resonance in the system.

A flowchart of the proposed optimization algorithm is provided in Fig. 3. The steps followed for the optimization in this analysis are summarized below:

Step 1: Stochastic variables and the design variables are identified first. Stochastic input variables are considered to follow uniform probability distributions which are defined by their upper and lower bounds. For Monte Carlo simulation based reliability analysis, it is more important to capture all the possible combinations of stochastic input variables within the design space than the type of probability distribution of those variables. In view of the above, uniform distribution is considered for all the stochastic input variables bounded by upper and lower limits. In this analysis, the design variables are considered to have uncertain characteristics i.e. the design variables are also stochastic variables. However, it is noteworthy that the design bounds for width (b) and thickness (t) (Table 1) are taken higher than the perturbation bounds of these two variables (Table 2). Basic idea of the proposed optimization algorithm in this article is as follows. First the range of variation in the fundamental natural frequency is quantified by randomly perturbing the stochastic variables following a Monte Carlo simulation. Then an optimization is performed as described in Eq. (14) to exclude a portion of the ZOR for achieving desired level of confidence in a particular design.

Step 2: After identifying the stochastic and design variables, the next step is to construct the surrogate model for fundamental natural frequency using D-optimal design. For details of formation of surrogate model using finite element code please refer to the work of Dey *et al.* [40]. In the present study the purpose of employing surrogate model is to eliminate the need of running expensive finite element model several times and thus to achieve computational efficiency.

Step 3: In this step, Monte Carlo simulation (10,000 samples) is carried out for combined variation of all the stochastic variables employing surrogate modelling approach.

Step 4: After carrying out Monte Carlo simulation different ZORs as depicted in Fig. 2 are defined according to required level of confidence in a particular design (refer to Table 4).

Tab. 1. Upper and lower control limits of design variables

Parameters	Symbol	Design Variables	
		UCL	LCL
Width	b	1.5 m	0.5 m
Thickness	t	0.007 m	0.003 m

Step 5: Volume optimizations are carried out corresponding to different level of desired confidence in design to exclude ZORs as described in Eq. (14).

Step 6: In this step probability of failures are obtained following Eq. (16) corresponding to different ZORs. Here N_{samp} is the total number of samples for Monte Carlo simulation and the numerator is the number of realizations that are not considered corresponding to a particular ZOR. From probability of failures respective reliability indexes can be obtained using Eq. (17). In the present article, optimized structural configurations are presented for different probability of failures as shown in Fig. 6 to Fig. 8.

4 Results and Discussion

In the present study, four layered graphite-epoxy angle-ply laminated composite cantilever shallow spherical shells are considered. Finite element formulation of the composite spherical shell structure is based on Mindlin's theory considering an eight noded isoparametric quadratic element. Table 3 represents the non-dimensional fundamental natural frequencies [refer to Eq. (18)] for isotropic, corner point-supported spherical shells [41, 42].

$$\omega = \omega_n L^2 [12\rho(1-\mu^2)/E_1 t^2]^{1/2} \quad (18)$$

The test of accuracy of surrogate model with respect to R^2, R^2_{adj}, R^2_{pred} and adequate precision values are furnished in Table 5. The scatter plot (refer to Fig. 4) represents the validation of present surrogate model with respect to finite element model. The surface plot for fundamental natural frequency with variations of thickness and width of composite shells is presented in Fig. 5.

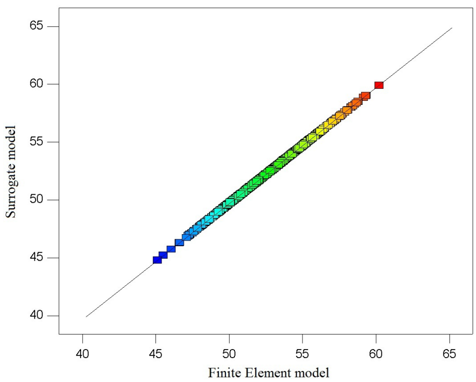

Fig. 4. Surrogate model validation with finite element model for fundamental natural frequencies

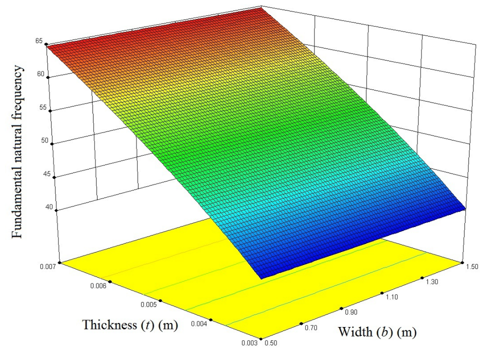

Fig. 5. Surface plot for fundamental natural frequency with variations of thickness and width of composite shells

Due to paucity of space, only a few important representative results of reliability based optimization are furnished in this article. The optimized width, thickness and volume for different probability of failures are furnished in Fig. 6, Fig. 7 and Fig. 8, respectively. The points shown in blue solid circles are corresponding to the minimum weight obtained at zero probability of failure. It is observed that as the probability of failure increases, the volume decreases with corresponding optimization of width and thickness of the spherical shell. Depending on the constraints of probability of failure the optimal solutions for width, thickness and volume can be found from these figures according to design requirements. The reliability index corresponding to different probability of failures can be obtained by using Eq. (17) as furnished in section 3.

5 Conclusions

This article proposes a novel reliability based optimization approach for weight minimization of spherical composite can-

Tab. 2. UCL and LCL of stochastic and variables

Parameters	Symbol	Stochastic Variables	
		Upper control limit (UCL)	Lower control limit (LCL)
Width	b	1.1 m	0.9 m
Thickness	t	0.0055 m	0.0045 m
Ply angle	θ	50°/40°/ 50°/-40°	40°/-50°/ 40°/-50°
Elastic modulus (longitudinal)	E_1	151.8 GPa	124.2 GPa
Elastic modulus (transverse)	E_2	9.79 GPa	8.01 GPa
Shear modulus (longitudinal)	G_{12}	7.81 GPa	6.39 GPa
Shear modulus (transverse)	G_{23}	3.1249 GPa	2.556 GPa
Poisson ratio	ν	0.33	0.27
Mass density	ρ	3522.2 kg/m^3	2881.8 kg/m^3

Tab. 3. Non-dimensional fundamental frequencies of isotropic, corner point-supported spherical shells considering a/b = 1, a/a = 1, a/t = 100, a/R = 0.5,$\mu = 0.3$.

R_x/R_y	Present FEM	Leissa and Narita [38]	Chakravorty et al. [39]
1	50.74	50.68	50.76

Tab. 4. Probability of failures corresponding to different Zone of resonance (Refer to Fig. 2)

i	Zone of Resonance		Sample no. satisfying failure criteria	Probability of failure (P_F)
	Upper Bound $(f_{\max,1})_i$	Lower Bound $(f_{\min,1})_i$		
1	53.99	45.24	10000	1.00
2	52.99	46.24	9600	0.96
3	51.99	47.24	7666	0.77
4	50.99	48.24	5000	0.50
5	49.99	48.30	1800	0.18
6	49.69	49.34	800	0.08
7	49.59	49.38	367	0.04
8	49.49	49.40	167	0.02

Tab. 5. Test for accuracy of surrogate model

Parameter	Ideal value	Present value
R^2value	1.0	0.997
R^2_{adj}value	1.0	0.999
R^2_{pred}value	1.0	0.992
Adequate Precision	>4.0	69923.46

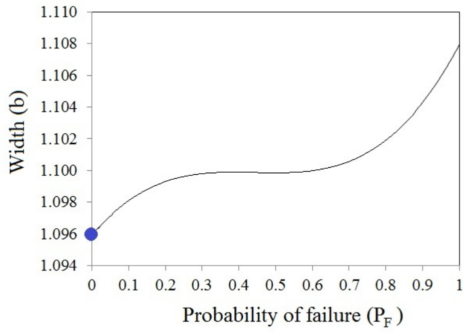

Fig. 6. Probability of failure with respect to width (m)

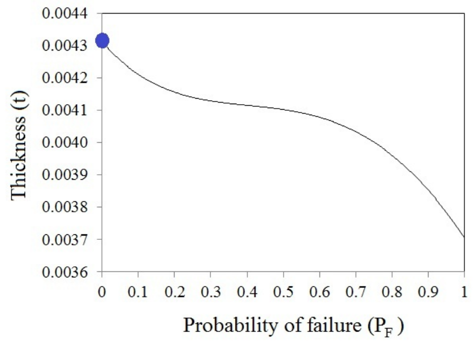

Fig. 7. Probability of failure with respect to thickness (m)

tilever shells with an attempt to avoid resonance. Genetic algorithm coupled with a local multivariate search function is employed to minimise the weight by optimising the width and thickness of the spherical shell corresponding to different probability of failures. In general, it is observed that as the probability of failure increases, the volumen of the composite shell decreases corresponding to optimized values of width and thickness. The optimised data obtained are the first known results for the type of analyses carried out here and the results could serve as reference solutions for future investigators. The proposed surrogate based approach of reliability based optimization can be extended to more complex system of laminated composite structures and optimization of material properties in addition to topology.

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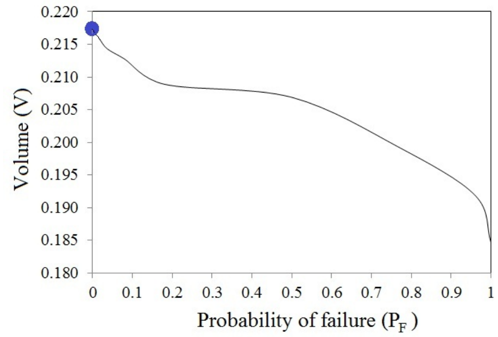

Fig. 8. Probability of failure with respect to volume (m^3)

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