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RESEARCH ARTICLE

Mixed Approaches to Handle Limitations and Execute Mutation in the Genetic Algorithm for Truss Size, Shape and Topology Optimization

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Abstract
A high-performance genetic algorithm for the optimal synthesis of trusses in discrete search spaces is developed. The main feature of the proposed computational procedure is the possibility of obtaining effective solutions without the violation of any constraint. In general, a varying of cross-sectional areas of bars, coordinates of nodes and topology system is provided. A group of individuals in the population can be accepted for further consideration only if all specified limitations have been fulfilled. Penalties that significantly change an objective function are introduced for other individuals. This mechanism of handling limitations provides for correction of inaccuracies that can introduce penalty functions for satisfying the problem conditions. Both a random change to the entire set of admissible values and a random choice of values among adjacent elements in this set can be performed during the mutation stage. Standard test examples for benchmark mathematical functions and trusses show high efficiency of the considered iterative procedure in terms of solution accuracy.

Keywords
trusses, optimization, genetic algorithms, mixed approaches

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1 Introduction

Genetic algorithms (GAs) [1] are one of the effective metaheuristic approaches to solve optimization problems. Simulated annealing algorithm [2], bat-inspired algorithm [3], magnetic charged system search method [4], ant colony optimization [5], particle swarm optimizer [6], harmony search algorithm [7, 8], charged system search method [9], and big bang-big crunch algorithm [10] are also in the class of such procedures. GAs have already found application in many areas of engineering and science [1, 11-13]. Different variants of GAs, adapted to the structural optimization problems, are presented in Refs. [14-23] and in many other works. Algorithms for optimal design of constructions combining GAs procedures with other methods of structure optimization were also considered [24-27]. A feature of many load-bearing structure optimization problems lies in the great number of conditions that significantly narrow the admissible search areas of parameters. When solving these problems with GAs, penalty functions became widely used in handle limitations [28, 29]. Other methods for taking into account limitations which may include repairing infeasible individuals [30], searching the boundaries of a feasible region [31], and homomorphous mapping [32], are less universal [29]. At the same time, it should be pointed out that penalty functions can lead to inaccurately satisfying limitations in the form of inequalities.

There was previously developed a GA for the optimization of deformable systems, where loading limitations are considered by means of the rigorous removal of unsatisfactory individuals [33-35]. During this, an auxiliary population of elite individuals is formed as well as the usual population. And it is used to refill the main population while removing unsatisfactory structure variants not corresponding to the set limitations. Such GA allows avoiding defects resulting from penalty functions. This procedure has helped to solve problems of synthesis of building industrial and civil frames, beam grillage, reticulated domes, trusses and various other structures. Nevertheless, in some cases this computation method allowed one to obtain individuals which would satisfy loading limitations only in case of several hundreds of performed iterations.

It is assumed that genetic algorithm can be made rather efficient if different approaches for taking into account limitations are used for individuals of a population. It is provided in this paper in respect to size, shape and topology discrete optimization of trusses that a part of individuals of one population can be allowed for selection and crossover only if all specified conditions have been met. At the same time, such rigorous rules are not specified for the remainder of individuals. Penalties are introduced for them, appreciably correcting the value of an objective function. When a couple of individuals are selected for the crossover procedure, all individuals of the population are considered on equal terms. As well, a mixed mutation procedure using one of two variants of random selection is realized: on the entire set of admissible values and only among several adjacent elements in this set. Efficiency of the offered GA is shown with the help of test examples for benchmark mathematical functions and truss structures which were considered in literature.

2 Statement of the truss optimization problem

Weight minimization of plane and space trusses which can generally vary on discrete sets of cross-sectional areas of bars and the coordinates of nodes is performed. Additionally, topology optimization is realized with the help of redundancy structures which are controlled using an ability to introduce “zero” (absent) construction elements by assigning relatively small cross-sectional areas for them. When calculating the weight of a construction, such elements are not taken into account. As a general matter, strength and stiffness limitations are considered. A truss is assumed to be discretized in terms of the finite element method according to a displacement approach [36]. Truss nodes coincide with nodes of the finite element model.

The stability of a geometrical shape condition for a load-bearing construction will be adduced to displacement limitations. For that purpose, first of all there must be provided stability of the geometrical shape of a redundant base design. Calculations show that in this case when fictitious cross-sectional areas of “zero” construction elements are $10^5\ldots10^6$ times less than the smallest allowed cross-sectional area of bars, both imitation of absence of these elements and ability to obtain a well-conditioned system of finite elements method equations if the object has a stable geometrical shape is provided. If a bar structure loses stability of its geometrical shape, this can be shown by relatively large fictitious displacements in a formal problem solving. If groups of finite elements become virtually isolated when low stiffness bars are introduced and these elements are not loaded, then solution of the system of equations of the finite elements method may not give great displacements, yet such individuals are excluded as irrational during the optimization process. An alternative of this approach to selecting geometrically unstable systems is evaluation of a matrix determinant for a system of equation. However, this way appears to be rather computationally expensive.

An optimization problem in general is formulated as follows: Find $\boldsymbol{A}$, $\boldsymbol{X}$, $\boldsymbol{Y}$ and $\boldsymbol{Z}$, which minimize

$$W\left(\boldsymbol{A},\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z}\right)=\sum_{i=1}^{I}\alpha_i\rho_i l_i A_i \qquad (1)$$

subject to

$$\boldsymbol{KU}_k=\boldsymbol{Q}_k,\ k=1,\ldots,k_0, \qquad (2)$$

$$P_{\sigma ik}=\left(\left|\sigma_{ik}\right|/\left[\sigma\right]_{ik}-1\right)\le 0,\ i=1,\ldots,I,k=1,\ldots,k_0, \qquad (3)$$

$$\begin{gathered}P_{\delta mk}^{(t)}=\left(\left|\delta_{mk}^{(t)}\right|/\left[\delta_{mk}^{(t)}\right]-1\right)\le 0,\ m=1,\ldots,M,\\ k=1,\ldots,k_0,t=1,2,3,\end{gathered} \qquad (4)$$

where W is the weight of bars, $A=\{A_1, \ldots, A_1\}^T$ is the numeric vector of cross-sectional areas A_j of bars, I is the number of bars, $X=\{X_1, \ldots, X_M\}^T$, $Y=\{Y_1, \ldots, Y_M\}^T$ and $Z=\{Z_1, \ldots, Z_M\}^T$ are the numeric vectors of x, y and z coordinates of truss nodes, respectively, M is the number of nodes, α_i is the Boolean value (α_i=1, if presence of the bar i is taken into consideration in the construction, α_i=0, if presence of the bar in the construction is not taken into account), ρ_i and l_i are the density and length of bar i, respectively, K is the stiffness matrix, U_k and Q_k are the vector of node displacements and vector of external forces for loading k, respectively, k_0 is the number of loadings, $P_{\sigma ik}$ and $P_{\delta mk}^{(t)}$ are the values that specify whether k loading satisfies the limitation or it does not satisfy limitation on stress in bar i and displacement in node m in the direction of axis under number t with consecutive numbering of x, y and z axes, respectively, σ_{ik} and $[\sigma_{ik}]$ are the calculated and working normal stresses in bar i for loading k, respectively, $\delta_{mk}^{(t)}$ and $\left[\delta_{mk}^{(t)}\right]$ are the calculated and allowed displacements of node m in t direction, respectively.

3 Genetic algorithm for trusses

Genetic operators in the search process are performed by the main population Π of fixed even size N. Additionally, an auxiliary elite population Ψ is used. Its size depends on the results of genetic algorithm operation, but does not exceed N. A set of admissible values is arranged in the order of their increasing for each varied parameter.

3.1 Selection of the initial population Π

Here N individuals of population Π are specified by means of assigning maximum admissible values for varied parameters. Population Ψ remains empty on this stage.

3.2 Iterative process

The following actions are performed at every iteration:

a) *Checking whether limitations for individuals of population Π are satisfied*. Calculations of the stress-strain state of construction variants of the given population are performed on the base of the solution of Eq. (2). The population is divided into groups Π_1 and Π_2. Group Π_1 includes the first N_1 individuals ($N_1 < 1$), group Π_2 includes the other individuals. If at least one of the limitations (3), (4) for an individual of group Π_1 is not satisfied, and there are individuals in auxiliary population Ψ which are missing from the main population, then the best of these individuals replaces the checked project. If there are no individuals in population Ψ for which this condition holds true, then a new individual of population Π is specified by means of random choice of design variable values. If limitations (3) and (4) for an individual from group Π_2 are not satisfied, then a penalty factor k_p is introduced for it:

$$W = k_p W_\alpha, \tag{5}$$

where W_α is the value of object function before penalty introduction. A ratio

$$k_p = \left(\prod_{k=1}^{k_0} \left(1 + \gamma_\sigma \chi\left(P_{\sigma k \max}\right) P_{\sigma k \max}\right) \right) \times \left(\prod_{k=1}^{k_0} \left(1 + \gamma_\delta \chi\left(P_{\delta k \max}\right) P_{\delta k \max}\right) \right) \tag{6}$$

is used, where $P_{\sigma \max}$ and $P_{\delta \max}$ are the maximum values of $P_{\delta ik}$ and $P_{\delta mk}^{(t)}$ for loading k, respectively, γ_σ, γ_δ are assigned positive values; $\chi(x)$ is the Heaviside function of some argument x: $\chi(x) = 0.5\ (1+\mathrm{sgn}(x))$.

b) *Modification of population Ψ*. Every individual in population Π is checked by two criteria: is there such an individual in population Ψ, does value W of this individual not exceed the value of the objective function for the worst individual in the population. If both answers are negative, the considered individual is placed into population Ψ. If the number of individuals in the auxiliary population was already equal to N, then an individual with the greatest objective function value is excluded from population Ψ.

c) *Mutation*. This procedure may be performed with the probability p_1 for a randomly selected

$$n_1 = \max\left(1, \lfloor \lambda n_0 \rfloor\right) \tag{7}$$

parameters for each individual of the population Π ($0 < \lambda < 1$), where p_1, λ are the specified values, n_0 is the total number of parameters. The next scheme to choose a value of parameter under number j is introduced. Values p_a, p_b are determined with the help of a random number generator on the segment (0, 1) with a uniform law of distribution, and then are compared with the mutation control numbers m_a ($0 < m_a < 1$), m_{bl}($l = 1,2,3, 0 < m_{b1} < m_{b2} < m_{b3} < 1$).

If the inequality $p_a > m_a$ holds true, then any of the admissible parameter values is chosen randomly with equal probability. Otherwise, for number r_j of the current position of this parameter in the set of its acceptable values any of the following modifications can be carried out:

$$\left(p_b < m_{b1}\right) \wedge \left(r_j \geq 3\right) \Rightarrow r_j = r_j - 2, \tag{8}$$

$$\left(\left(p_b < m_{b1}\right) \wedge \left(r_j = 2\right)\right) \vee \left(\left(m_{b1} \leq p_b < m_{b2}\right) \wedge \left(r_j \geq 2\right)\right) \Rightarrow r_j = r_j - 1, \tag{9}$$

$$\left(\left(m_{b2} \leq p_b < m_{b3}\right) \wedge \left(r_j \leq w_j - 1\right)\right) \vee \left(\left(p_b \geq m_{b3}\right) \wedge \left(r_j = w_j - 1\right)\right) \Rightarrow r_j = r_j + 1, \tag{10}$$

$$\left(p_b \geq m_{b3}\right) \wedge \left(r_j \leq w_j - 2\right) \Rightarrow r_j = r_j + 2, \tag{11}$$

where w_j is the number of elements in the set of admissible values of the variable parameter.

d) *Operations of steps a and b are repeated.*

e) Selection and crossover. Here N / 2 couples of individuals are consecutively chosen from individuals of population Π by the roulette-wheel method with regard to W value. So, a numerical interval is selected for every individual s on a unitary numerical interval. The length Δ_s of this interval is defined by the value of the objective function for this individual:

$$\Delta_s = \varphi_s \Big/ \sum_{i=1}^{N} \varphi_i, \tag{12}$$

where

$$\varphi_i = 1 / \left(\alpha W_i^\beta\right). \tag{13}$$

Here W_i is the value of the objective function for individual i, α, β are prescribed constants. Then, a change of parameters is performed for every considered couple according to the scheme of a single-point crossover with a random choice of cut point. We do not allow re-entering of an individual into a couple during crossover process. At the same time, one project may be included in several pairs. The research of the functioning of the provided algorithm showed for trusses that in order to obtain an efficient iteration procedure, it is useful to accept $p_1 = 0,8...\ 1$; $\lambda = 0,1...0,2$; $N = 15...25$; $\alpha = 5...15$; $\beta = 1...3$; $m_a = 0.9$, $m_{b1} = 0.5$; $m_{b2} = 0.75$; $m_{b2} = 0.9$; $\gamma_\sigma = 10$; $\gamma_\delta = 100$, assume the chance of mutation for every individual, and to take the part of individuals Π_1 in population Π equal to 60%. These recommendations were taken into account in the considered examples for trusses with the specified values $p_1 = 1$; $\lambda = 0,1$; $N = 20$; $\alpha = 10$; $\beta = 1$.

4 Numerical examples

To analyse the efficiency of the provided evolutionary strategy, optimization calculations for a number of standard mathematical functions and examples of size, size/topology and size/shape/topology optimization of trusses were carried out. Values of kips and inches for structures were used for a suitable comparison of obtained results with the data published by other researchers.

4.1 Benchmark mathematical functions

Some unimodal, multimodal, rotated, shifted and shifted and rotated functions taken from Ref. [37] are applied to test the efficiency of the proposed genetic algorithm (Table 1). Here $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)$ is the vector of continuous variables, M is the orthogonal rotation matrix, $O=(o_1, o_2, ..., o_n)$ is the vector of shifts, $\tilde{Y} = (\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_n)$ is the auxiliary vector [37, 38]. The task was to minimize each function within the specified range.

In order to optimize such functions some adjustments were introduced into the presented in Section 3 discrete algorithm. Several octal digits are used to represent each continuous variable. Each octal digit varied in the discrete optimization as an independent variable. Then $n_o = nl_{(8)}$, where $l_{(8)}$ is the number of octal digits. In order to operate with negative values of the objective function the following scheme for calculating values φ_i was used:

$$\varphi_i = 1 \Big/ \left(\alpha \left(W_i - W_{\min} + 0.0001 W_{avg} \right)^{\beta} \right) \tag{14}$$

where $W_{\min}$ and W_{avg} are the minimum and the average values of the objective function for the population, respectively.

Unlike the optimization of trusses we have put $\lambda = 0.18$ and the parameter p_1 varies depending on the value $j = \lfloor r / r_\alpha \rfloor +$, where r is the number of iteration and r_α is the integer. It was set that $r_\alpha = 50$. For odd values j we have put $p_1 = 1$, for even j $p_1 = 0.1$ value was used.

For functions 1–13 $n = 8$ and the maximum number of iterations was assumed to be 500, for functions 14 and 15 we have put $n = 10$ and the maximum number of iterations was assumed to be 800 [37]. When $n = 8$ it was taken into account that $l_{(8)} = 5$, when $n = 10$ the value $l_{(8)} = 7$ was used. Here 30 runs of the algorithm were performed for each function.

The results of optimization for the considered benchmark functions are given in Table 2. These results are compared with new binary particle swarm optimization (NMBPSO) algorithm [39], binary gravitational search algorithm (BGSA) [40] and recently developed high-performance binary hybrid topology particle swarm optimization quadratic interpolation (BHTPSO-QI) algorithm [37]. Data for comparison are taken from [37]. Table 2 shows that in general the proposed genetic algorithm allowed to achieve slightly better results in comparison with BHTPSO-QI and significantly better results in comparison with NBPSO algorithm and BGSA on the average values of best solutions.

4.2 10-bar truss

The truss shown in Fig. 1 is a standard example for optimization problems and is used by researchers to evaluate performance of algorithms for efficiency analysis of the evolutionary modelling strategies. This truss was considered in Refs. [15], [16], [28], [41] and in many other works. We accepted the following characteristics for the material of bars: density $\rho = 0{,}1$ lb / in^3, coefficient of elasticity $E = 10{,}000$ ksi. Force $P = 100$ kips . A limitation 2.0 in. on displacements along x and y axis for every node of the deformed system was introduced. It was supposed that the stress modulus in truss bars would not exceed 25 ksi.

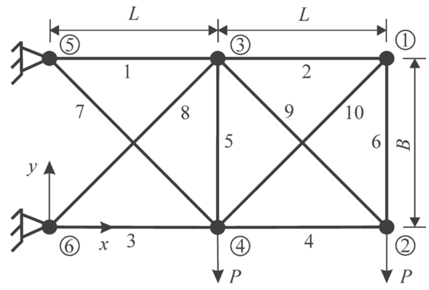

Fig. 1 10-bar truss

4.2.1 Size optimization

Let distance L = B = 360 in. A cross-sectional area of every member was varied independently. Two sets of values of varied parameters, shown in Table 3 (Case 1 and Case 2), were considered. Here 50 runs of the genetic algorithm were held with every set of admissible cross-sectional areas to estimate convergence.

The weight of the determined rational construction in every run was equal to 5490.7 lb in a discrete set of areas taken from Ref. [28]. The obtained vector of values of designed variables is {33.5, 1.62, 22.9, 14.2, 1.62, 1.62, 7.97, 22.9, 22, 1.62}. This is the best result at the exact compliance by the set limitations among those considered in the literature sources [42], [43]. Our algorithm required here less than 250 iterations to obtain this solution in more than 80% of runs. The longest calculation took 407 iterations. How quickly this project is found in the first 16 runs is shown in Fig. 2.

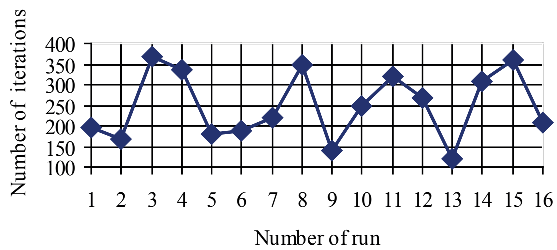

Fig. 2 Number of iterations required to determine solution of the problem in the first 16 runs at size optimization of the 10-bar truss in Case 1

For a set of cross-sectional areas which was given in Ref. [15], the weight in all runs was equal to 5130.20 lb. The maximum number of iterations required to determine this result was equal to 2683. It required less than 1500 iterations in more than 80% of runs. The solution was compared in Table 4 with the results from corresponding literature sources for this problem which used the same discrete set of admissible area values. The weight which was determined via the proposed algorithm appeared to be smaller than in Refs. [15] (Solution 2) and [45], yet bigger than in Refs. [15] (Solution 1) and [44], however the results of this problem in Refs. [15] (Solution 1) and [44] do not exactly satisfy the displacement limitation.

Table 1 Bencmark functions

Function	Name	Type	[Range]^n	Global minimum
$f_1=\sum_{i=1}^{n}\tilde{x}_i^2$	Sphere Model	Unimodal	$[-100, 100]^n$	0
$f_2=\sum_{i=1}^{n}\lvert\tilde{x}_i\rvert+\prod_{i=1}^{n}\lvert\tilde{x}_i\rvert$	Schwefel's Problem 2.22	Unimodal	$[-10, 10]^n$	0
$f_3=\sum_{i=1}^{n}\left(\sum_{j=1}^{i}\tilde{x}_j\right)^2$	Schwefel's Problem 1.2	Unimodal	$[-100, 100]^n$	0
$f_4=\frac{1}{n}\sum_{i=1}^{n}\left(\tilde{x}_i^4-16\tilde{x}_i^2+5\tilde{x}_i\right)$	2^n Minima Function	Multimodal	$[-5, 5]^n$	-78.3323
$f_5=-\sum_{i=1}^{n}\tilde{x}_i\sin\left(\sqrt{\lvert\tilde{x}_i\rvert}\right)$	Generalized Schwefel's Problem 2.26	Multimodal	$[-500, 500]^n$	$-418.9829\times n=$ -3351.86
$f_6=\frac{1}{4000}\sum_{i=1}^{n}\tilde{x}_i^2-\prod_{i=1}^{n}\cos\left(\frac{\tilde{x}_1}{\sqrt{i}}\right)+1$	Generalized Griewank Function	Multimodal	$[-5.12, 5.12]^n$	0
$f_7=\sum_{i=1}^{n}\lvert\tilde{y}_i\rvert+\prod_{i=1}^{n}\lvert\tilde{y}_i\rvert,\ \tilde{Y}=M\times\tilde{X}$	Rotated Schwefel's Problem 2.22	Rotated Unimodal	$[-10, 10]^n$	0
$f_8=\frac{1}{n}\sum_{i=1}^{n}\left(\tilde{y}_i^4-16\tilde{y}_i^2+5\tilde{y}_i\right),\ \tilde{Y}=M\times\tilde{X}$	Rotated 2n Minima Function	Rotated Multimodal	$[-5, 5]^n$	-78.3323
$f_9=\sum_{i=1}^{n}\left[\tilde{y}_i^2-10\cos\left(2\pi\tilde{y}_i\right)+10\right],\ \tilde{Y}=M\times\tilde{X}$	Rotated Rastrign Function	Rotated Multimodal	$[-5.12, 5.12]^n$	0
$f_{10}=\sum_{i=1}^{n}\left(\sum_{j=1}^{i}\tilde{y}_j\right)^2-450,\ \tilde{Y}=\tilde{X}-O$	Shifted Schwefel's Problem 1.2	Shifted Unimodal	$[-100, 100]^n$	-450
$f_{11}=\max_i\left\{\lvert\tilde{y}_i\rvert,\ 1\le i\le n\right\}-450,\ \tilde{Y}=\tilde{X}-O$	Shifted Schwefel's Problem 2.21	Shifted Unimodal	$[-100, 100]^n$	-450
$f_{12}=\left[\frac{1}{4000}\sum_{i=1}^{n}\tilde{y}_i^2-\prod_{i=1}^{n}\cos\left(\frac{\tilde{y}_i}{\sqrt{i}}+1\right)\right]-180,\ \tilde{Y}=\tilde{X}-O$	Shifted Generalized Griewank Function	Shifted Multimodal	$[-600, 600]^n$	-180
$f_{13}=\sum_{i=1}^{n}\left[\tilde{y}_i^2-10\cos\left(2\pi\tilde{y}_i\right)+10\right]-330,\ \tilde{Y}=\tilde{X}-O$	Shifted Rastrign Function	Shifted Multimodal	$[-5.12, 5.12]^n$	-330
$f_{14}=\sum_{i=1}^{n-1}\left[100\left(\tilde{y}_i^2-\tilde{y}_{i+1}\right)^2+\left(\tilde{y}_i-1\right)^2\right]+400,$ $\tilde{Y}=M\left(\frac{2.048\left(\tilde{X}-O\right)}{100}\right)+1$	Shifted and Rotated Rosenbrock's Function	Shifted and Rotated Multimodal	$[-100, 100]^n$	400
$f_{15}=\sum_{i=1}^{n}\left[\tilde{y}_i^2-10\cos\left(2\pi\tilde{y}_i\right)+10\right]+900,$ $\tilde{Y}=M\left(\frac{5.12\left(\tilde{X}-O\right)}{100}\right)$	Shifted and Rotated Rastrigin's Function	Shifted and Rotated Multimodal	$[-100, 100]^n$	900

Table 2 Minimization results for the bencmark functions

Function		NBPSO	BGSA	BHTPS-QI	This work
f_1	Avg. best solution	6.414	1.088	**0.0008**	0.3463
	SD	12.32	2.658	**0.0021**	0.6625
	Median best solution	2.563	0.0149	**0.0001**	0.1331
f_2	Avg. best solution	0.1303	0.0631	**0.0082**	0.0309
	SD	0.1126	0.1417	**0.0153**	0.0284
	Median best solution	0.0971	0.0147	**0.0031**	0.0168
f_3	Avg. best solution	105.4	773.3	**35.02**	36.609
	SD	163.3	696.3	78.35	58.993
	Median best solution	54.30	531.6	**6.559**	16.817
f_4	Avg. best solution	-76.95	-77.07	-77.41	**-78.26**
	SD	0.6794	0.9648	0.5434	**0.1888**
	Median best solution	-77.01	-77.26	-77.43	**-78.31**
f_5	Avg. best solution	-3150	-3283	-3311	**-3316**
	SD	118.9	63.82	48.72	63.68
	Median best solution	-3177	-3305	-3317	**-3351.50**
f_6	Avg. best solution	5.668	7.680	4.434	**0.1319**
	SD	1.629	1.944	1.736	**0.1269**
	Median best solution	5.318	7.005	5.000	**0.1747**
f_7	Avg. best solution	5.191	7.211	5.709	**0.0841**
	SD	1.421	1.465	1.077	**0.0756**
	Median best solution	5.338	6.896	5.896	**0.0565**
f_8	Avg. best solution	-63.52	-63.25	-64.30	**-66.40**
	SD	2.621	2.082	2.53	3.187
	Median best solution	-63.04	-63.49	-64.11	**-67.52**
f_9	Avg. best solution	30.93	34.57	28.85	**12.42**
	SD	4.439	4.652	4.504	5.140
	Median best solution	30.72	34.97	29.01	**11.68**
f_{10}	Avg. best solution	-244.1	833.3	**-386.9**	-377.0
	SD	340.1	936.0	111.3	60.48
	Median best solution	-342.7	879.1	**-421.2**	-392.8
f_{11}	Avg. best solution	-443.3	-439.8	-446.00	**-448.1**
	SD	4.624	7.583	1.748	**1.369**
	Median best solution	-443.9	-443.2	-445.7	**-448.7**
f_{12}	Avg. best solution	-179.2	-179.6	-179.4	**-179.7**
	SD	0.4436	0.3698	0.4111	**0.1127**
	Median best solution	-179.3	-179.7	-179.6	**-179.7**
f_{13}	Avg. best solution	-321.8	-323.6	**-324.4**	-323.8
	SD	2.834	2.559	**2.132**	2.203
	Median best solution	-321.6	-323.8	**-324.8**	-323.5
f_{14}	Avg. best solution	438.0	450.1	433.0	**416.5**
	SD	19.80	15.47	15.029	24.72
	Median best solution	438.4	447.5	436.0	**409.8**
f_{15}	Avg. best solution	919.3	913.3	**912.0**	917.0
	SD	5.896	3.435	5.420	4.193
	Median best solution	919.6	913.6	**911.9**	917.1
Avg. rank		3.40	3.40	1.73	1.47

Table 3 Cross-sectional areas of members for the size optimization of a 10-bar truss

Case	Areas (in.^{2})
1: Nanakorn and Meesomklin [28]	{1.62, 1.8, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.5, 13.5, 13.9, 14.2, 15.5, 16.0, 16.9, 18.8, 19.9, 22, 22.9, 26.5, 30.0, 33.5}
2: Jenkins [15]	{0.1, 0.347, 0.44, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.8, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.3, 10.85, 13.33, 14.29, 17.17, 19.18, 23.68, 28.08, 33.7}

Table 4 Comparison for the size optimization of the 10-bar truss in Case 2

Variables	Optimal cross-sectional areas (in.^{2})				
	Jenkins [15]		Toğan & Daloğu [44]	Grierson [45]	This work
	Solution 1	Solution 2			
A_1	28.08	33.7	28.08	33.7	28.08
A_2	0.1	0.1	0.1	0.347	0.1
A_3	23.68	23.68	23.68	19.18	23.68
A_4	17.17	14.29	17.17	19.18	19.18
A_5	0.347	0.347	0.1	0.347	0.1
A_6	0.1	0.1	0.1	0.539	0.44
A_7	7.192	7.192	7.192	10.85	7.192
A_8	19.18	19.18	19.18	23.68	19.18
A_9	23.68	23.68	23.68	19.18	23.68
A_{10}	0.1	0.1	0.1	0.347	0.1
Weight (lb)	5054	5153	5054.6	5356	5130.2
Max. def. (in.)	2.02	2.00	2.0046	1.97	1.9971

4.2.2 Size/topology and size/shape/topology optimization

Two problems were solved where the 10-bar truss was considered as a base system. In the first task (A), studied in Refs. [41, 46], topology optimization with a fixed construction shape was held, and in the second task (B), which was considered in Refs. [8, 41, 46], size/shape/topology optimization was held. A set of 32 discrete values (1.62, 1.8, 2.38, 2.62, 2.88, 3.09, 3.13, 3.38, 3.63, 3.84, 3.87, 4.18, 4.49, 4.8, 4.97, 5.12, 5.74, 7.22, 7.97, 11.5, 13.5, 13.9, 14.2, 15.5, 16.0, 18.8, 19.9, 22.0, 22.9, 26.5, 30.0, and 33.5) (in.^{2}) given in Ref. [41] was used for varied areas of every actual bar.

Problem A W*e provided the possibility* to remove any bar. Here 50 runs of the program were held. The same result obtained in all runs is shown in Fig. 3. The weight of bars of the truss was 4962.1 lb. It took from 133 to 467 iterations. This project completely matches in weight, topology, and sizes with the solution of Ref. [41]. The same result of optimization in weight and topology was published in Ref. [46].

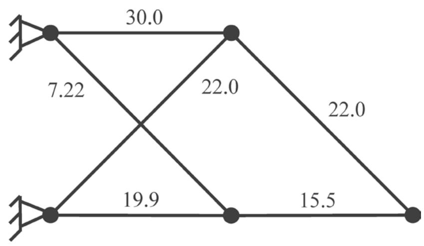

Fig. 3 Resultant topology for the size/topology design of a 10-bar base truss with cross-sectional areas (in.^{2})

Problem B In addition to the condition of problem *A*, the positions of nodes 1, 3 and 5 of the truss were varied vertically between b=180 in. and b=1,000 in., with a 10 in. interval. In 50 runs, during execution of 1000 iterations, more than 30 different solutions with the weight ranging from 2.74 to 2.94 kips were obtained. Two best results are shown on Fig. 4. Topologies for these solutions were also presented in Refs. [8, 47]. Solution 1 in topology and weight corresponds to the best result among those considered in the literature sources [47].

4.3 A 200-bar plane truss

This truss (Fig. 5) was considered by many researchers [7, 44, 48], when solving the problem of size optimization with a different limitation and grouping of members. We accepted the conditions of the problem in accordance with Ref. [44]. Distances L_1 = 240 in, L_2 = 144 in, L_3 = 360 in. Values $\rho = 0.283$ b/in^3 and $E = 30{,}000$ ksi were specified for the material of the bars. It was assumed that stress in every bar during compression and tension did not exceed 10 ksi in modulus. Displacements for unfixed nodes were not limited. Here 3 loadings were considered. In the first loading, forces of 1 kip were applied in a positive x direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62 and 71. In the second loading forces of 10 kips were applied in a negative y direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19 … 71, 72, 73, 74 and 75. In the third loading, the two previous loadings were combined. The members of the truss were linked into 29 groups. The numbers of members inserted in these groups are shown in Table 5. To vary the cross-sectional areas, the values from the set of Case 2 (see Table 3) were taken into account. Here 30 runs of the algorithm were held with 4000 iterations. The received solutions ranged from 27701.7 lb to 29053.51 lb. It took from 377 to 3927 iterations. Comparison of two individuals with a minimum weight using some data from literature sources is shown in Table 5. The results obtained by us appeared to have a better value of an objective function than those determined with the help of discrete design variables in [44] and [48].

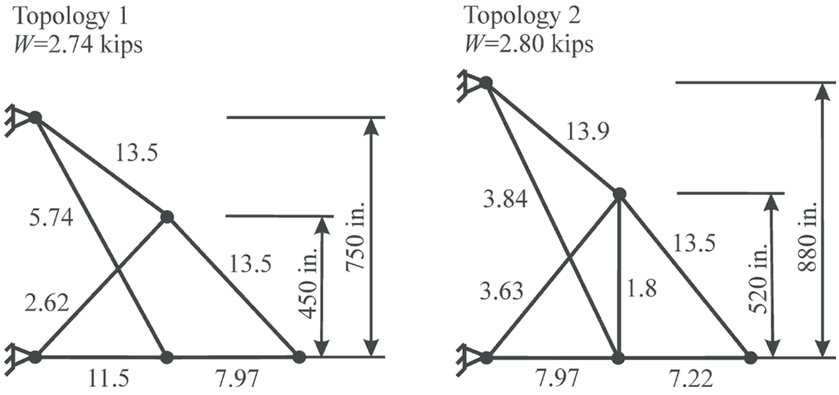

Fig. 4 Resultant topology for the size/shape/topology design of a 10-bar base truss with cross-sectional areas (in.^{2})

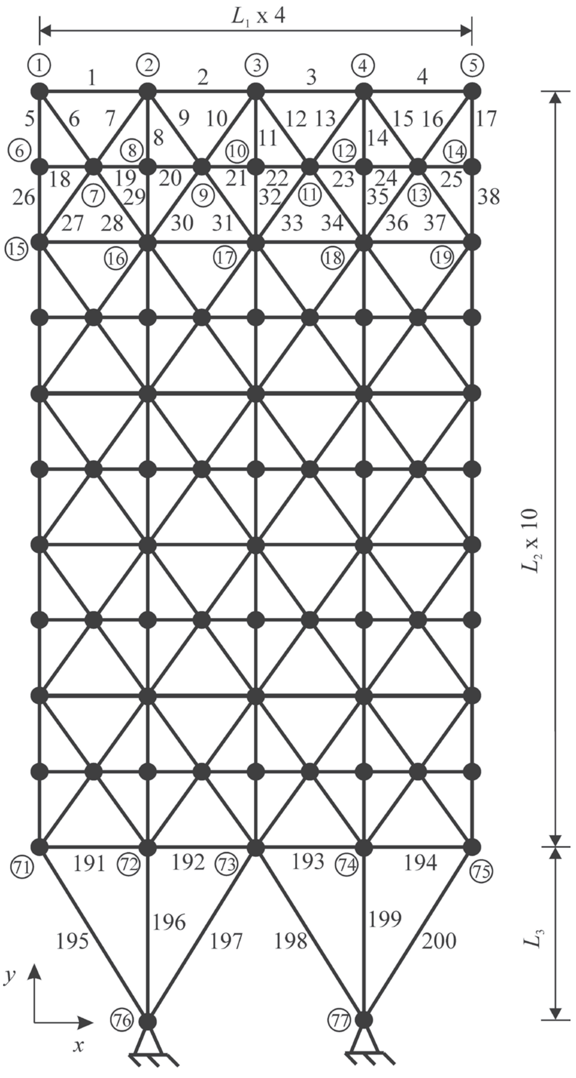

Fig. 5 200-bar truss

4.4 25-bar space truss

Optimal design of this object (Fig. 6) in different variants was performed in Refs. [7, 10, 16, 44, 49-51] and in many other articles. This paper gives consideration to the condition of a 25-bar truss size optimization problem according to Ref. [16]. It was supposed that L_1 = 75 in., L_2 =100 in., L_3 = 200 in. Loading was taken into account, which is provided in Table 6.

Cross-sectional areas of the truss were linked into eight groups: (1): A1, (2): A2- A5, (3): A6- A9, (4): A10- A11, (5): A12- A13, (6): A14- A17, (7): A18- A21, (8): A22- A25. Here 34 discrete values were used for every group, and these values were uniformly distributed on the interval [0.1-3.4] in.2, ρ = 0,1 lb/in.3 and E = 10,000 ksi were taken for the material of bars. It was assumed that stress in bars did not exceed 40 ksi in modulus. Displacements along x and y axis for every unfixed node did not exceed 0.35 in. Optimal synthesis process gave the same result in all 50 runs. It took from 163 to 3278 iterations. The optimum solution vector is presented in Table 7. This is identical to the best design developed in [10, 51]. It performs better than others when the number of average weight for 50 runs are compared.

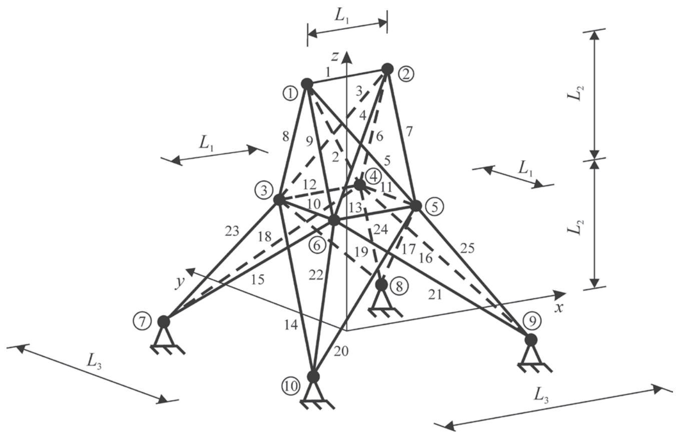

Fig. 6 25-bar space truss

4.5 Kirsh's Example

The example of size/topology optimization of 15-member truss (Fig. 7) suggested by Kirsch [52] was considered. Here L=40 in., H=30 in., E=10,000 ksi, P=20 kips. Absolute values of axial stresses did not have to exceed 50 ksi. Weight optimization is deduced to the minimization of total material volume V of members. Limitation of displacements is not specified. The cross-sectional area of every member according to Refs. [46, 47] was selected on a discrete set of 16 values ranging from 0.1 in.^{2} to 1.6 in.^{2}. Each member may be deleted. Members

Table 5 Comparison for the 200-bar truss

Group	Optimal cross-sectional areas (in²)				
	Members	Toğan & Daloğu [44]	Thierauf & Cai [48]	This work	
				Solution 1	Solution 2
1	1, 2, 3, 4	0.347		0.1	0.1
2	5, 8,11,14,17	1.081		0.954	1.081
3	19, 20, 21, 22, 23, 24	0.1		0.1	0.1
4	18, 25, 56, 63, 94, 101, 132, 139, 170, 177	0.1		0.347	0.1
5	26, 29, 32, 35, 38	2.142		2.142	2.142
6	6, 7, 9, 10, 12, 13, 15, 16, 27, 28, 30, 31, 33, 34, 36, 37	0.347		0.347	0.347
7	39, 40, 41, 42	0.1		0.539	0.1
8	43, 46, 49, 52, 55	3.565		2.8	3.565
9	57, 58, 59, 60, 61, 62	0.347		0.539	0.1
10	64, 67, 70, 73, 76	4.805		3.813	4.805
11	44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75	0.44		0.954	0.347
12	77, 78, 79, 80	0.44		0.1	0.347
13	81, 84, 87, 90, 93	5.952		5.952	5.952
14	95, 96, 97, 98, 99, 100	0.347		0.1	0.347
15	102, 105, 108, 111, 114	6.572		6.572	6.572
16	82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110, 112, 113	0.954		0.539	0.954
17	115, 116, 117, 118	0.347		0.954	0.1
18	119, 122, 125, 128, 131	8.525		8.525	8.525
19	133, 134, 135, 136, 137, 138	0.1		0.1	0.1
20	140, 143, 146, 149, 152	9.3		9.3	9.3
21	120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151	0.954		1.174	0.954
22	153, 154, 155, 156	1.764		0.44	0.1
23	157, 160, 163, 166, 169	13.33		13.33	10.85
24	171, 172, 173, 174, 175, 176	0.347		1.081	0.1
25	178, 181, 184, 187, 190	13.33		13.33	13.33
26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189	2.142		2.142	0.954
27	191, 192, 193, 194	4.805		3.565	6.572
28	195, 197, 198, 200	9.3		8.525	13.33
29	196, 199	17.17		17.17	13.33
	Weight (lb)	28544.0	29737	27701.7	27910.7

were not separated into groups. It is known that the theoretical minimum value of V for this discrete problem is 240 in.^{3} [49]. As a result of 100 runs, we obtained only this value of V. It required from 104 to 158 iterations in more than 90 % of runs; in the remainder of cases it required more iterations, but less than 1200. Here 6 topologies were obtained in total. They are presented in Fig. 8 and in Table 8.

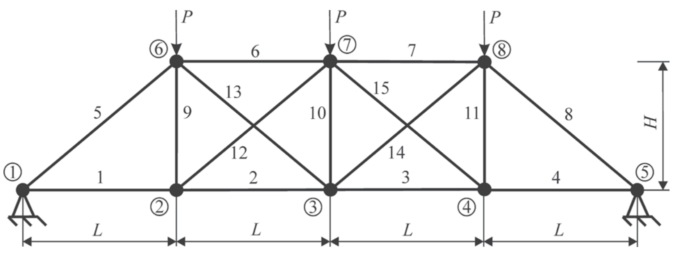

Fig. 7 Base structure for a 15-bar truss

Only one variant of the cross-sectional areas of bars was obtained for every topology. It should be pointed out that topologies 1 and 2 are practically the same. Topology 6 was obtained also in Ref. [46]. In Ref. [47], 19 topologies including those given in Fig. 8 were obtained for this task, but the volume of members there was ranging from 240 in.^{3} to 255 in.^{3}.

Table 6 Loading for the 25-bar space truss

Node	Axial force (kips)		
	x	y	z
1	1.0	-10.0	-10.0
2	0	-10.0	-10.0
3	0.5	0.0	0.0
6	0.6	0.0	0.0

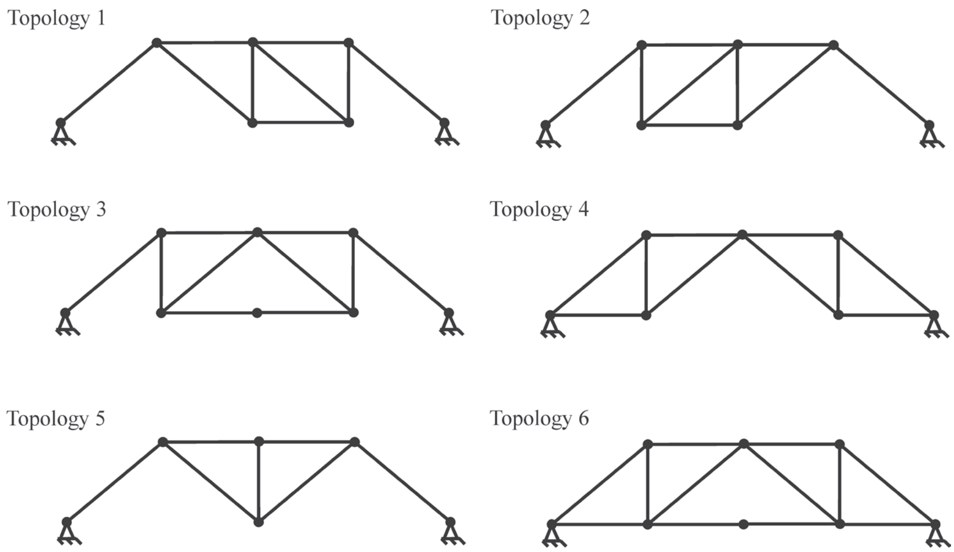

Fig. 8 Optimal topologies for Kirsh's example

Table 7 Comparison for the 25-bar space truss

Variables	Optimal cross-sectional areas (in.^{2})				
	Coello Coello et al. [49]	Erbatur et al. [16]	Camp [10]	Kaveh et al. [51]	This work
A_1	0.1	0.1	0.1	0.1	0.1
A_2	0.7	1.2	0.3	0.3	0.3
A_3	3.2	3.2	3.4	3.4	3.4
A_4	0.1	0.1	0.1	0.1	0.1
A_5	1.4	1.1	2.1	2.1	2.1
A_6	1.1	0.9	1.0	1.0	1.0
A_7	0.5	0.4	0.5	0.5	0.5
A_8	3.4	3.4	3.4	3.4	3.4
Best weight (lb)	493.94	493.8	484.85	484.85	484.85
Average weight (lb)			485.10	484.90	484.85

5 Conclusions

A mixed strategy of constructing a genetic algorithm for the optimization of trusses has been developed. For various individuals of a population, removal is performed if specified limitations are not satisfied. Significant penalties are provided for the remaining individuals in this case. A combined mutation scheme is also applied by means of random selection of parameter values of all admissible quantities or random transfer to values which are close to the current state of varied variables. Then, the general procedures of selection according to the roulette-wheel method and crossover are carried out. Such approach to form a genetic algorithm allows one to provide exact satisfying of the limitations, with relatively high stability and convergence rate. Two sets of problems have been considered to evaluate the performance of the algorithms: optimization of 15 unimodal, multimodal, rotated, shifted and shifted and rotated benchmark mathematical functions and several standard tasks for size, size/topology and size/shape/topology optimization of trusses. The average results for the benchmark functions have been compared with the three effective discrete optimization algorithms: NMBPSO, BGSA and BHTPSO-QI. Solutions on trusses are compared with the well-known numerical results. On the basis of these comparisons, we can conclude that the proposed approach is better in terms of accuracy for benchmark functions than other considered algorithms and provides the obtaining of new results for trusses having for discrete design variables less value of an objective function than results of other authors, or the best of those published in literature.

Table 8 Optimization results for Kirsch's example

Number of topology	**Cross-sectional areas (in.^{2}) for the member**															**V (in.^{3})**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	
1	-	-	0.3	-	1.0	1.1	0.8	1.0	-	0.2	0.2	-	0.4	-	0.4	240
2	-	0.3	-	-	1.0	0.8	1.1	1.0	0.2	0.2	-	0.4	-	0.4	-	
3	-	0.3	0.3	-	1.0	0.8	0.8	1.0	0.2	-	0.2	0.4	-	-	0.4	
4	0.3	-	-	0.3	1.0	0.8	0.8	1.0	0.2	-	0.2	0.4	-	-	0.4	
5	-	-	-	-	1.0	1.1	1.1	1.0	-	0.4	-	-	0.4	0.4	-	
6	0.1	0.2	0.2	0.1	1.0	0.8	0.8	1.0	0.2	-	0.2	0.4	-	-	0.4	

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