

Layout Optimization of Braced Frames Using Differential Evolution Algorithm and Dolphin Echolocation Optimization

Ali Kaveh, Neda Farhoudi

Received 14-04-2015, revised 18-05-2015, accepted 22-05-2015

Abstract

In this study, topology optimization is applied to concentrically braced frames in order to find economical solutions for conventional structural steel frames. Differential Evolution Algorithm and Dolphin Echolocation Optimization are applied for structural optimization. Numerical examples are studied and results of comparison with other meta-heuristic algorithms, including Genetic Algorithm, Ant colony optimization, Particle Swarm, and Big Bang-Big Crunch are presented.

Keywords

Differential Evolution Algorithm · Dolphin Echolocation Optimization · Layout optimization · Steel braced frames · Meta-heuristic algorithms

1 Introduction

Structural optimization helps engineers to design structures economically with less human effort. Structural optimization can be performed using various methods. There are different meta-heuristic optimization methods; Genetic Algorithms (GA) [1, 2], Simulated Annealing (SA) [3], Ant Colony Optimization (ACO) [4], Differential Evolution (DE) [5], Harmony Search algorithm (HS) [6], Particle Swarm Optimizer (PSO) [7], Charged System Search method (CSS) [8], Bat algorithm [9], Water Cycle Algorithm [10], Ray optimization algorithm (RO) [11], Krill-herd algorithm [12], Dolphin Echolocation Optimization (DEO) [13], Colliding Bodies Optimization (CBO) [14] are some of such meta-heuristic algorithms. These methods and their hybrid versions are extensively applied to structural optimization by researchers [15–18].

In the present study, dolphin echolocation optimization and differential evolution are applied to layout optimization (simultaneous size and topology optimization) of steel braced frames with dual systems.

In the first part of this paper, dolphin echolocation optimization and differential evolution (DE) are discussed. In the second section, formulation for the optimization problem is presented. In the third section, numerical examples are presented. In the fourth section, results of different optimization methods are discussed. In the last section concluding remarks are presented.

2 Optimization methods

Two optimization methods consisting of Differential Evolution and Dolphin Echolocation Optimization, implemented in structural optimization are briefly presented in this section.

2.1 Differential evolution

Main steps of the differential evolution algorithm are as follows:

1 Initiate search variable vectors randomly as:

$$x_i = [x_{1i}, x_{2i}, \dots, x_{Di}] \quad i = 1, 2, \dots, N \quad (1)$$

Ali Kaveh

Centre of Excellence for Fundamental Studies in Structural Engineering, School of Civil Engineering, Iran University of Science and Technology, Narmak, Tehran-16, Iran
e-mail: alikaveh@iust.ac.ir

Neda Farhoudi

Centre of Excellence for Fundamental Studies in Structural Engineering, School of Civil Engineering, Iran University of Science and Technology, Narmak, Tehran-16, Iran

2 Define upper and lower bounds for each parameter:

$$x_j^L \leq x_{ji} \leq x_j^U \quad (2)$$

3 Randomly select the initial parameter values uniformly on the intervals $[x_j^L, x_j^U]$.

For a given parameter x_i , randomly select three vectors x_{r1} , x_{r2} and x_{r3} , add the weighted difference of two of the vectors to the third to create the donor vector v_i .

$$v_i = x_{r1} + F(x_{r2} - x_{r3}) \quad (3)$$

The mutation factor F is a constant selected from $[0, 2]$.

4 Develop trial vector $u_{j,i}$ from the elements of the target vector x_i and the elements of the donor vector v_i . In this case, elements of the donor vector enter the trial vector with probability CR .

$$u_{j,i} = \begin{cases} v_{j,i} & \text{if } rand(0, 1) \leq CR \text{ or } j = I_{rand} \\ x_{j,i} & \text{otherwise} \end{cases} \quad (4)$$

CR is a crossover control parameter or factor within the range $[0,1)$ and presents the probability of creating parameters for a trial vector from the donor vector. Index I_{rand} is a randomly chosen integer within the range $[1, NP]$. This ensures that the trial vector contains at least one parameter from the mutant vector [5].

2.2 Dolphin echolocation optimization

Steps of the DEO for discrete optimization are as follows [13]:

1 Initiate NL locations for a dolphin randomly.

2 Calculate the PP of the loop using Eq. (5).

$$PP(Loop_i) = 10 + 90 \frac{Loop_i^{Power} - 1}{(LoopsNumber)^{Power} - 1} \quad (5)$$

Changes in PP in an optimization with 200 numbers of loops is presented in Fig. 1 by altering the power in the above equation.

3 Calculate the fitness of each location. Fitness should be defined in a manner that the better answers get higher values. In other words the optimization goal should be to maximize the fitness.

4 Distribute fitness of each location to its neighbors according to a symmetric triangular distribution (Fig. 2) or any symmetric distribution. It should be added that where the base of triangle exceeds the borders, AF should be calculated using a reflective characteristic. In other word, a mirror should be assumed on the edges to reflect whatever is placed beyond borders.

5 Add all devoted fitnesses to form accumulative fitness.

6 Add a small value of ε to AF matrix. ε should be chosen according to the way the fitness is defined. It is better to be less than minimum possible fitness.

$$AF = AF + \varepsilon \quad (6)$$

7 Find the best location achieved and set its AF to zero.

8 Calculate the probability by normalizing AF as:

$$P_{ij} = \frac{AF_{ij}}{\sum_{i=1}^{MaxAj} AF_{ij}} \quad (7)$$

Where P_{ij} is the probability of the i th alternative to appear in the j th dimension; AF_{ij} is the accumulative fitness of the i th alternative to be in the j th dimension; $MaxAj$ is the maximum number of alternatives available for the j th dimension.

9 Select $PP(Loop_i)$ percent of next step locations from best location dimensions. Distribute other values according to P_{ij} .

10 Repeat steps 2 to 8 for as many times as the Loops Number.

Flowchart of the DEO is depicted in Fig. 3.

3 Formulation of the optimization problem

In this study, minimizing the weight of steel braced frames with dual system is studied. Both placement of bracings and size of members are considered as optimization variables. Problem definition is as follows:

Minimize:

$$w = \rho \sum_{i=1}^M A_i L_i \quad (8)$$

Subjected to:

$$KU - P = 0 \quad (9)$$

$$g_1 \geq 0, g_2 \geq 0, \dots, g_n \geq 0$$

Where $g_1, g_2 \dots g_n$ are constraint functions and K, U and P are the stiffness matrix, nodal displacement and force vectors, respectively. In this study, the members should satisfy the following constraint on drift, deflection, compaction, strength and stability coefficients according to the Specification for Structural Steel Buildings [19], Minimum Design Loads for Buildings and Other Structures [20], International Building Code 2006 [21] and Seismic Provisions for Structural Steel Buildings [22]:

• Drift

$$Drift \leq 0.02h_{sx} \quad (10)$$

• Deflection

$$\begin{cases} \Delta_L < l/360 \\ \Delta_{D+L} < l/240 \end{cases} \quad (11)$$

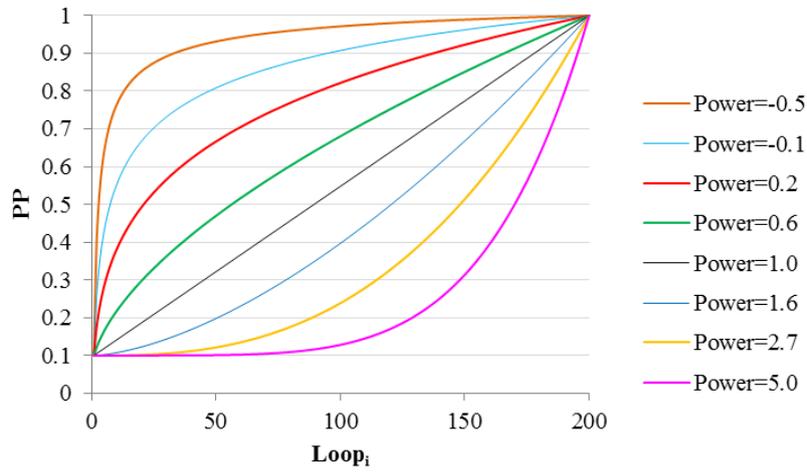


Fig. 1. Changes in PP in an optimization with 200 number of loops by altering the power in Eq. (5)

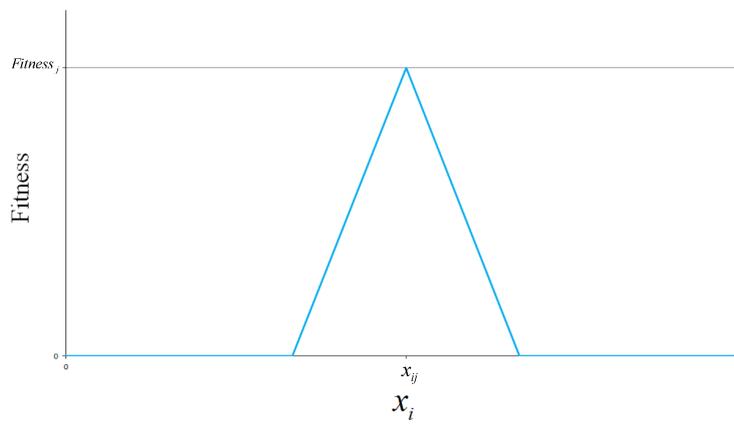


Fig. 2. Triangular distribution of fitness for i^{th} variable of j^{th} location

- **Compactness**
Requirements of Table I-8-1 (Limiting Width-Thickness Ratios for Compression Elements) of Seismic Provisions for Structural Steel Buildings for SLRS members are satisfied [22].
- **Strength:**
Requirements of both AISC 360-05 specification [19] and Seismic Provisions for Structural Steel Buildings are satisfied [22].
- **Stability:**

$$\theta_{\max} < \frac{0.5}{\beta C_d} \quad (12)$$
- **Irregularity**
There is no horizontal irregularity, but vertical irregularity limits are taken into consideration according to the Table 12.3-2 (Vertical Structural Irregularities) of the ASCE/SEI 7-05 [20]. In order not to restrict feasible bracing placement, vertical geometric irregularity has not been considered.
- **Slenderness**
As a practical consideration, slenderness ratio or KL/r is considered to be less than 200 [23].

By applying a penalty function, final formulation in an unconstrained form is as follows:

$$F = -w * (1 + K_p \cdot V) \quad (13)$$

$$V = \sum_{NLC} (\max(g^d, 0) + \max(g^s, 0)) \quad (14)$$

where K_p is the penalty coefficient and V denotes the total constraints' violation considering all nLC load combinations.

Calculation of displacements, forces and stresses are based on the second-order elastic behavior of the structure using a finite element analysis software with amplified first-order elastic analysis.

4 Simultaneous design

According to ASCE 7-05, a dual building frame system is a structural system with an essentially complete space frame providing support for vertical loads. Seismic force resistance is provided by the moment resisting frames and the shear walls, or braced frames.

Considering these requirements, one is not permitted to design all the members simultaneously. The method presented in this study which is called "simultaneous design of structure for

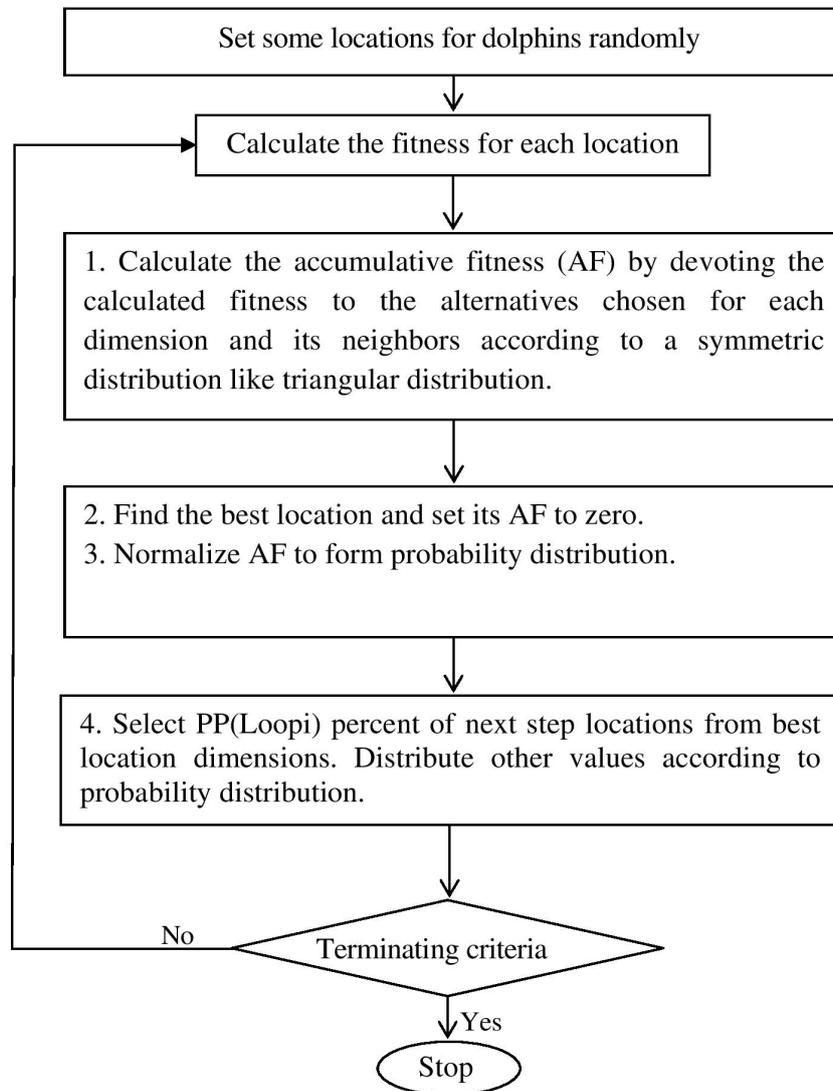


Fig. 3. Flowchart of the DEO algorithm

all loads and frame for gravity loads” helps to assure the building code requirements. In this method, analysis outputs are achieved in two different steps: one when only essential frame exists and one for the entire structure, including frame and bracings. After each step, the requirements of the building code are checked.

Layout optimization of braced frames in this study includes finding the best placement for bracings and the best section for elements of a dual system of moment frames having X-bracings.

5 Numerical examples

Three frames of 3-, 5- and 10-story are studied in the present work. The following features are common in all these examples:

5.1 Geometry

Height of each floor = 3.0 m

Width of the frame = 5.0 m

Three degrees of freedom for each joint (x, y-translations and z-rotation)

All connections and supports are rigid.

5.2 Loading condition

- 1 Uniform distributed dead load of 6.3 kN/m^2 in negative y-direction on all beam elements.
- 2 Uniform distributed live load of 1.96 kN/m^2 in negative y-direction on all beam elements.
- 3 Earthquake concentrated loads are calculated according to the ASCE 7-05 [20], by considering, $R=7$, $I=1$, $S_s = 1.32$, $S_1 = 0.535$ and seismic design category = E; Earthquake loads acting on the given examples are provided in Table 1.

5.3 Material properties

The 50 ksi steels are the predominant ones in use today. In fact some of the steel mills charge extra for W-sections if they consist of A36. On the other hand, A992 and A500 are preferred material for W-shapes and HSS Rectangular, respectively [23]. Material properties are according to Table 2 and the following data:

$$E = 2e8(\text{kN/m}^2), \quad \rho = 76.82(\text{kN/m}^3), \quad \text{and} \quad \nu = 0.3$$

Tab. 1. Earthquake loads acting on different frames in the numerical examples

Floor	Earthquake loads (kN)		
	3-story	5-story	10-story
1	120.12	80.08	32.73
2	240.24	160.16	68.372
3	360.36	240.24	105.2
4		320.32	142.83
5		400.4	181.05
6			219.76
7			258.88
8			298.35
9			338.14
10			378.2
Base shear	720.72	1201.2	2023.5

Tab. 2. Section types selected for numerical examples

member type	shape	ASTM designation	F _y (MPa)	F _u (MPa)
Column	W	A992	344.70	448.20
Beam	W	A992	344.70	448.20
Bracing	HSS Rect.	A500	317.20	399.90

Tab. 3. List of the W-shape profiles

Number	Profile	Number	Profile	Number	Profile	Number	Profile
1	W6X8.5	25	W8X24	49	W14X43	73	W10X60
2	W6X9	26	W6X25	50	W21X44	74	W18X60
3	W8X10	27	W10X26	51	W12X45	75	W14X61
4	W10X12	28	W12X26	52	W10X45	76	W21X62
5	W6X12	29	W16X26	53	W16X45	77	W24X62
6	W4X13	30	W14X26	54	W18X46	78	W12X65
7	W8X13	31	W8X28	55	W8X48	79	W18X65
8	W12X14	32	W12X30	56	W14X48	80	W8X67
9	W10X15	33	W10X30	57	W21X48	81	W10X68
10	W8X15	34	W14X30	58	W10X49	82	W14X68
11	W6X15	35	W8X31	59	W12X50	83	W16X67
12	W5X16	36	W16X31	60	W16X50	84	W21X68
13	W12X16	37	W10X33	61	W18X50	85	W24X68
14	W6X16	38	W14X34	62	W21X50	86	W18X71
15	W10X17	39	W8X35	63	W12X53	87	W12X72
16	W8X18	40	W12X35	64	W14X53	88	W21X73
17	W5X19	41	W18X35	65	W10X54	89	W14X74
18	W12X19	42	W16X36	66	W18X55	90	W18X76
19	W10X19	43	W14X38	67	W21X55	91	W24X76
20	W6X20	44	W10X39	68	W24X55	92	W10X77
21	W8X21	45	W8X40	69	W21X57	93	W16X77
22	W12X22	46	W12X40	70	W16X57		
23	W10X22	47	W16X40	71	W12X58		
24	W14X22	48	W18X40	72	W8X58		

Tab. 4. List of the HSS-shape profiles

Number	Profile
1	HSS1-1/4X1-1/4X.125
2	HSS1-1/2X1-1/2X.125
3	HSS2X1X.125
4	HSS1-5/8X1-5/8X.125
5	HSS1-1/4X1-1/4X.1875
6	HSS2X2X.125
7	HSS2-1/2X1-1/2X.125
8	HSS3X1X.125
9	HSS1-1/2X1-1/2X.1875

Table 3 contains the list of W-sections and Table 4 contains list of HSS-sections used for optimization of the frames. Sections of columns and beams are selected from W-shaped sections and sections of the bracings are selected from HSS-shaped ones.

In this study, all members are selected using optimization methods.

6 Results

Optimum design of numerical examples of this study, using GA, ACO, PSO, BB-BC, modified GA, modified ACO, modified PSO, modified BB-BC was studied in the work of Kaveh and Farhodi [24]. Results of the previously studied methods, DE and DEO for optimization of numerical examples are depicted in Tables 5 to 7 which include minimum or optimum weight, maximum weight, and the standard deviation achieved for each method. In metaheuristic optimization methods, where the optimum answer is the same, standard deviation of the results in different runs of an algorithm shows the performance of the algorithm, in other words, if an algorithm results in lower standard deviation, its performance is considered to be better.

6.1 Results of the 3-story braced frame

According to the results provided in Table 5, it can be seen that except GA, all other methods reached the same result as the optimum answer but maximum weight and the standard deviation of methods are different.

In terms of the standard deviation of results of different methods which are depicted in Table 5, DEO, modified ACO, modified GA, modified PSO, DE, modified BB-BC, ACO, PSO, BB-BC and GA showed better performance to solve this problem, respectively. Fig. 4 shows convergence curves of different methods for optimizing 3-story braced frame, and Fig. 7 illustrates the optimum placement of the bracings of the considered 3-story braced frame.

6.2 Results of the 5-story braced frame

According to the results provided in Table 6, it can be seen that except GA and BB-BC, all the other methods attained the same result as the optimum answer; however the maximum weight and standard deviation of the methods are different. In

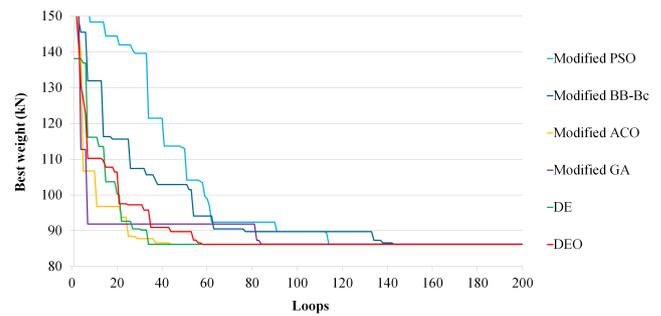


Fig. 4. Convergence curves of optimization methods for 3-story

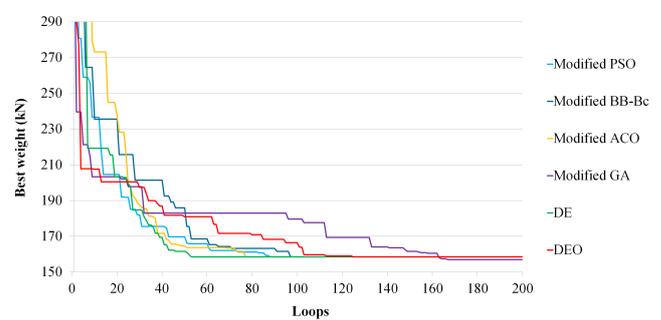


Fig. 5. Convergence curves of optimization methods for 5-story braced frame.

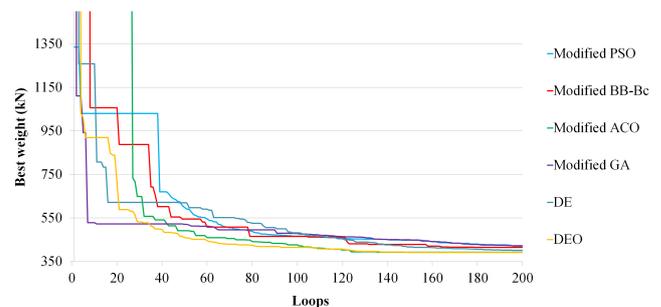


Fig. 6. Convergence curves of optimization methods for 10-story braced frame.

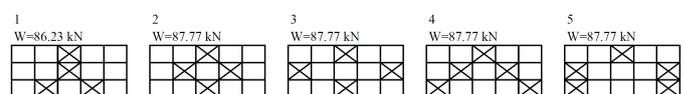


Fig. 7. Optimum placement of bracings in 3-story frame

Tab. 5. Optimum results of the 3-story braced frame

Element group	GA	ACO	PSO	BB-BC	Modified GA	Modified ACO	Modified PSO	Modified BB-Bc	DE	DEO
1	12	12	12	12	12	12	12	12	12	12
2	17	17	17	17	17	17	17	17	17	17
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	1	1	1	1	1	1
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1
min weight(kN)	90.2	86.2	86.2	86.2	86.2	86.2	86.2	86.2	86.2	86.2
max weight(kN)	226.8	110.1	143.3	129.5	94.1	90.2	94.3	98.0	92.4	87.8
standard deviation	22.2	5.9	8.6	9.5	1.4	0.9	2.0	3.2	2.4	0.6

Tab. 6. Optimum results of the 5-story braced frame

Element group	GA	ACO	PSO	BB-BC	Modified GA	Modified ACO	Modified PSO	Modified BB-Bc	DE	DEO
1	22	22	22	22	22	22	22	22	22	22
2	16	12	12	12	12	12	12	12	12	12
3	17	17	17	17	17	17	17	17	17	17
4	0	0	0	1	1	1	0	1	1	1
5	0	0	0	0	1	1	0	1	0	0
6	1	1	1	0	0	0	1	0	1	1
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	1	1	1	0	0	0	1	0	0	0
10	1	1	1	1	0	0	1	0	1	1
11	0	0	0	1	1	1	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0
17	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1
min weight(kN)	161.22	158.54	158.54	161.22	158.54	158.54	158.54	158.54	158.54	158.54
max weight(kN)	295.26	270.24	183.27	392.17	197.82	176.59	178.22	187.24	161.22	160.08
standard deviation	28.44	19.68	6.74	40.35	6.99	4.61	5.19	7.05	0.77	0.46

Tab. 7. Optimum results of the 10-story frame

Element group	GA	ACO	PSO	BB-BC	Modified GA	Modified ACO	Modified PSO	Modified BB-Bc	DE	DEO
1	42	34	36	36	41	34	34	34	34	36
2	31	28	28	28	28	28	28	26	28	28
3	30	18	18	18	20	16	21	21	18	18
4	22	22	22	23	23	23	22	22	23	23
5	17	17	17	17	17	17	17	17	17	17
6	0	0	0	1	0	3	2	1	0	1
7	1	0	2	1	0	0	0	1	1	1
8	0	0	0	0	1	0	1	0	0	0
9	2	3	4	0	2	0	0	0	0	0
10	0	0	2	1	0	0	0	1	0	1
11	0	0	0	0	0	2	1	0	2	0
12	0	0	0	0	0	2	2	0	0	0
13	0	1	0	1	0	0	0	1	1	1
14	0	1	0	0	1	0	0	0	0	0
15	0	1	0	0	0	0	0	0	0	0
16	3	0	1	0	0	0	0	0	3	0
17	0	2	0	0	1	0	4	0	0	0
18	1	2	1	1	0	2	0	1	2	1
19	0	0	0	1	0	2	1	1	2	1
20	1	2	0	0	2	2	1	0	1	0
21	2	0	2	1	0	0	0	1	0	1
22	0	1	2	1	1	0	0	1	1	1
23	0	0	4	0	0	1	0	0	0	0
24	0	0	0	0	0	0	1	0	0	0
25	0	0	0	0	0	0	0	0	0	0
26	0	6	3	4	7	0	1	3	0	4
27	2	0	1	1	2	5	0	2	2	1
28	1	0	1	2	1	0	4	3	0	2
29	1	0	1	2	1	0	3	1	0	2
30	1	0	2	2	0	0	1	5	1	2
31	0	4	1	0	5	0	3	0	0	0
32	2	0	0	0	0	0	0	0	0	0
33	2	0	0	0	4	0	5	0	0	0
34	1	0	1	1	0	1	0	3	1	1
35	1	0	1	1	1	1	1	4	1	1
min weight(kN)	451.89	400.08	424.50	417.06	422.92	392.52	420.75	415.16	401.70	391.95
max weight(kN)	796.56	798.45	491.03	778.45	628.48	509.23	480.68	495.19	465.95	437.12
standard deviation	90.85	106.53	17.12	76.66	47.35	27.90	14.06	15.00	19.70	8.27

terms of the standard deviation, the results of different methods which is depicted in Table 6, DEO, DE, modified ACO, modified PSO, modified GA, PSO, modified BB-BC, ACO, GA and BB-BC showed better performance to solve this problem, respectively. Fig. 5 shows the convergence curve of different methods for optimizing the 5-story braced frame, and Fig. 8 illustrates the optimum placement of bracings of the 5-story braced frame.

6.3 Results of the 10-story braced frame

According to the results of Table 7, DEO, Modified ACO, ACO, DE, Modified BB-BC, BB-BC, Modified PSO, Modified GA, PSO and GA achieved better optimum results respectively. According to standard deviation, the results of different methods as depicted in Table 7, DEO, Modified BB-BC, Modified PSO, PSO, DE, Modified ACO, Modified GA, BB-BC, GA and ACO showed better performance to solve this problem, respectively. Fig. 6 shows convergence curves of different methods for optimizing 10-story braced frame, and Fig. 9 illustrates the optimum placement of the bracings of the 10-story braced frame.

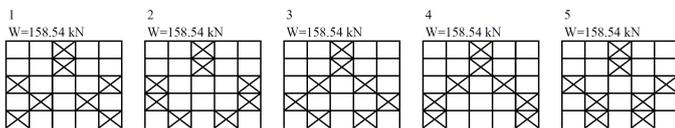


Fig. 8. Optimum placement of bracings in 5-story frame

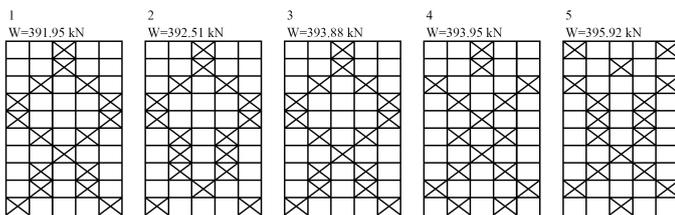


Fig. 9. Optimum placement of bracings in 10-story frame

6.4 Concluding remarks

In this study, Dolphin Echolocation Optimization (DEO) and Differential Evolution (DE) are applied to layout optimization of braced frames. The results show that both DE and DEO show good performance in discrete structural topology optimization. Also DEO leads to better results with less standard deviation in comparison to GA, ACO, PSO, BB-BC and DE in the numerical examples studied in the present research.

References

- 1 Holland JH, *Adaptation in natural and artificial systems*, University of Michigan Press; Ann Arbor, 1975.
- 2 Goldberg DE, Holland JH, *Genetic algorithms and machine learning*, Machine Learning, **3**(2/3), (1988), 95–99, DOI 10.1023/A:1022602019183.
- 3 Kirkpatrick S, Gelatt CD, Vecchi MP, *Optimization by Simulated Annealing*, Science, **220**(4598), (1983), 671–680, DOI 10.1126/science.220.4598.671.
- 4 Dorigo M, Maniezzo V, Colomi A, *Ant system: optimization by a colony of cooperating agents*, In: Part B: IEEE Transactions on System, Man, and Cybernetics, 1996, pp. 29–41.

- 5 Storn R, Price K, *Differential Evolution—a simple and efficient heuristic for global optimization over continuous spaces*, Journal of Global Optimization, **11**(4), (1997), 341–359, DOI 10.1023/A:1008202821328.
- 6 Geem ZW, Kim JH, Loganathan GV, *A New Heuristic Optimization Algorithm: Harmony Search*, Simulation, **76**(2), (2001), 60–68, DOI 10.1177/003754970107600201.
- 7 Eberhart R, Kennedy J, *A new optimizer using particle swarm theory*, In: Micro Machine and Human Science, Proceedings of the IEEE Sixth International Symposium, 1995, pp. 39–43.
- 8 Kaveh A, Talatahari S, *A novel heuristic optimization method: charged system search*, Acta Mechanica, **213**(3-4), (2010), 267–289, DOI 10.1007/s00707-009-0270-4.
- 9 Yang XS, *Bat algorithm for multi-objective optimisation*, International Journal of Bio-Inspired Computation, **3**(5), (2011), 267–274, DOI 10.1504/IJBIC.2011.042259.
- 10 Eskandar H, Sadollah A, Bahreininejad A, Hamdi M, *Water cycle algorithm – A novel metaheuristic optimization method for solving constrained engineering optimization problems*, Computers & Structures, **110-111**, (2012), 151–166, DOI 10.1016/j.compstruc.2012.07.010.
- 11 Kaveh A, Khayatazad M, *A new meta-heuristic method: Ray Optimization*, Computers & Structures, **112**, (2012), 283–294, DOI 10.1016/j.compstruc.2012.09.003.
- 12 Gandomi AH, Alavi AH, *Krill herd: A new bio-inspired optimization algorithm*, Communications in Nonlinear Science and Numerical Simulation, **17**(12), (2012), 4831–4845, DOI 10.1016/j.cnsns.2012.05.010.
- 13 Kaveh A, Farhoudi N, *A new optimization method: Dolphin echolocation*, Advances in Engineering Software, **59**, (2013), 53–70, DOI 10.1016/j.advengsoft.2013.03.004.
- 14 Kaveh A, Mahdavi V R, *Colliding bodies optimization: A novel meta-heuristic method*, Computers & Structures, **139**, (2014), 18–27, DOI 10.1016/j.compstruc.2014.04.005.
- 15 Kaveh A, Zolghadr A, *A multi-set charged system search for truss optimization with variables of different natures; element grouping*, Periodica Polytechnica Civil Engineering, **55**(2), (2011), 87–98, DOI 10.3311/pp.ci.2011-2.01.
- 16 Kaveh A, Mahdavi VR, *Optimal design of structures with multiple natural frequency constraints using a hybridized BB-BC/Quasi-Newton algorithm*, Periodica Polytechnica Civil Engineering, **57**(1), (2013), 27–38, DOI 10.3311/PPci.2139.
- 17 Kaveh A, Ilchi Ghazaan M, Bakhshpoori T, *An improved ray optimization algorithm for design of truss structures*, Periodica Polytechnica Civil Engineering, **57**(2), (2013), 97–112, DOI 10.3311/PPci.7166.
- 18 Kaveh A, Javadi SM, *An efficient hybrid particle swarm strategy, ray optimizer, and harmony search algorithm for optimal design of truss structures*, Periodica Polytechnica Civil Engineering, **58**(2), (2014), 155–171, DOI 10.3311/PPci.7550.
- 19 ANSI/AISC 360-05, *Specification for Structural Steel Buildings*, American Institute of Steel Construction; Chicago, Illinois 60601-1802, March 9, 2005.
- 20 ASCE/SEI 7-05, *Minimum Design Loads for Buildings and Other Structures*, American Society of Civil Engineers; Chicago, Illinois 60601-1802, 2009.
- 21 International Building Code 2006, International Code Council, INC.; USA, 2006.
- 22 ANSI/AISC 341-05, *Seismic Provisions for Structural Steel Buildings*, American Institute of Steel Construction; Chicago, Illinois 60601-1802, March 9, 2005.
- 23 McCormac J, *Structural Steel Design*, Pearson International Edition; UK, 2008.
- 24 Kaveh A, Farhoudi N, *A unified approach to parameter selection in meta-heuristic algorithms for layout optimization*, Journal of Constructional Steel Research, **67**(10), (2011), 1453–1462, DOI 10.1016/j.jcsr.2011.03.019.