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RESEARCH ARTICLE

Determination of the Punching Cross-Section of Reinforced Concrete Flat Slabs

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Abstract

In this paper, the punching shear resistance of slabs without punching shear reinforcement is investigated. We assumed that the punching shear resistance can be characterized by the shear resistance of the concrete compression zone, and that the load bearing near the column head can be investigated by the theory of bent shallow shells. With these assumptions the punching control perimeter can be calculated. For analyzing the bent shallow shell, the method of the generator function was applied. This method is based on the generalization of the determinants and cofactors of quadratic matrices. We also assumed that the shape of the shell can be approximated by a paraboloid of revolution. We compared the calculated result with the value according to Eurocode 2. The good agreement between these values shows the efficiency of the above-mentioned assumptions.

Keywords

punching · reinforced concrete slabs · generator function

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1 Introduction

The design codes typically give empirical expressions for the punching shear capacity of flat slabs, which are based on experimental investigations. In these expressions the punching shear strength and the defined control perimeter are both determined by statistical methods. According to the code rules the control perimeter is located at a distance of $1,50 \ldots 2,00\,d$ from the face of the column.

In the following we assumed that the punching shear resistance can be characterized by the shear resistance of the concrete compression zone. Moreover, we assumed that the load bearing near the column head can be investigated by the theory of bent shallow shells, where the shape of the shell can be represented as a paraboloid of revolution.

Based on the abovementioned assumptions, the punching control perimeter can be calculated.

2 Punching tests for flat slabs and the punching shear resistance

The punching shear resistance of a flat slab supported by columns of a square mesh is investigated by considering a representative slab element surrounding a column. The theory of thin elastic plates shows that, in the case of small values of c / L, where c is the radius of a circular column and L is the axis-to-axis spacing of the columns (Fig. 1.a), the bending moments in the radial direction practically form a zero circle of radius $r = 0,22\,L$, as shown in Fig. 1.b. Thus the plate around the column and inside such a circle can be approximated as a circular plate simply supported along the circle $r = 0,22\,L$ [1].

Since the beginning of the 20^{th} century a great number of experiments on punching have been based on this rotationally symmetric case. Concrete specimens with and without shear reinforcement have been investigated with additional parameters [2].

In the expressions of code rules the punching shear resistance of flat slabs emerges as a product of the punching shear stress, the critical perimeter, and the effective depth. Eurocode 2 gives

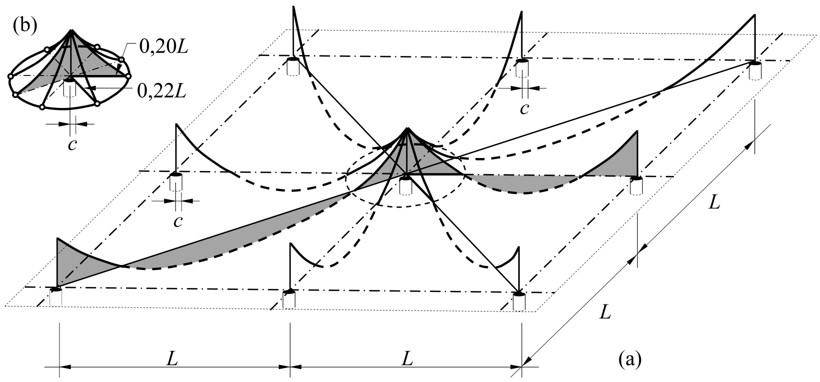

Fig. 1. Distribution of the radial moments of a flat slab under dead load [2]

the following expression

$$V_{Rc} = v_{Rc} ud = 0,18k\,(100\rho_l\, f_{ck})^{\frac{1}{3}}\;u\,d \qquad (1)$$

where v_{Rc} is the shear strength, u is the punching control perimeter located $2d$ from the face of the column, d is the effective depth of the slab, k is a factor accounting for size effect defined by a function of the effective depth of the slab, ρ_l is the flexural reinforcement ratio, and f_{ck} is the characteristic compressive strength of concrete [3]. Actually, expression (1) is equal to the shear resistance of a beam without shear reinforcement according to Eurocode 2, but there the width of the cross-section b_w emerges instead of the control perimeter u.

In the last two decades a new trend emerged in calculating the punching shear resistance, where the semi-empirical failure criterion is a function of the width of the critical crack. According to [4] the failure criterion is formulated as follows

$$\frac{V_R}{b_0\, d\sqrt{f_c}} = \frac{\frac{3}{4}}{1 + 15\frac{\psi d}{d_{g0}+d_g}} \qquad (2)$$

where ψ is the rotation, from which the width of the critical crack can be assumed to be proportional to the product ψd, b_0 is the perimeter of the critical section located $d/2$ from the face of the column, f_c is the concrete compressive strength, d_g is the maximum aggregate size, and d_{g0} is a reference size equal to 16 mm.

Due to the fact, that u and b_0 have the same meaning, expression (2) can be given as $V_{Rc} = v_{Rc}\, u\, d$, thus the shear strength can be expressed as

$$v_{Rc} = \frac{\frac{3}{4}}{1 + 15\frac{\psi d}{d_{g0}+d_g}}\sqrt{f_c} \qquad (3)$$

In these expressions the value of v_{Rc} and the defined control perimeter are both determined by statistical methods, on the basis of the available experimental data.

By defining the control perimeter, the value of the punching shear capacity includes effects which cannot be interpreted by traditional plate theory of small deflections.

In [5] we assumed that the shear resistance of a beam without shear reinforcement can be well characterized by the shear resistance of the concrete compression zone, thus the shear resistance can be defined as a function of the curvature of the cross-section. Considering that the distribution of the shear stresses is parabolic, we proposed the following expression for the shear resistance

$$V_{Rc} = \frac{2}{3}\tau_{c,MOHR}\, b x_{II} \qquad (4)$$

where b is the width of the cross-section, x_{II} is the depth of the compressive zone, assuming that steel and concrete are both in the elastic state, and $\tau_{c,MOHR} = 0,5\sqrt{f_c\, f_{ct}}$. For the values of shear slenderness $\lambda = a\,/\,d \geq 3$, where $a\,/\,d$ is the shear span-to-depth ratio, the results of expression (4) are in good agreement with the test results reported by Walther [6]. Depending on the flexural reinforcement ratio, expression (4), is well approximated by the function of $\sqrt[3]{\,}$. Based on the test parameters by Walther, for $f_{cm} = 30$ MPa and $k = 1 + \sqrt{200\,/\,d} = 1,86 \leq 2$, where k is the size effect factor, we obtained the following approximation

$$V_{Rc} = 0,17\,k\;(100\,\rho_l\, f_{ck})^{1/3}\;b\,d \qquad (5)$$

Expression (5) is practically the same as the shear resistance of a rectangular beam according to Eurocode 2. This good agreement shows that our assumption was correct.

A similar conclusion was drawn by [7], in which the shear resistance of a beam without shear reinforcement is given by the following expression

$$V_{Rc} = \frac{2}{3}\sqrt{f_t^2 + f_t\frac{\sigma_m}{2}}\; b_w c \qquad (6)$$

where c is the depth of the concrete compression zone, $f_t = 0,5\sqrt{f_c'}$, $\sigma_m = 0,625\sqrt{f_c'}$ and f_c' is a specified concrete strength.

In the following, according to the above mentioned assumptions, we suppose that the punching shear resistance of a flat slab is determined by the shear resistance of the concrete compression zone, and that the punching shear stress according to

Eurocode 2 is derived from this resistance. In this case, the control perimeter must be determined by the cracked cross-section along the perimeter of the column, hence the initial control perimeter is $u_0 = 2\pi c$.

Based on this assumption, using the punching tests results of slabs without shear reinforcement of [2] with the value of v_{Rc} according to Eurocode 2, we calculated the punching control perimeters. For these values, we determined the distances between the calculated control cross-sections and the cracked cross-section. The calculations were based on the test data of the appendix I of bulletin [2], which contains 200 punching tests of slabs without shear reinforcement from the year 1956 to 2000. These test results are reported by authors such as Elstner/Hognestad (1956), Kinnunen/Nylander (1960), Manterola (1966), Regan (1986), Lovrovich/McLean (1990), Ramdane (1993) and Hallgren (1996). The relative frequency histogram of the location of punching control perimeters is shown in Fig. 2.

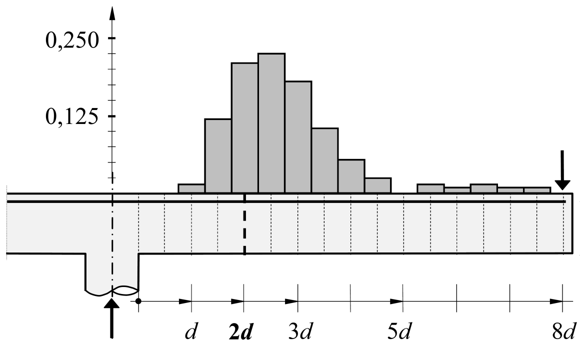

Fig. 2. The relative frequency histogram of the location of punching control perimeters

As shown in Fig. 2, the cracked cross-section does not match the calculated control cross-sections; instead the location of control cross-sections are typically farther than a distance 1,5d from the face of the column. Based on these differences, we can conclude that a portion of the loads is balanced without shearing forces; thus, the membrane forces have a significant effect around the column.

Hence the load bearing around the column can be investigated more adequately on the basis of the theory of bent shallow shells, than that of thin plates. The assumed shell around the column and its geometry is shown in Fig. 3.

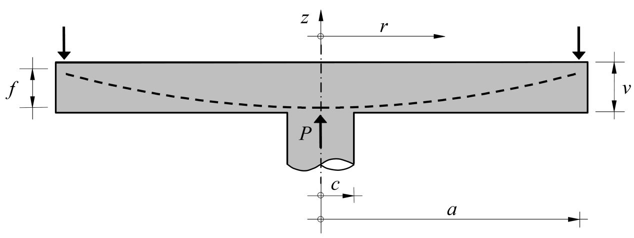

Fig. 3. Geometry of shell around the column head

Based on the above-mentioned assumptions, the membrane action can be calculated. From this action the punching control perimeter can also be determined. Actually, the critical perimeter is not directly related to the punching failure mechanism, because it is defined on the basis of the test results, in order to simplify the standard equations, and to make the punching resistance independent of the column's dimensions. However, if our assumptions are right, with necessarily fixed parameters, the model should predict this quantity in accordance with Eurocode 2.

The emerging membrane forces of flat slabs are shown in [8], through nonlinear finite element simulation.

3 Analysis of the bent shallow shell

For analyzing the bent shallow shell, the method of the generator function was applied. The application of the method of the generator function is shown in detail according to [9]. Let the shape function of the flat shell in an r, ϑ, z cylindrical coordinate system

$$z = \frac{\alpha}{2} r^2 \tag{7}$$

in which α is the parameter of the shell, and

$$\alpha = \frac{2f}{a^2} \tag{8}$$

where a is the boundary radius, and f is the depth of the shell (Fig. 3). The load and the supports of this paraboloid are assumed axisymmetric. The material is assumed as homogenous and isotropic with elastic constants E and ν.

The differential equation system of bent shallow shells is the following [10]

$$\begin{aligned} -P(z,F) + K\Delta\Delta w &= p \\ \frac{1}{Et}\Delta\Delta F + P(z,w) &= 0 \end{aligned} \tag{9}$$

in which w is the displacement normal to the middle surface, F is the stress function of membrane forces, and p is the function of external loads. P is called Kármán's shell operator, Δ is the two-dimensional Laplace operator, which in cylindrical coordinate system takes the form

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \vartheta^2} \tag{10}$$

and the flexural stiffness of the shell is

$$K = \frac{Et^3}{12(1-\nu^2)} \tag{11}$$

where t is the thickness of the shell. The first row of the differential equation system enforces equilibrium, and shows, that the load p can be subdivided into two parts, namely $p_{membrane} + p_{plate} = p$, in such a manner that $p_{membrane}$ is balanced by membrane forces, and p_{plate} is balanced by the bending moments and shear forces calculated by means of the theory based on small deflections. The second row of the differential equation system shows enforces compatibility, and shows that the membrane-like and the plate-like deformations are corresponding.

The second order derivatives of function z in operator P, in case of the paraboloid of revolution, are

$$\frac{\partial^2 z}{\partial r^2} = \alpha, \quad \frac{1}{r}\frac{\partial z}{\partial r} = \alpha \quad \text{and} \quad \frac{\partial z}{\partial \vartheta} = 0 \tag{12}$$

Thus, the following differential equation system is obtained:

$$\begin{aligned} K\Delta\Delta w - \alpha\Delta F &= p \\ \alpha\Delta w + \frac{1}{Et}\Delta\Delta F &= 0 \end{aligned} \tag{13}$$

The homogeneous linear differential equation system for unknowns $y_1 = w$ and $y_2 = F$ can be written in vectorial form as

$$\Theta \begin{bmatrix} w \\ F \end{bmatrix} = \Theta\mathbf{y} = \mathbf{0} \tag{14}$$

where Θ is the operator matrix

$$\Theta = \begin{bmatrix} K\Delta\Delta & -\alpha\Delta \\ \alpha\Delta & \frac{1}{Et}\Delta\Delta \end{bmatrix} \tag{15}$$

The operator determinant and the cofactor matrix of Θ are

$$\det(\Theta) = \frac{K}{Et}\Delta\Delta\Delta\Delta + \alpha^2\Delta\Delta \tag{16}$$

$$Cof(\Theta) = \begin{bmatrix} \frac{1}{Et}\Delta\Delta & -\alpha\Delta \\ \alpha\Delta & K\Delta\Delta \end{bmatrix} \tag{17}$$

Using the generator function H the following characteristic equation emerges

$$\frac{K}{Et}\Delta\Delta\Delta\Delta\{H\} + \alpha^2\Delta\Delta\{H\} = 0 \tag{18}$$

After a further factorization of the determinant and introducing the characteristic length

$$L = \sqrt[4]{\frac{K}{\alpha^2 Et}} \tag{19}$$

we obtain

$$\det(\Theta) = \frac{K}{Et}\Delta\Delta\left(\Delta\Delta + \frac{\alpha^2 Et}{K}\right) \tag{20}$$

$$\begin{aligned} \det(\Theta) &= \alpha^2 L^4\Delta\Delta\left(\Delta\Delta + \frac{1}{L^4}\right) = \\ &= \alpha^2 L^4\Delta\Delta\left(\Delta + \sqrt{-\frac{1}{L^4}}\right)\left(\Delta - \sqrt{-\frac{1}{L^4}}\right) \end{aligned} \tag{21}$$

Eq. (21) shows that the solution of the eighth order characteristic differential equation can be reduced to those of one fourth order and two second order differential equations as follows:

$$\begin{aligned} &\Delta\Delta H^{(1)} = 0 \\ &\left(\Delta + i\sqrt{\frac{1}{L^4}}\right)H^{(2)} = 0 \\ &\left(\Delta - i\sqrt{\frac{1}{L^4}}\right)H^{(3)} = 0 \end{aligned} \tag{22}$$

The solutions of Eq. (22) are biharmonic functions, and a combination of two solutions of the Bessel differential equations. The general solution can be represented in the following form:

$$\begin{aligned} &H = A(r) + c_5 ber(x) + c_6 bei(x) + c_7 kei(x) + c_8 \ker(x) \\ &A(r) = c_1 + c_2 r^2 + c_3 \ln r + c_4 r^2 \ln(r) \end{aligned} \tag{23}$$

where *ber(x), bei(x), ker(x)* and *kei(x)* are the zero order Thomson functions, and

$$x = \frac{r}{L} \tag{24}$$

is a dimensionless radial coordinate.

Generating functions w and F from $Cof(\Theta)\{H\} = \mathbf{y}$ using the second row of the cofactor matrix (17), we find

$$\begin{aligned} w = \alpha\Delta\{H\} = \alpha\frac{1}{L^2}\,&[4c_2 + 4c_4[1 + \ln(x)] - \\ &-c_5 bei(x) + c_6 ber(x) + c_7\ker(x) - c_8 kei(x)] \end{aligned} \tag{25}$$

$$\begin{aligned} F &= K\Delta\Delta\{H\} = \\ &= -K\frac{1}{L^4}[c_5 ber(x) + c_6 bei(x) + c_7 kei(x) + c_8\ker(x)] \end{aligned} \tag{26}$$

For the displacement at point $x = 0$ $(r = 0)$ becomes infinitely large, since $1 + \ln(0) = -\infty$ and $\ker(0) = \infty$, the coefficient c_4 and c_7 must vanish. Thus, the solution functions are reduced to

$$w = \frac{\alpha}{L^2}[4c_2 - c_5 bei(x) + c_6 ber(x) - c_8 kei(x)] \tag{27}$$

$$F = -\frac{K}{L^4}[c_5 ber(x) + c_6 bei(x) + c_8\ker(x)] \tag{28}$$

The four constants can be determined on the basis of the corresponding boundary conditions. In fact, only three constants can be calculated, because if there is no concentrated load P at the point $x = 0$, the value of the shear force is $(Q_r)_{x=0} = 0$, thus we obtain $c_8 = 0$. In the case of a shell carrying a concentrated load P at the point $x = 0$, $c_2 = 0$ because c_2 represents a constant displacement of the shell. The remaining constants can be calculated from the boundary conditions as follows:

$$w_{r=a} = 0, \quad (M_r)_{r=a} = 0, \quad P = -\lim_{r\to 0}(2r\pi Q_r) \tag{29}$$

From the expression of displacement, for the radial bending moment and the radial shear force, the following two expressions are given:

$$M_r = -K\left[\frac{\partial^2 w}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \vartheta^2}\right)\right] \tag{30}$$

$$Q_r = -K\frac{\partial}{\partial r}(\Delta w) \tag{31}$$

For the expression of displacement w, the radial bending moment, and the radial shear force, we obtain

$$w = \frac{\alpha}{L^2}\left[-c_5 bei(x) + c_6 ber(x) - c_8 kei(x)\right] \tag{32}$$

$$\begin{aligned} M_r = -\frac{K\alpha}{L^4}\Bigg\{ & -c_5\left[ber(x) - \frac{1-\nu}{x}bei'(x)\right] + \\ & + c_6\left[-bei(x) - \frac{1-\nu}{x}ber'(x)\right] - \\ & - c_8\left[\ker(x) - \frac{1-\nu}{x}kei'(x)\right]\Bigg\} \end{aligned} \tag{33}$$

$$Q_r = \frac{K\alpha}{L^5}\left[c_5 ber'(x) + c_6 bei'(x) + c_8 \ker'(x)\right] \tag{34}$$

Introducing Eqs. (32), (33) and (34) in Eq. (29), we find

$$\begin{aligned} c_5 = & -\frac{PL^4}{2\pi K\alpha}\cdot \\ & \cdot \frac{\left(bei\,\xi + \frac{1-\nu}{\xi}ber'\,\xi\right)kei\,\xi + \left(\ker\,\xi - \frac{1-\nu}{\xi}kei'\,\xi\right)ber\,\xi}{\left(ber\,\xi - \frac{1-\nu}{\xi}bei'\,\xi\right)ber\,\xi + \left(bei\,\xi + \frac{1-\nu}{\xi}ber'\,\xi\right)bei\,\xi} \end{aligned} \tag{35}$$

$$\begin{aligned} c_6 = & \frac{PL^4}{2\pi K\alpha}\cdot \\ & \cdot \frac{\left(ber\,\xi - \frac{1-\nu}{\xi}bei'\,\xi\right)kei\,\xi - \left(\ker\,\xi - \frac{1-\nu}{\xi}kei'\,\xi\right)bei\,\xi}{\left(ber\,\xi - \frac{1-\nu}{\xi}bei'\,\xi\right)ber\,\xi + \left(bei\,\xi + \frac{1-\nu}{\xi}ber'\,\xi\right)bei\,\xi} \end{aligned} \tag{36}$$

$$c_8 = \frac{PL^4}{2\pi K\alpha} \tag{37}$$

where

$$\xi = \frac{a}{L} \tag{38}$$

By substituting these values of the constants in expression (32), the final expression of the deflection is obtained. When f tends to zero, this solution approaches the solution of a concentrated load acting at the center of a circular plate, where the solution is determined by the theory of small deflections. This gives

$$w_0 = \frac{Pa^2}{16\pi K}\left[\frac{3+\nu}{1+\nu}\left(1-\rho^2\right) + 2\rho^2\ln\rho\right] \tag{39}$$

where

$$\rho = \frac{r}{a} \tag{40}$$

is a relative radial coordinate [11].

4 Membrane action in the bent shallow shell

From expressions (39) and (32) with the constants c_5, c_6 and c_8, the membrane action can be expressed in the form

$$\beta_{r=0} = \frac{p_{membrane}}{p} = 1 - \left(\frac{w}{w_0}\right)_{r=0} \tag{41}$$

where p is the total load, and $p_{membrane}$ is the part of p equilibrated by membrane forces. Substituting in expression (41), we conclude, that the membrane action only depends on the relative depth of the shell, denoted f/t. When $f = t$, we find $\beta_{r=0} = 0,62$. The variations of the membrane action and the plate action with the ratio f/t are shown in Fig. 4, where the plate action is determined by the expression $1 - \beta_{r=0}$.

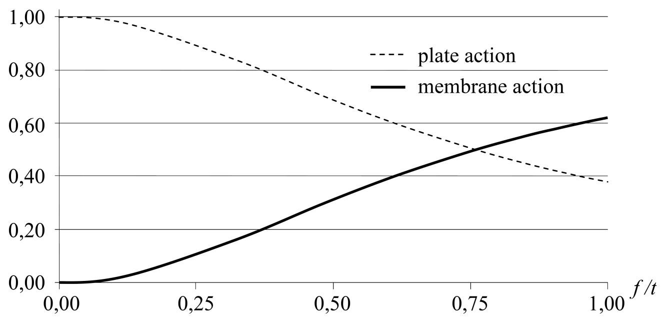

Fig. 4. Membrane action and plate action as a function of the relative depth of the shell

The membrane action can be expressed by the following approximate formulas

$$\beta_{r=0} \approx 0,62\frac{f}{t} - 0,041\sin\left(2\pi\frac{f}{t}\right) \tag{42}$$

or

$$\beta_{r=0} \approx 1 - \frac{1}{1 + 1,753\frac{f^2}{t^2}} \tag{43}$$

The membrane action can be represented by means of a circular plate on elastic foundation, where the intensity of the reaction of the subgrade is given by the curvature of the middle surface (Fig. 5).

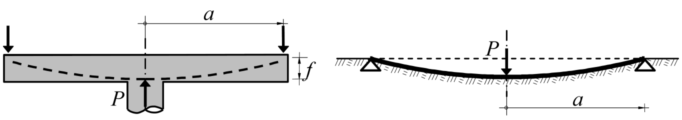

Fig. 5. Bent shallow shell and its analogy as circular plate on elastic foundation

This analogy as circular plate on elastic foundation can be made visible by simplifying the cofactor matrix of Θ (17) with the operator Δ. After this simplification and using the generator function H, the characteristic equation is reduced to a fourth order differential equation as follows

$$K\Delta\Delta\{H\} + C\{H\} = 0 \tag{44}$$

where the constant C is the modulus of a fictitious Winkler-type foundation, assuming $C = \alpha^2 Et$. The differential equation for the deflections of a circular plate on elastic foundation is the following:

$$K\Delta\Delta w + Cw = p \tag{45}$$

where K is the flexural rigidity of the plate and C is the modulus of the foundation. Comparing Eq. (44) with Eq. (45) the analogy becomes evident.

5 Determination of punching cross-section

For determining the punching or control cross-section, all effects, which increase the punching shear resistance, were interpreted as an increase of the control perimeter. The control radius r_{cont} can be determined from the calculated perimeter, where the control radius is the distance of the control cross-section from the centroid of the column. In the calculations we assumed that the shear resistance is determined by the shear resistance of the concrete compression zone; therefore, the basic control section $u_0 = 2\pi c$ is at the column face.

5.1 Effect of membrane action

The calculations are performed with the mean value of the data on slabs without shear reinforcement of [2]. In these tests for flat slabs when $d \cong 0,8\,v$, we find $a\,/\,v = 1,00 \ldots \underline{5,88} \ldots 12,0$ and $c\,/\,a = 0,03 \ldots \underline{0,128} \ldots 0,50$ (mean values are underlined), where v is the total height of slab, the additional symbols are according to Fig. 1 and Fig. 3. In the case of $v = 25$ cm and $d = 20$ cm, we obtain $c = 18{,}82$ cm. Thus, we can calculate $L = 6{,}50$ m column distance and 30 x 30 cm quadratic column. These obtained values are in good agreement with the practical usage.

In our calculations we assumed that the slab-column connections are monolithic, therefore we determined the value of the membrane action at the point $r = c$. For $f = d$, we find $\beta_{r=c} = 1 - (w\,/\,w_0)_{r=c} = 0,623 \ldots \underline{0,637} \ldots 0,689$.

In various test results that part of load, which is balanced by membrane forces (without bending moments and shear), seems like an effect increasing the punching shear resistance. Using the mean value, we obtain

$$r_{cont} = \frac{p}{p_{plate}}c = \left(\frac{w_0}{w}\right)_{r=c} c = \frac{1}{1-\beta_{r=c}}c = 2,755\,c \tag{46}$$

For $c = 0,941\,d$, we find

$$r_{cont} = c + 1,65\,d \tag{47}$$

If the control perimeter is at a distance 1,65 d from the face of the column, and the punching shear stress is defined according to Eurocode 2, then we obtain the lower limit value of the punching shear resistance, because the ultimate punching loads are larger than those obtained by calculation in 96,5% of cases.

5.2 Effect of the failure criteria of the concrete compression zone

The concrete compressive zone in a reinforced concrete beam subjected to bending is a typical case of plane stress conditions, whose behaviour can be adequately studied by Mohr's criterion [12]. For Mohr envelope usually a parabolic form or a simple straight line is used in practice. The straight-line envelope is called Coulomb's line. From the different envelopes of the Mohr circles, different $\tau(\sigma)$ failure criteria and different values of concrete shear strength can be calculated. The shear strength of concrete, which is calculated from the parabolic envelope, is named $\tau_{c,WALTHER}$ [6] and that, which is calculated from the straight line envelope, is named $\tau_{c,MOHR}$ [5]. For $\sigma_{\max} = f_c$ the value of $\tau_{c,WALTHER}$ is 42,3 percent more than the value of $\tau_{c,MOHR}$. The test results reported by Kármán [13] show $\tau_{c,MOHR}$ as the lower limit value and $\tau_{c,WALTHER}$ as the upper limit value.

Based on expression (4) the following lower limit to the punching shear resistance is obtained:

$$V_{R,\min} = \frac{2}{3}\tau_{c,MOHR}\,u_0\,x_{II} \tag{48}$$

Assuming that the maximum value of the concrete shear strength is calculated from the parabolic envelope, the upper limit to the punching shear resistance is given by

$$V_{R,\max} = \frac{2}{3}\tau_{c,WALTHER}u_0 x_{II} \tag{49}$$

When the upper limit value of the ultimate shear stress $\tau_{c,WALTHER} = 1,423\,\tau_{c,MOHR}$ is interpreted as an increase of the control perimeter, for $c = 0,941\,d$, we obtain

$$r_{cont} = c + 0,40\,d \tag{50}$$

6 Calculation of the punching resistance

The value of the punching shear resistance, considering the effect of the membrane action and the effect of the failure criteria of the compressive zone, can be expressed as

$$V_{Rc} = \frac{C}{1-\beta_{r=c}}\frac{2}{3}\tau_{c,MOHR}\,u_0\,x_{II} \tag{51}$$

Using the former notations, $C = 1{,}000 \ldots 1{,}423$ and $\tau_{c,MOHR} = 0,5\,\sqrt{f_c\,f_{ct}}$.

We compared the results of expression (51) with test results of flat slabs without punching shear reinforcement by [2]. For $C = 1{,}000$ the expression (51) gives a lower limit value of the punching resistance. For $C = 1{,}423$ the mean square error of the results of expression (51) is 14% worse than the value of the punching resistance according to Eurocode 2.

The results of expression (51) versus the value of the characteristic compressive strength of concrete f_{ck} are represented graphically in Fig. 6. For the purpose of comparison, the figure also includes test results reported by Ramdane [2]. In these experiments, with constant geometry and reinforcement ratio, the compressive strength of concrete is varied.

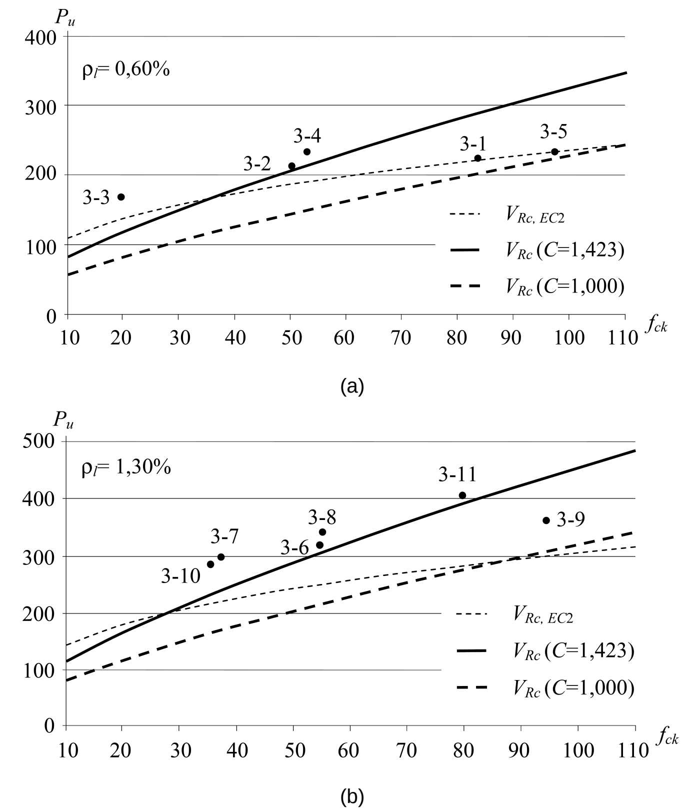

Fig. 6. Comparison of expression (51) with test results reported by Ramdane [2].

As shown in Fig. 6, the values of V_{Rc} (51) for $C = 1{,}000$ represent lower limit values to the punching resistance. These test results suggest a relationship between the values of C and the compressive strength of concrete, which actually means a relationship between the failure criteria of the concrete and the compressive strength of concrete.

7 Conclusions

The final results of the investigation show that our assumptions were correct. For the distance from the centroid of the column to the control section we obtained $r_{cont} = c + 1,65 \ldots 2,05\,d$. This result is in good agreement with the value according to Eurocode 2, where the control perimeter at a distance $2d$ should be considered. The uncertainty of the calculated value is due to the uncertainty of failure criteria of the concrete compression zone.

The good agreement of the calculated result of the punching control perimeter shows that the membrane effect is an important part of the load bearing around a column. For calculating the punching shear resistance, the shape of the shell can be approximated as a paraboloid of revolution and the height of the shell can be determined with $f = d$.

On the basis of the above mentioned assumptions a simple mechanical model and an expression for the punching shear capacity can be given. This simple mechanical model can be made more complex by taking into account the effect of the displacement, the effect of the normal forces, the effect of the flexural reinforcement and the effect of the bending cracks, in order to achieve a lower coefficient of variation.

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