

Maximum Rainfall Model Based on Archival Pluviographic Records – Case Study for Legnica (Poland)

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Abstract

The scope of this study is to develop a probabilistic model for maximum rainfall in Legnica, based on a 50-year series of pluviographic records. The present authors use Fréchet, Gamma, Generalized Exponential Distribution (GED), Gumbel, Log-Normal and Weibull distributions to describe the measurement data. Distributions parameters are estimated using maximum likelihood method. Coincidence of the analyzed theoretical distributions with measured data are inspected using the Anderson-Darling test, while the best fitting distribution is chosen by Bayesian information criterion (BIC) of Schwartz as well as by the relative residual mean square error. Among others distributions Fréchet, Gamma, GED, and Weibull distributions fulfill the compliance criterion for each of the 20 analyzed rainfall durations. BIC criterion indicates for a GED, but differences between GED, Gamma and Weibull is minor. Only RRMSE analysis revealed that in comparison to other distribution GED best describes the measurement rainfall data. At first glance maximum rainfall model was well described by the generalized exponential distribution. However, there is a substantial inconvenience to use it for engineering purposes. Generalization of the shape parameter α depended on the rainfall duration, by averaging and then recalculating remaining parameters λ and γ brought relatively simpler form of model.

Keywords

precipitation amount · probabilistic models · pluviograph

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1 Introduction

Dimensioning of urban drainage systems is mainly based on the maximum predicted rainfall data. Equations describing the dependence on the rainfall amount (h , mm), the duration (t , min), and probability exceedance (p) are called rainfall models. The height of the rainfall can be converted to the intensity (I , mm/min or q , dm³/s·ha), depending on the model application purpose. Actual Polish law status imposes on the sewer system designers obligation to secure dimensioning, according to the best available techniques (BAT). European Standard EN 752 allows the frequency ($C = 1/p$) of sewer flooding to rare and socially acceptable repeatability: once for every 10 years in the case of rural areas, and once for every 20 to 50 years in urban areas – according to the type of spatial development of the area (Table. 1).

In the case of expansion or modernization of sewer systems applying the principle of BAT currently involves the use of modern tools for the hydrodynamic simulation [33, 34]. Simulation studies of functioning of the storm water drainage with accompanying facilities, such as storm water overflows [19, 48], separators and reservoirs are becoming essential tools for use in engineering practice [32, 35, 36]. In recent years, many studies have focused on the assessment of the reliability and risks associated with the functioning of urban infrastructure systems [5, 14, 22, 42, 43, 50], as well as the impact of changes climate for their functioning [16, 21, 23, 25, 28, 29, 37, 41].

The primary obstacle to the dimensioning of drainage systems may be the lack of a reliable maximum rainfall model - applicable in the urban areas. This problem has been properly solved in Germany, where the unit of the reference rainfall intensity can be found in KOSTRA atlas individually for each urban basin. Much earlier similar work was carried out in the USA [11]. So far in Poland, the maximum rainfall models for a given duration and exceedance probability have been developed only for a few urban catchments among others in Wrocław, Kielce, Krakow or Lodz [15, 39, 44, 45, 49]. For other urban basins designers are doomed to use nationwide models characterized by lower accuracy due to the large spatial variability of rainfall [3].

The main reason for the lack of availability of local models

Tab. 1. Recommended frequency of designed computational rain and limit the frequency of spill in accordance to EN 752

The area drainage standard category	Design rainfall frequency [1 per <i>C</i> years]	Flooding occurrence frequency [1 per <i>C</i> years]
I. Out of town areas (rural)	1 per 1 year	1 per 10 years
II. Residential areas	1 per 2 years	1 per 20 years
III. City centers, service and industry areas	1 per 5 years	1 per 30 years
IV. Underground transportation facilities, underpasses, etc.	1 per 10 years	1 per 50 years

in Poland is limited access to archival pluviographic records. Although the measurements are carried out by the Institute of Meteorology and Water Management - National Research Institute (IMWM-NRI) across the whole country for decades, but only for recent few years a uniform digital recording of rain was made. Therefore longterm period measurement series are available only in the paper strips those preparation for sewer design consumes much time due to the need to use rainfall with specific durations [39]. In order to develop a reliable maximum rainfall model for designing and modeling drainage systems (for *C* from 1 to 50 years) there should be used at least 50 years of rainwater series (relatively 30 years with subsequent extrapolation). However, it should be noted that the short data series, for example 30-years, do not always include a wide range of natural variability of rainfall.

The scope of this study was to develop a probabilistic model for maximum rainfall in Legnica, based on a 50-year series of pluviographic records. Six probability distributions were used to describe the measurement data. Parameters of each distribution was determined by the maximum likelihood method. Selection of the best distribution was made using the Bayesian information criterion of Schwartz and the relative mean square residual error, respectively.

2 Pluviographic research material

Archived pluviographs from meteorological station of IMWM-NRI in Legnica from the time span 1961–2010 were used as research data. Pluviographs illustrated the daily mileage of rainfall on the 10 minutes scale which was also a basis for calculation of the hourly and daily totals.

Measuring station in Legnica, as part of a national measurement and observation network at hydrological and meteorological service, is a synoptic station which is participating in the international weather monitoring program (Weather World Watch) as part of the World Meteorological Organization (WMO), of which Poland is a member. Station building is located on the south-eastern outskirts of the city of Legnica, at elevation of 122 m above the sea level. Physiographical Legnica is situated temporarily on the edge of the Sudeten Foothills and the Silesian Lowland in the fork of the river Kaczawa and its left bank is tributary of the Black Water River. The predominant land use in both the municipality and rural area around the station are fields and wasteland.

To implement the national measuring program, station in Legnica uses standard equipment, typical for synoptic stations: meteorological instruments connected to the automatic MAWS workstation. Rainfall measurement is carried out in parallel with the automatic SEBA rain gauge that records 10 minutes values and with the participation of a meteorological observer who collects rainfall data using traditional Hellman rain gauge in the 6 hours checksums, and the daily totals. Collected data are compared and verified after each measurement. Rainfall data in digital form is stored in the IMWM-NRI database since 1999 (first launch of the automatic stations). Previously continuous recording of rainfall data was perpetuated on paper strips used in clockwise pluviograph gauges that functioned in Poland continuously since the 60s. Standard pluviograph recorded continuous rainfall pattern that occurred during one day, on a 10-minute pluviographic grid and 1 millimeter severed ordinate. At the end of the day (at 6 UTC) pluviographic strip was changed, and the measurement results were analyzed and noticed in the "pluviographic summary" as two values: the amount of daily sum and the total duration of observed rain episode.

In order to ensure long-term period rainfall data series and establish appropriate digital and analog measurement comparison, pluviograph in Legnica were held up to 2010. Despite to the official withdrawal of device from IMWM-NRI measuring program, there were built a rare and extremely valuable measurements data set, in terms of quality and accuracy of comparative rainfall. At the same time, the development of archival material of rain coming exclusively from the float rain gauge helped to maintain genetic homogeneity of the data series. National literature clearly indicates that the results of measurement sequences taking into account the use of automatic rain gauges disorder were characterized by considerable homogeneity of data [39].

In Legnica in the period from January 1961 to December 2010 there were recorded 8043 days with precipitation - those in which daily total of rain or snow exceeded 0.1 mm. In 2902 days there were recorded 0.1 mm value. In the long-term period the number of days with precipitation took from 120 to 191 which is 32.9% to 52.3% of all days in the year. Analyzing the warm half-years (V–X) rainfall days varied between 57 and 94, taking percentage of the entire year from 15.6% to 25.8%, and in respect only to wet days in the year, from 47.5% to 49.2% (Fig. 1). The average number of wet days in analyzed period

Tab. 2. The maximum, minimum and average monthly rainfall totals in long-term period of 1961–2010

Month	Maximum monthly rainfall, mm	Minimum monthly rainfall, mm	Average monthly rainfall, mm
I	85.6	3.5	24.1
II	47.5	1.2	23.0
III	70.3	9.8	29.6
IV	87.7	1.0	34.6
V	144.0	15.9	60.9
VI	154.5	18.6	67.7
VII	263.1	3.9	81.9
VIII	207.3	11.9	70.5
IX	122.5	3.2	43.3
X	105.0	2.5	35.0
XI	77.8	9.3	34.1
XII	82.5	5.9	29.1

was 161 (44%), while the warm half-years 76 days (respectively 20.8% and 47.4% of the year relative to the total wet days).

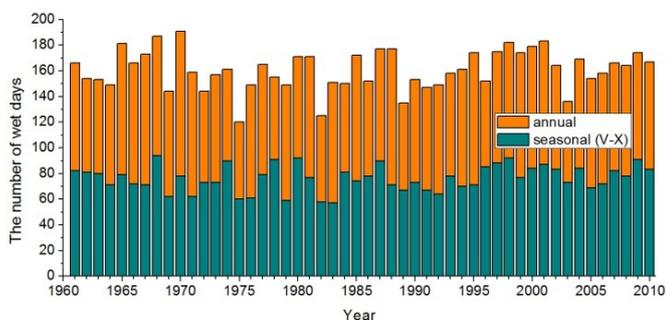


Fig. 1. Number of days with precipitation exceeding 0.1 mm in each year and the warm season (V–X) in long-term of 1961–2010

Annual totals in Legnica varied and have ranged between 351 mm (in 2003) and 765 mm (in 1977). The average value of the long-term period 1961-2010 was 521 mm. The share of the warm season V–X in each year was between 53.9% and 82.5%, reflecting the typical climatic conditions of Lower Silesia precipitation patterns. An increasing share of warm season rainfall at the background of whole year was viewable. The largest monthly sum covered the period from May to September, with the maximum values in July, as shown in Table 2.

The maximum daily totals in Legnica varied between 19.9 mm (in 2007) and 85.9 mm (in 2001). The heaviest rainfall values were recorded in warm season V–X of 1961-2010 long-term period. Mileage of largest daily amounts are shown in Fig. 2.

Detailed analysis of the 50 years pluviographic material indicates an increase of frequency of maximum daily amounts despite declining value of annual precipitation totals and the annual number of days with precipitation. In the case of data from Legnica extremely high daily values generally were affected by short-term episodes of rain.

A descending precipitation trend is observed in the long-term course of variability which confirms the increasing amount of rainfall events in recent years. This situation is of great scientific

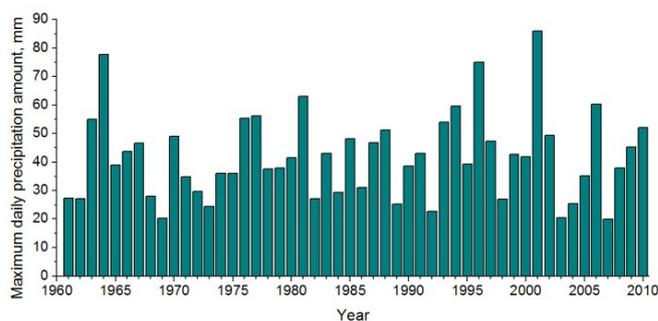


Fig. 2. Maximum daily rainfall totals patterns in the years 1961 to 2010 in Legnica

and engineers interest, especially results of studies related to the probabilistic description of meteorological phenomena [1, 26].

3 Depth-Duration-Frequency model

In order to determine the relationship between amount of rainfall from duration and probability of exceedance $h(t,p)$, there must be done a selection of data on which the relationship will be developed. Elaborating archival pluviographs authors limited period of analysis to months from May to October (V–X). Indeed, as demonstrated in the research [3], based on all investigated 63 meteorological stations in Poland in the 30-year period 1961–1990, the largest daily amount of rainfall occurred in the winter season (November to April) only occasionally and it was much lower than the average of the highest daily rainfall.

For the purpose of this paper, using total review method [40] there were isolated from the tested 50-years period top 50 maximum amount (h , mm) of rainfall for each of the 20 following rainfall durations (t), i.e.: 5, 10, 20, 30, 40 and 50 minutes, 1, 1.5, 2, 3, 6, 12 and 18 hours and at 1, 1.5, 2, 3, 4, 5 and 6 days.

In the first place the top 50 amount of rainfall was ordered decreasing (in 20 groups of a time duration from 5 minutes to 6 days). Then there were successively assigned to it empirical probability of exceedance according to (1) from $p = 0.020$ (for

Tab. 3. Recommended frequency of designed computational rain and limit the frequency of spill in accordance to EN 752

<i>t</i> , min	<i>m</i> = 1	<i>m</i> = 5	<i>m</i> = 10	<i>m</i> = 25	<i>m</i> = 50
	<i>p</i> = 0.020	<i>p</i> = 0.098	<i>p</i> = 0.196	<i>p</i> = 0.490	<i>p</i> = 0.980
5	16.2	11.2	9.8	7.6	4.6
10	23.5	17.5	14.1	10.9	8.3
20	31.7	25.1	20.1	14.5	11.3
30	32.0	26.9	24.5	16.0	12.5
40	37.4	28.2	25.2	17.6	13.0
50	39.8	31.0	25.2	18.2	13.7
60	40.6	31.6	25.9	19.9	14.2
90	41.0	36.2	29.4	21.6	16.5
120	49.6	39.6	32.2	22.8	18.2
180	57.6	40.7	34.5	26.3	20.0
360	57.7	47.2	40.9	30.6	23.9
720	74.9	51.8	44.6	35.8	28.6
1080	77.3	57.7	51.6	41.3	32.4
1440	77.3	66.3	57.7	46.4	35.7
2160	114.8	77.3	61.1	49.4	38.7
2880	129.3	97.8	74.3	53.0	41.3
4320	143.1	97.8	77.6	57.4	41.3
5760	157.0	116.0	83.9	61.9	47.1
7200	158.5	121.1	86.0	69.9	51.7
8640	167.9	132.3	91.9	71.4	53.7

the highest value) to *p* = 0.980 (for the lowest value):

$$p(mn) = \frac{m}{n + 1} \tag{1}$$

where *m* is the sequence number within a decreasing ordered string of the number of *n*.

It should be noted that the greatest probability estimation errors should be expected for extreme elements of the investigated data series [12, 38, 46, 47]. The amount of rainfall recorded for selected values of empirical probability are shown in the Table 1.

Theoretical distributions: Fréchet, Gamma, generalized exponential (GED), Gumbel, Log-Normal and Weibull were used to describe the measurement data [2, 4, 7–10, 18, 20, 23, 27, 30, 31, 40, 47]. Likelihood functions of these designated distributions are shown in Table 4.

Estimators parameters of particular distributions were determined by maximum likelihood method (MLM), through a numeric maximizing likelihood function (or its logarithm), taking into account the range of variability of investigated parameters.

Coincidence of theoretical distributions with measured data was examined using the Anderson-Darling test for statistics [6, 13]:

$$A^2 = -n - \frac{1}{n} \sum_{i=1}^n (2i - 1) [\ln F(x_i) + \ln(1 - F(x_{n-i+1}))] \tag{2}$$

where:

- x_i* *i*-th value in the decreasing ordered random sample,
- F(x)* cumulative distribution function for the theoretical distribution.

Tab. 4. Log-likelihood function for the investigated distributions (*α, β, γ, λ, μ* - the parameters of particular distributions)

Likelihood function
Fréchet distribution: $\ln L = \alpha^n \beta^{n\alpha} \prod_{i=1}^n (x_i - \gamma)^{-(\alpha+1)} \exp \left[- \sum_{i=1}^n \left(\frac{\beta}{x_i - \gamma} \right)^\alpha \right]$
Gamma distribution: $\ln L = (\alpha - 1) \sum_{i=1}^n \ln(x_i - \gamma) - n \ln \gamma(\alpha) - n \alpha \ln \beta - \frac{1}{\beta} \sum_{i=1}^n (x_i - \gamma)$
GED distribution: $\ln L = n \ln \alpha + n \ln \lambda - \sum_{i=1}^n (\lambda (x_i - \gamma)) + (\alpha - 1) \sum_{i=1}^n \ln(1 - e^{-(x_i - \mu)^\lambda})$
Gumbel distribution: $\ln L = -n \ln \sigma - \sum_{i=1}^n \left(\frac{x_i - \gamma}{\sigma} \right) - \sum_{i=1}^n \exp \left(- \frac{x_i - \gamma}{\sigma} \right)$
Log-Normal distribution: $\ln L = - \sum_{i=1}^n \ln(x_i - \gamma) - n \ln \sigma - \frac{n}{2} \ln(2\pi) - \frac{1}{2\sigma^2} \sum_{i=1}^n (\ln(x_i - \gamma) - \mu)^2$
Weibull distribution: $\ln L = n \ln \alpha - n \alpha \ln \beta + (\alpha - 1) \sum_{i=1}^n \ln(x_i - \gamma) - \sum_{i=1}^n \left(\frac{x_i - \gamma}{\beta} \right)^\alpha$

The null hypothesis *H*₀ (when the measurement data were suitable for tested theoretical distribution), were taken on a significance level of 0.05 if the *A*² test statistic was less than the critical value *A*²_{*kr*}. The alternative hypothesis was taken otherwise. The critical values were read from the statistical tables [6]. The calculation results for Anderson-Darling statistics are shown in Table 5. To increase the clarity of the results, *A*² values higher than the critical value *A*²_{*kr*} were bolded.

Four of the analyzed distributions, i.e. Fréchet, Gamma, GED, and Weibull distributions fulfill the compliance criterion for each of the 20 analyzed rainfall durations. Log-Normal distribution only in 1 case, and the Gumbel distribution even in 15

Tab. 5. Anderson-Darling statistics values for analyzed distributions (the critical values was given in brackets in the heading of the table)

$t, \text{ min}$	Fréchet (0.757)	Gamma (0.762)	GED (0.723)	Gumbel (0.757)	Log-Normal (0.752)	Weibull (0.757)
5	0.152	0.176	0.178	0.201	0.156	0.204
10	0.379	0.262	0.261	0.884	0.344	0.261
20	0.582	0.500	0.491	1.499	0.492	0.596
30	0.514	0.320	0.319	1.761	0.421	0.339
40	0.403	0.138	0.139	1.140	0.295	0.142
50	0.383	0.194	0.197	0.867	0.359	0.186
60	0.261	0.275	0.311	0.536	0.261	0.220
90	0.325	0.414	0.422	1.114	0.262	0.369
120	0.467	0.404	0.405	1.719	0.369	0.403
180	0.489	0.177	0.176	1.085	0.421	0.185
360	0.436	0.289	0.292	1.248	0.370	0.281
720	0.292	0.408	0.416	0.708	0.262	0.350
1080	0.386	0.249	0.255	0.752	0.340	0.217
1440	0.322	0.699	0.722	0.491	0.303	0.565
2160	0.359	0.474	0.474	1.317	0.377	0.479
2880	0.305	0.163	0.166	1.490	0.198	0.150
4320	0.232	0.423	0.424	1.327	0.207	0.405
5760	0.581	0.366	0.370	1.714	0.392	0.351
7200	0.609	0.410	0.396	1.277	0.776	0.527
8640	0.399	0.266	0.268	1.372	0.375	0.261

of the 20 cases did not fulfill undertook criteria. Gumbel distribution was rejected.

The best of remaining distributions was determined using the Bayesian information criterion of Schwartz, BIC [17, 24, 51], in the form of:

$$BIC = -2\ln L + k\ln n \quad (3)$$

where:

- L likelihood function of the tested distribution,
- k number of estimated parameters,
- n number of observations.

BIC criterion consists of two parts. The first describes the measure of fit of the model, and the second determines its simplicity. For the best it is deemed to such a model, for which the information criterion obtained the lowest value.

BIC criterion indicates a GED as the distribution that best representing the data distribution (the lowest value of BIC for 17 of the 20 analyzed rainfall durations). It should be noticed that differences between the three top distributions, i.e. GED Gamma and Weibull were minor.

Relative residual mean square error (RRMSE) was also used to evaluate the aptitude of investigated distributions and to describe the measurement data:

$$RRMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n \left(\frac{h_{t,i} - h_{m,i}}{h_{m,i}} \right)^2} \cdot 100\% \quad (4)$$

where:

- h_t the theoretical amount of rainfall (mm),

h_m amount of rainfall from measurements (mm).

Applying selected criterion, 9 (of 20 analyzed) rainfall durations, the best turns out to be a Weibull distribution, for 7 rainfall durations – GED, for 3 – Log-Normal distribution, but only 1 for Gamma distribution.

There were also calculated the RRMSE statistics, covering the entire range of data, i.e. all 20 durations. In this case, the best fit was characterized, in the indicated order: Weibull (RRMSE = 3.166%), Gamma (3.172%) and GED (3.173%). It should be noted that, as in the case of BIC criterion, the differences between the three top distributions, i.e. Weibull, Gamma, and GED, were very low. Other distributions slightly worse described the measurement data: Log-Normal (RRMSE = 4.558%), Fréchet (6.448%) and Gumbel (6.792%).

Considering both used criteria, GED was pointed as the best distribution. Quantile of the random variable GED distribution takes the form of the following formula:

$$h(p) = \gamma - \frac{1}{\lambda} \ln \left(1 - (1 - p)^{\frac{1}{\alpha}} \right) \quad (5)$$

The quality of this distribution fit to the empirical distribution of the amount of rainfall in Legnica, with different parameters for particular rainfall durations, shown in $h-h$ plot (Fig. 3).

As shown in Fig. 3, the amount of rainfall from Legnica was well described by generalized exponential distribution, while there was a substantial inconvenience in the use of (especially for engineering purposes) model (Eq. (5)) – with a number of parameters, depended on the rainfall duration. Moreover, there is no possibility to determine the amount of rainfall for the duration not included in the statement of parameters (e.g. there is

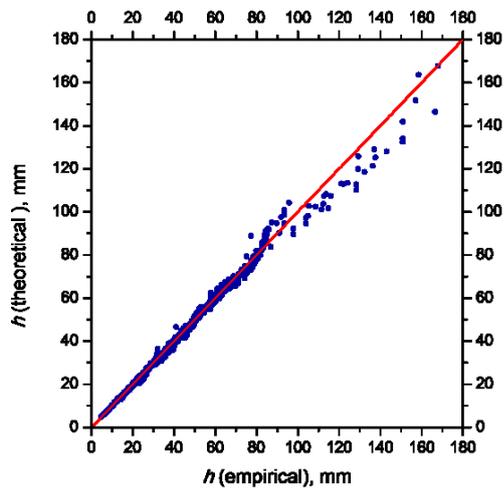


Fig. 3. $h-h$ plot for the GED distribution for measurement data for Legnica in the years 1961–2010

no possibility of calculating the rainfall with a duration of 15 minutes).

Therefore, authors attempted to generalize the results, i.e. designated the formula for the amount of rainfall for Legnica with specified duration $t \in [5; 8640]$ min and exceedance probability $p \in (0.02; 1]$.

In the absence of the parameter α depending on the rainfall duration, the parameter was averaged ($\bar{\alpha} = 0.963$) and then estimators of parameters λ and γ were recalculated. The calculation results are presented in Table 6.

Based on the calculated GED distribution parameters (Table 6) there was prepared plots showing their dependence on the rainfall duration (Fig. 4 and 5).

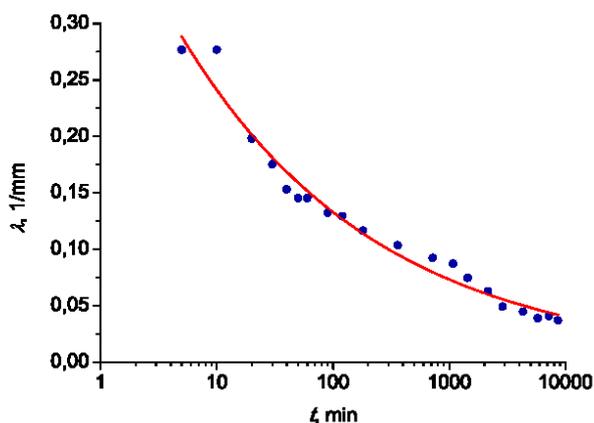


Fig. 4. The dependence of the parameter λ of the rainfall duration

The relationship of parameters λ and γ of the rainfall duration are described as a functions:

$$\lambda = 0.438t^{-0.259} \quad (6)$$

$$\gamma = 5.074t^{0.260} \quad (7)$$

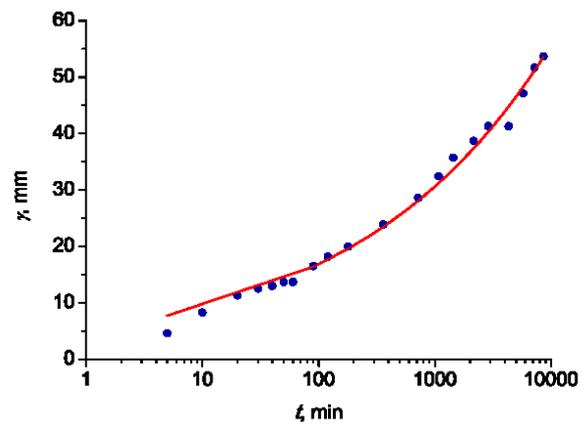


Fig. 5. The dependence of the parameter γ of the rainfall duration

for the coefficient of determination R^2 of 0.973 and 0.992, respectively.

Finally, a model for Legnica describing the dependence of the amount of rainfall on its duration and a specified exceedance probability, based on the GED distribution, takes the form of:

$$h(tp) = 5.074t^{0.260} - 2.283t^{0.259} \ln(1 - (1 - p)^{1.038}) \quad (8)$$

The fit quality of the equation (Eq. 8) for rainfall data from Legnica is shown in the $h-h$ plot (Fig. 6).

Analyzing results obtained from the model (Eq. 8) it is clear that the model qualitatively differs from the model (Eq. 5), especially for the several days rainfall for the highest amount (above 100 mm). Distinct differences, to the disadvantage of the model (Eq. 8), were also seen for the rainfall values up to 20 mm. On the other hand, the model (Eq. 8) has a relatively simple form.

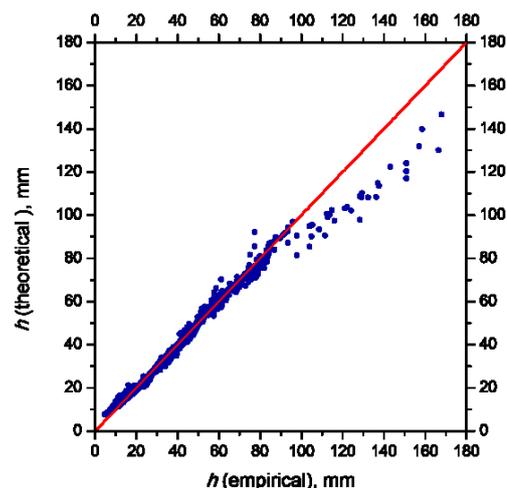


Fig. 6. $h-h$ plot for the GED distribution for measurement data for Legnica in the years 1961–2010

4 Conclusions

Fréchet, Gamma, (GED), Gumbel, Log-Normal and Weibull distributions were used to describe dependent variable $h(t, p)$ for

Tab. 6. Recommended frequency of designed computational rain and limit the frequency of spill in accordance to EN 752

t , min	α	λ , 1/mm	γ , mm	t , min	α	λ , 1/mm	γ , mm
5	0.963	0.2766	4.59	360	0.963	0.1036	23.89
10		0.2766	8.29	720		0.0924	28.59
20		0.1980	11.29	1080		0.0872	32.39
30		0.1753	12.49	1440		0.0746	35.69
40		0.1531	12.99	2160		0.0628	38.69
50		0.1454	13.69	2880		0.0491	41.29
60		0.1454	13.69	4320		0.0447	41.29
90		0.1323	16.49	5760		0.0390	47.09
120		0.1294	18.19	7200		0.0407	51.69
180		0.1165	19.99	8640		0.0370	53.69

rainfall from Legnica. All distributions parameters were estimated using maximum likelihood method.

Conformity of the analyzed theoretical distributions with measured data was investigated using the Anderson-Darling test, while choosing the best distribution was made using Bayesian information criterion of Schwartz (BIC) and also by the relative residual mean square error (RRMSE). Considering these two used criteria as the best distribution was considered GED (Eq. 5).

Towards the large inconvenience in the use of (especially for engineering purposes) formula (Eq. 5), as well as its limitations (the possibility of setting the amount of rainfall only for the days for which parameters was estimated) present authors attempted to generalize the results. Final outcome was to design the formula (Eq. 8) on the amount of rainfall for Legnica with specified duration $t \in [5; 8640]$ minutes and exceedance probability $p \in (0.02; 1]$.

Formula (Eq. 8) was characterized by a simpler form, but also significantly lower accuracy according to the output model (Eq. 5). Therefore, authors stipulate to create in the future an atlas of maximum rainfall in Poland, with tabular list (for each urban area) of amount of rainfall for the specified time duration and exceedance probability, like it was done for KOSTRA atlas in Germany.

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