Periodica Polytechnica Civil Engineering

61 (4), pp.780–793, 2017
https://doi.org/10.3311/PPci.8532

RESEARCH ARTICLE

Design of Pin Jointed Structures under Stress and Deflection Constraints Using Hybrid Electromagnetism-like Mechanism and Migration Strategy Algorithm

Shahin Jalili[1*], Yousef Hosseinzadeh[2]

Received 29 August 2015; Revised 13 December 2016; Accepted 31 January 2017

Abstract

Hybrid electromagnetism-like mechanism and migration strategy (EM-MS) algorithm is a recently developed optimization method which tries to benefit from both electromagnetism-like mechanism (EM) algorithm and migration strategy (MS). The EM algorithm is a population-based meta-heuristic method that simulates the attraction and repulsion mechanism between the charged particles to move them around the solution space. In the EM-MS method, the EM algorithm has a role of the global optimizer, while migration strategy (MS) helps the particles to exploit search space efficiently. In the current study, this hybrid method is utilized for optimum design of pin jointed structures under stress and deflection constraints. The efficiency of the EM-MS algorithm is demonstrated through four benchmark design examples. The results obtained confirm the potential and effectiveness of the EM-MS algorithm compared to other methods published in the recent state-of-the art literatures for the optimum design of pin jointed structures under stress and deflection constraints.

Keywords

electromagnetism-like mechanism, migration strategy, optimum design, pin jointed structures

[1]Young Researchers and Elite Club, Urmia Branch, Islamic Azad University, Urmia, Iran

[2]Department of Structural Engineering, The Faculty of Civil Engineering, University of Tabriz, Tabriz, Iran

[*]Corresponding author, email: shahinjalili@tabrizu.ac.ir

1 Introduction

Attaining optimum designs for structures has been in the focus of wide attention over past years and has established its position as one of the main optimization problems in structural engineering domain. However it is very widely believed that, for many structures with the large number of elements, searching optimum designs is very extreme hardness and sometimes completely time consuming procedure. Hence, extensive studies have been carried out to develop different optimization methods, ranging from gradient-based search techniques to derivative-free global optimization algorithms. As an alternative to the classical optimization approaches, meta-heuristic optimization techniques such as harmony search (HS) algorithm [1], particle swarm optimization (PSO) [2], big bang-big crunch (BB-BC) [3] algorithm, teaching-learning-based optimization (TLBO) [4], Biogeography-Based Optimization (BBO) [5], League Championship Algorithm (LCA) [6], and Cultural Algorithm (CA) [7] have been widely utilized and improved to solve structural optimization problems characterized by non-convex, dis-continuous, and non-differentiable [8–16].

The meta-heuristic algorithms have some advantages such as a simple framework and ease of implementation. Therefore, these algorithms have been adopted by researchers so far and are well suited to solve various structural optimum design problems including the sizing, layout, and topology optimization problems [17–21]. Due to probabilistic nature of the meta-heuristics, they do not guarantee finding global optimum solutions for any kind of the problems. However, if they properly implemented, meta-heuristics can provide near-optimal or optimal solutions with higher qualities. In designing efficient meta-heuristic algorithms, the exploration and exploitation are extremely important mechanisms. The exploration mechanism is related to the ability of exploring many and different regions of the search space, while the exploitation mechanism is related to the reduction of the diversity by focusing on the individuals with higher fitness to obtain high quality solutions. Therefore, the adequate balance between the exploration and exploitation mechanisms is a vital issue for these algorithms to be effectively executed. To this end, numerous standard and hybrid meta-heuristic algorithms have been applied and developed to optimum design of structures. Some of them will be mentioned below.

The harmony search (HS) algorithm was first introduced to optimize various mathematical functions by Geem et al. [1]. This algorithm simulates the improvisation process that occurs when a musician searches for a better state of harmony. Lee and Geem [22] developed a structural optimization method based on the HS algorithm. In another work, Degertekin [23] proposed efficient HS algorithm (EHS) and self-adaptive HS algorithm (SAHS) for sizing optimization of pin jointed structures.

The particle swarm optimizer (PSO) originally developed by Kennedy and Eberhart [2] is inspired by social behavior of bird flocking or fish schooling. Based on this algorithm, Li et al. [24] proposed a heuristic particle swarm optimizer (HPSO) for optimization of pin connected structures. The method is based on the particle swarm optimizer with passive congregation (PSOPC) and a harmony search (HS) scheme.

The big bang-big crunch (BB-BC) algorithm was proposed by Erol and Eksin [3]. It is based on the theory of the evolution of the universe; namely, the big bang and big crunch theory. Camp [25] utilized standard BB-BC algorithm to sizing of truss structures.

Teaching-learning-based optimization (TLBO) was introduced by Rao et al. [4]. It is a relatively new population-based optimization algorithm which is inspired by the social interaction between the teacher and the learners in a class. Togan [26] utilized this algorithm to design of planar frame structures. Recently, Degertekin and Hayalioglu [27] employed the TLBO algorithm to sizing of pin jointed structures and the numerical results showed that it is a promising method for benchmark design examples.

In some cases, researchers have been utilized novel optimization algorithms such as ray optimization (RO) [28] and colliding bodies optimization (CBO) [29], to optimum design of pin jointed structures. Kaveh and Khayatazad [30] employed RO to size and shape optimization of truss structures. In RO, each solution is modeled as a ray of light that moves in the search space in order to find optimum solution. In another work, Kaveh and Mahdavi [31] utilized CBO algorithm to size optimization of pin jointed structures. The method is based on one-dimensional collisions between two bodies, where each agent solution is modeled as the body [31].

The electromagnetism-like mechanism (EM) algorithm is a population-based meta-heuristic method developed by Birbil and Fang [32] based on the behavior of the charged particles in the electromagnetism field. It simulates the attraction and repulsion mechanism between the charged particles in the field of the electromagnetism to find optimal solutions, in which each particle is a solution candidate for the optimization problem. Recently, authors proposed a hybrid EM and migration strategy (EM-MS) algorithm for layout and size optimization of pin-jointed structures with frequency constraints [19]. In the EM-MS algorithm, the EM algorithm has a role of the global optimizer, while migration strategy (MS) helps the particles to exploit search space efficiently.

Following previous successful application of the EM-MS algorithm to solve layout and size optimization of truss structures under frequency constraints [19], this study utilizes the EM-MS algorithm to design optimization of pin jointed structures with stress and deflection constraints. As mentioned before, this algorithm benefits from both exploration and exploitation abilities of the EM algorithm and migration strategy. A set of four well-known design examples are considered to validate the efficiency of the EM-MS algorithm. The numerical results validate the efficiency of this hybrid approach in obtaining optimal designs as compared with other methods.

The remainder of the paper is organized as follows. Section 2 formulates the optimum design problem of pin jointed structures. In Section 3, the EM algorithm is introduced and then the EM-MS algorithm will be described in detail. The numerical examples are solved by the hybrid EM-MS algorithm and the results obtained are given in Section 4. Finally, Section 5 concludes the paper.

2 Mathematical description of the optimum design problem

The main aim of optimum design problem of a pin jointed structure is to minimize the weight of the structure while satisfying some constraints on stresses and deflections. In this class of the optimization problems, cross-sectional areas are taken as design variables. The optimal design of a pin jointed structure can be formulated as follows:

$$\begin{gathered}\text{Find } X=\left[x_1,x_2,\ldots,x_{nd}\right]\\ \text{To minimize } W\left(X\right)=\sum_{i=1}^{m}\gamma_i x_i L_i\\ \text{Subjected to:}\\ \sigma_i^c\le\sigma_i\le\sigma_i^t,\quad i=1,2,\ldots,m\\ x_{min}\le x_j\le x_{max}\quad j=1,2,\ldots,nd\\ \delta_{\min}\le\delta_k\le\delta_{\max},\quad k=1,2,\ldots,n\end{gathered}\tag{1}$$

where X is the vector containing the design variables; nd is the number of design variables; m is the number of members making up the structure; $W(.)$ demonstrates the weight of the structure; γ_i is the material density of member i; x_i is the cross-sectional area of the member i which is between x_{min} and x_{max}; L_i is the length of the member i; σ_i is the existing axial stress in the member i; σ_i^t and σ_i^c are the allowable tension and compressive stresses for member i, respectively; δ_k is the displacement of node k; δ_{min} and δ_{max} are the lower and upper limits for displacement at node k, and n is the number of nodes.

As demonstrated above, the optimal design of a pin jointed structure should satisfy some constraints on stress and deflection. In this study, the constraints of the problem are handled by using a simple penalty function method. So, for each solution candidate, following penalized weight is calculated:

$$W^p(X) = W(X) \times f_{penalty} \quad (2)$$

where:

$$f_{penalty} = (1+\varepsilon_1.\varphi)^{\varepsilon_2}, \quad \varphi = \sum_{k=1}^{nc} \varphi_k \quad (3)$$

where $W^p(.)$ is the penalized wieght, $f_{penalty}$ is the penalty function, nc is the number of constraints, and φ is the penalty factor which is related to the violation of constraints. In order to obtain the values of the penalty function for each solution, the axial stresses and nodal displacements of the structure are compared to the corresponding upper or lower bounds as follows.

$$\begin{cases} \varphi_i = \left| \dfrac{\sigma_i^{t,c} - \sigma_i}{\sigma_i^{t,c}} \right| & for \quad \sigma_i \langle \sigma_i^c \; or \, \sigma_i \rangle \sigma_i^t \\ \varphi_i = 0 & for \quad \sigma_i^c \le \sigma_i \le \sigma_i^t \end{cases} \quad (4)$$

$$\begin{cases} \varphi_i = \left| \dfrac{\delta_{\max,min} - \delta_i}{\delta_{\max,min}} \right| & for \quad \delta_i \langle \delta_{\min} \; or \; \delta_i \rangle \delta_{\max} \\ \varphi_i = 0 & for \quad \delta_{\min} \le \delta_i \le \delta_{\max} \end{cases} \quad (5)$$

As it can be seen from Eq.(4) and Eq.(5), if the constraints are not violated, the value of the penalty function will be zero. Otherwise, it has a positive value for penalization of objective function. In addition, in Eq.(3), the values of parameters ε_1 and ε_2 are selected by considering the exploration and exploitation mechanisms. In this study ε_1 is taken as unity, and ε_2 starts from 1.5 and gradually increases to 2.5.

3 Optimization technique

In this section, the electromagnetism-like mechanism algorithm and migration strategy are briefly described, and then details of the EM-MS method will be discussed.

3.1 The electromagnetism-like mechanism (EM) algorithm

The EM algorithm is a nature inspired optimization technique which is introduced by Birbil and Fang [32]. The EM algorithm is a population-based meta-heuristic algorithm motivated by the electromagnetism theory of physics, in which each electrically charged particle is a solution candidate for the optimization problem. The algorithm simulates the attraction and repulsion mechanism between the charged particles to move particles around the solution space.

In order to better explain, the detailed steps of the basic EM algorithm can be summarized as below:

Step 1: Initialization of the particles:

Define the problem as: minimize $f(x)$. Every particle x_i(i = 1, 2,…, N_p) is a n-dimensional vector, where n denotes the dimension of the problem. Initialize the positions of the N_p particles randomly within the given search space.

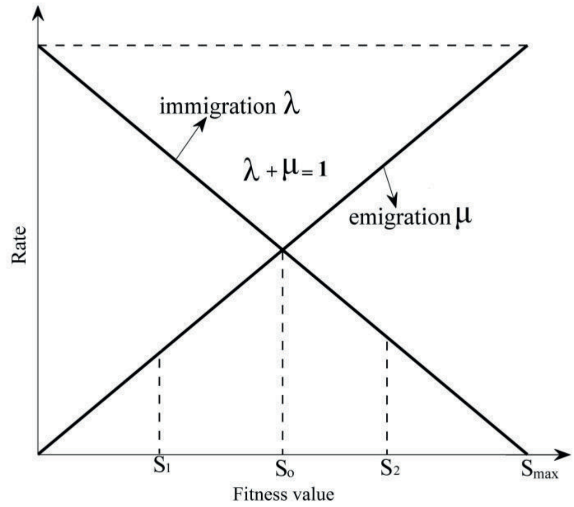

Fig. 1 The simple linear migration model

Step 2: Evaluation of the objective function for each particle:

Calculate the objective function for each particle and also calculate the best objective function value related to the best particle as follows.

$$best(t) = min(f(x_i)), \quad i = 1,2,...,N_p \quad (6)$$

where $best(t)$ is the best objective function value that obtained by particles at iteration t.

Step 3: Computation of the charge and total force for each particle:

Compute the charge of each particle and the total force acted on particle i, using the following equations.

$$q_i = \exp\left(-n \frac{best(t) - f(x_i)}{\sum_{k=1}^{N_p} best(k) - f(x_k)} \right), \quad i = 1,2,...,N_p \quad (7)$$

$$F_i = \begin{cases} \sum_{j=1, j\ne i}^{m} (x_j - x_i) \dfrac{q_i \times q_j}{\|x_j - x_i\|^2}; f(x_j) < f(x_i) \\ \sum_{j=1, j\ne i}^{m} (x_i - x_j) \dfrac{q_i \times q_j}{\|x_j - x_i\|^2}; f(x_j) \ge f(x_i) \end{cases} \quad i = 1,2,\ldots,N_p \quad (8)$$

where n is the dimension of the problem; q_i and q_j denote the charges for particles i and j at iteration t. According to the Eqs. (7–8) following points can be concluded:

- It can be seen from Eq. (7), the particle with better objective function value has a bigger charged value.
- According to the Eq. (8), the particle with the better objective function value attracts others with the worse objective function, whereas the particle with the worse objective function repulses others with the better objective function.
- The particle with the best objective function value in the population attracts other particles, while it is never attracted or repulsed by the others.

Step 4: Position update of particles:

Update the positions of particles for next iteration (t + 1) by employing the following equation:

$$x_{ik}^{t+1} = \begin{cases} x_{ik}^t + \lambda \frac{F_{ik}}{\|F_i\|}\left(u - x_{ik}^t\right); & F_{ik} > 0 \\ x_{ik}^t + \lambda \frac{F_{ik}}{\|F_i\|}\left(x_{ik}^t - l\right); & F_{ik} \le 0 \end{cases} \tag{9}$$

where $x_{ik}^{\,t}$ is the kth variable of ith particle at iteration t; u and l are the upper and lower bounds for the variables, respectively; λ is assumed to be uniformly distributed between 0 and 1.

Step 5:

Repeat from Steps 2–4 until iterations reaches their maximum limit or the stopping criterion is met and output the best solution.

3.2 The migration strategy

An important issue in providing better exploitation ability is which particles from the population should be selected to undergo local improvement. To this end, a migration strategy is used in this study. The main idea of the migration strategy is borrowed from the biogeography-based optimization (BBO) method. The BBO is a simple and efficient optimization algorithm originally proposed and shown effective for finding global optima for some optimization problems by Simon [5]. In fact, BBO is a population-based meta-heuristic algorithm motivated by migration behavior of species between the habitats in the nature, in which each habitat is a solution candidate for the problem.

In the BBO, the emigration and immigration processes are done by migration operator between the good and poor habitats to share information about the appropriate habitats. This information sharing depends on the immigration rate λ_i and emigration rate μ_i of each habitat, which are functions of the fitness values. Fig. 1 illustrates a simple linear migration model. As it can be seen from Fig.1, the habitat which has worse fitness (poor solution) like S_1 has a low emigration rate and a high immigration rate. This means that, the habitat with worse fitness value have a greater chance to take information about the good habitats. On the other hand, the habitat which has better fitness value (good solution) like S_2 has a low immigration rate and a high emigration rate. In this way, the habitat with better fitness value tends to share its good information among the other habitats. Moreover, the particle with medium fitness value, like point S_0, both immigration and emigration rates are equal, in which the probability of taking or giving information from or to other habitats is equal. In fact, the point S_0 is the equilibrium point. The migration process for the ith habitat can be described as follows:

$$x_{ik} \leftarrow x_{jk} \tag{10}$$

where x_i and x_j are kth variable of the immigrating and emigrating habitats, respectively. The emigrating habitat is the probabilistically selected habitat based on the emigration rates. Fig. 2 depicts the migration procedure of the BBO algorithm. It is worth mentioning that the roulette wheel selection method is implemented to select emigrating habitat.

3.3 The EM-MS algorithm

In this section, the procedures of the electromagnetism-like mechanism with migration strategy (EM-MS) algorithm for optimal design of pin jointed structures are described in detail. In the EM-MS algorithm, the required balance between exploration and exploitation mechanisms is achieved by using the modified electromagnetism-like mechanism (EM) algorithm as a global optimizer for global exploration and the migration strategy (MS) as an auxiliary tool for the local exploitation. This hybrid algorithm effectively uses the advantages of both the EM and MS techniques and avoids their weaknesses.

```
Habitat migration
Select Hi with probability α λi
 if Hi selected
   If rndreal(0,1)< λi
      For k=1:n
      Select Hj with probability α μi
      If Hj is selected
            Randomly select an variable from xj
            Replace a random variable in xi
     end if
     end for
  end if
 end if
```

Fig. 2 The migration procedure of the BBO algorithm

3.3.1 The modified EM algorithm

At the initial steps of the optimization process, meta-heuristic algorithm needs to explore the whole search space and identify the optimal regions (exploration mechanism), while whatever the algorithm closes to the final iterative process, the algorithm should search to find the solutions with the higher qualities (exploitation mechanism). In the EM-MS algorithm, the modified electromagnetism-like mechanism is utilized as an exploration tool to explore search space effectively. In the EM algorithm, the movement formula has a major effect in convergence behavior of this algorithm. The basic EM algorithm utilizes Eq. (9) to move particles in search space, which often implies a rapid loss of diversity in the population. In order to enhance the exploration ability of the standard EM algorithm, the following movement formula is proposed [19].

$$x_{ik}^{t+1} = x_{ik}^t + \lambda \frac{F_{ik}}{\|F_i\|} \frac{\left(U_k - L_k\right)}{t} \tag{11}$$

where t is the iteration number. As it can be seen, in initial steps of optimization process, the particles move with a bigger step size and this value is decreased to zero as the iteration process gets closer to final steps. It is important to note that, whenever the updated position of a particle goes beyond its lower or upper bound, the particle will take the value of its corresponding lower or upper bound.

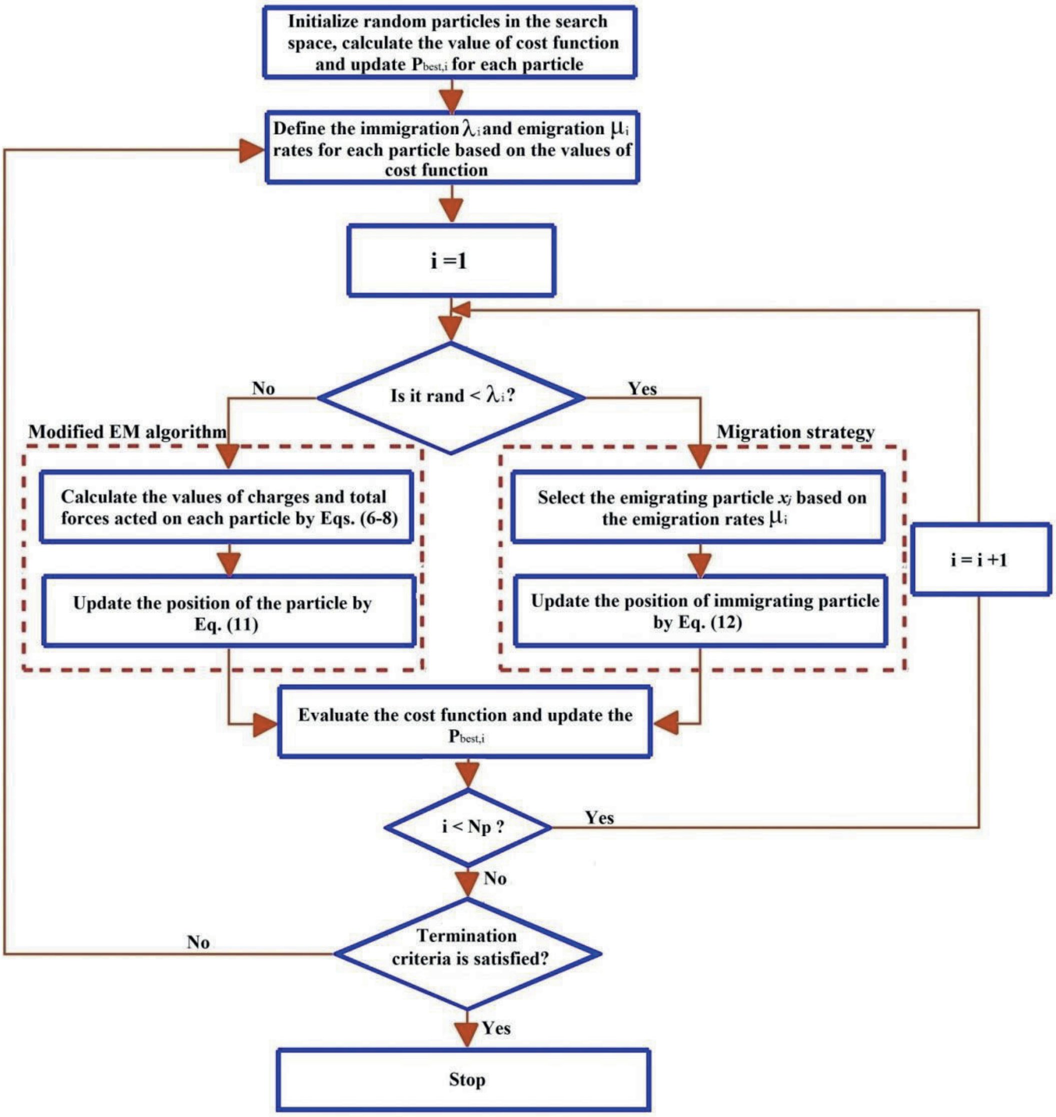

Fig. 3 The flowchart of the EM-MS algorithm

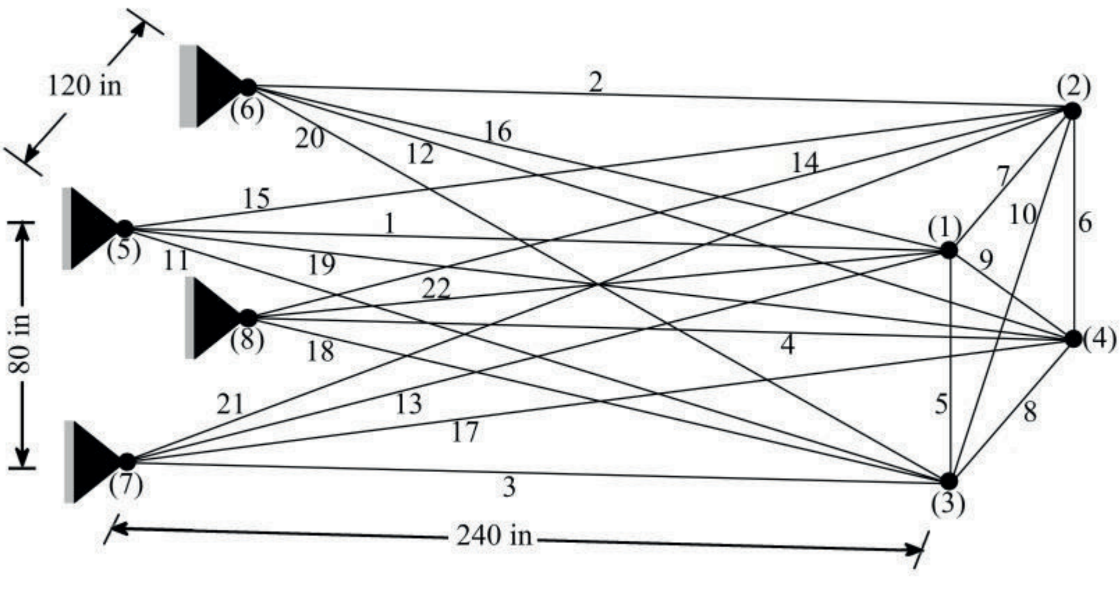

Fig. 4 Schematic of the 22-bar spatial truss structure

3.3.2 The migration strategy (MS)

In order to simulate the migration procedure between the particles in the solution space, the immigration λ_i and emigration μ_i rates are defined for each particle based on the simple linear migration model shown in Fig. 1. According to the simple migration operator of the BBO algorithm which is described as Eq. (10), the new position of a particle would be generated by moving the previous one towards another solution selected randomly from the population based on the emigration rate. However, this simple migration operator may leads to weak exploitation ability. In order to get a better performance including the better exploitation ability, following migration scheme is used [19]:

$$x_{ik} \leftarrow x_{ik} + rand\left(x_{jk} - x_{ik}\right) + rand\left(\boldsymbol{P}_{best,jk} - x_{ik}\right) \quad (12)$$

where x_{ik} and x_{jk} are kth variable of the immigrating and emigrating particles, respectively, and $\boldsymbol{P}_{best,jk}$ is kth variable of the best position experienced by the emigrating particle. According to this migration scheme, both of the current and best positions of the emigrating particle affect the migration process. This modification has a significant role in enhancing exploitation ability.

Fig. 3 shows the flowchart of the EM-MS algorithm [19]. Based on the flowchart of the EM-MS algorithm, following points can be stated:

- The EM-MS algorithm has two main mechanisms to update the position of each particle in the solution space: modified EM algorithm and migration strategy (MS).
- Each particle migrates toward another particle with the probability of λ_i.
- The immigration (λ_i) and emigration (μ_i) rates for each particle are calculated based on the objective function values. So, the quality of particles is considered during migration process.
- Since the immigration rate λ_i is inversely proportional to the objective function value, the particles with worst objective function values have more chance to migrate toward another particle.
- When the migration condition for a particle is not satisfied, its new position will be determined by the modified EM algorithm.
- In fact, the migration strategy treats as a local exploiter, while the modified EM algorithm provides a global optimizer mechanism to prevent premature convergence to the local optimums.
- The EM-MS algorithm has no any internal parameter to adjust expect the number of particles (N_p).

4 Design examples

In this section, four design examples have been conducted to assess the performance of the EM-MS algorithm for the optimum design of pin-jointed structures: 22-bar spatial truss, 25-bar spatial truss, 72-bar spatial truss, and 582-bar tower truss. In the all design examples, the number of particles (N_p) is set to 30. In order to assess the effect of different initial solution vector on the final result and because of the random nature of the algorithm, all design examples are independently optimized 30 times and the best, worst, average, and the standard deviation of trial runs are given in the tables. As it mentioned in introduction, metaheuristics do not guarantee finding exact optimal solution for the problem at hand. Hence, the best reported results are not necessary exact optimal solutions for the investigated problems. Each run stops when the maximum structural analyses are reached. The maximum structural analyses are set to 20,000 for the first three examples and 6000 for the last design example. The EM-MS algorithm and direct stiffness method for the analysis of pin jointed structures was coded in Matlab program and all executions were made on a Dell Vostro 1520 with Intel Core2 Duo CPU T9550 @ 2.66 GHz.

4.1 A 22-bar spatial truss structure

The 22-bar spatial truss shown in Fig. 4 is the first design example. The Young's modulus and material density of truss members are 10^4 ksi and 0.1 lb/in^3, respectively. This structure is subjected to three loading conditions as shown in Table 1.The members of the structure are categorized into seven groups. For each group element, the allowable tension and compressive stresses are listed in Table 2. In addition, the maximum nodal displacements in the all directions are limited to ±2.0 in for the all free nodes and the minimum permitted cross-sectional area is 0.1 in^2.

The results obtained by the EM-MS algorithm are summarized in Table 3 and compared to those reported previously. From Table 3, it can be concluded that the EM-MS algorithm gives lightest design as compared to the results obtained by Refs [22, 33, 34], and relatively same design when compared with the PSO [35], MSPSO [35] and HPSSO [36] methods. Although the EM-MS algorithm requires more structural analyses than the MSPSO [35] and HPSSO [36] methods, but the EM-MS algorithm is more efficient than these methods in terms of standard deviation value and stability of results. In addition, Table 3 shows that the average and worst weights obtained by the EM-MS algorithm are much better than the same values for the PSO [35], MSPSO [35], and HPSSO 36] methods. Moreover, the convergence behavior of the EM-MS algorithm is presented in Fig. 5. As it can be seen, the EM-MS algorithm reaches to the vicinity of the final result after about 10,000 analyses.

4.2 A 25-bar spatial truss structure

The second design example deals with the size optimization of a 25-bar spatial truss structure shown in Fig. 6. The Young's modulus and material density of truss members are 10^4 ksi and 0.1 lb/in^3, respectively. The twenty five members are categorized into eight groups, as follows:

A_1, (2) $A_2 - A_5$, (3) $A_6 - A_9$, (4)$A_{10} - A_{11}$, (5)$A_{12} - A_{13}$, (6) $A_{14} - A_{17}$, (7) $A_{18} - A_{21}$, and (8) $A_{22} - A_{25}$.

The spatial truss structure is subjected to the multiply loading conditions as shown in Table 4. The maximum nodal

Table 1 Multiply loading conditions for the 22-bar spatial truss structure.

Node	Condition 1			Condition 2			Condition 3		
	PX	PY	PZ	PX	PY	PZ	PX	PY	PZ
1	-20.0	0.0	-5.0	-20.0	-5.0	0.0	-20.0	0.0	35.0
2	-20.0	0.0	-5.0	-20.0	-50.0	0.0	-20.0	0.0	0.0
3	-20.0	0.0	-30.0	-20.0	-5.0	0.0	-20.0	0.0	0.0
4	-20.0	0.0	-30.0	-20.0	-50.0	0.0	-20.0	0.0	-35.0

Table 2 Allowable stress values for the 22-bar spatial truss structure

Element group		Allowable tension stress (ksi)	Allowable compression stress (ksi)
1	A_1-A_4	24.0	36.0
2	A_5-A_6	30.0	36.0
3	A_7-A_8	28.0	36.0
4	A_9-A_{10}	26.0	36.0
5	A_{11}-A_{14}	22.0	36.0
6	A_{15}-A_{18}	20.0	36.0
7	A_{19}-A_{22}	18.0	36.0

Table 3 Comparison of optimum designs obtained by various methods for 22-bar spatial truss structure.

Design variable (in^2)	Lee and Geem [22]	Khan et al. [33]	Sheu and Schmit [34]	Li et al. [24]	Talatahari et al. [35]		Kaveh et al. [36]	Present work
	HS			HPSO	PSO	MSPSO	HPSSO	EM-MS
1	2.588	2.5630	2.629	1.657	2.580	2.632	2.620593	2.64791
2	1.083	1.5530	1.162	0.716	1.131	1.195	1.206836	1.17990
3	0.363	0.2810	0.343	0.919	0.347	0.354	0.355719	0.35681
4	0.422	0.5120	0.423	0.175	0.421	0.415	0.419223	0.42000
5	2.827	2.6260	2.782	4.576	2.833	2.764	2.783028	2.76075
6	2.055	2.1310	2.173	3.224	2.095	2.030	2.082686	2.07069
7	2.044	2.2130	1.952	0.450	2.021	2.091	2.029553	2.04131
Best weight (lb)	1022.23[a]	1,034.74	1024.8	1057.14	1024	1024	1023.9857	1023.99
Mean weight (lb)	N/A	N/A	N/A	N/A	1033.790	1028.550	1027.599	1025.76
Standard deviation (lb)	N/A	N/A	N/A	N/A	17.29	6.63	6.357	2.16
No. of analyses	10,000	N/A	N/A	N/A	25,000	12,500	14,406	17,000
Worst weight (lb)	N/A	N/A	N/A	N/A	1093.120	1049.180	1052.048	1032.61

[a]Some of constraints are violated

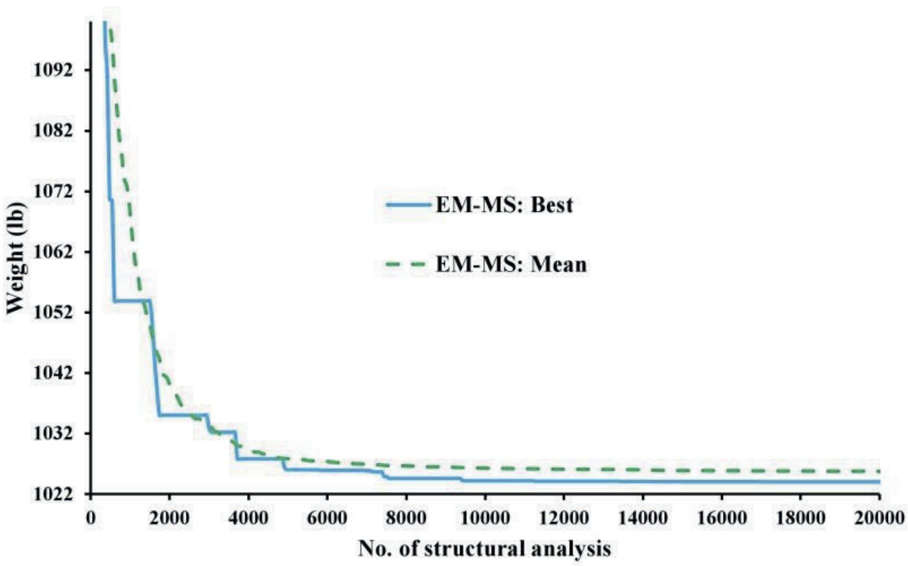

Fig. 5 The convergence diagrams of the EM-MS algorithm for the 22-bar spatial truss structures.

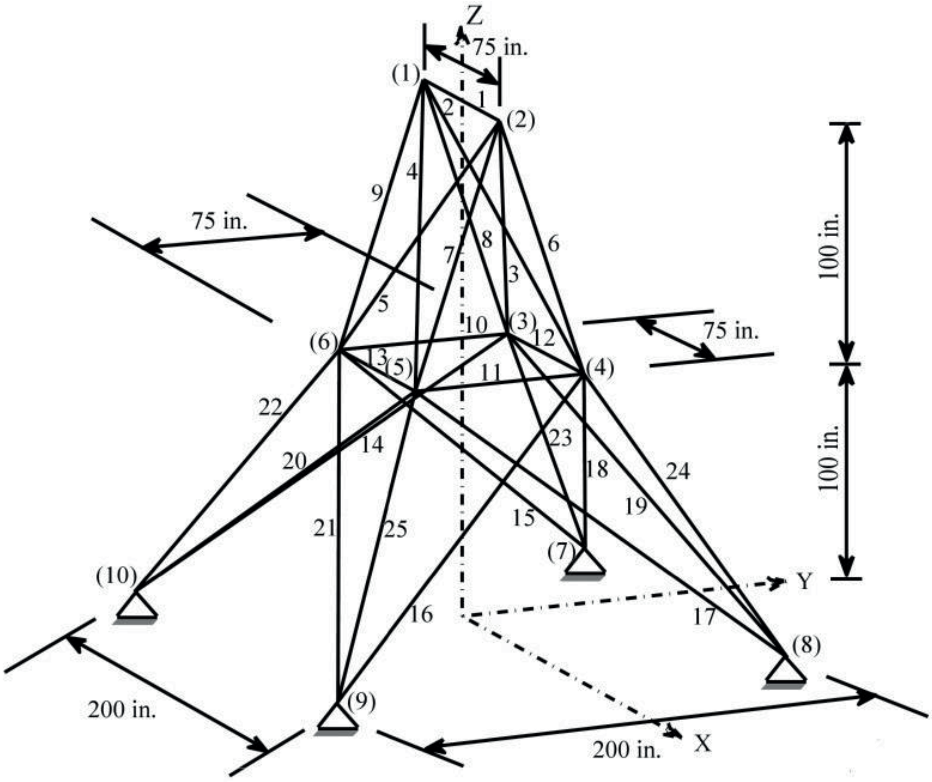

Fig. 6 Schematic of the 25-bar spatial truss structure.

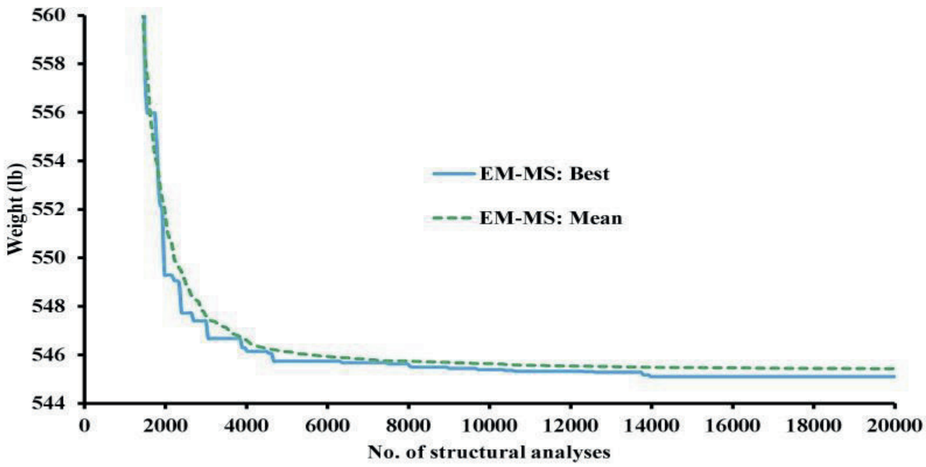

Fig. 7 The convergence diagrams of the EM-MS algorithm for the 25-bar spatial truss structure.

displacements in all directions are limited to ±0.35 in for all free nodes and the allowable stresses are different for each design group as shown in Table 5. In addition, the range of cross sectional areas varies from 0.01 in^2 to 3.4 in^2.

To show the effectiveness of the EM-MS algorithm, the obtained results for the 25-bar spatial truss are compared with those reported in the literature like the PSO [24], PSOPC [24], HPSO [24], BB-BC [25], EHS [23], SAHS [23], TLBO [27], and HPSSO [36] methods. From Table 6, it is evident that the EM-MS yields better design than the PSO [24], PSOPC [24], HPSO [24], BB-BC [25], EHS [23], SAHS [23], and HPSSO [36] methods and slightly heavier design than the TLBO [27] method. In addition, it can be clearly seen form Table 6 that the EM-MS algorithm yields less standard deviation value. This issue shows that the EM-MS algorithm has a relatively stable behavior during 30 independent runs. Moreover, the convergence diagrams of the best result and average of 30 independent runs are illustrated in Fig. 7. It can be seen that the EM-MS algorithm converges to the near-optimum design after about 14,000 analyses.

4.3 A 72-bar spatial truss structure

A 72-bar spatial truss shown in Fig. 8 is the third design example. The Young's modulus and material density of truss members are 10^4 ksi and 0.1 lb/in^3, respectively. The 72 members of this spatial truss are divided into 16 groups using symmetry, as follows:

(1) $A_1 - A_4$, (2) $A_5 - A_{12}$, (3) $A_{13} - A_{16}$, (4) $A_{17} - A_{18}$, (5) $A_{19} - A_{22}$, (6) $A_{20} - A_{30}$,
(7) $A_{31} - A_{34}$, (8) $A_{35} - A_{36}$, (9) $A_{37} - A_{40}$, (10) $A_{41} - A_{48}$, (11) $A_{49} - A_{52}$, (12) $A_{53} - A_{54}$, (13) $A_{55} - A_{58}$, (14) $A_{59} - A_{62}$, (15) $A_{63} - A_{70}$, (16) $A_{71} - A_{72}$.

The spatial truss structure is subjected to the loading conditions given in Table 7. The maximum nodal displacements in all directions are limited to ±0.25 in for all free nodes. In addition, the minimum and maximum cross-sectional areas are considered as 0.1 in^2 and 4 in^2, respectively.

The optimum designs obtained by the EM-MS algorithm and some other previous studies reported in the literature such as the PSO [37], BB-BC [25], RO [30], EHS [23], SAHS [23], TLBO [27], and CBO [31] methods are presented in Table 8. From the comparison, it is noticed that the EM-MS algorithm gives better design as compared to the PSO [37], BB-BC [25], RO [30], EHS [23], SAHS [23], and CBO [31] , but slightly heavier design when compared with the TLBO [27] method. However, the number of structural analyses for the EM-MS algorithm is relatively less than the TLBO [27] method. In addition, the EM-MS algorithm is much better than all other methods in terms of average, standard deviation, and worst results. As it can be seen, the worst design yielded by the EM-MS algorithm is also lighter than the designs obtained by the PSO [36], BB-BC [25], RO [30], EHS [23], and SAHS [23] methods. This issue demonstrates that this algorithm is more stable and reliable than other methods. Also, the convergence diagrams of best run for two cases are presented in Fig. 9.

Table 4 Multiply loading conditions for 25-bar spatial truss structure.

Node	Condition 1 (kips)			Condition 2 (kips)		
	P_x	P_y	P_z	P_x	P_y	P_z
1	0.0	20.0	-5.0	1.0	10.0	-5.0
2	0.0	-20.0	-5.0	0.0	10.0	-5.0
3	0.0	0.0	0.0	0.5	0.0	0.0
6	0.0	0.0	0.0	0.5	0.0	0.0

Table 5 Allowable stress values for the each element group of 25-bar spatial truss structure.

Element group	Allowable compressive stress (ksi)	Allowable tension stress (ksi)
1	35.092	40.0
2	11.590	40.0
3	17.305	40.0
4	35.092	40.0
5	35.092	40.0
6	6.759	40.0
7	6.959	40.0
8	11.082	40.0

Table 6 Comparison of optimum designs obtained by various methods for 25-bar spatial truss structure.

Design variables (in^2)		Li et al. [24]			Camp [25]	Degertekin [23]		Degertekin and Hayalioglu [27]	Kaveh et al. [36]	Present work
		PSO	PSOPC	HPSO	BB-BC	EHS	SAHS	TLBO	HPSSO	EM-MS
1	A_1	9.863	0.010	0.010	0.010	0.010	0.010	0.0100	0.0100	0.0100
2	$A_2 - A_5$	1.798	1.979	1.970	2.092	1.995	2.074	2.0712	1.9907	2.0159
3	$A_6 - A_9$	3.654	3.011	3.016	2.964	2.980	2.961	2.9570	2.9881	3.0170
4	$A_{10} - A_{11}$	0.100	0.100	0.010	0.010	0.010	0.010	0.0100	0.0100	0.0100
5	$A_{12} - A_{13}$	0.100	0.100	0.010	0.010	0.010	0.010	0.0100	0.0100	0.0100
6	$A_{14} - A_{17}$	0.596	0.657	0.694	0.689	0.696	0.691	0.6891	0.6824	0.6994
7	$A_{18} - A_{21}$	1.659	1.678	1.681	1.601	1.679	1.617	1.6209	1.6764	1.6384
8	$A_{22} - A_{25}$	2.612	2.693	2.643	2.686	2.652	2.674	2.6768	2.6656	2.6450
Best weight (lb)		627.08	545.27	545.19	545.38	545.49	545.12	545.09	545.164	545.10
Average weight (lb)		N/A	N/A	N/A	545.78	546.52	545.94	545.41	545.556	545.42
Standard deviation (lb)		N/A	N/A	N/A	0.491	1.05	0.91	0.42	0.432	0.37
No. of structural analyses		150,000	150,000	125,000	20,566	10,391	9051	15,318	13,326	13,980
Worst weight (lb)		N/A	N/A	N/A	N/A	548.04	546.60	546.33	546.990	546.46

Table 7 Multiply loading conditions for the 72-bar spatial truss structure.

Node	Condition 1 (kips)			Condition 2 (kips)		
	Px	Py	Pz	Px	Py	Pz
17	5.0	5.0	-5.0	0.0	0.0	-5.0
18	0.0	0.0	0.0	0.0	0.0	-5.0
19	0.0	0.0	0.0	0.0	0.0	-5.0
20	0.0	0.0	0.0	0.0	0.0	-5.0

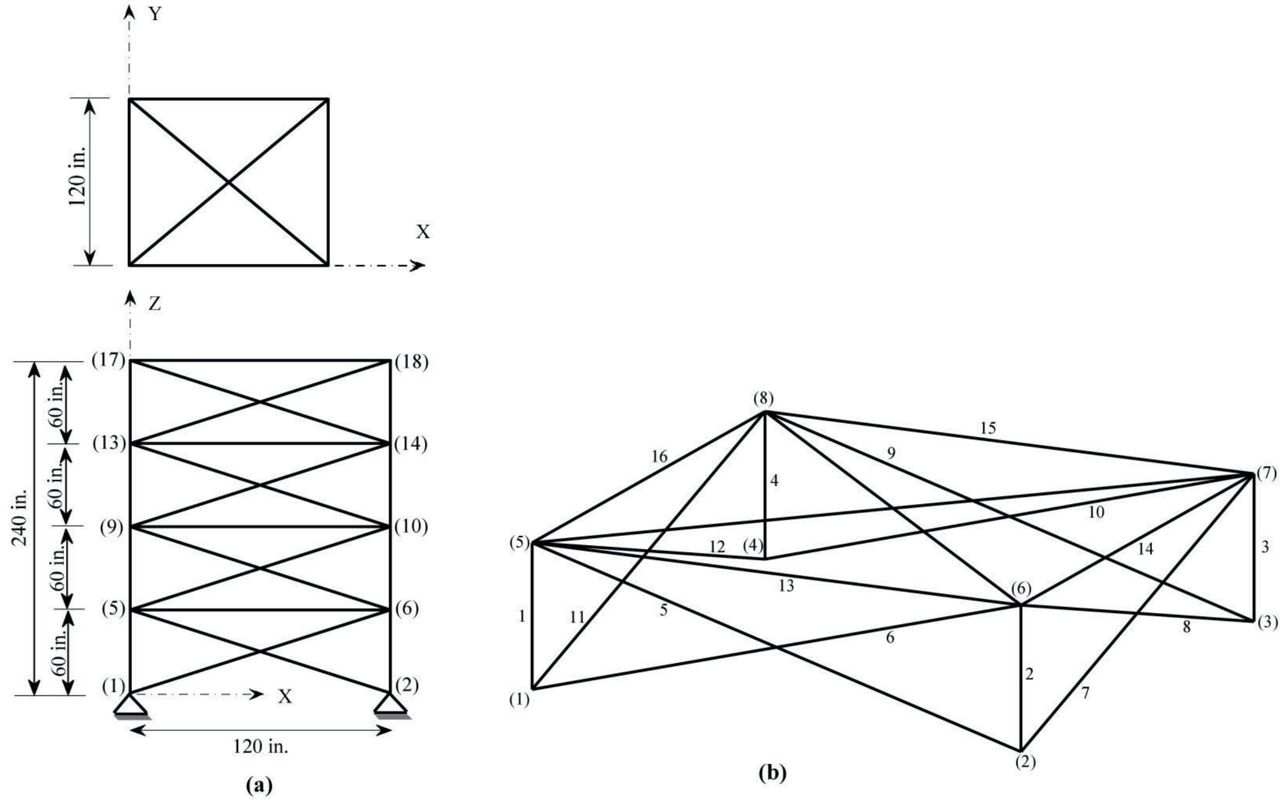

Fig. 8 Schematic of the 72-bar spatial truss structure: (a) Top and side view (b) Element and nodal numbering patterns for first story.

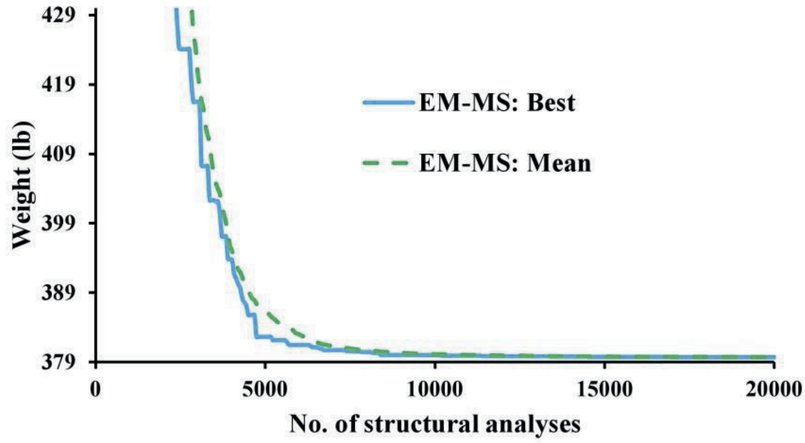

Fig. 9 The convergence diagrams of the EM-MS algorithm for the 72-bar spatial truss structure.

4.4. A 582-bar tower truss structure

The last design example is the size optimization of a 582-bar tower truss shown in Fig. 10. Hasancebi et al. [17] has been done some studies to optimize this design example with discrete variables. However, Kaveh and Mahdavi [31] optimized this design example with continuous sizing variables. As seen in Fig. 10, the elements of the structure are categorized in 32 groups with respect to symmetry. The Young's modulus is 29,000 ksi and the yield stress of steel is 36 ksi. The tower is subjected to the single load condition as follows: 1.12 kips acting in the X and Y directions and -6.74 kips acting in the Z direction at all free nodes of the tower. In addition, the range of cross sectional areas varies from 3.1 in^2 to 155 in^2. The stress and displacement constraints are considered as follows:

(1) Stress constraint (according to the AISC ASD code [38]):

$$\begin{cases} \sigma_i^+ = 0.6Fy & for\ \sigma_i \geq 0 \\ \sigma_i^- & for\ \sigma_i < 0 \end{cases} \tag{13}$$

where σ_i^- is calculated according to the slenderness ratio:

$$\begin{cases} [\left(1-\frac{\lambda_i^2}{2C_c^2}\right)F_y]/\left(\frac{5}{3}+\frac{3\lambda_i}{C_c}-\frac{\lambda_i^3}{8C_c^3}\right) & for\ \ \lambda_i < C_c \\ \frac{12\pi^2 E}{23\lambda_i^2} & for\ \ \lambda_i \geq C_c \end{cases} \tag{14}$$

where E = the modulus of elasticity; F_y = the yield stress of steel; C_c = the slenderness ratio (λ_i) dividing the elastic and inelastic buckling regions $\left(C_c = \sqrt{2\pi^2 E / f_y}\right)$; λ_i = the slenderness ratio (λ_i=kL_i / r_i); k = the effective length factor; L_i = the member length; and r_i = the radius of gyration.

Table 8 Comparison of optimum designs obtained by various methods for 72-bar spatial truss structure.

Design variables (in^2)	Perez and Behdian [37]	Camp [25]	Kaveh and Khayatazad [30]	Degertekin [23]		Degertekin and Hayalioglu [27]	Kaveh and Mahdavi [31]	Present work
	PSO	BB-BC	RO	EHS	SAHS	TLBO	CBO	EM-MS
1 $A_1 - A_4$	1.7427	1.8577	1.83649	1.967	1.860	1.90640	1.9028	1.8973
2 $A_5 - A_{12}$	0.5158	0.5059	0.502096	0.510	0.521	0.50612	0.5180	0.5079
3 $A_{13} - A_{16}$	0.1000	0.1000	0.100007	0.100	0.100	0.10000	0.1001	0.1002
4 $A_{17} - A_{18}$	0.1000	0.1000	0.10039	0.100	0.100	0.10000	0.1003	0.1001
5 $A_{19} - A_{22}$	1.3079	1.2476	1.252233	1.293	1.271	1.26170	1.2787	1.2580
6 $A_{20} - A_{30}$	0.5193	0.5269	0.503347	0.511	0.509	0.51110	0.5074	0.5202
7 $A_{31} - A_{34}$	0.1000	0.1000	0.100176	0.100	0.100	0.10000	0.1003	0.1000
8 $A_{35} - A_{36}$	0.1000	0.1012	0.100151	0.100	0.100	0.10000	0.1003	0.1003
9 $A_{37} - A_{40}$	0.5142	0.5209	0.572989	0.499	0.485	0.53170	0.5240	0.5065
10 $A_{41} - A_{48}$	0.5464	0.5172	0.549872	0.501	0.501	0.51591	0.5150	0.5222
11 $A_{49} - A_{52}$	0.1000	0.1004	0.100445	0.100	0.100	0.10000	0.1002	0.1002
12 $A_{53} - A_{54}$	0.1095	0.1005	0.100102	0.100	0.100	0.10000	0.1015	0.1000
13 $A_{55} - A_{58}$	0.1615	0.1565	0.157583	0.160	0.168	0.15620	0.1564	0.1568
14 $A_{59} - A_{62}$	0.5092	0.5507	0.52222	0.522	0.584	0.54927	0.5494	0.5454
15 $A_{63} - A_{70}$	0.4967	0.3922	0.435582	0.478	0.433	0.40966	0.4029	0.4011
16 $A_{71} - A_{72}$	0.5619	0.5922	0.597158	0.591	0.520	0.56976	0.5504	0.5674
Best weight (lb)	381.91	379.85	380.458	381	380.62	379.63	379.6943	379.69
Average Weight (lb)	N/A	382.08	382.5538	383.5	382.42	380.20	379.8961	379.72
Standard deviation (lb)	N/A	1.912	1.2211	1.92	1.38	0.41	0.0791	0.03
No. of analyses	N/A	19,621	19,084	15,044	13,742	19,778	15,600	17,100
Worst weight (lb)	N/A	N/A	N/A	385.50	383.89	380.83	N/A	379.79

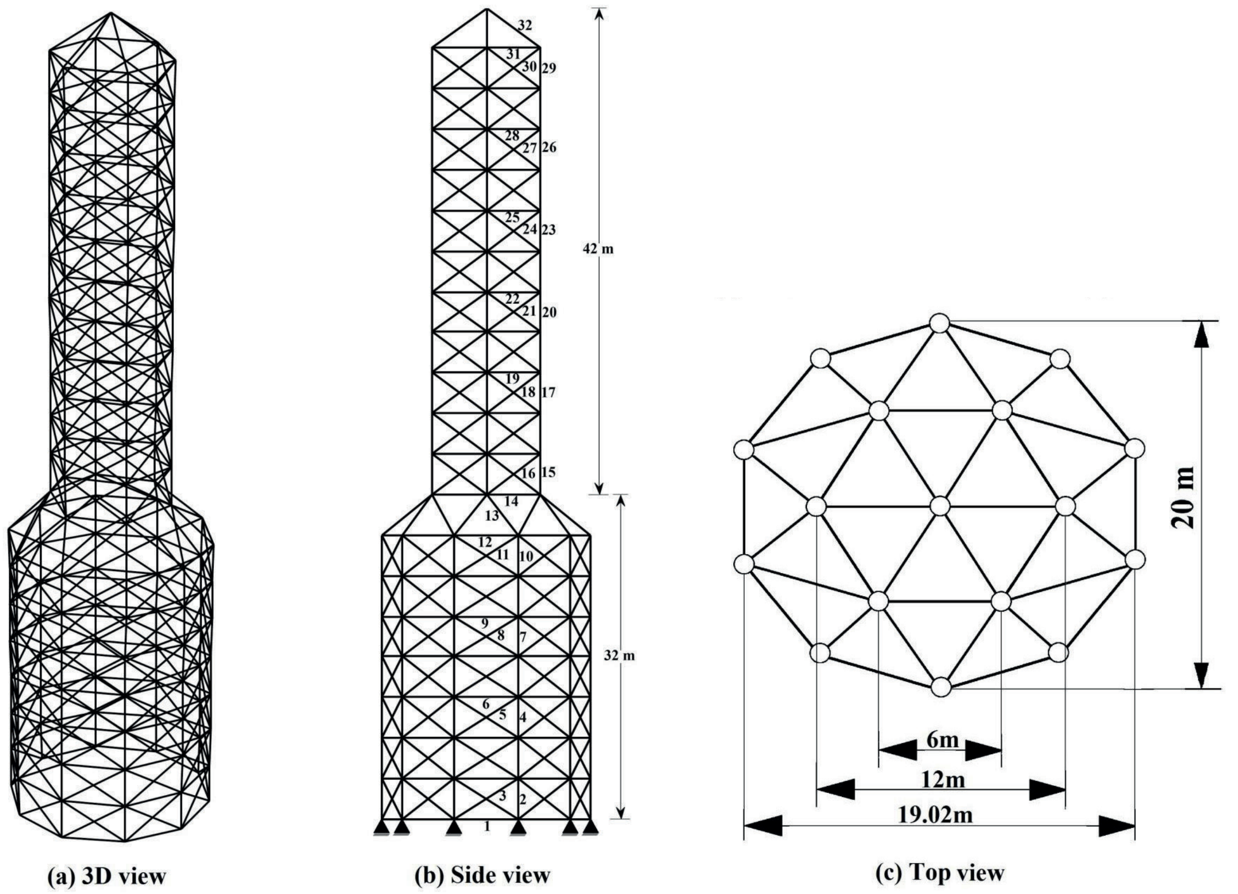

Fig. 10 Schematic of the 582-bar tower truss structure: 3D, Side and top views.

Table 9 Comparison of optimum designs obtained by various methods for 582-bar tower truss structure.

Design variables (cm^2)	Kaveh and Mahdavi [31]	Present work	Design variables (cm^2)	Kaveh and Mahdavi [31]	Present work
	CBO	EM-MS		CBO	EM-MS
1	20.5526	20.0000	17	155.6601	143.7971
2	162.7709	164.4115	18	21.4951	20.1022
3	24.8562	20.4508	19	25.1163	20.0000
4	122.7462	134.7547	20	94.0228	93.8866
5	21.6756	20.6472	21	20.8041	20.1140
6	21.4751	20.0000	22	21.2230	20.0914
7	110.8568	112.0897	23	53.5946	55.6278
8	20.9355	20.3045	24	20.6280	20.0734
9	23.1792	20.0094	25	21.5057	20.0476
10	109.6085	95.0446	26	26.2735	26.9963
11	21.2932	20.2939	27	20.6069	20.0392
12	156.2254	158.4182	28	21.5076	20.0000
13	159.3948	167.1142	29	24.1394	20.1617
14	107.3678	114.1851	30	20.2735	20.0366
15	171.9150	179.7715	31	21.1888	20.4490
16	31.5471	28.2502	32	29.6669	20.3688
			Best volume (m^3)	16.152	15.88
			Mean (m^3)	N/A	15.898
			Standard deviation (m^3)	N/A	0.0106
			Worst (m^3)	N/A	15.922
			No. of analyses	20,000	10,000

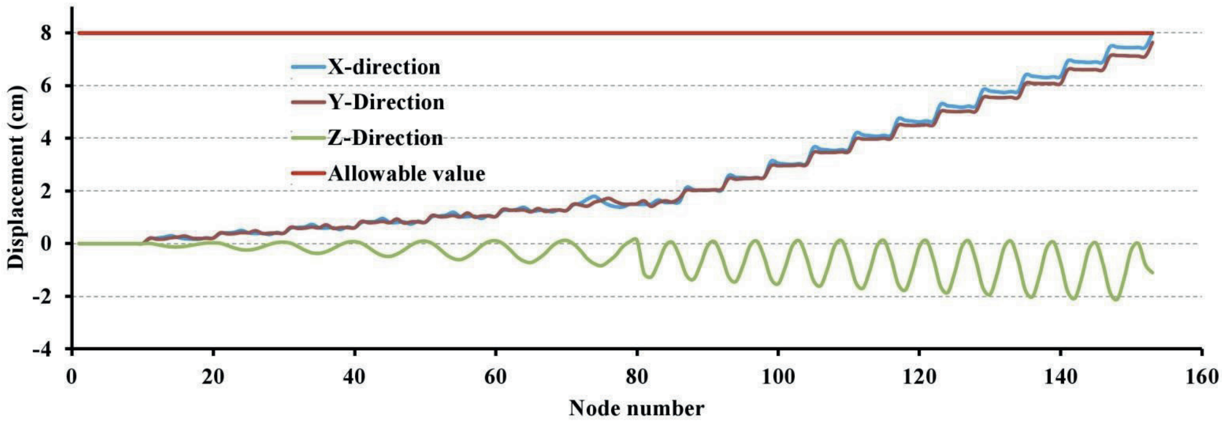

Fig. 11 Comparison of existing and allowable displacements in X, Y and Z directions in the 582-bar tower truss structure.

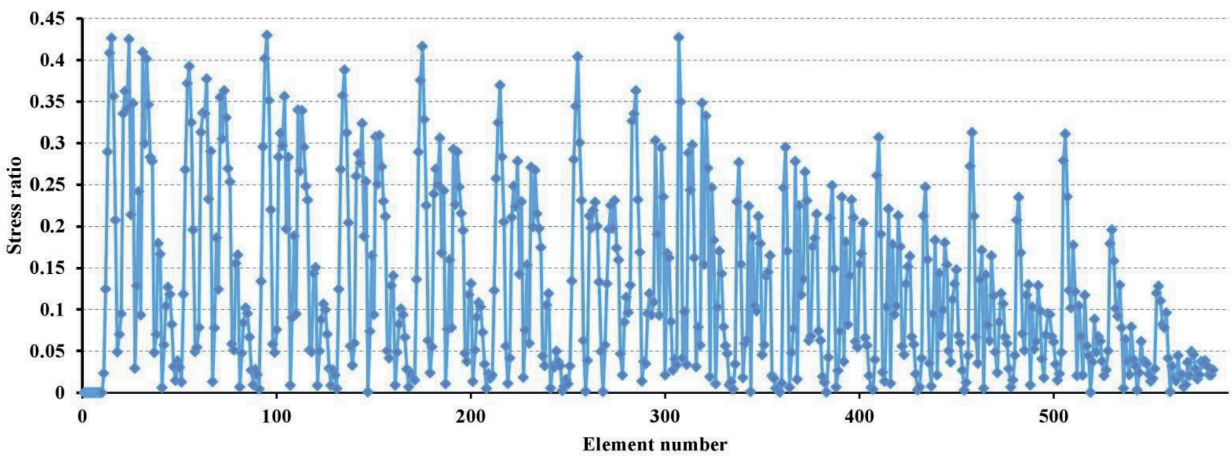

Fig. 12 The stress ratio of members in 582-bar tower truss structure.

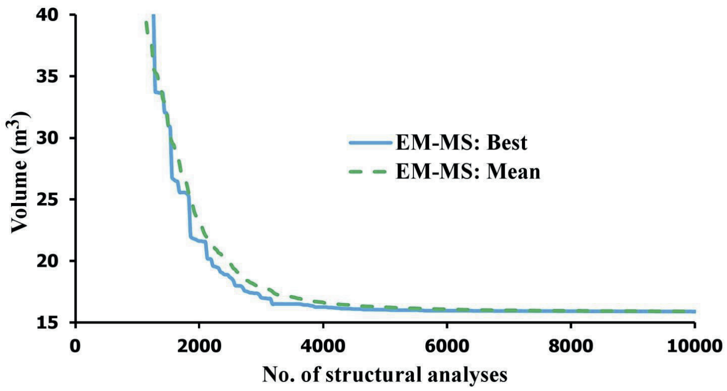

Fig. 13 The convergence diagrams of the EM-MS algorithm for the 582-bar tower truss structure.

(2) Displacement constraint:

For all free nodes, the maximum nodal displacements are limited to 3.15 in in all directions.

Table 9 compares the optimum designs obtained by the EM-MS and CBO [31] methods. The CBO [31] method developed a minimum volume of 16.1520 m^3 after 20,000 structural analyses while the EM-MS algorithm obtained the same volume after 4500 analyses. It is worth mentioning that the EM-MS obtains better designs than the CBO [31] in all independent runs. Moreover, the standard deviation of the final results during 30 independent runs is 0.0106 m^3, which is about %0.07 of structural volume. It means that the EM-MS algorithm can provide higher quality and more robust designs.

Fig. 11 compares the existing values and allowable values of the displacement constraints at nodes. As it can be seen, the displacement at top node of the tower in X direction controls the optimization process and the nodal displacements in Y and Z directions have not major effect. The value of maximum displacement in X direction is 7.9985 cm. Also, Fig. 12 shows the stress ratio in the members of the structure. The maximum stress ratio in the members is equal to 0.4296. In addition, Fig .13 shows the convergence curves of the EM-MS algorithm.

5 Concluding remarks

In this paper, a hybrid electromagnetism-like mechanism with migration strategy (EM-MS) algorithm is applied for optimum design of pin jointed structures under stress and deflection constraints. In the EM-MS algorithm, the adequate balance between exploration and exploitation mechanisms is achieved by using the electromagnetism-like mechanism (EM) algorithm as a global optimizer for the global exploration and the migration strategy (MS) as an auxiliary tool for the local exploitation. The performance of the EM-MS algorithm is evaluated using a set of four well-known benchmark design examples. The EM-MS algorithm has no any internal parameter to adjust except the number of particles (N_p), and it needs not any sensitivity analysis. In the all design examples, the number of particles is set to 30. The numerical results show the efficiency and capabilities of the EM-MS algorithm in finding better designs for the examined pin jointed structures. Moreover, in most cases, the average and standard deviation of 30 independently runs are relatively small when compared with those reported in literature. This means that the EM-MS algorithm has a relatively stable convergence behavior than other methods.

References

[1] Geem, Z. W., Kim, J. H., Loganathan, G. V. "A new heuristic optimization algorithm: harmony search.". *Simulations*, *76*(2). pp 60–68. 2001. https://doi.org/10.1177/003754970107600201

[2] Kennedy, J., Eberhart, R. C. "*Swarm intelligence*.". Morgan Kaufmann. San Francisco. 2001.

[3] Erol, O. K., Eksin, I. "New optimization method: big bang–big crunch.". *Advances in Engineering Software*, *37*(2). pp. 106–111. 2006. http://doi.org/10.1016/j.advengsoft.2005.04.005

[4] Rao, R. V., Savsani, V. J.,Vakharia, D. P. "Teaching-learning-based optimization: a novel method for constrained mechanical design optimization problems.". *Computer Aided Design*, *43*(3). pp. 303–315. 2011. https://doi.org/10.1016/j.cad.2010.12.015

[5] Simon, D. "Biogeography-Based Optimization." IEEE Transactions on Evolutionary Computation. 12(6), pp. 702–713. 2008. DOI: https://doi.org/10.1109/TEVC.2008.919004

[6] Kashan, A. H. "League Championship Algorithm (LCA): An algorithm for global optimization inspired by sport championships.". *Applied Soft Computing*, *16*. pp. 171–200. 2014. http://doi.org/10.1016/j.asoc.2013.12.005

[7] Reynolds, R. G. "Cultural Algorithms: Theory and Application.". In: *New Ideas in Optimization,* (Corne, D., Dorigo, M., Glover, F. (eds.)). pp. 367–378. McGraw-Hill Ltd., UK Maidenhead, UK, England. 1999.

[8] Jalili, S., Hosseinzadeh, Y., Taghizadieh, N. "A biogeography-based optimization for optimum discrete design of skeletal structures.". *Engineering Optimization*, *48*(9). pp. 1491–1514. 2016. https://doi.org/10.1080/0305215X.2015.1115028

[9] Jalili, S., Kashan, A. H., Hosseinzadeh, Y. "League championship algorithms for optimum design of pin-jointed structures.". *Journal of Computing in Civil Engineering (ASCE)*, *31*(2), 2016. https://doi.org/10.1061/(ASCE)CP.1943-5487.0000617

[10] Jalili, S., Hosseinzadeh, Y. "A cultural algorithm for optimal design of truss structures.". *Latin American Journal of Solids & Structures*, *12*(9), pp. 1721–1747. 2015. https://doi.org/10.1590/1679-78251547

[11] Taheri, S. H. S., Jalili, S. "Enhanced biogeography-based optimization: a new method for size and shape optimization of truss structures with natural frequency constraints.". *Latin American Journal of Solids & Structures*, *13*(7), pp. 1406–1430. 2016. https://doi.org/10.1590/1679-78252208

[12] Jalili, S., Hosseinzadeh, Y., Kaveh, A., "Chaotic biogeography algorithm for size and shape optimization of truss structures with frequency constraints.". *Periodica Polytechnica Civil Engineering*, *58*(4). pp. 397–422. 2014. https://doi.org/10.3311/PPci.7466

[13] Kaveh, A., Mahdavi, V. "Optimal design of structures with multiple natural frequency constraints using a hybridized BB-BC/Quasi-Newton algorithm.". *Periodica Polytechnica Civil Engineering*, *57*(1). pp. 27–38. 2013. https://doi.org/10.3311/PPci.2139

[14] Kaveh, A., Javadi, S. "An efficient hybrid particle swarm strategy, ray optimizer, and harmony search algorithm for optimal design of truss structures.". *Periodica Polytechnica Civil Engineering*, *58*(2). pp. 155–171. 2014. https://doi.org/10.3311/PPci.7550

[15] Kaveh, A., IlchiGhazaan, A. "Optimum design of skeletal structures using PSO-Based algorithms.". *Periodica Polytechnica Civil Engineering, 61*(2), pp. 184–195. 2017. https://doi.org/10.3311/PPci.9614

[16] Kaveh, A., Khanzadi, M., Alipour, M., Naraki, M. R. "CBO and CSS algorithms for resource allocation and time-cost trade-off.". *Periodica Polytechnica Civil Engineering*, *59*(3). pp. 361–371. 2015. https://doi.org/10.3311/PPci.7788

[17] Hasancebi, O., Carbas, S., Dogan, E., Erdal, F., Saka, M. P. "Performance evaluation of metaheuristic search techniques in the optimum design of real size pin jointed structures.". *Computers & Structures*, *87*(5–6). pp. 284–302. 2009. https://doi.org/10.1016/j.compstruc.2009.01.002

[18] Kaveh, A., Zolghadr, A. "Shape and size optimization of truss structures with frequency constraints using enhanced charged system search algorithm.". *Asian Journal of Civil Engineering, 12*(4). pp. 487–509. 2011.

[19] Hosseinzadeh, Y., Taghizadieh, N., Jalili, S. "Hybridizing electromagnetism-like mechanism algorithm with migration strategy for layout and size optimization of truss structures with frequency constraints.". *Neural Computing & Application, 27*(4). pp. 953–971. 2016. https://doi.org/10.1007/s00521-015-1912-1

[20] Kaveh, A., Zolghadr, A. "Topology optimization of trusses considering static and dynamic constraints using the CSS.". *Applied Soft Computing*, *13*(5). pp. 2727-2734. 2013. http://doi.org/10.1016/j.asoc.2012.11.014

[21] Tao, Xu.,Wenjie, Zuo., Tianshuang, Xu. "An Adaptive Reanalysis Method for Genetic Algorithm with Application to Fast Truss Optimization.". *Acta Mechanica Sinica*, *26*(2). pp. 225–234. 2010. https://doi.org/10.1007/s10409-009-0323-x

[22] Lee, K S., Geem, Z. W. "A new structural optimization method based on the harmony search algorithm.". *Computers & Structures*, 82(9–10). pp. 781–798. 2004. http://doi.org/10.1016/j.compstruc.2004.01.002

[23] Degertekin, S. O. "Improved harmony search algorithms for sizing optimization of truss structures.". *Computers & Structures*, 92–93. pp. 229–241. 2012. http://doi.org/10.1016/j.compstruc.2011.10.022

[24] Li, L. J., Huang, Z. B., Liu, F., Wu, Q. H. "A heuristic particle swarm optimizer for optimization of pin connected structures.". *Computers & Structures*, *85*(7–8). pp. 340–349. 2007. http://doi.org/10.1016/j.compstruc.2006.11.020

[25] Camp, C. V. "Design of space trusses using big bang–big crunch optimization.". *Journal of Structures Engineering ASCE, 133*(7). pp. 999–1008. 2007. https://doi.org/10.1061/(ASCE)0733-9445(2007)133:7(999)

[26] Toğan,V. "Design of steel frames using teaching-learning based optimization.". *Engineering Structures*. 34. pp.225–232. 2012. http://doi.org/10.1016/j.engstruct.2011.08.035

[27] Degertekin, S. O., Hayalioglu, M. S. "Sizing truss structures using teaching-learning-based optimization.". *Computers & Structures*, *119*. pp. 177–188. 2013. http://doi.org/10.1016/j.compstruc.2012.12.011

[28] Kaveh, A., Khayatazad, M. "A new meta-heuristic method: ray optimization.". *Computers & Structures*, *112–113*. pp. 283–294. 2012. http://doi.org/10.1016/j.compstruc.2012.09.003

[29] Kaveh, A., Mahdavi, V. R. "Colliding bodies optimization: A novel meta-heuristic method.". *Computers & Structures*, *139*. pp. 18–27. 2014. http://doi.org/10.1016/j.compstruc.2014.04.005

[30] Kaveh, A., Khayatazad, M. "Ray optimization for size and shape optimization of truss structures.". *Computers & Structures, 117*. pp. 82–94. 2013. http://doi.org/10.1016/j.compstruc.2012.12.010

[31] Kaveh, A., Mahdavi, V. R. "Colliding Bodies Optimization method for optimum design of truss structures with continuous variables.". *Advances in Engineering Software*, *70*. pp. 1–12. 2014. http://doi.org/10.1016/j.advengsoft.2014.01.002

[32] Birbil, I., Fang, S. C. "An electromagnetism-like mechanism for global optimization.". *Journal of Global Optimization, 25*(3). pp. 263–282. 2003. https://doi.org/10.1023/A:1022452626305

[33] Khan, M. R., Willmert, K. D., Thornton, W. A. "An optimality criterion method for large-scale structures.". *AIAA Journal*, *17*(7). pp. 753–761. 1979. https://doi.org/10.2514/3.61214

[34] Sheu, C. Y., Schmit, L. A. "Minimum weight design of elastic redundant trusses under multiple static load conditions.". *AIAA Journal*, *10*(2). pp. 155–162. 1972. https://doi.org/10.2514/3.50078

[35] Talatahari, S., Kheirollahi, M., Farahmandpour, C., Gandomi, A. H. "A multi-stage particle swarm for optimum design of truss structures.". *Neural Computing & Application, 23*(5). pp. 1297–309. 2013. https://doi.org/10.1007/s00521-012-1072-5

[36] Kaveh, A., Bakhshipur, T., Afshari, E. "An efficient hybrid Particle Swarm and Swallow Swarm Optimization algorithm.". *Computers & Structures*, *143*. pp. 40–59. 2014. http://doi.org/10.1016/j.compstruc.2014.07.012

[37] Perez, R. E., Behdinan, K. "Particle swarm approach for structural design optimization.". *Computers & Structures, 85*(19–20), pp. 1579–1588. 2007. http://doi.org/10.1016/j.compstruc.2006.10.013

[38] American Institute of Steel Construction (AISC). "Manual of steel construction allowable stress design". Chicago.