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RESEARCH ARTICLE

# Optimal Design of the Monopole Structures Using the CBO and ECBO Algorithms

Ali Kaveh[1*], Vahid Reza Mahdavi[1], Mohammad Kamalinejad[1]

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***Abstract***
*Tubular steel monopole structure is widely used for supporting antennas in telecommunication industries. This research presents two recently developed meta-heuristic algorithms, which are called Colliding Bodies Optimization (CBO) and Enhanced Colliding Bodies Optimization (ECBO), for size optimization of monopole steel structures. The design procedure aims to obtain minimum weight of monopole structures subjected to the TIA-EIA222F specification. Two monopole structure examples are examined to verify the suitability of the design procedure and to demonstrate the effectiveness and robustness of the CBO and ECBO in creating optimal design for this problem. The outcomes of the enhanced colliding bodies optimization (ECBO) are also compared to those of the standard colliding bodies optimization (CBO) to illustrate the importance of the enhancement of the CBO algorithm.*

***Keywords***
*monopole structures, optimal design, metaheuristic algorithms*

[1]Centre of Excellence for Fundamental Studies in Structural Engineering, Iran University of Science and Technology,
Narmak, Tehran, P.O. Box 16846-13114, Iran
[*]Corresponding author, e-mail: alikaveh@iust.ac.ir

## 1 Introduction

Over the last decade there has been ever increasing use of cellular telephones, including new smart phones, for voice and data communication, and wireless internet access, which drives increased demand for wireless data transmission bandwidth. As a result, there has been a large increase in the number of monopoles installed around populated areas to support antennas. Monopoles have become an important part of our communications infrastructure [1-4]. Therefore, optimal design of the monopole structures can be an interesting and challenging issue in the structural engineering research.

The monopole structures can be categorizes based on cross-sectional variations along height into two types: the tapered type and stepped type. In tapered type the cross-section is continuously decreasing from bottom to top of monopole, and in stepped type the structure is divided into some piece or part with abrupt change between sections [1]. The sections of stepped monopoles can be made circular and polygonal sections [5]. Figure 1 shows schematic shape of a treble-part-monopole with circular sections. The main objective of this paper is to find the optimum size of sections of the steel circular stepped monopoles. Here, the CBO and ECBO algorithms are utilized for optimization and weight of the monopole is considered as the objective function. The design method used in this study is also consistent with TIA-EIA222F [6] specifications.

The optimization algorithms can be divided into two categories: 1. Local optimizers; 2. Global optimizers. Local optimizer algorithms which often utilize the gradient information or iterative method to search the solution space near an initial starting point by local changes and are hard to apply and time-consuming in these optimization problems. Hereupon, global optimizers such as meta-heuristic algorithms are proposed for difficult optimization problems by global change ([7, 8]). Meta-heuristic algorithms are proposed for difficult optimization problems. In recent years, many meta-heuristics have been developed based on or have been inspired by natural phenomena from a variety of scientific fields (for example: [9-12]). Applications are also extended to include different problems in optimization [13-16]. Colliding bodies optimization (CBO) belongs to a family of meta-heuristic algorithms which recently developed by the authors [8, 17]. This algorithm can be considered as a multi-agent method, where each agent is a Colliding Body (CB). Simple formulation and no internal parameter tuning are advantages of this algorithm. The enhanced colliding bodies optimization (ECBO) was introduced by Kaveh and Ilchi Ghazaan [18] and it used memory to save some historically best solution to improve the CBO performance without increasing the computational cost and some components of agents was also changed to jump out from local minimum.

In this study, two design examples are considered to optimize by CBO and ECBO algorithms. Comparison of the optimal solution of the ECBO algorithm with those of the CBO method demonstrate the capability of CBO in solving the present type of design problem. It is also observed that optimization results obtained by the ECBO algorithm for two design examples have less weight in comparison to the results of the standard CBO algorithm. From the results obtained in this paper, it can be concluded that the optimum structures obtained by meta-heuristic algorithms requires smaller amount of steel material.

The remainder of this paper is organized as follows: In subsequent section, firstly, the mathematical formulations of the structural optimization of monopole structures problems are presented and a brief explanation of the TIA-EIA222F [6] is provided. After an explanation of the CBO, the ECBO algorithm is presented. Next section, includes two standard examples. The last sections present discussion on the examples results and concludes the study.

## 2 The Monopole structure optimization problems

The optimization problem can formally be stated as follows:

$$\begin{aligned} &\text{Find} \quad X = [x_1, x_2, x_3 \ldots, x_n] \\ &\text{to minimizes} \quad Mer(X) = f(X) \times f_{\text{penalty}}(X) \\ &\text{subjected to} \quad g_i(X) \le 0, \ i = 1,2,\ldots,m \\ &x_{imin} \le x_i \le x_{imax} \end{aligned} \tag{1}$$

where $X$ is the vector of design variables with $n$ unknowns, $g_i$ is the $i$th constraint from $m$ inequality constraints and $Mer(X)$ is the merit function; $f(X)$ is the cost; $f_{penalty}(X)$ is the penalty function which results from the violations of the constraints corresponding to the response of the monopole structures. Also, $x_{imin}$ and $x_{imax}$ are the lower and upper bounds of design variable vector, respectively.

Exterior penalty function method is employed to transform the constrained dam optimization problem into an unconstrained one as follows:

$$f_{penalty}(X) = 1 + \gamma_p \sum_{i=1}^{m} \max\left(0, g_j(x)\right) \tag{2}$$

where $\gamma_p$ is penalty multiplier.

### 2.1 Design variables

The most effective parameters for creating the monopole structure geometry were showed in Fig. 1. The parameters can be adopted as design variables:

$$X = \{D_1 \quad D_2 \quad \cdots \quad D_n \quad t_1 \quad t_2 \quad \cdots \quad t_n\} \tag{3}$$

where $X$ vector of design variables contains $2n$ shape parameters of the monopole structures, $n$ is number of monopole parts, $D_i$ and $t_i$ are the diameter and thickness of pipe cross section of the $i$th part.

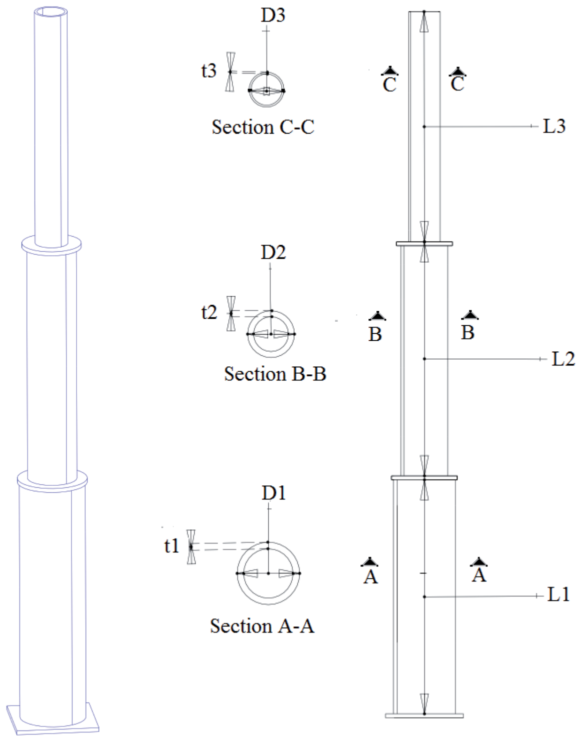


**Fig. 1** The circular treble-part-monopole: (a) 3D view, (b) front view.

### 2.2 Design constraints

Design constraints are divided into some groups including the operational, stress and stability constraints. The operational constraint is the restricted rotation at the top of pole structure that is limited as 1.5 degree. The stress constraint is considered as the ASICE-LRFD [19] manual. The local stability of cross sections constraint is achieved as follow:

$$\frac{D_i}{t_i} \le 0.11 \frac{E}{F_y} \Rightarrow \frac{D_i}{t_i} \le 96.25 \tag{4}$$

where $E$ and $F_y$ respectively are the modulus of elasticity and minimum yield stress of the material. Here, it is assumed that the material type is st-37($E$=2100000 kg/cm$^2$, $Fy$=2400 kg/cm$^2$ and $\rho$=7928.5 kg/m$^2$).

### 2.3 Cost function

The cost function is the weight of monopole structure, which may be expressed as:

$$f(X)=\sum_{i=1}^{n}\rho V_i=\sum_{i=1}^{n}\rho A_i l_i=\sum_{i=1}^{n}\rho\left(2\pi r_i t_i\right)l_i \tag{5}$$

where $\rho$ is the weight per volume of monopole material, $V_i$, $A_i$ and $l_i$ are the volume, cross section area and length of *i*th part of monopole structure, respectively.

### 2.4 The applied loads

In this study, TIA-EIA222F [6] specification is used for considering the wind and ice loading and influence of them on structures. The applied loads on the monopole structures consist of the vertical and horizontal loads, which are described in the follow subsections.

#### 2.4.1 The vertical loads

The most effective vertical loads, which should be considered in analysis process, consist of the self-weight of structure, the weight of ice and the weight of appurtenance (i.e. dish, light rod and cable). For considering the load of ice weight, it is assumed that type of ice is solid and density of it ($\rho_{ice}$) is equal to 897.043 kg/m$^3$ and thickness of attached ice on structure ($t_{ice}$) is 0.0127 m (0.5 in). Thus, the weight of ice on unit length of *i*th part of pole structure ($W_i^{ice}$) is calculated as:

$$W_i^{ice}=\rho_{ice}S_i t_{ice}=897.043*\left(\pi D_i\right)*0.0127=35.790D_i \tag{6}$$

where $S_i$ and $D_i$ are the circumference and diameter of cross section of the *i*th part. The load is a uniform load which is vertically assigned to the *i*th part.

In the load case of attached appurtenance weight at the top of pole structure, the weight of feedle cable of monopole is assumed as 2721.6 kg. The weight of dish and light rod with and without ice weight is also assumed as Table 1. It should be noted that these concentrated loads assigned to top point of pole structure.

**Table 1** Weight of the appurtenance loading with and without the influence of ice.

| Description | Weight (kg) |
|---|---|
| The light rod | 16 |
| The dish | 1235 |
| Sum of the weights | 1251 |
| Sum of the weights with considering the ice | 1625 |

#### 2.4.2 The horizontal loads

The wind load is considered a lateral load which we applied to pole structure. The applied distributed wind load to unit length of the *i*th part ($w_i^{wind}$) is calculated as:

$$\omega_i^{wind}=F_i Z_i \tag{7}$$

where $Z_i$ is the elevation of center of *i*th part, and $F_i$ is related to coefficient of wind force of the *i*th part which calculated as:

$$F_i=G_h Qz_i Ae_i CF \tag{8}$$

where $G_h$ is gust response factor for fastest mile basic wind speed and it assumed as 1.69 for pole structures. The structure force coefficients *CF* is determined 0.59 based on Table 1 of TIA/EIA-222-F. *Qz* is the velocity pressure and determined as:

$$Qz_i=0.613Kz_i V^2 \tag{9}$$

where $V$ is the basic wind speed of the structure location that it is assumed 36.1 m/s (130 km/h), and $Kz$ is the exposure coefficients:

$$Kz_i=\left(Z_i/10\right)^{0.285}\geq 1 \tag{10}$$

Also, $Ae_i$ is effective projected area of the *i*th part cross section in one face:

$$Ae_i=1.03Ag_i=1.03L_i D_i \tag{11}$$

where $Ag_i$, $L_i$ and $D_i$ are the projected area, length and diameter of the *i*th part.

Moreover, the ice effect is ignored in above equation. If we consider the ice thickness (i.e. 0.0254 m or 1 in) on the diameter of pole structure, $Ae_i$ is modified as:

$$Ae_i^{ice}=1.03Ag_i^{ice}=1.03L_i\left(D_i+0.0254\right) \tag{12}$$

The applied wind load to appurtenance at the top pole structure is similarity calculated. In this case, the coefficient of wind force (*F*) calculated as:

$$F=G_h QzAaCa \tag{13}$$

where $Aa$ and $Ca$ are the projected area and force coefficients of appurtenance, respectively. The appurtenance force coefficients (*Ca*) is assumed as 1.20 based on table-3 of TIA/EIA-222-F. The *Aa* is assumed as 1.45 and 1.50 m$^2$ with and without the effect of ice thickness on the appurtenance, respectively.

### 2.5 Loading combinations

In this study, two loading combinations have been considered based on existence of the ice load effect. Then, two loading combinations are defined:

The load combination 1 (without considering of the ice load effect): dead load (consisting of the self-weight of structure and weight of the appurtenance) + wind load (consisting of the applied wind load to the face of the pole structure and appurtenance without the ice thickness).

The load combination 2 (with considering the ice load effect): dead load (consisting of the self-weight of structure, weight of the appurtenance and ice thickness) + wind load (consisting of the applied wind load to face of pole structure and appurtenance with considering the ice thickness).

## 3 Enhanced colliding bodies optimization algorithm

The optimization of monopole structure problem is a complex problem because of having a large search space, multiple local optima and corresponding constraints. In this paper we apply a simple and efficient meta-heuristic algorithm, so-called enhanced colliding bodies optimization (ECBO), to solve this problem. For comparative study and showing the complexity problem, the standard colliding bodies optimization (CBO) is also utilized. In the following, both standard CBO and ECBO algorithms are briefly introduced.

### 3.1 Colliding Bodies Optimization algorithm

The colliding bodies optimization is based on momentum and energy conservation law for 1-dimensional collision [17]. This algorithm contains a number of Colliding Body (CB) where each one is treated as an object with specified mass and velocity which collide to others. After collision, each CB moves to a new position with new velocity with respect to old velocities, masses and coefficient of restitution. CBO starts with a set of agents determined with random initialization of a population of individuals in the search space. Then, CBs are sorted in an ascending order based on the value of cost function (see Fig. 2a). The sorted CBs are divided equally into two groups. The first group is stationary and consists of good agents. This set of CBs is stationary and their velocity before collision is zero. The second group consists of moving agents which move toward the first group. Then, the better and worse CBs, i.e. agents with upper fitness value, of each group collide together to improve the positions of moving CBs and to push stationary CBs towards better positions (see Fig. 2b). The change of the body position represents the velocity of the CBs before collision as:

$$v_i = \begin{cases} 0, & i = 1,\ldots,n \\ x_i - x_{i-n}, & i = n+1,\ldots,2n \end{cases} \tag{14}$$

where, $v_i$ and $x_i$ are the velocity vector and position vector of the $i$th CB, respectively. $2n$ is the number of population size.

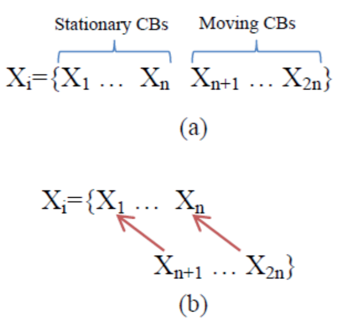


**Fig. 2** (a) The sorted CBs in an increasing order, (b) The pairs of objects for the collision.

After the collision, the velocity of bodies in each group is evaluated using momentum and energy conservation law and the velocities before collision. The velocity of the CBs after the collision is:

$$v_i = \begin{cases} \dfrac{\left(m_{i+n} + \varepsilon m_{i+n}\right) v_{i+n}}{m_i + m_{i+n}}, & i = 1,\ldots,n \\ \dfrac{\left(m_i - \varepsilon m_{i-n}\right) v_i}{m_i + m_{i-n}}, & i = n+1,\ldots,2n \end{cases} \tag{15}$$

where, $v_i$ and $v_i'$ are the velocities of the $i$th CB before and after the collision, respectively; is the mass of the $i$th CB defined as:

$$m_k = \frac{\dfrac{1}{fit(k)}}{\sum_{i=1}^{n} \dfrac{1}{fit(i)}}, \quad k = 1,2,\ldots,2n \tag{16}$$

where *fit*($i$) represents the objective function value of the $i$th agent. Obviously a CB with good values exerts a larger mass and fewer moves than the bad ones. Also, for maximizing the objective function, the term $\frac{1}{fit(i)}$ is replaced by *fit*($i$). $\varepsilon$ is the coefficient of restitution (COR) and is defined as the ratio of the separation velocity of the two agents after collision to approach velocity of two agents before collision. In this algorithm, this index is defined to control of the exploration and exploitation rates. For this purpose, the COR decreases linearly from unit value to zero. Here, $\varepsilon$ is defined as:

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \tag{17}$$

where $iter$ is the actual iteration number, and $iter_{\max}$ is the maximum number of iterations. Here, COR is equal to unity and zero representing the global and local search, respectively.

In this way a good balance between the global and local search is achieved by increasing the iteration.

The new positions of CBs are evaluated using the generated velocities after the collision in the position of stationary CBs:

$$x_i^{new} = \begin{cases} x_i + rand \circ v_i', & i = 1,\ldots,n \\ x_{i-n} + rand \circ v_i', & i = n+1,\ldots,2n \end{cases} \tag{18}$$

where, $x_i^{new}$ and $v_i'$ are the new position and the velocity after the collision of the $i$th CB, respectively.

### 3.2 Enhanced Colliding Bodies Optimization algorithm

In order to improve the CBO to obtain faster and more reliable solutions, Enhanced Colliding Bodies Optimization (ECBO) is developed which uses memory to save a number of historically best CBs and also utilizes a mechanism to escape from local optima [18]. The steps of this technique are given as follows:

***Level 1: Initialization***

**Step 1:** The initial positions of all the CBs are determined randomly in the search space.

***Level 2: Search***

**Step 1:** The value of mass for each CB is evaluated according to Eq. (16).

**Step 2:** Colliding memory (CM) is utilized to save a number of historically best CB vectors and their related mass and objective function values. Solution vectors which are saved in CM are added to the population and the same number of current worst CBs are removed. Finally, CBs are sorted according to their masses in a decreasing order.

**Step 3:** CBs are divided into two equal groups: (i) stationary group, (ii) moving group (Fig. 2).

**Step 4:** The velocities of stationary and moving bodies before collision are evaluated by Eq. (14).

**Step 5:** The velocities of stationary and moving bodies after the collision are evaluated using Eq. (15).

**Step 6:** The new position of each CB is calculated by Eq. (18).

**Step 7:** A parameter like **Pro** within (0, 1) is introduced and it is specified whether a component of each CB must be changed or not. For each colliding body **Pro** is compared with $rn_i$ ($i$=1, 2, …, $n$) which is a random number uniformly distributed within (0, 1). If $rn$ < **Pro**, one dimension of the $i$th CB is selected randomly and its value is regenerated as follows:

$$x_{ij} = x_{i,\min} + random \cdot \left( x_{j,\max} - x_{j,\min} \right) \tag{19}$$

where $x_{ij}$ is the $j$th variable of the $i$th CB, and $x_{j,\min}$ and $x_{j,\max}$ are the lower and upper bounds of the $j$th variable, respectively. In order to protect the structures of CBs, only one dimension is changed.

***Level 3: Terminal condition check.***

**Step 1:** After a predefined maximum evaluation number, the optimization process is terminated.

## 4 Design Examples

In this section, two recently developed optimization algorithms consisting of the CBO and ECBO are utilized for optimization of two monopole structures. The number of design variables for the first example and second example is 10 and 12, respectively. Similarly, the number of Colliding Bodies (CB) or agents for these examples is considered 30. For both examples, the maximum number of iteration is considered as 200. For the sake of simplicity, the penalty approach is used for constraint handling. The optimization algorithms and the analysis and design of monopole structures are coded in Matlab and SAP200 software, respectively.

### 4.1 A 30 meter high monopole structure

As the first example, a monopole structure with a height of 30 m is considered. The height of the structure is divided into five equal parts. For this test example, the weight of structure is the objective function. The monopole structure is modeled by 10 shape design variables as:

$$X = \{ D_1 \quad D_2 \quad D_3 \quad D_4 \quad D_5 \quad t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \} \tag{20}$$

Design variables can be selected from a discrete list of available values set $D$ = {20, 21, 22, …, 89, 90} cm and $t$ = {0.4, 0.45, 0.5, 0.6, 0.8, 0.9, 1} cm, which have 121 discrete values.

Table 2 compares the results obtained by the both algorithms with engineering design values, whose appropriate values that designer has determined using trial-error method [20]. The constraints values are also shown in Table 2, it can be seen that all constraints of the both algorithms results are satisfied. Moreover the evolution processes of best fitness value obtained by both algorithms are shown in Fig. 3.

**Table 2** Optimum design variables (cm) for the 30-m height monopole using different methods

| Design variables | Engineering design | CBO | ECBO |
|---|---|---|---|
| D5 | 40 | 38 | 38 |
| D4 | 47 | 50 | 55 |
| D3 | 60 | 57 | 59 |
| D2 | 70 | 73 | 69 |
| D1 | 80 | 75 | 76 |
| t5 | 0.45 | 0.6 | 0.4 |
| t4 | 0.5 | 0.6 | 0.6 |
| t3 | 0.8 | 0.6 | 0.8 |
| t2 | 0.8 | 0.8 | 0.8 |
| t1 | 1 | 1 | 0.8 |
| Weight (kg) | 3329.4 | 3253.4 | 3123.1 |
| Rotation | 1.3454 | 1.3469 | 1.3499 |
| Maximum stress ratio | 0.4194 | 0.4416 | 0.4574 |
| Maximum (D/t) | 94.00 | 95.00 | 95.00 |

### 4.2 A 36 meter high monopole structure

We now consider a monopole structure with a high of 36 m. The height of the structure is divided into six equal parts. Similarity, for this test example the weight of structure is the objective function. All assumptions and definitions are same to first example. The monopole structure is modeled by 12 shape design variables as:

$$X = \{ D_1 \quad D_2 \quad D_3 \quad D_4 \quad D_5 \quad D_6 \quad t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \} \tag{21}$$

Table 3 compares the results obtained by the both algorithms with engineering design values. All constraints of the both algorithms results are satisfied as the first example. Moreover the evolution processes of best fitness value obtained by both algorithms are shown in Fig. 4.

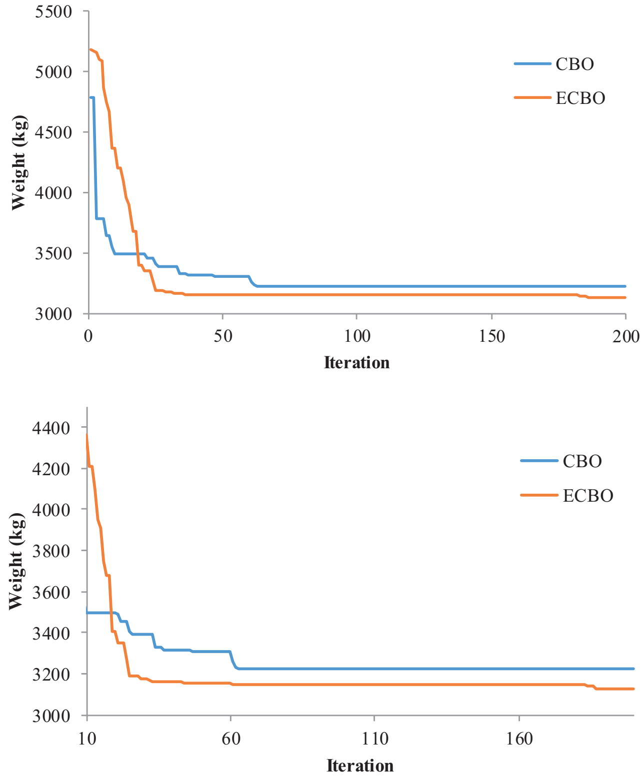


**Fig. 3** Comparison of the convergence rates between the two algorithms for the first example. (a) All iterations (b) 10-200 iterations.

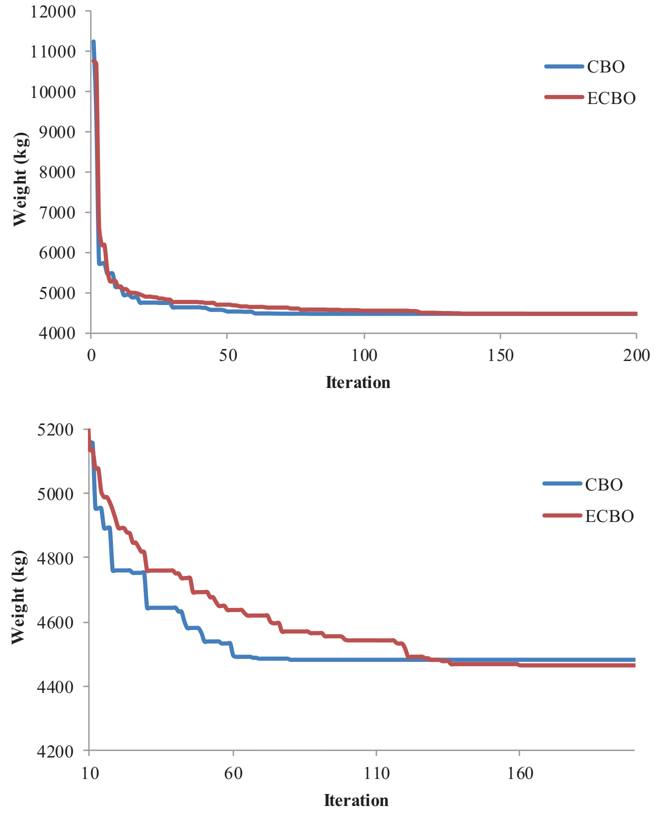


**Fig. 4** Comparison of the convergence rates between the two algorithms for the second example. (a) All iterations (b) 10-200 iterations.

**Table 3** Optimum design variables (cm) for the 36 m height monopole using different methods

| Design variables | Engineering design | CBO | ECBO |
|---|---|---|---|
| D6 | 43 | 40 | 39 |
| D5 | 57 | 56 | 56 |
| D4 | 66 | 65 | 64 |
| D3 | 73 | 74 | 74 |
| D2 | 75 | 76 | 76 |
| D1 | 85 | 86 | 86 |
| t6 | 0.5 | 0.45 | 0.45 |
| t5 | 0.6 | 0.60 | 0.60 |
| t4 | 0.8 | 0.80 | 0.80 |
| t3 | 0.8 | 0.80 | 0.80 |
| t2 | 0.8 | 0.80 | 0.80 |
| t1 | 1 | 1 | 0.90 |
| Weight (kg) | 4608.55 | 4557.59 | 4430.80 |
| Rotation | 1.4115 | 1.4449 | 1.4951 |
| Maximum stress ratio | 0.6247 | 0.6060 | 0.6041 |
| Maximum (D/t) | 95.00 | 95.00 | 95.00 |

Rotation: Rotation at top pole structure (degree)

## 5 Discussion on the results of the examples

In this section, the results obtained in the examples are discussed. We firstly should be noted that the optimization problem of monopole structure is a non-convex and nonlinear optimization problem, because the stiffness and applied loads (consisting the self-weight, ice and wind load as describe in Eqs. (6)–(13)) are simultaneously increased with increasing the cross section diameters of parts.

Tables 2 and 3 compare the results obtained using the CBO and ECBO algorithms with the engineering design ones for both examples, respectively. As discussion before and shown in these tables, the constraints of outcome of both algorithms are satisfied, and then we can comparison these results with the engineering design ones. As anticipated the results obtained using both algorithms are better than the engineering design values for both examples. The outcomes of the ECBO algorithm results are also better than the CBO algorithm with the same number of objective function evaluations.

It can be seen from Figs. 3 and 4, though the CBO algorithm is considerably faster in the early optimization iterations, the ECBO algorithm converged to a significantly better design in the latter optimization iterations without being trapped in local optima.

## 6 Conclusions

An efficient optimization method is proposed for optimal design of the steel circular stepped monopole structures, based on Colliding Bodies Optimization (CBO) and Enhanced Colliding Bodies Optimization (ECBO) algorithms. The CBO mimics the laws of collision between objects. The very simple implementation and parameter independency are definite strength points of CBO. In the ECBO, some strategies have been achieved to promote the exploitation ability of the CBO. In order to finding the optimal cross section sizes of monopole structure, the weight of monopole and cross section sizes are respectively defined as objective function and variables in the optimization process. Then, the cross section sizes are selected based on optimization algorithms from available discrete variables.

The validity and efficiency of the proposed method are shown through two test problems. The results of the proposed algorithms are compared to those of the engineering design values. The outcomes are that both algorithms could decrease the weight of engineering design monopole structures without appearing any violation. Moreover, the ECBO algorithm clearly outperforms the CBO algorithm with a same computational time, which it indices importance of selecting the effective optimization algorithm in this problem. Future researches can investigate problems such as, optimization of other type of monopole structures using recently meta-heuristic optimization algorithms.

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## References

[1] Dicleli, M. "Computer-aided optimum design of steel tubular telescopic pole structures." *Computers & Structures*. 62(6), pp. 961-973. 1997. DOI: 10.1016/S0045-7949(96)00298-2

[2] Hawkins, D. W. "Discussion of current issues related to steel telecommunications monopole structures." In: Structures Congress 2010, American Society of Civil Engineers, Orlando, Florida, May 12-15, 2010.

[3] Reese, B. R., Hawkins, D. W. "Protecting your investment: what every tower owner should know about upgrading a monopole's most important connection." *Above Ground Level Magazine*, 3(4), pp. 30-35. 2006.

[4] Gönen, T. "Electric Power Transmission System Engineering: Analysis and Design." Wiley, New York.1988.

[5] Hu, J. "A study on cover plate design and monopole strengthening application." *Thin-Walled Structures*. 49(9), pp. 1098-1107. 2011. DOI: 10.1016/j.tws.2011.04.002

[6] Telecommunication Industry Association. Structural standards for steel antenna towers and antenna supporting structures (ANSI/TIA/EIA-222-F-1996). Arlington, VA, Telecommunications Industry Association, 1996.

[7] Kaveh, A. "*Advance in metaheuristic algorithms for optimal design of structures*." Springer International Publishing, Switzerland. 2014. DOI: 10.1007/978-3-319-05549-7

[8] Kaveh, A., Mahdavi, V. R. "Colliding Bodies Optimization:Extensions and Applications." Springer International Publishing, Switzerland. 2015. DOI: 10.1007/978-3-319-19659-6

[9] Eberhart, R. C., Kennedy, J. P. "A new optimizer using particle swarm theory." Proceedimgs of the sixth international symposium on Micro Machine and Human Science, Nagoya, Japan, Oct. 4-6, 1995. DOI: 10.1109/MHS.1995.494215

[10] Dorigo, M., Maniezzo, V., Colorni, A. "The ant system: optimization by a colony of cooperating agents." *IEEE Transactions on Systems, Man, and Cybernatics, Part B (Cybernetics)*. 26(1), pp. 29-41. 1996. DOI: 10.1109/3477.484436

[11] Kaveh, A., Talatahari, S. "A novel heuristic optimization method: charged system search." *Acta Mechanica*. 213(3), pp. 267-289. 2010. DOI: 10.1007/s00707-009-0270-4

[12] Mirjalili, S., Mirjalili, S. M., Lewis, A. "Grey Wolf Optimizer." *Advances in Engineering Software*. 69, pp. 46-61. 2014. DOI: 10.1016/j.advengsoft.2013.12.007

[13] Csébfalvi, A. "A hybrid meta-heuristic method for continuous engineering optimization." *Periodica Polytechnica Civil Engineering*, 53(2), pp. 93-100. 2009. DOI: 10.3311/pp.ci.2009-2.05

[14] Csébfalvi, A. "A new theoretical approach for robust truss optimization with uncertain load directions." *Mechanics Based Design of Structures and Machines: An International Journal*. 42(4), pp. 442-453. 2014. DOI: 10.1080/15397734.2014.880064

[15] Kaveh, A., Javadi, S. "An efficient hybrid particle swarm strategy, ray optimizer, and harmony search algorithm for optimal design of truss structures." *Periodica Polytechnica Civil Engineering*. 58(2), pp. 155-171, 2014. DOI: 10.3311/PPci.7550

[16] Kaveh, A., Khanzadi, M., Alipour, M., Naraky, M. R. "CBO and CSS algorithms for resource allocation and time-cost trade-off." *Periodica Polytechnica Civil Engineering*. 59(3), pp. 361-371. DOI: 10.3311/PPci.7788

[17] Kaveh, A., Mahdavi, V. R. "Colliding bodies optimization: A novel meta-heuristic method." *Computers & Structures*. 139, pp. 18-27. 2014. DOI: 10.1016/j.compstruc.2014.04.005

[18] Kaveh, A., Ilchi Ghazaan, M. "Enhanced colliding bodies optimization for design problems with continuous and discrete variables." *Advances in Engineering Software*. 77, pp. 66-75. 2014. DOI: 10.1016/j.advengsoft.2014.08.003

[19] American Institute of Steel Construction. *Steel construction Manual*. (AISC Specification), 13th edition. Chicago, IL, 2005.

[20] Kamalinejad, M. Constructing of cell towers for the IRANCELL operator, PARTO SANAT VIRA company, registration no. 315839, Limited liability company, 2005.