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RESEARCH ARTICLE

Three Dimensional Finite Element Analysis of Deep Excavations' Concave Corners

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Abstract

The behavior of deep excavation's retaining structures may become rather complex near the corners. The vicinity of concave corners is an especially problematic zone, and causes numerous problems in everyday practice. Conventional 2D analyses considering axisymmetric or plane strain conditions cannot be used in such cases due the spatial nature of the problem. A set of three dimensional finite element analyses have been performed aiming better understanding of soil and structure behavior near excavations' concave corners and evaluating the influence of corner angle. The bending moment arising in anisotropic diaphragm walls near the concave corners are summarized, and compared to 2D plane strain and axisymmetric results.

Keywords

deep excavation · retaining structures · 3D finite-element analysis · concave corner

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1 Introduction

Geotechnical problems are often complex in geometry and therefore 3D analysis is required for reliable design. 3D finite element modelling are more widespread in some special geotechnical fields, such as tunnelling. Nevertheless special factors, such as time dependent material models are still only analysed in 2D [1]. Concave corners of deep excavations are a typical example where 3D effects cannot be neglected. Additionally this type of problem is often faced during the everyday practice. Although commercially available 3D finite element softwares became available in the last decade; everyday geotechnical design is still mostly restricted to 2D analyses. A commercial FE software (Midas GTS) was used to create 3D models of excavations retained by diaphragm walls having concave corners of different angles. A set of analyses has been performed to investigate the diaphragm wall behaviour near the concave corner of a deep excavation. This paper summarizes the results of this study.

2 Analysis of retaining walls

The design methods of retaining walls can be classified into 3 groups [2, 3]: simplified calculation of the earth pressures according to the assumed movement directions of the wall; calculations based on beam on elastic springs theory; and finite element analysis. The first two approaches are not really capable of taking into account the non-linearity of soil behaviour or for complex soil-structure interaction problems. In such situation finite element analyses are used, at which the structure and the soil can be modelled using finite elements and their interaction can be considered by defining interface elements. Such analyses may provide realistic predictions of excavation performance, when appropriate constitutive model and carefully calibrated input parameters are used [4]. Due to the complexity of the problem, finite element back analyses are often performed to enable deeper understanding of the soil behaviour [5].

In case of deep excavations, design practice generally assumes plane strain conditions for the middle parts and in some cases axisymmetric conditions for the proximity of the corners. Such assumption may provide reliable results in case of excavations with long sides and consistent load distributions, but for problems with more complex geometry or soil conditions may lead to unreliable results. Based on the results of 150 FE analyses Finno et al. [6] found that surface settlements obtained by 2D analyses are nearly equal to the result of 3D analyses results only, when the width of the excavation is greater than six times the excavation depth.

A detailed study based on 2D and 3D finite element analyses of a 40 m deep and 35 m wide excavation [7] pointed out that anisotropy of wall stiffness may also have a significant effect on the computed results. Different types of retaining walls used in underground construction, such as pile walls or diaphragm walls, have much lower horizontal stiffness compared to the vertical one. Thus, the load bearing occurs mainly by resisting bending moments rotating around the horizontal axis. Fig. 1 shows the comparison of the bending moments rotating around the horizontal axis obtained by Zdravkovic using different considerations. A quite good correlation of the curves from different models can be observed. Nevertheless, there is a significant difference in the upper 15 - 20 m of the wall when isotropic wall stiffness is considered in 3D model.

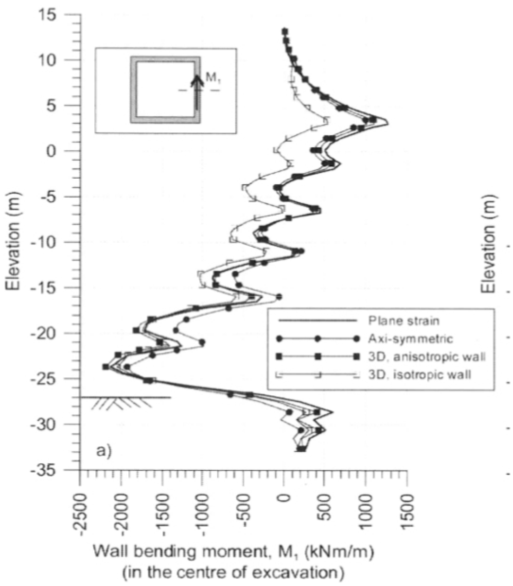

Fig. 1. Bending moments obtained from the different models [7]

3 Behaviour of concave corners

Many study aimed to analyse the behaviour of deep excavations' retaining structures near the corners [8–10], but these studies focus on "positive" (convex) corners.

An excavation can be interpreted as a steep earth mass retained by a wall structure. Thus the stabilizing actions are amended with the reaction of the supporting structures. The sketches in Fig. 2 show a straight part and a concave corner of an excavation and the potential slip surfaces. In the case of concave corners the volume of sliding wedge is somewhat smaller than that in the case of an infinite wall (i.e. plane strain), but the difference is not significant. On the other hand the area of the sliding surface decreases dramatically especially in the zones of high stress level, where significant shear resistance can develop. Thus a wall retaining such distribution of earth mass must resist higher destabilizing actions compared to straight wall sections of the same height. Furthermore, the concave corner in an excavation must usually be constructed because there is an existing structure which represents a high surface load. Thus we can state that the destabilizing actions are in a similar range in both cases.

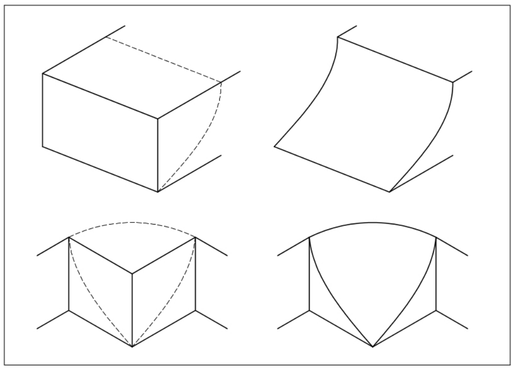

Fig. 2. The analysis of a sliding wedge

The possible solutions are different in case of different retaining structure types. The most frequently used types; pile wall and diaphragm wall are discussed here.

In case of non-contiguous pile walls, the single piles are connected by a capping beam and possibly by some braces for the struts or anchors. Therefore, the horizontal stiffness of the wall is usually quite low and the wall behaviour can be interpreted by the resistance of single piles. The major difference in case of a pile near the concave corner, compared to a straight wall section, is that the main direction of the bending moments acting on the piles is not perpendicular to the wall plane. However the construction experiences show that cracking of piles, larger displacements and problems with the waterproofing occur frequently. Auxiliary solutions are usually necessary, such as installation of extra props or grouting in between the piles.

The case of diaphragm walls is more complex and special, and the details of construction may cause non-negligible differences in excavation performance [11, 12]. The connection of the adjoining panels cannot resist tension and bending moments and its shear resistance in the direction perpendicular to the wall plane is also limited. Due to wall deformations caused by earth pressure, the panels joining along the edge of the concave corner would split up without modification of the usual joint design. Thus a possible construction solution can be to link the reinforcement of these panels and to form a rigid corner. Despite this technique, occasional cracking of the wall cannot be avoided. Therefore similar auxiliary methods can be utilized as mentioned previously. With respect to the special geometry and structural distribution, the application of traditional design procedures is inaccurate, the usual design of concave corners is based on previous experiences, and generally higher factors of safety are applied.

A short remark must be made for the very rapid technological development of deep excavation construction techniques: so-called deep mixing walls made of soil-binder mixtures with steel profiles are used in practice for shallower excavations. The application of such innovative construction techniques can be expected for deeper excavations, as well. Therefore future research shall focus on realistic modelling of concave corners of such wall structures which can be totally different from diaphragm-wall behaviour analysed in present paper. Prediction of deep mixing materials characteristics are a present-day research topic [13] making the issue even more complicated.

As a conclusion it can be stated, that no special design technique exists to find an economic solution for walls retaining concave corners. Instead application of higher safety factors or auxiliary construction procedures is used.

4 Finite element analysis

4.1 The 3D analysis series

Three dimensional FE analyses were carried out to analyse the internal forces and displacements of the wall retaining the concave corners having different corner angles. Fig. 3 shows the four different layouts created. The wall sections around the corner are 6 m long while the adjoining wall sections are 7, 8, 9 and 10 m long respectively. The total height of the wall is 18 m, the depth of the excavation is 8 m thus the embedment depth is 10 m. To avoid the possible negative effect of larger displacements on the accuracy of the calculations, the excavation is carried out in four steps.

As it was discussed previously the application of a correct stiffness distribution and realistic modelling of structural joints can strongly influence the results. During the analyses 80 cm thick and 3 m wide cantilever diaphragm wall panels were modelled with isotropic plate elements. The joints of the adjacent panels are hinged thus rotation around the vertical axis is allowed. The only exception is the joint in the edge of the concave corner where fixed connections have been defined. To obtain a correct soil-structure interaction, interface elements were defined among the solid elements of the soil and the plate elements of the wall.

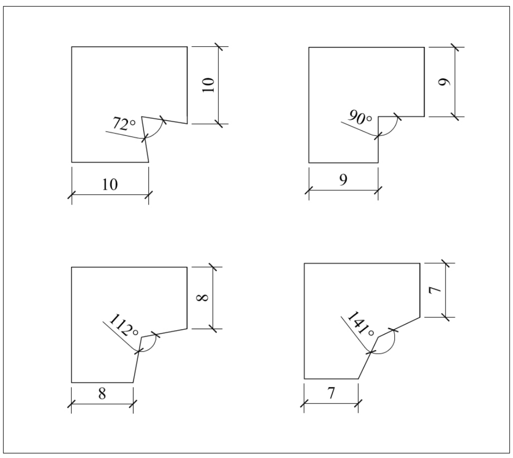

Fig. 3. The layout of the excavations

Modelling soil behaviour generally requires the use of advanced soil models, however their application increases the calculation time significantly, especially in case of three dimensional models [14]. Therefore, a linear elastic – perfectly plastic ("Mohr-Coulomb") model was used for the calculations. In order to simplify the problem and the evaluation, a single sand layer was considered, the properties defined for the diaphragm wall, for the soil and for the interface elements are summarized in Table 1 and Table 2.

The finite element model of the diaphragm wall were built of 0.5×0.5 m size rectangle shaped elements. The solid elements of the soil inside the excavation were fitted to this size while outside the excavation their sizes were increased linearly with increasing distance from the wall. Thus the size of the solid elements at the boundary is 5 m. The finite element mesh is shown in Fig. 4, the number of elements and nodes are summarized in Table 3.

The diaphragm wall installation and the soil excavation were defined in six construction stages, considering four excavation steps. A distributed surface load of 10 kPa was applied to avoid the possible numerical problems, which may arise from the detachment of the soil and wall elements in shallow regions due to the small soil cover.

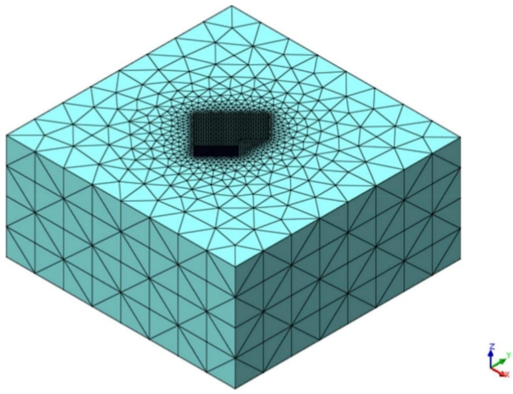

Fig. 4. The finite element mesh

4.2 The results of the model with a concave corner of right angle

Fig. 5 presents the total displacements (DXYZ) of the model in hundredfold magnification. The panel joints behave as expected: the hinged joints are rotated while the edge of the concave corner behaves rigidly. The red shade of the bottom means about 4 cm heave. This unrealistic result is caused by the considered linear elastic soil model, which does not differentiate between primary loading and unloading-reloading. Such errors can be avoided by using more sophisticated soil models. A relatively large embedment depth was used in the analysis to minimize the effect of base heave on the retaining wall deformations.

The same results from a different point of view can be seen on Fig. 6. This shows the total displacements of the half model intersected in its symmetry plane. The wall panels and the soil mass inside the walls are hidden. The shades of the concave

Tab. 1. Material properties

Type	Material model	Young's modulus E [kPa]	Poisson's ratio ν [-]	Unit weight γ [kN/m^3]	Cohesion c [kPa]	Internal friction angle φ [°]	K_0 [-]
Wall	Elastic	35000000	0.2	24	-	-	-
Soil	Mohr Coulomb	30000	0.3	20	1	30	0.5

Tab. 2. Interface properties

Element	Normal Stiffness Modulus K_n [kN/m^3]	Shear Stiffness Modulus K_t [kN/m^3]	Internal friction angle φ [°]	Cohesion c [kPa]
Interface	6000000	60000	15	1

corner shows similar slipping earth mass as it was described on Fig. 2. The displacements are quite low, the red shades show the highest values of about 1.4 cm.

The bending moments rotating around the horizontal axis are shown on Fig. 7. The positive values (green, yellow and red shades) illustrate the case when the outer side of the wall is in tension while the negative values (blue shades) mean the opposite direction of the bending moments. Fig. 7 only shows one half of the wall structure to obtain better visibility. Contrary to the expectations the maximum values of these bending moments are much lower around the concave corner (~ 40 kNm) than at the longer, straight wall section (~ 135 kNm). Although the linear plastic soil model may overestimate the soil expansion at the excavation base, thus underestimate the positive moments, the tendencies are clear; the moments rotating around the horizontal axis do not seem to be the major problem in case of concave corners.

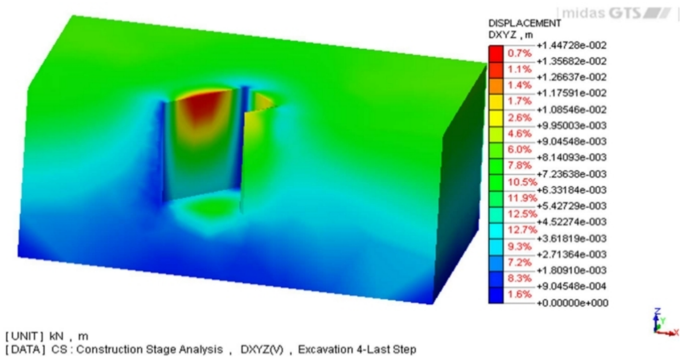

Fig. 6. Total displacements (DXYZ) at the final stage

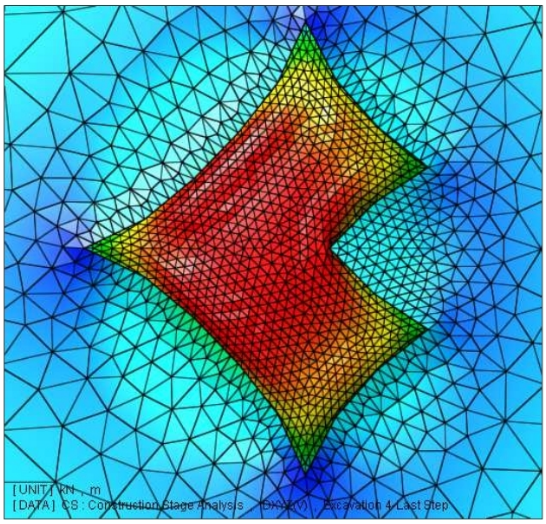

Fig. 5. Total displacements (DXYZ) at the final stage

Two dimensional finite element analyses have also been performed to enable a better evaluation of the results of 3D analyses. Plane strain and axisymmetric models were created using the same soil and interface properties

Fig. 8 shows the comparison of the computed bending moments of the 2D and 3D models. The inner forces resulting from the plane strain analyses are compared with the bending moments arising at the mid span of the 15 metre long, straight wall section. The results of the axisymmetric calculations are compared with the bending moments obtained from the proximity of the convex corner in the same straight wall section.

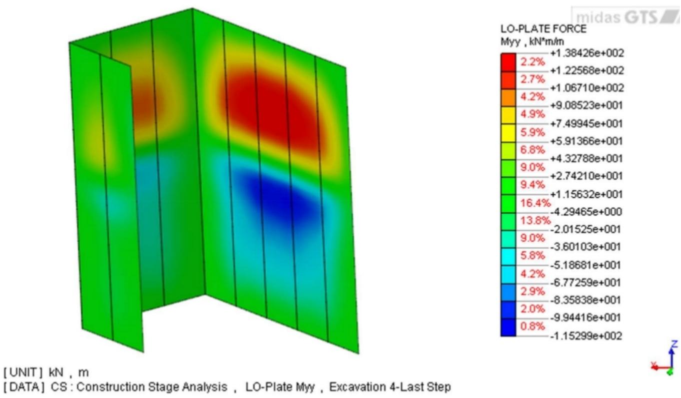

Fig. 7. Bending moments rotating around the horizontal axis at the final stage

The 3D model in the proximity of the convex corner and the axisymmetric models result in bending moments and deformations varying over similar range, but showing different tendencies. The two curves are nearly the opposite of each other, implying that analysis of an axisymmetric silo-like structure could provide results that are more reliable. However, due to the extremely small deformations and bending moments here, the importance of this topic is negligible.

Tab. 3. Mesh characteristics

Element/Node		72°	90°	112°	142°
Number of elements	Wall (2D)	4464	4320	4176	4032
	Soil (3D)	11391	11020	10824	10706
Number of nodes		33141	32095	31366	30778

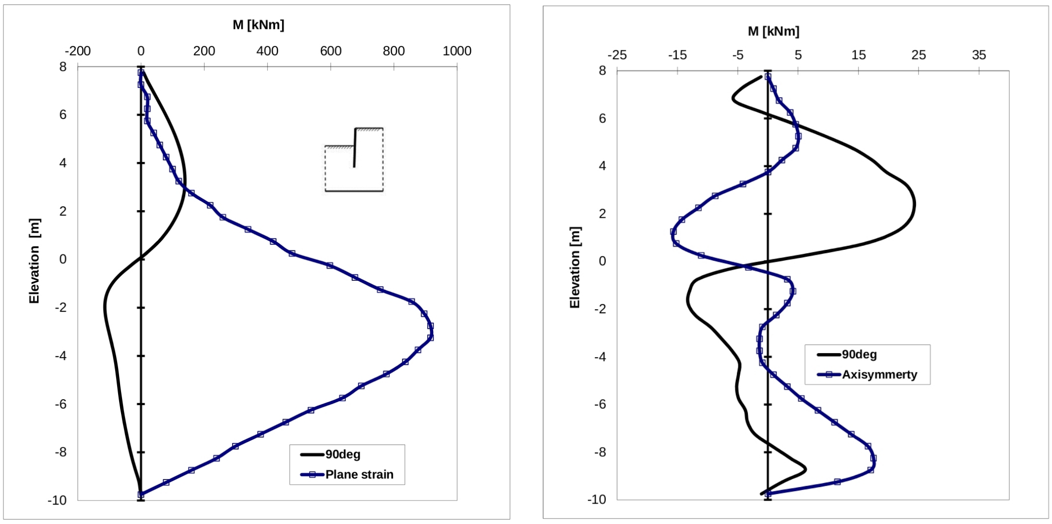

Fig. 8. Comparison of the results obtained by 2D and 3D analysis

The plane strain analyses showed ~7.5 cm horizontal displacement at the top of the wall while the 3D model resulted ~1.4 cm only. Such significant difference appears in the bending moment results too: neither the maximum value nor the shape of the curve show similarity in the case of the two assumptions. The 3D model changes sign around the bottom part of the excavation while the 2D model presents that the outer side of the wall is in tension all along. This implies that simplifications, which are generally used for 2D calculations (such as neglecting arching effect, anisotropic wall behaviour) are not acceptable in case of short walls and their application leads to significant overdimensioning.

The other important question related to the 3D analyses results is the distribution of bending moments rotating around the vertical axis. Fig. 9 shows these results. The shades can be interpreted as it was described in case Fig. 7. These results reveal the tendency that may lead to problems during construction. The bending moments around the concave corner are significantly higher than the values at straight wall section divided by hinged joints. The extreme maximum of bending moments arises at the edge of the concave corner presented by the blue shades on Fig. 9. Such moments lead to the structural problems that are experienced in the everyday construction practice.

4.3 Effect of corner angle

The previous figures showed that in this case the bending moments rotating around the horizontal axis are not as high as the bending moments rotating around the vertical axis. Nevertheless a short examination must be made. Fig. 10 shows four vertical sections where the bending moments were analysed in all the layouts. Here the bending moments arose in section number 3 are shown only as the highest values are experienced here. According to the diagram on Fig. 11, the bending moments increase with the increasing corner angles. With respect to the previous figures it is reasonable due to the fact that higher angle means higher mass of retained earth.

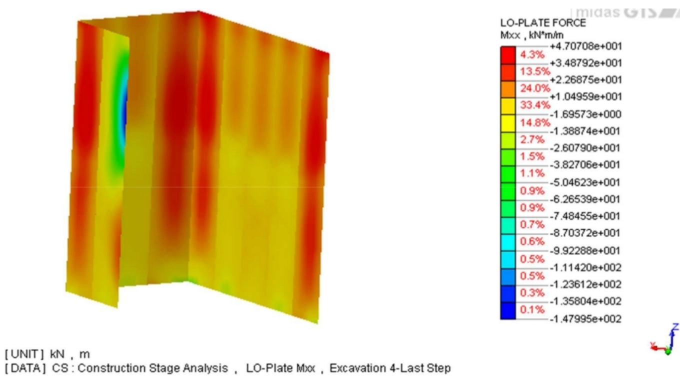

Fig. 9. Bending moments rotating around the vertical axis at the final stage

The bending moments rotating around the vertical axis were compared in 8 different horizontal segments marked in Fig. 12. The origin of z axis is at the excavation bottom, sections with positive z coordinates are located above this level, negative values below. Section number 1 (Fig. 13), 4 (Fig. 14) and 6 (Fig. 15) are presented in this paper only.

Hinged joints are defined along the both sides of the second wall panel retaining the concave corner. Accordingly, the bending moments rotating around the vertical axis are zero at the connections as it can be noticed in Fig. 13, 14 and 15.

However, the absolute value of the bending moment in the

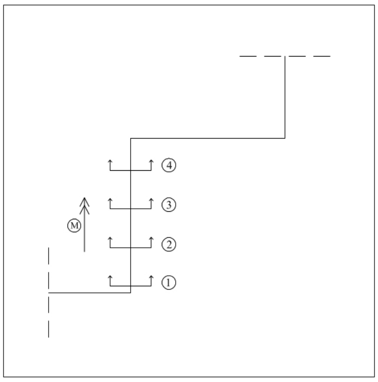

Fig. 10. The analysed vertical sections of the wall retaining the concave corner

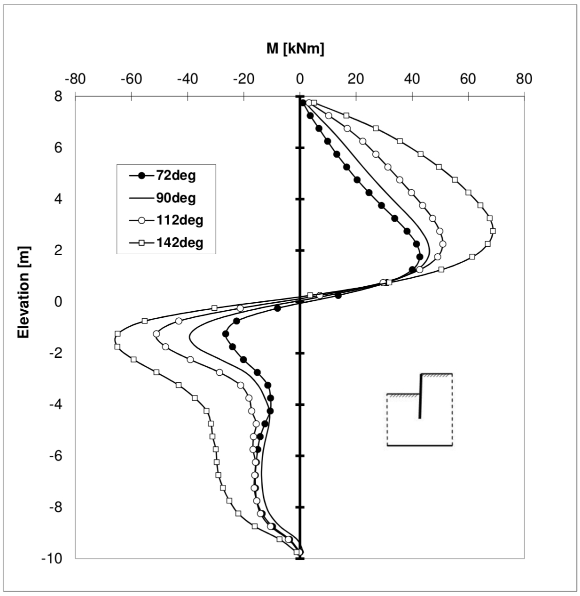

Fig. 11. The bending moments arose in section 3 at the final stage

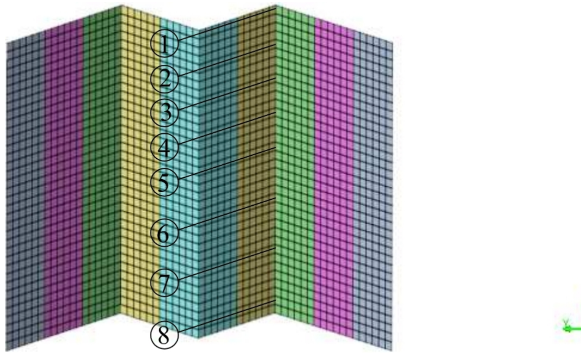

Fig. 12. The examined horizontal sections of the wall

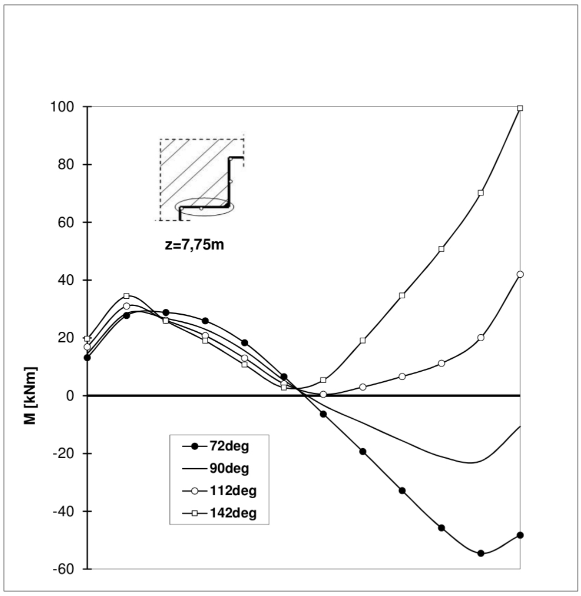

Fig. 13. The bending moments in section number 1 ($z = 7{,}75$ m) at the final stage

middle of this panel changes with the angle of the corner. The models with the angle of 112°and 142°show approximately 30 kNm in section number 1 and their magnitude changes only slightly in deeper sections. Beneath the bottom of the excavation, these forces disappear nearly linearly. In the case of models with the angle of 72°and 90°the maximal bending moments are also around 30 kNm but their value increase with depth. The absolute maximum is given in section 4 where the model with the right angle shows 50 kNm and the model with the angle of 72°shows 60 kNm. Beneath the bottom of the excavation these values disappear linearly. It can be seen that the bending moments are higher with lower angle of the concave corner.

The wall panels adjoining in the edge of the corner are more of practitioners' interest. The bending moments change sign in all the models except the model with the angle of 72°. The bending moments tend to be negative with increasing depth. The models provide their maximum value of bending moment in section 4, except the model with the angle of 142°. This tendency fits the previous expectations as section 4 is a bit above the bottom of the excavation. This maximum values are 86, 135 and 187 kNm in the models with the angle of 112°, 90°and 72°respectively.

In the embedded section the bending moments tend to have positive values again but extreme values cannot be found there. Thus it can be stated that the lower angles of the concave corner cause higher bending moments.

Fig. 16 illustrates these tendencies in a different way a section was taken at the corner and moments rotating around the vertical axis are plotted against depth as well. The curves moves to the negative part as the corner angle decreases and the negative extreme increases significantly. The same tendency results in decreasing positive extreme value.

5 Conclusions

A diaphragm wall retaining an 8 m deep excavation has been analysed by 2D and 3D finite element models. Special attention has been paid to the behaviour of concave corners of different angles. Joints have been defined at the connection of the diaphragm wall panels to take into account their anisotropic behaviour and rigid connection has been chosen for the concave corner.

The computed bending moments rotating around the horizontal axis are significantly smaller in case of 3D analysis than in the case of 2D plane strain model. This is in good agreement

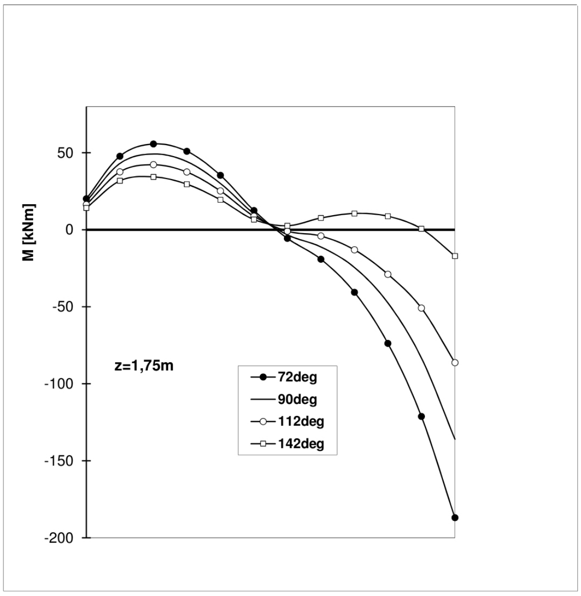

Fig. 14. The bending moments at the final stage in section 4 ($z = 1{,}75$ m)

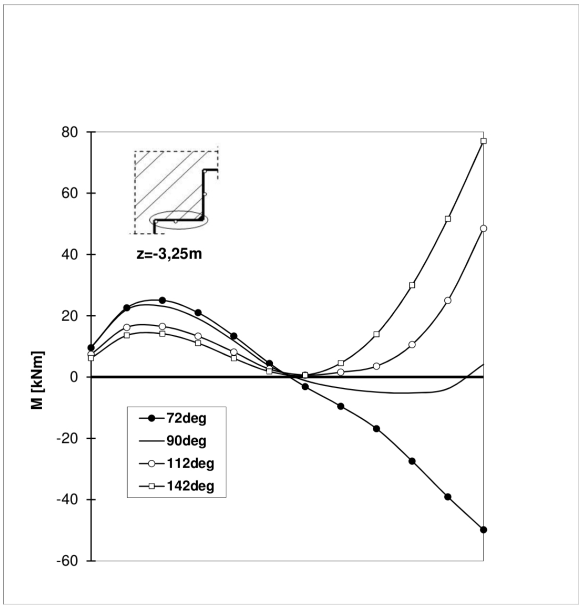

Fig. 15. The bending moments at the final stage in section 6 ($z = -3{,}25$ m)

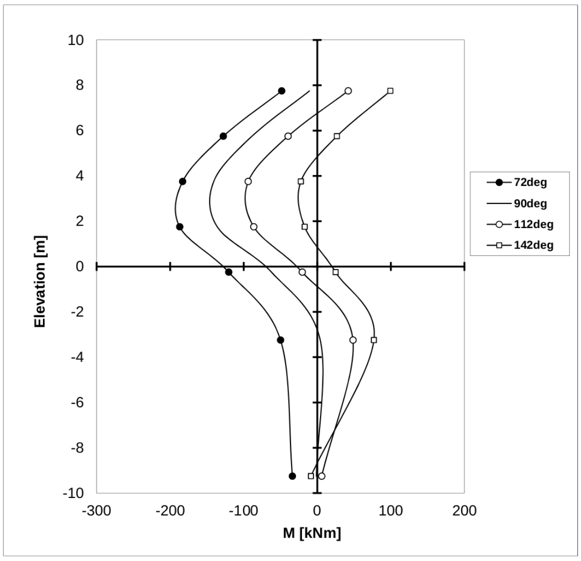

Fig. 16. Bending moments rotating around vertical axis vs. depth along the edge of the concave corner

with earlier experiences, considering that the chosen excavation width is less than 20 m, therefore the arching effect and anisotropic wall behaviour causes significant reduction in bending moments. It also reconfirmed that this internal force is not the cause of the problem that is generally experienced at concave corners.

Results of bending moments rotating around the vertical axis revealed the basic problem of concave corners. They have shown that the moments increase significantly in the vicinity of the concave corners, and they can be as large as or even larger than the moments rotating around the horizontal axis. As this component cannot be computed using 2D analyses the dimensioning of this part is still a problematic point of design in many cases, and it is handled with a lot of empiricism. The critical parts of the structure can be easily located, however the magnitude of the bending moments is complicated to estimate. The magnitude and even the sign of this moment is strongly influenced by the depth; the extreme moments, that causes tension on the excavation side, developed around the bottom third of the excavation depth ($z = 2.5 - 3.0$ m). This area has been reported the critical zone in case of similar excavations earlier by contractors too.

A clear tendency has been observed regarding the corner angle as well: the larger the corner angle, the bending moments move to the positive part and in the cases of smaller angles negative bending moments arise (Fig. 16). Thus significant bending moments can develop in case of sharp corner angles.

The analysis of this simplified problem helps to understand the nature of the basic characteristics of soil and structure behaviour near concave corners. Recent results helped in locating the critical parts of the structure and find the critical internal force components. However, further work is needed to give recommendation on estimating the value of the bending moments without 3D finite element analyses.

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