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RESEARCH ARTICLE

Modeling the Effect of Waves on the Diurnal Temperature Stratification of a Shallow Lake

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Abstract
In this paper a three-dimensional circulation model has been applied to determine the spatial distribution of the temperature and mixed-layer depth (MLD) of a medium-sized shallow lake. In order to reproduce the vertical thermal structure of the lake, the impact of waves has been incorporated using several recently published schemes. These schemes approximate the additional mixing due to the orbital motion of waves, the turbulent kinetic energy produced by waves, or both. The reference solver is the Reynolds-Averaged Navier-Stokes equations with two-equation Mellor-Yamada 2.5 turbulence closure. The wave field is characterized by bulk parameters, e.g. significant wave height. The sensitivity to the choice of the wave mixing scheme is analyzed by means of an academic test case. The accuracy of the scheme is explored through data collected in Lake Balaton. The modeled temperature profiles and MLDs match measurements much better when the effect of non-breaking surface waves is included.

Keywords
shallow lake, non-breaking waves, wave-turbulence interaction, mixed-layer depth, lake hydrodynamics

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1 Introduction

The water quality and the ecological status of lakes are strongly governed by hydrodynamics through the transport of contaminants, suspended solids, nutrients, temperature and light (affected by turbidity). The latter three are among the most important factors that determine the biological production in natural waters [1]. Their spatial distribution in the waterbody is highly non-homogeneous in both horizontal and vertical sense. It is important to understand the role of wind in mixing shallow lakes through the induced water motions, i.e. currents and surface waves. During strong winds wave action penetrates to the bottom and the intensive vertical mixing nearly homogenises the whole water column, while during low and moderate winds water stratification can easily evolve owing to the solar radiation [2, 3]. In the latter case only the upper layer of the water column is well-mixed, while in the lower layers turbulent mixing is impeded by stable stratification. At night, the cooling of the water surface induces convective overturning, thus the mixed layer reaches the lake bottom. At daytime, the mixed layer depth (MLD) is affected by the wind, surface heat fluxes and solar radiation. MLD has primary importance in water quality and can directly affect the biological production [4] because e.g. stratification can separate sediment- and nutrient-rich lower layers from the warmer upper layers, to where light can still penetrate.

In natural freshwater lakes water density is determined mainly by the temperature since salinity differences can be neglected. In many studies the evolution of the temperature of shallow lakes and reservoirs is simulated for the water column using vertical, horizontally averaged one-dimensional models, implying that horizontal advective and diffusive transport can be disregarded [3, 5, 6]. In case of large lakes horizontal uniformity is rarely valid, hence the application of three-dimensional (3D) hydrodynamic models is required for proper calculations. The necessity of the 3D description is already pointed out by many researchers even for very shallow depths [7]. The 3D Reynolds-averaged Navier-Stokes (RANS) equations are closed by turbulence models among which the two-equation type models represent a good balance between computational cost and accuracy and are therefore used widely. Two-equation turbulence models are traditional concepts based on transport equations, one of which usually gives the evolution of the turbulent kinetic energy (TKE). In case of the Mellor-Yamada 2.5 level (MY) model it takes the following form [8]:

$$\frac{\partial q^2}{\partial t} + u_i \frac{\partial q^2}{\partial x_i} = 2\left(P_s + P_b - \varepsilon\right) + \frac{\partial}{\partial x_3}\left(K_q \frac{\partial q^2}{\partial x_3}\right), \quad (1)$$

where q^2 is the TKE, $i = 1,2,3$ is the index of coordinate direction, u_i is the velocity vector, x_i is the coordinate vector, P_s and P_b is the TKE production by mean shear and buoyancy, respectively, K_q is the vertical diffusion coefficient of TKE, and ε is the dissipation rate of TKE and t is the time. The other transport equation governs dissipation formulated as a function of the dissipation rate, length scale (l) or some other related variable. The vertical evolution of momentum and scalars depends on eddy viscosity (K_m) and diffusivity (K_h), respectively that are determined by the turbulence closure equations. In the MY model the two take the following forms:

$$K_m = qlS_m; \quad K_h = qlS_h, \quad (2)$$

where S_m and S_h are stability functions. For its calculation we refer to [9].

In many studies these two-equation prognostic turbulence models are applied with the assumption that the Kármán-Prandtl law-of-the-wall is valid at the free surface, and the dissipation rate of TKE depends only on the water-side friction velocity [10]. Furthermore, the commonly used expression for the production of TKE takes into account only the shear caused by the mean (= wave phase-averaged) flow (P_{curr}):

$$P_s = P_{curr} = K_m\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]. \quad (3)$$

The evolution of MLD and of the vertical structure of temperature or other scalars is determined by the balance of buoyancy and diffusion and hence it is explicitly related to the TKE through eddy viscosity and diffusivity. By ignoring the contribution of waves, classical turbulence closures may not provide an accurate description of the wave-affected surface layer (WASL). In fact, it has been recognized by many researchers that these classical turbulence models produce a too shallow MLD and thus water surface temperatures are overestimated, see e.g. [11].

The ultimate aim of our work is to model accurately the thermal structure and the lake-wide MLD distribution of Lake Balaton. The lake is shallow thus its temperature field is characterized by diurnal stratification in the whole water volume. It has been recognized from the observed temperature data that even in case of light and moderate wind conditions vertical temperature profiles are significantly affected by the wave motion even though in such cases waves are not steep enough to break or whitecap. In our work we have studied the sensitivity of heat transport to mixing due to non-breaking waves and explore the ability of various schemes to incorporate this effect in a three-dimensional lake circulation model.

The paper is organized as follows: in Section 2, the basic theory of wave-induced turbulence and mixing is briefly summarized. In Section 3, the applied lake circulation model is described. The sensitivity of modeled flow and temperature to extending the turbulence model by wave action is studied in Section 4 by means of an idealized academic test case. In Section 5 the application of the extended turbulence model to Lake Balaton is presented.

2 Wave-induced turbulence and mixing

Wind shear stress acting on the water surface induces currents by its tangential component and generates wind waves owing to the normal stress component. Most of the wind energy is transferred to wave generation and only a smaller part is conducted into the mean flow [12]. Wave motion has a great influence on the momentum, heat and mass exchange, so the application of coupled wave-circulation models is increasingly common. Wave motion can interact with the mean flow and turbulence in a variety of ways. One way of coupling and transferring energy from the wave motion to the mean flow is through radiation stresses that enter the RANS equations as momentum source terms [13]. This method alters significantly the turbulence field but is not able to describe accurately and fully the wave-induced turbulence mixing [14].

Wave energy dissipation has been extensively studied in case of breaking waves [15, 16]. If the wave grows and becomes too steep or reaches shallow regions, it breaks or whitecaps, creating strong turbulence throughout a surface layer whose thickness scales by the wave height. This source of TKE diffuses to deeper regions very slowly.

As to non-breaking waves, it has been shown experimentally and theoretically that they introduce turbulent mixing due to their orbital motion [11, 17, 18, 19]. This action is less intensive that of breaking waves but it occurs more frequently and penetrates deeper, so it also warrants attention. The subject is not new, but surprisingly few parameterizations have been published to account for mixing due to non-breaking waves in lake or ocean circulation models. These schemes are based on the fundamental characteristics of wave motion and some of them are scaled by the wind stress. Where these schemes have been applied they improved model results significantly in seas and oceans [11, 14, 20, 21].

Two main approaches of modeling mixing by non-breaking waves can be distinguished.

One is to directly increase turbulent eddy viscosity and turbulent diffusivity in the Reynolds-averaged momentum and transport equations, respectively. In case of the MY model, the enhanced coefficients take the form [22]

$$K_m = qlS_m + B_v; \quad K_h = qlS_h + B_v, \quad (4)$$

where B_v is the wave-induced component of eddy viscosity / turbulent diffusivity expressed following Prandtl's mixing length theory as

$$B_v = l_w^2 S_w, \tag{5}$$

where l_w is the length scale, S_w is the vertical shear, both associated with the wave orbital motion and thus proportional to particle displacement and orbital velocity, respectively. These quantities attenuate with depth nearly exponentially according to the linear wave theory. The prevailing length scale associated with wave motion (l_w) is similar in the test case and in Lake Balaton as both the wind, the fetch and the water depth fall into the same range, respectively. However, in Lake Balaton, long fetches may develop, extending the studied length scales upwards.

The another way to account for mixing by non-breaking waves is to introduce into the TKE equation the production by waves (P_{wave}) in addition to the production by the mean shear [21, 23]:

$$P_s = P_{curr} + P_{wave}. \tag{6}$$

In this approach P_{wave} is expressed based on the wave orbital motion, similarly to how P_{curr} is linked to the mean current shear.

Pleskachevsky et al. [14] argue that these two additional terms (B_v and P_{wave}) have to be incorporated simultaneously, while Ghantous and Babanin [19] opined that this is wrong: there should be no need to invoke any explicit addition to turbulent viscosity with an explicit production term because the effect of wave-induced turbulence would be counted twice. According to Kantha and Clayson [10], in addition to a TKE production term, the recalibration of the empirical turbulence closure constants should be the other possibility to improve the modeling of mixing due to non-breaking waves. It is important to note that in many numerical experiments the effect of P_{wave} was moderate and the direct increase of eddy viscosity with B_v lead to better results [19].

3 Model description

In this study, as a necessary step towards the development of an accurate 3D model of shallow lake thermodynamics, our aim was to explore the sensitivity of the temperature distribution to wave mixing and to select the most accurate approach from those outlined in the previous section. In the present work we focus on the relative impact of wave mixing on the thermal structure of a shallow lake and for this purpose, we have applied a simple and computationally efficient method that is based on bulk wave properties, such as the significant wave height, H_s and the average time period, T_{avg}. In other words, we approximate the full wave-energy spectrum by a monochromatic wave. Bulk wave parameters represent integrals over the wave motion and thus provide a general description of it. For example the spatial distribution of the Stokes drift field depends on the shape of wave spectrum [24]. If the wave spectrum is narrow enough bulk parameters can indeed characterize the effect of the irregular wave motion with sufficient accuracy, as opposed to deriving turbulent mixing and shear production from the spectrum [19]. In the absence of a wave spectrum (modeled or measured), many researchers successfully applied mixing parametrizations based on bulk characteristic [14, 23] but it has to be underlined that several studies reported that model results improve significantly by using a 2D spectral wave model since wave mixing can be overestimated by bulk parametrizations [25, 26].

Eddy viscosity and turbulent diffusivity terms have been enhanced by B_v (Eq. (4)) following the work of Qiao et al. [11]:

$$B_v = \alpha A u_{s0} \left[\frac{\sinh k\left(H+z\right)}{\sinh kH} \right]^3, \tag{7}$$

where H is the water depth, z is the upward vertical coordinate, A is the wave amplitude ($H_s/2$), $k = 2\pi/L$ is the wave number, L is the wave length, α is a dimensionless constant to calibrate, $u_{s0} = c\,(H_s/2 \cdot k)^2$ is the Stokes-drift, $c = L/T_{avg}$ is the wave phase speed.

The TKE production transferred from wave motion, P_{wave} is based on the theoretical consideration that its value should be equal to the energy loss (ε_w) of the wave field. We have been applied the expression for ε_w following Huang et al [22]:

$$P_{wave} = \varepsilon_w = 148\beta\sqrt{\delta}\,\frac{u_{s0}u_*^2}{L}e^{2kz}, \tag{8}$$

where u_* is the water-side friction velocity, $\delta = H_s/L$ is the wave steepness, β is an empirical parameter, which can be calibrated from observations and which does not exceed 1 according to [23].

In our simulations α and β are not calibrated in absence of measured turbulence data; both are set to 1.0. For α this set up is in agreement with most of the studies, where it also is treated as 1.0 [11, 14, 25]. The empirical constant β ranges between 0.15 and 1.5 by fitting Eq. (8) to observations [26]. Huang and Qiao [23] determined its value as 1 for a medium-size lake which is featured by wave motion similar to that of Lake Balaton. The observed wave height and period were 0.25 m and 4 s [27]. These dimensionless parameters may be associated with characteristics of surface waves and depend on the accuracy of the wave field calculation. It can be also expected that their values differ if the full wave energy spectrum is used instead of bulk wave properties.

As it was mentioned, breaking or whitecapping waves cause strong turbulence at the surface. In order to clarify the mixing effect of breaking waves against that induced by non-breaking waves, we considered the parameterization of Mellor and Blumberg [16] to describe turbulence energy flux at the surface. In case of wave breaking only the boundary condition for q^2 and l had to be modified as follows:

$$K_q \frac{\partial q^2}{\partial z} = 2\alpha_{CB} u_*^3; \quad l = \kappa z_w, \tag{9}$$

where $\alpha_{CB} = 100$ according to the measurement data of Craig and Banner [15], κ is the von Kármán constant and z_w is the roughness length on the water side of the air-water interface. It can be calculated by a Charnock type equation, as follows:

$$z_w = a \frac{u_*^2}{g}, \tag{10}$$

where $a = 2 \cdot 10^5$ following the literature [28, 29].

In our study we implemented these wave mixing schemes into the FVCOM model [9]. FVCOM is a three-dimensional, primitive equation, hydrostatic, free-surface circulation model that solves the governing equations using a finite volume scheme. The model consists of six Reynolds-averaged integral equations: three for momentum, and three others for incompressible continuity, temperature and salinity. A constitutive equation links density to temperature and salinity. The extra nonlinear terms arising from Reynolds-averaging are closed using the Boussinesq approximation, in which the turbulent eddy viscosities and diffusivities are determined with the modified Mellor and Yamada 2.5 level, and the Smagorinsky turbulence schemes for vertical and horizontal mixing, respectively. The computational domain is covered horizontally with an unstructured triangular mesh. A second-order accurate upwind scheme is used for the advective terms and a modified fourth-order Runge-Kutta scheme for time integration. The model applies the so-called "mode-splitting" method: the displacement of the free surface is calculated by vertically integrated 2D equations, then the 3D flow field is computed under that free surface. We have extended the MY 2.5 turbulence model in FVCOM to account for wave mixing.

The wave height and period are calculated using the depth-limited formulas of the Shore Protection Manual (SPM) in the nodes of the mesh of the circulation model as a function of wind speed, fetch and water depth. This local equilibrium approach provides good accuracy despite its simplicity [30]. See the formulas there.

4 Numerical experiments with idealized test case

Before applying any of the turbulence model modifications to a real lake, we analyzed the effect of the different schemes on the vertical profiles by means of an academic test case. This test was performed on elliptic basin shown in Fig. 1. During the simulation spatially uniform wind stress was prescribed with a constant magnitude of 0.05 Pa and direction of NNW. For simplicity the assumed wave field was also set to uniform, with bulk parameters: $H_s = 0.1$ m, $T_{avg} = 1.5$ s and $L = 2$ m. The same radiative and turbulent heat flux forcings were imposed as for the Lake Balaton simulation to be presented in the following section (Fig. 9). The sediment heat flux was neglected.

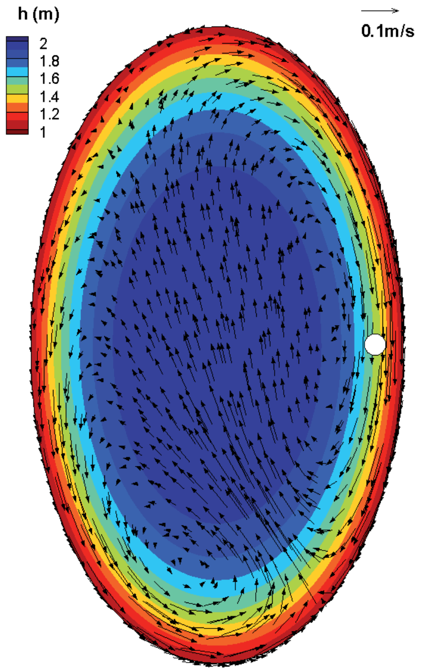

Fig. 1 Plan view of the elliptic lake for the academic test. The contours show the bathymetry, the vector field represents depth-averaged velocities in steady state, for NNW wind. The white point at the E shore is where the vertical profiles are analyzed.

In this paper we present the results of five basic schemes. In case '*orig*' we used the original MY 2.5 model without any modification. In scheme 'B_v' only the eddy viscosity (and turbulent diffusivity) term was incorporated in the model (Eq. (4), (7)) , whereas in scheme 'ε_w' only the TKE production shear term (Eq. (6), (8)). The fourth scheme, 'W_{br}' models the action of breaking waves onto turbulence by injecting TKE at the surface (Eq. (9) and (10)) following the work of Craig and Banner [15] and Stacey [29]. In the fifth scheme the modifications of the 'B_v' and 'ε_w' schemes are applied in the model to see their joint effect following the suggestion of Pleskachevsky et al. [14].

All simulations cover a time period of 24 hours. Initial velocities were zero and temperature was set to 24.1 °C for the whole lake. For the academic test case the Coriolis force was neglected. For all schemes we show here the results at a point near the east coast, where the mean depth is 1.43 m. The location is indicated with a white circle in Fig. 1. The elliptic lake is covered horizontally with a uniform mesh. The cell size is 20 m. In the vertical direction a total of 12 sigma layers is used. The vertical resolution is higher near the free surface and the bottom than at mid-depths. The maximum layer thickness is 30 cm, which can be found in the deepest part of the lake at mid-depth, while the thinnest layers (of about 3 cm) are along the shoreline, at the free surface.

The eddy viscosity and the TKE profiles generated by the modified models are all quite different towards the free surface (Fig. 2 and Fig. 3). TKE at the surface is zero owing to the law-of-the-wall boundary condition except when wave breaking is accounted for. Indeed, by accounting for wave breaking ('W_{br}') the eddy viscosity is nonzero due to the TKE source, but its slightly increased value drops exponentially and wave-induced mixing evolves only in a thin surface layer.

When wave mixing is taken into account through the shear production term (scheme 'ε_w') the shape of the q profile (i.e. the square root of TKE) follows the one obtained with the original MY model (scheme '*orig*'), but with a slight increment. In both schemes K_m and K_h have their maximum value at the same depth. In contrast, in the 'B_v' scheme, eddy viscosity and turbulent diffusivity have a maximum at the surface and decrease exponentially.

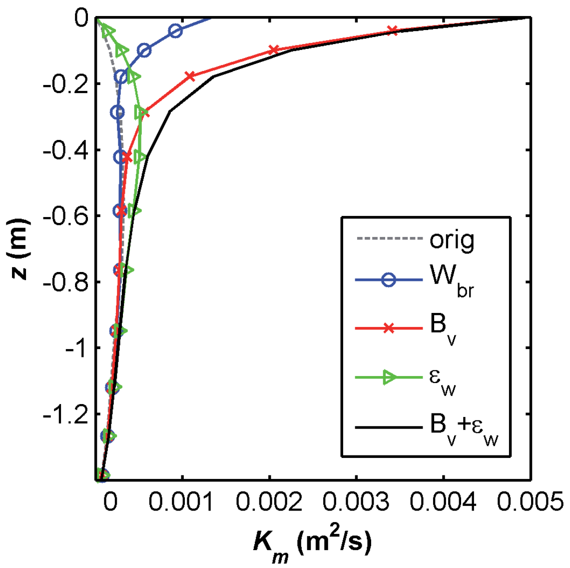

Fig. 2 Vertical turbulent viscosity profiles at the eastern shore in the elliptical lake, after reaching the quasi-steady state, using various wave mixing schemes: '*orig*' = original; 'W_{br}' = wave-breaking; 'B_V' = direct modification of turbulent viscosity/diffusivity; 'ε_w' = modification of TKE production; '$B_V + \varepsilon_w$' = joint usage of the two approaches.

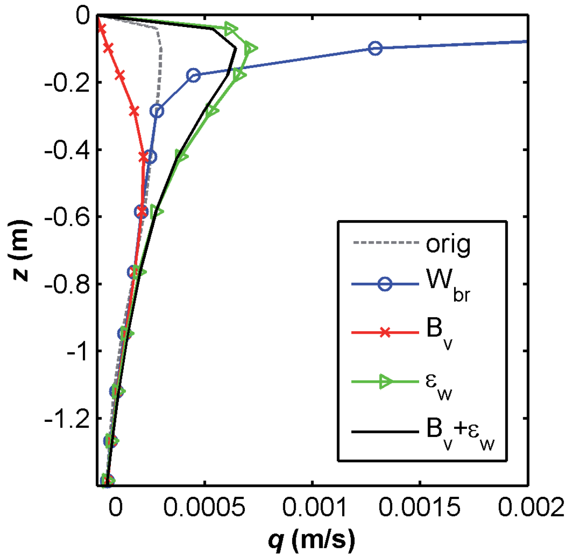

Fig. 3 Vertical profiles of the square root of turbulent kinetic energy at the eastern shore of the elliptic lake, after reaching the quasi-steady state, using various wave mixing schemes.

The vertical turbulent mixing determines the evolution of velocity profiles (Fig. 4) and temperatures (Fig. 5). Compared to the original MY model, incorporation of wave breaking has just a weak effect on these profiles, flattening them only very near the surface. By adding the wave shear production term (scheme 'ε_w'), the shape of the velocity profile does not change, but it is characterized by lower values. For the 'B_v' scheme, as it can be expected from the K_m profile, the velocity profile differs significantly: vertical gradients are reduced not only near the surface, but also to greater depths.

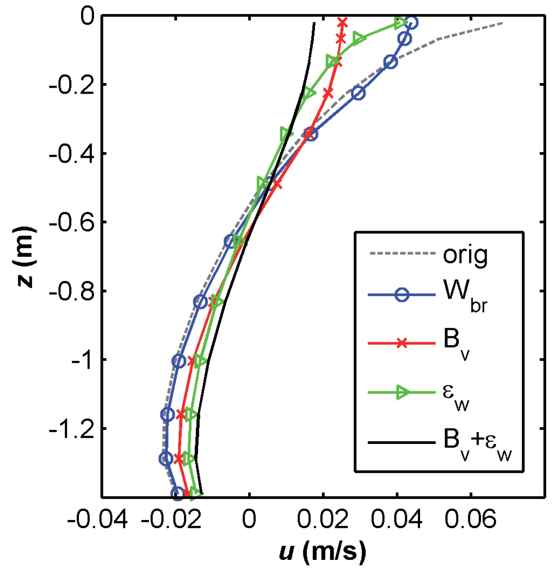

Fig. 4 Vertical profiles of the velocity magnitude at the eastern shore of the elliptical lake, after reaching the quasi-steady state, using various wave mixing schemes.

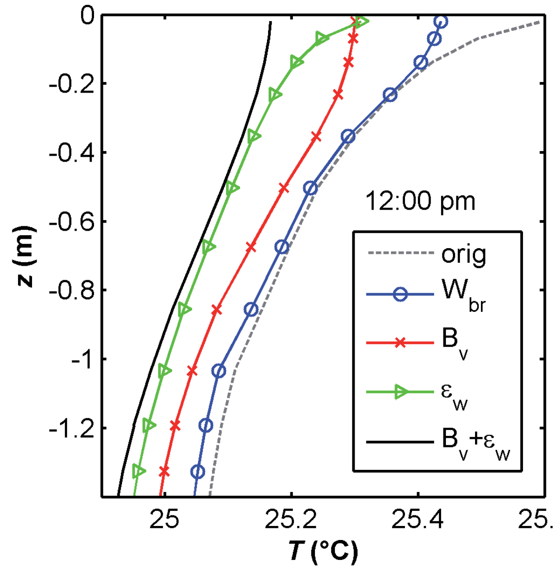

Fig. 5 Vertical profiles of water temperature at the eastern shore of the elliptical lake at 12:00 pm, using various wave mixing schemes.

Currents and surface waves play an important role on lake's mixing through many processes. In our sensitivity analyses four main processes were compared to reveal their impacts: shear production of currents, TKE input of wave breaking, contribution of non-breaking waves to shear production and stirring by non-breaking waves.

The results indicate that non-breaking waves play the most important role in mixing. Non-breaking waves influence mixing in two ways: through the wave-induced stirring (direct enhancement of eddy viscosity), and through the contribution to total shear production. The effect of stirring of non-breaking waves is decisive for the eddy viscosity profile and so the main impact is linked to it and in consequence to the B_v term. In turn, the enhanced viscosity has a feedback by reducing temperature and velocity gradients in the upper layer. In contrary to the direct influence of non-breaking waves on mixing, the shear production of these waves is much smaller compared to the contribution of the mean flow leading to very small increment in TKE and thus in eddy viscosity. Furthermore the shear production of non-breaking waves also decreases exponentially with depth and its effect vanishes in mid-depth. The total shear production is reduced by taking account of non-breaking wave induced mixing compared to a simulation that ignores non-breaking waves.

The role of wave breaking can be also significant but it is confined to a thin surface layer. It leads to a very high increment of TKE near the free surface that is able to increase eddy viscosity as a feedback but this intensive mixing also vanishes quickly with depth.

As to the temperature profile, in the non-breaking wave schemes temperatures are decreased, but while the additional shear production term (scheme 'ε_w') cannot influence the shape of the profile, 'B_v' can. This can be more clearly seen in Fig. 6 by plotting non-dimensional temperature profiles [6]:

$$\theta = \frac{T_s - T(z)}{\Delta T}, \quad \zeta = -\frac{z}{H} \tag{11}$$

where θ is the non-dimensional temperature, ζ is the non-dimensional depth, T_s is the surface temperature and ΔT is the temperature difference between the surface and the bottom layer.

Mixed-layer depth is determined by finding the depth at which the temperature differs from T_s by more than ±0.1°C. MLD evolution during the first 24 h period is shown in Fig. 7. Mixing due to wave-breaking is confined only to the near-surface zone and cannot affect vertical temperature distribution and thus MLD in merit. In contrast, the applied non-breaking wave mixing schemes provide more realistic evolutions regarding the temperature distribution. As it was noted by Burchard [31], a wave-enhanced layer cannot be reproduced by a circulation model when only its boundary conditions are adjusted without modifying the governing equations.

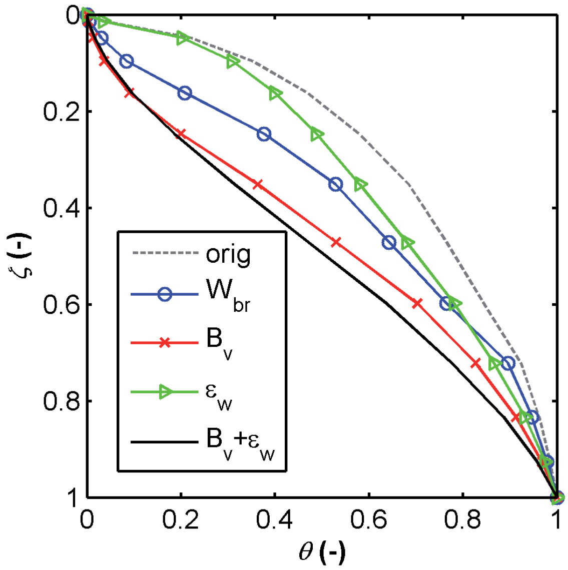

Fig. 6 Vertical profiles of non-dimensional day-averaged water temperature at the eastern shore of the elliptical lake as a function of the non-dimensional depth, after reaching the quasi-steady state, using various wave mixing schemes.

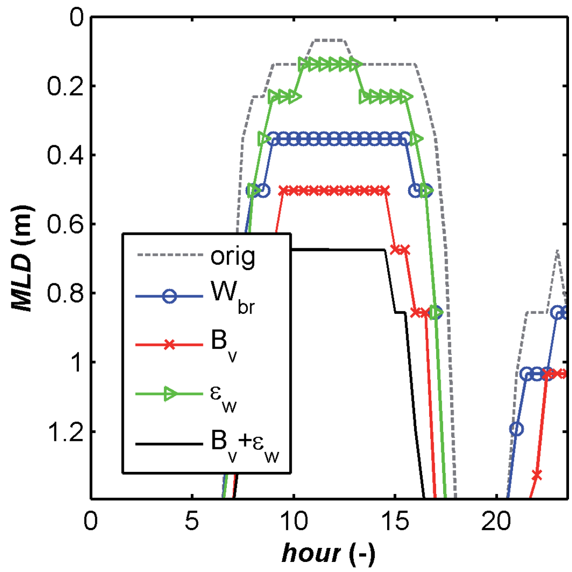

Fig. 7 Temporal evolution of the mixed layer depth at the eastern shore of the elliptical lake, using various wave mixing schemes.

The sensitivity analysis revealed that the effect of the various wave mixing schemes on the thermal structure differs significantly. Adding a complementary shear production term leads to similar temperature profiles as obtained by the original model. The scheme that directly enhances eddy viscosity and turbulent diffusivity has the opportunity to yield a temperature distribution that matches the observed ones in Lake Balaton (see next section). However, the enhanced eddy viscosity and turbulent diffusivity also reduce TKE in the upper layers, which is contrary to reality. In summary these results suggest that the simultaneous inclusion of the two different schemes ('ε_w' and 'B_v') is justified, confirming findings of Pleskachevsky et al. [14].

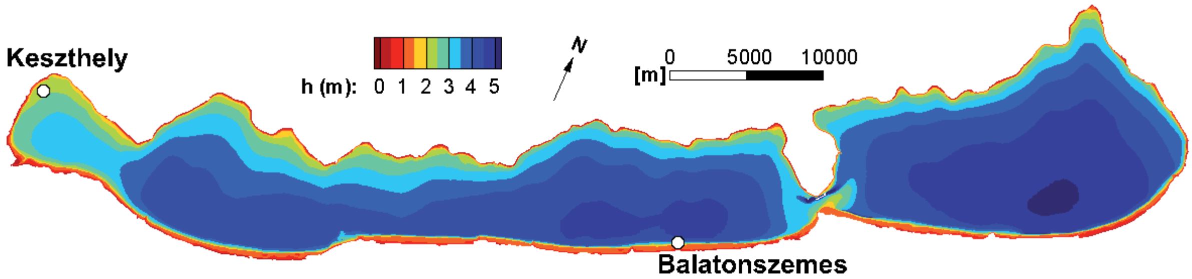

Fig. 8 Plan view of Lake Balaton, Hungary. Contours = bathymetry, white dots = location of the two hydrometeorological monitoring stations.

5 Modeling temperature distribution in Lake Balaton

In shallow lakes stratification evolves diurnally producing an MLD highly variable in space and time, which has many implications from ecological point of view. Lake Balaton is a large and shallow freshwater lake located in Hungary, Central-Europe. Its surface area is approximately 600 km^2 while the mean depth is only 3.5 m. The longitudinal axis of the lake is oriented ENE and the elongated shape is narrowed by a peninsula separating the lake into two main sub-basins (Fig. 8). Lake Balaton is thus large enough so that horizontal advection should not be neglected. Unless a 3D model is used, results of slice or column models will be less reliable. Vörös et al. [3] used a 1D water column model to predict vertical evolution of temperature in Lake Balaton. While surface temperatures were reproduced properly thanks to the energy budget approximation, the frequency and duration of stratification were overestimated.

Our aim was to model temperature profiles and MLD evolution correctly. Adopting the 3D circulation model with the original MY turbulence model, we realized that the mixing by (non-breaking) waves may not be neglected even in case of light and moderate winds, otherwise only the surface and bottom temperatures can be modeled with tolerable accuracy, but the vertical distribution and thus the MLD is clearly inaccurate.

5.1 Hydrometeorological forcing and measurements

Basic measurements were obtained from the hydrometeorological monitoring station (Keszthely) operated by the MTA-BME Water Management Research Group [32] in the western bay of the lake, near the shoreline. Here, the mean depth is 1.4 m. Water temperature is monitored at 5 depths (surface, bottom and 3 probes in between). Sediment temperature is also measured in approximately 5 and 15 cm deep under the bed surface. Global radiation and wind data are also collected at this station, while air temperature, relative humidity was obtained from the Balatonszemes station located on south shore and operated by the Water Authorities. Cloud cover was obtained from the Sármellék synoptic station located 10 km west of Keszthely. Water temperatures are recorded every 1 min, while meteorological parameters have an averaging time step of 10 min. Every variable was averaged to 20 min, which was also the time resolution of further atmospheric forcing calculations. The period chosen for this study is a summer period of 16 days, from July 13 to 29, 2013.

Shortwave radiation was determined based on global radiation measurement, while upward and downward longwave radiation was calculated using cloud cover, air and water surface temperature data following Holtslag and Van Ulden [33]. Sensible and latent heat fluxes were calculated using the Monin-Obukhov flux-gradient similarity theory [34, 35, 36]. Parameters of the atmospheric forcing model were calibrated based on the observations of Kiss and Józsa on Lake Neusiedl [37]. From analyzing sediment and water temperatures at the bottom, we believe that the sediment heat flux may not be negligible. We set up and calibrated a 1D heat conduction model using the temperature time-series to estimate heat conductivity from which heat flux can be derived. The daily average of the radiative and turbulent heat fluxes for this 16-day period is shown in Fig. 9.

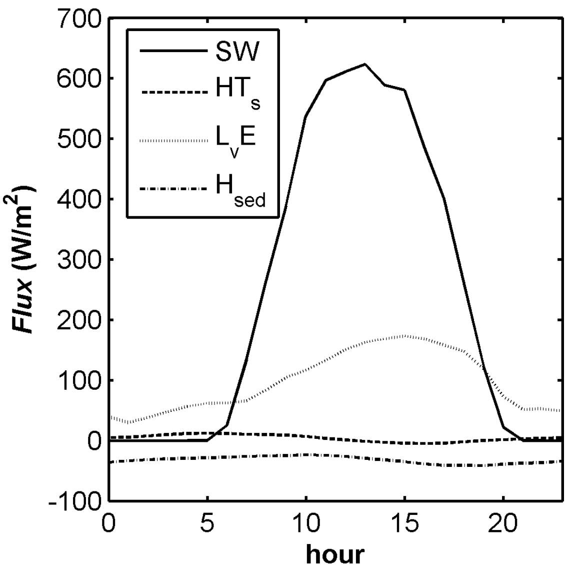

Fig. 9 Shortwave radiation, turbulent and sediment heat fluxes at Lake Balaton. Hourly averages of the simulated 16-day period.

The wind stress field over the lake surface was calculated using a semi-empirical model which incorporates the internal boundary layer, namely the strengthening of the wind along the fetch. For further details we refer to [38]. The determined wind velocity components and fetch distances over each computational cell were also the input parameters for the SPM wave field calculations.

So, in our simulations wind and wave fields are spatially variable, while heat fluxes are assumed to be spatially homogeneous. The computational domain is discretized with a non-uniform triangular grid in the horizontal. The resolution near the measurement station is 20 m, while it is 80 m in the open region of the western bay and 500 m in the eastern bay. The horizontal mesh has 33k cell. The vertical resolution is similar to that of the test case with 12 layers. The distribution of layers satisfies a parabolic function with high vertical resolution near the surface and the bottom. The applied external time step is 1 s and the 3D flow field is determined at every 5th time step. Using 4 cores the simulation takes 2 hours for a 1 day period.

5.2 Results

The different wave mixing schemes were evaluated for this real case by comparing modeled temperature time-series and profiles, as well as MLDs to observed data. As we also did with the elliptical lake, we have performed simulations using the different schemes. While the modeled temperature profiles and MLDs showed only a slight improvement using the wave-breaking boundary condition or the TKE production shear term in the turbulence scheme, the 'B_v' scheme brought significantly better results. The direct calculation of the wave-induced eddy viscosity term has the disadvantage that it reduces velocity gradients and thus the TKE production near the surface by the mean shear (Eq. (3)). To compensate this artificial effect we applied B_v and ε_w terms at the same time (Eq. (4) and (6)), as it was theoretically confirmed and also done by Pleskachevsky et al. [14].

When wave mixing is incorporated into the model the most outstanding improvement is gained in the temperature field. Modeled and observed water temperature time series are shown at three different depths in Fig. 10 for the first five days of the simulation. The diurnal variation of water temperature is reproduced well by the model (RMSE = 0.49 °C). At daytime, stratification can establish itself when solar heating is strong enough to overcome the mixing induced by waves and currents. While on 15 July a weak stratification for a few hours long could evolve, until 17 July higher temperature differences between the surface and bottom layers have been observed and modeled. The radiative flux has a steep gradient along the depth because of the high turbidity characteristic to Lake Balaton [39]. The light attenuation coefficient is in range of 1.6 m, which was set to constant in time. During daytime the strong shortwave radiation heats the whole water column, while during nighttime only the surface layer cools due to the latent heat. The negative heat flux at the water surface leads to an unstable lake stratification and convective overturning during nighttime.

An important effect of wave mixing can be derived by means of the vertical structure of temperature. A series of observed daily averaged non-dimensional temperature profiles for each day are shown in Fig. 11 as a cartoon strip, together with those modeled by the original and improved MY turbulence model. The shape of the observed profiles has consistently the opposite curvature of what would result from vertical light attenuation. The low gradients near the surface occurred clearly due to the wave-enhanced eddy diffusivity. Near the bottom, steeper gradients are present, what suggests that the conventional insulating bottom boundary condition for temperature might not be used, the effect of sediment heat flux is not negligible.

As a consequence of improved vertical thermal structure, the model-based MLD also shows better match with MLD calculated from the observations (Fig. 12). The modeled MLD time series follows nicely the observed one, their evolutions within the day are in good agreement (RMSE = 0.37 m). It can be concluded that even during light and moderate winds a WASL can establish which can penetrate into deeper layers. If wave mixing was not accounted for (green dotted line), the steep temperature gradients would lead to the underestimation of MLDs (RMSE = 0.61 m).

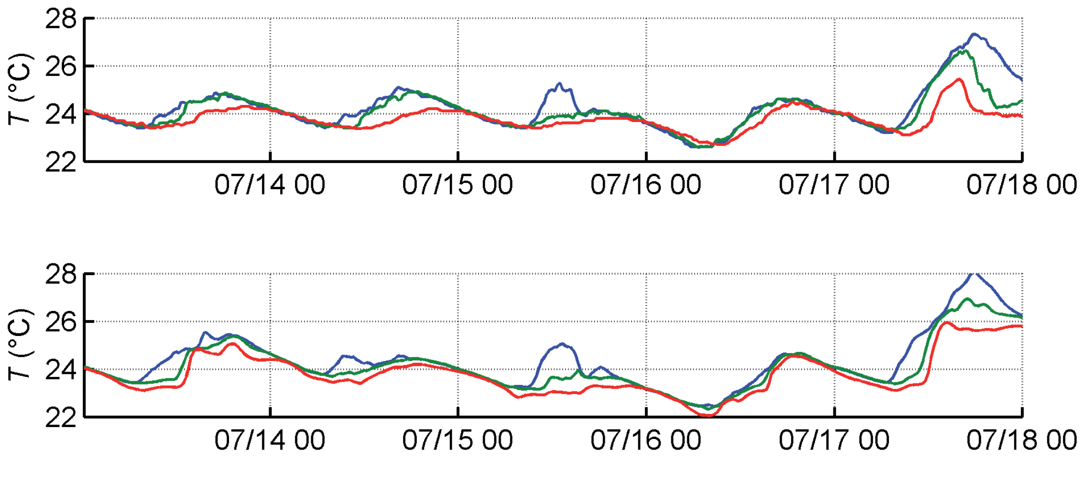

Fig. 10 Measured (upper panel) and modeled (lower panel) water temperature time series of surface (blue), mid-depth (green) and bottom (red) layers at the western (Keszthely) monitoring station in Lake Balaton.

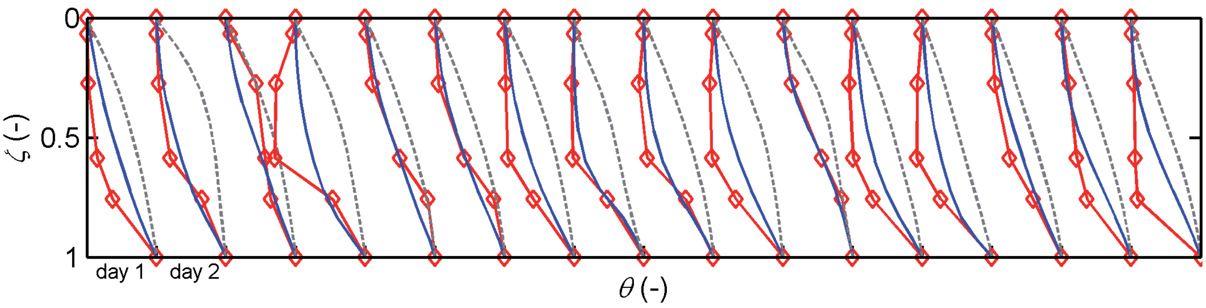

Fig. 11 Daily average non-dimensional temperature profiles at the western (Keszthely) monitoring station shown for 16 consecutive days. Red line and diamonds = observation; blue line = model including wave mixing scheme; gray dotted line = model using original MY 2.5 turbulence model.

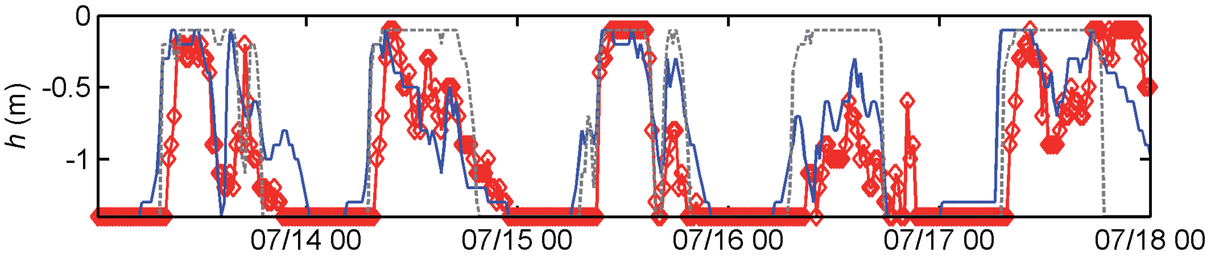

Fig. 12 Mixed-layer depth time series at the western (Keszthely) monitoring station. Red line and diamonds = observation, blue line = model including wave mixing scheme (RMSE = 0.37 m); gray dotted line = model using original MY 2.5 turbulence model (RMSE = 0.61 m).

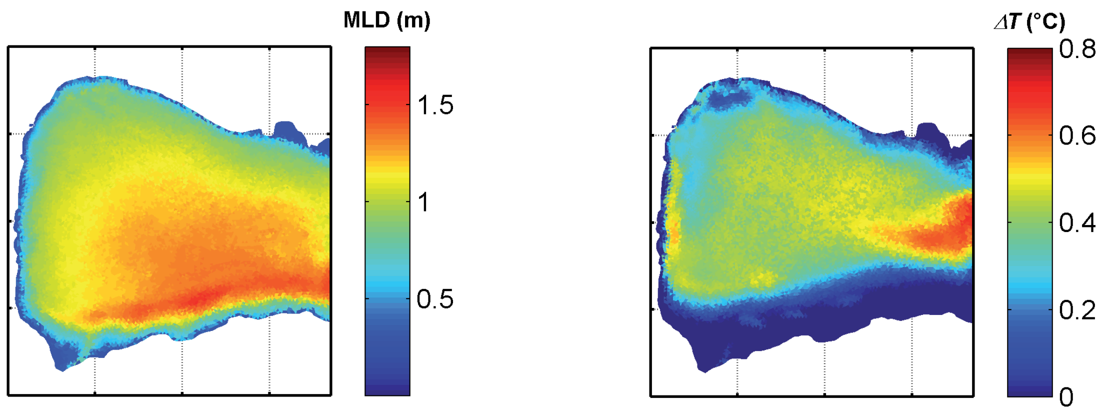

Fig. 13 Day-averaged MLD and surface-to-bottom temperature difference distribution in Keszthely Bay, modeled for July 18, 2013.

Based on the promising model results at the monitoring station we believe that our model is able to predict with acceptable accuracy the spatial distribution of temperature and thus MLD provided that the meteorological forcing, i.e. turbulent and radiative heat fluxes, can be prescribed accurately. Here, we present as a representative example the daily averaged MLDs and temperature differences between the surface and bottom layers for the western bay of Lake Balaton on 18 July, 2013 (Fig. 13). As it was mentioned only heat fluxes are homogeneous, wind stress and wave fields are fetch-dependent. Vertical mixing in our case arises from the combination of buoyancy forces, wave mixing, and the vertical shear of horizontal currents. In addition, the bathymetry can also affect directly heat exchange, thus a strong and hardly predictible spatial variability can characterize the lake. On the presented day, the mixing layer is thicker near the deeper region of the wave-exposed south shore. More extensive and stronger stratification can set in the eastern basin of the lake which is the deepest and the most open.

6 Discussion and conclusions

In summary, diurnal stratification can establish in shallow lakes such as Lake Balaton depending on the intensity of heat fluxes, vertical shear of mean currents and the turbulence production of wave motion. The mixing by non-breaking waves is a few magnitudes greater than those created by the mean flow and thus it is not negligible. It suggests that wave models should be coupled to circulation models not only through the radiation stress and the bottom shear stress but also through turbulence. The wave energy dissipation has to be transferred towards the mean flow as a source of turbulence and mixing.

In our paper we have analyzed the effect of non-breaking wind waves on mixing. Wave action on turbulence was modeled using simple, but efficient bulk parameterization schemes found in the recent literature. The well-known two-equation Mellor-Yamada 2.5 turbulence closure model was extended by the appropriate terms according to the analyzed schemes. The main difference between the schemes is whether we introduce TKE into the water column via the production term in the TKE transport equation [19, 40] or we enhance vertical mixing through the direct increase of eddy viscosity and turbulent diffusivity based on the wave spectrum [12, 20]. In the latter case eddy viscosity is strengthened without regard to TKE production and destruction. Although this usage is not preferred, some studies [12, 19] found that it performed better against the other more fundamental modifications of turbulence closures. The third scheme is based on the theoretical considerations of Pleskachevsky et al. [14], according to which the governing equations have to be complemented by both types of terms, i.e., enhanced eddy viscosity, turbulent diffusivity and TKE production.

We have compared these schemes by means of an academic test case. Based on the results we concluded that the modeled eddy viscosity and TKE distributions differ greatly between the wave mixing schemes, and so, there is a chance to find the best by validating with accurate turbulence measurements.

The original MY turbulence model underestimates MLDs consistently, even if its boundary conditions are modified in order to incorporate the effect of wave breaking [15, 31]. Based on the sensitivity analysis we can conclude that the TKE injection at the surface owing to breaking waves cannot cause such an intensive mixing which can be responsible for the evolving thermal structure during light and moderate wind conditions. The analyzed experiment period is characterized by moderate winds when the effect of breaking waves is local in Lake Balaton and thus only a thin layer is affected. This means also that the available resolution of temperature measurement is also not suitable to calibrate the parameters in the wave breaking scheme. By including the mixing of non-breaking waves into the turbulence model, not only the surface and bottom water temperatures are approximated with good accuracy but also the vertical structure of the temperature and thus the evolution of MLD is captured correctly. It has to be noted that in accordance with the earlier findings of other researchers, our model performs better when eddy viscosity and turbulent diffusivity is also directly increased as a function of orbital wave motion parameters.

As to the evolution of temperature and MLD in Lake Balaton, both have high spatial variability. It is worth mentioning that since heat fluxes were assumed spatially constant, the simulated thermal and MLD distributions are so variable because of the fetch-dependence of the wave and wind stress fields, as well as the bathymetry. The complexity emerging from these fields confirms the usage of the 3D models in case of lakes of the size of Lake Balaton even if they are shallow.

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