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RESEARCH ARTICLE

Project Duration Evaluated Using Affine Arithmetic

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Abstract

A civil engineering work can be performed by organizing the available resources (manpower, equipment and materials) in many different ways. Each different configuration results in a realization time and a cost that a building company has to bear. To produce reliable duration forecasts and money savings, it is essential to take into account all the uncertainties involved in the project operations. Generally, since it is impractical to process numerous uncertain variables - also undefined from a statistical point of view -, traditional probabilistic methods involve application difficulties for complex environments such as construction sites. To properly handle this issue, the authors propose in this paper the application of the Affine Arithmetic technique. This method treats the variables as intervals and returns reliable results, even when the variables are mutually dependent. The numerical example presented in the paper proves the efficiency of the procedure, even if some analytical complications are included in the analysis (dependency between variables, non-linear functions, etc.). Comparisons with Interval Analysis and traditional procedures are also provided. Adopting Affine Arithmetic, the results are reported in terms of intervals, avoiding the definition of unrealistic deterministic values that can strongly affect the operation organization. Furthermore, without increasing the problem complexity, the model admits continuous modifications (interval amplitudes, new variable dependencies, etc.) to correct and optimize the durations.

Keywords

project management, construction operations, planning and control, scheduling

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1 Introduction

Project managers combine their knowledge, analytical techniques and various tools to achieve the goals of a proper design in compliance with specific constraints - such as cost, time, and quality of the building. Then, a good result is obtained only if all the resources (manpower, mechanical equipment and materials) are appropriately allocated and if the risks related to the workers' health are minimized or avoided. The traditional instruments used for a project planning - such as PERT, CPM, or Gantt - have been introduced several decades ago. Although they are nowadays obsolete, these methods are still widely used and accepted, because of their remarkable simplicity (Wilson, 2003). Moreover, an abrupt replacement of these techniques with more performing but complex tools might produce problems of interpretation, understanding, and dialogue between the different involved operators.

After the 80's, the traditional approaches were apparently threatened by the general development and diffusion of innovative computers, able to handle complex operation research problems in an easy way. As a result, analysts preferred to apply sophisticated analyses rather than traditional methods with preliminary acceptable results (Faghihi et al., 2014). However, although the traditional methods usually involve specific operational complications (for example, complex time-cost relationships), they are widely adopted in practice because of the advantages related to an immediate graphical visualization of the scheduling (Fatemi et al., 2002; White and Fortune, 2002).

Each method shows different advantages and drawbacks in relation to the specific application field. In civil engineering projects, for example, the choice between a deterministic or probabilistic based methodology needs some reflections. Forecasting and evaluating the duration of each individual working phase is impractical, because of the dependence on several parameters, such as site efficiency, workers' ability, adopted machines and their state, topography, etc. As a result, the differences in duration cannot be reported as deterministic values (Mummolo, 1997; Dawson & Dawson, 1998). Unfortunately, a probabilistic characterization is also extremely complex, due to environmental condition variability, both within one site and between different sites. Consequently, the distribution functions assigned to each uncertain variable should be properly calibrated through appropriate reliability tests, but these are generally very time consuming and valid in particular scenarios only (Trietsch and Baker, 2012). Despite this limitation, literature indications provide coefficients derived from general distributions. Since these distributions should be valid for all the work phases and detailed considerations concerning critical resources like manpower, materials, and machinery are not included (Pleguezuelo et al., 2003), the related indications are widely imprudent.

In order to overcome these problems, in recent years numerous researchers have proposed simple programming tools that can also represent very composite reality (Pontrandolfo, 2000; Maylor, 2001). Moreover, a stochastic time determination can only be done assuming the independence of the involved variables (Arunava & Anand, 2008) and a strict sequence of activities, but this schematization is very far from reality (Hardie, 2001). If this assumption is not valid, more approximations should be considered in the analysis or the random variable number should be minimized (Fatemi Ghomi & Rabani, 2003; Mouhoub et al., 2011).

The time and cost ratio is also very significant (Castro et al., 2008a, 2008b). Different configurations of activities and resources heavily modify the economic aspects of the project (Chretienne and Sourd, 2003). The ultimate solution should be the result of an appropriate optimization analysis (Azaron et al., 2005), including all the involved parameters (also those related to workers' safety) (Madadi and Iranmanesh, 2012). In recent years, for this purpose, Azaron and Fatemi Ghomi (2008) have proposed a novel probabilistic methodology. They have also realized that if the results are presented as limit values, they are more clear and suitable for practical application than classical probabilistic variables. This concept is contrary to a more rigorous probabilistic analysis, but can the uncertain information can be easily handled by all the figures involved in the project (company, workers, mid-level cultural technicians, etc.) and contain a reserve of caution otherwise impractical.

Finally, it is clear that the time variable uncertainty is a weak point of all the tools proposed so far. However, it is impractical to define a rigorous method to handle this issue and that could be also easily implemented and understood by the operators involved in the operation. For this reason, in this paper the authors propose a different approach to manage the project uncertainties, described by lower and upper limits, using the Affine Arithmetic (AA) technique (Comba and Stolfi, 1993; Stolfi and de Figueiredo, 2003). This technique represents a novel analytical approach derived from the Interval Analysis (IA) (Moore, 2009). AA is a more advanced method than IA, because it can consider the dependencies existing between variables. In this context, the method provides for each project an execution time, expressed in an interval form, allowing the operator to be aware of its uncertainty and of the related reliability.

In the following, the authors will present brief analytical details concerning the method (Section 2), then the procedure and the example will be discussed (Sec. 3, 4). Finally, the analytical results are presented in Sec. 5 and interesting considerations provided in Sec. 6.

2 Method

2.1 Brief note about Interval Analysis

The Interval Analysis assigns each uncertain variable a possible range within which there is the real value of the variable. In general, an interval is defined by Eq. (1), according to IA theory (Dennis *et al.*, 1998; Neumaier, 2001).

$$x=\left[\inf(x),\sup(x)\right]=\left\{x\,|\inf(x)\le x\le\sup(x),\quad \inf(x),\sup(x),x\in R\right\} \tag{1}$$

Where inf(x) and sup (x) denote the lower and the upper bounds of x respectively.

The interval width $w(x)$ indicates the value uncertainty. Intervals with zero thickness are called crisp intervals. The intervals could be described in terms of radius and midpoint, as in Eq. (2).

$$x=\langle mid;rad\rangle \tag{2}$$

where:

$$rad(x)=w(x)/2 \tag{3}$$

$$mid(x)=\left(\sup(x)+\inf(x)\right)/2 \tag{4}$$

Let $a=[\underline{a},\overline{a}]$ and $b=[\underline{b},\overline{b}]$ be real compact intervals and $\circ$ represent one of the basic operations for real numbers - addition, subtraction, multiplication and division -, so that $\circ\in$ {+, –, ×, ÷}.

Then, the corresponding operations for intervals a and b can be defined by expression (5):

$$[a]\circ[b]=\left\{a\circ b\,|\,a\in[a],b\in[b]\right\} \tag{5}$$

where $0\notin[b]$ is assumed in case of division.

More specifically, it is possible to write the analytical rules represented in equations (6), (7), (8):

$$[a]+[b]=\left[\underline{a}+\underline{b},\overline{a}+\overline{b}\right] \tag{6}$$

$$[a]-[b]=\left[\underline{a}-\overline{b},\overline{a}-\underline{b}\right] \tag{7}$$

$$[a]\times[b]=\left[\min\left\{\underline{ab},\underline{a}b,a\overline{b},\overline{ab}\right\},\max\left\{\underline{ab},\underline{a}b,a\overline{b},\overline{ab}\right\}\right] \tag{8}$$

If $1/[b]$ it is defined according to Eq. (9), it is also possible to define the division of $[a]$ by $[b]$ (Eq. 10).

$$\frac{1}{[b]}=\left\{\frac{1}{[b]}\,|\,b\in[b]\right\}\ if\,0\notin[b] \tag{9}$$

(10)

The weak point of this analysis is the so called "error explosion" or "dependence phenomenon" (Muhanna and Mullen, 2001; Moens and Vandepitte, 2005; Moore et al., 2009), occurring when an expression contains numerous times one or more interval quantities, determining an overestimated uncertainty interval for the output.

In recent years, in order to limit these effects, some novel procedures called "Generalized Interval Analysis" (Hansen, 1975), "Affine Arithmetic" (Comba and Stolfi, 1993; Stolfi and de Figueiredo, 2003) or "Improved Interval Analysis" (Muscolino and Impollonia, 2011; Muscolino and Sofi, 2012) have been proposed. In these techniques, each intermediate result is represented by a linear function with a small interval remaining (Nedialkov *et al.*, 2004). Because of the development of these novel methods, the IA has been not used in specific complex problems, but it is now applied only in very simple conditions in which the error affecting the results can be rapidly verified.

2.2 Brief note about Affine Arithmetic

The Affine Arithmetic uses affine combinations of uncertain variables. This method is considered an evolution of Interval Analysis since it derives automatically the first-order approximations of the interested formulas.

IA does not consider any correlation between the uncertain quantities, generating overly precautionary results. One of the most effective solution to reduce this phenomenon is related to AA. According to AA, each intermediate result can be represented by a first-degree polynomial (Eq. 11):

$$\hat{x} = x_0 + x_1 \cdot \varepsilon_1 + x_2 \cdot \varepsilon_2 + \ldots + x_n \cdot \varepsilon_n \quad (11)$$

Where x_0 is the central value of the examined quantity, x_i represents the partial deviations and thus the deviation from the central value of the i-th component, and he coefficients ε_i are the noise symbols (-1 or 1) representing an independent component of the total uncertainty of the x quantity.

If $\hat{x}$ and $\hat{y}$ are two different affine forms, the same ε can be assigned to both. As a result, a partial dependency among these variables can be introduced in the analysis, solving the major limitation of IA. Any operation involving affine forms has to return a further affine form $\hat{z}$ to preserve information contained in the original variables, apart from overflows and rounding errors. The only fully affine operations are addition / subtraction, scaling and translation (Eq. 12, 13, 14).

$$\hat{x} \pm \hat{y} = \left(x_0 \pm y_0\right) + \left(x_1 \pm y_1\right) \cdot \varepsilon_1 \ldots + \left(x_n \pm y_n\right) \cdot \varepsilon_n \quad (12)$$

$$\alpha \hat{x} = \left(\alpha x_0\right) + \left(\alpha x_1\right) \cdot \varepsilon_1 \ldots + \left(\alpha x_n\right) \cdot \varepsilon_n \quad (13)$$

$$\hat{x} \pm \zeta = \left(x_0 \pm \zeta\right) + x_1 \cdot \varepsilon_1 + \ldots x_n \cdot \varepsilon_n) \quad (14)$$

Other operations involve some inaccuracies, that can be explained in detail considering a generic not affine operation as $z \neg f(x, y)$, represented by expression (15).

Since f^* does not represent an affine form, z cannot be defined as an affine combination of the noise symbols ε_i Then,

$$\begin{aligned} z &= f\left(x_0 \circ x_1 \cdot \varepsilon_1 \circ \ldots \circ x_n \cdot \varepsilon_n, \quad y_0 \circ y_1 \cdot \varepsilon_1 \circ \ldots \circ y_n \cdot \varepsilon_n\right) \\ &= f^*\left(\varepsilon_1 \ldots, \varepsilon_n\right) \end{aligned} \quad (15)$$

it is necessary to find an affine function f^a of the ε_i sufficiently similar to the f^* (Eq. 16). This causes the addition of an extra term $z_k \times \varepsilon_k$ representative of the approximation error, as evidenced in Eq. (17).

$$f^a = \left(\varepsilon_1, \ldots, \varepsilon_n\right) = z_0 \circ z_1 \cdot \varepsilon_1 \circ \ldots \circ z_n \cdot \varepsilon_n \quad (16)$$

$$z = f^a\left(\varepsilon_1, \ldots, \varepsilon_n\right) + z_k \cdot \varepsilon_k = z_0 \circ z_1 \cdot \varepsilon_1 \circ \ldots \circ z_n \cdot \varepsilon_n + z_k \cdot \varepsilon_k \quad (17)$$

Even in this case, ε_k must be distinct from all previous noise symbols. Moreover, the coefficient z_k is an upper limit of the error of approximation $f^* - f^a$ and involves rounding errors occurred in the computation of the other coefficients z_i (Eq. 18).

$$\left|z_k\right| \geq \max\left\{\left|f^*(\varepsilon_1, \ldots, \varepsilon_n) - f^a(\varepsilon_1, \ldots, \varepsilon_n)\right| : \varepsilon_1, \ldots, \varepsilon_n \in U\right\} \quad (18)$$

However, Eq. (17) contains a loss of information, measured by z_k. In detail, as ε_k cannot represent a dependency among different quantities, it is assumed independent. In order to have sufficiently simple analytical equations, the authors will express affine combinations of affine quantities $\hat{x}$ and $\hat{y}$ as defined in Eq. (19).

$$f^a(\varepsilon_1, \ldots, \varepsilon_n) = \alpha \hat{x} + \beta \hat{y} + \zeta \quad (19)$$

Where α, β and ζ represent the unknows.

The committed error in the affine form approximation depends quadratically on the extent of the input variable ranges, but some authors showed that it is essentially zero, if the function f depends on a single variable (Stolfi and de Figueiredo, 2003), as in Eq. (20).

$$f^a(\varepsilon_1, \ldots, \varepsilon_n) = \alpha \hat{x} + \zeta \quad (20)$$

Chebyshev's approximation theory can minimize the maximum absolute error. This theory is valid if:

- the function f is bounded and defined in an interval $I = [a\_b]$;
- f is twice differentiable and the second derivative f'' does not change sign on I.

In order to derive f^a, the unknowns α and ζ must be deduced through the following conditions:

- the coefficient α is provided by the slope of the line r(x) that interpolates the points $[a, f(a)]$ and $[b, f(b)]$: $\alpha = [f(b) - f(a)]/(b - a)$.
- the maximum error occurs twice with the same sign at the extreme points a and b of the considered range, while it occurs once (with opposite sign) at a point "u" into the interval where $f'(u) = \alpha$.
- the independent term z is deduced by $\alpha u + \zeta = [f(u) + r(u)]/2$.
- the maximum absolute error is $\delta = |f(u) - r(u)|/2$.

The multiplication of affine forms deserves some consideration. It provides a quadratic polynomial that can be represented by Eq. (21). Its best approximation can be divided in an affine term, generally indicated as A(ε1,…,ε_n) and an affine approximation Q(ε1,…,ε_n), respectively represented by Eq. (22) and Eq. (23).

$$f^*(\varepsilon_1,...,\varepsilon_2) = \hat{x} \cdot \hat{y} \tag{21}$$

$$A(\varepsilon_1,...,\varepsilon_n) = x_0 \cdot y_0 + \sum_{i=1}^{n}(x_0 \cdot y_i + y_0 \cdot x_i) \cdot \varepsilon_i \tag{22}$$

$$Q(\varepsilon_1,...,\varepsilon_n) = (\sum_{i=1}^{n} x_i \cdot \varepsilon_i) \cdot (\sum_{i=1}^{n} y_i \cdot \varepsilon_i) = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i \cdot y_i \cdot \varepsilon_i \cdot \varepsilon_j \approx \approx \pm rad(\hat{x}) \cdot rad(\hat{y}) \cdot \varepsilon_k \tag{23}$$

The last term on the left of (23) is a simplification of its most rigorous expression that is often difficult to calculate. This can cause approximations at least four times larger than the one affecting the original affine forms, because its range is obtained without any correlation between the two factors - essentially the same value derived from the traditional IA. However, , since the latter part is only used for the determination of the quadratic residue, while the linear part perfectly handles possible dependencies among variables, the multiplication in AA is much more precise than that performed with the IA.

Finally, concerning the division, the easiest way to manage this operation is to rewrite it as a multiplication (Eq. 24).

$$\hat{x} / \hat{y} = \hat{x} \cdot (1/\hat{y}) \tag{24}$$

The rigorous treatment of this operation between affine forms would be very complex. In particular, especially with ranges of the same order of the central value or when the interval of the divisor includes zero, it may lead to very obvious inaccuracies.

3 Problem formulation

To prove the possible benefits of the procedure, the authors present the results of a practical example, considering, for clarity, a simple and typical road construction work.

Example scenario

The reference design is related to the earth works involved in a road trench section building. The planned activities are the following:

1. site preparation: work area delimitation, track preparation, construction of workers' boxes, etc;
2. deforestation: elimination of shrubs and vegetation using a loader.
3. ditch excavation: generally performed using an excavator before starting the road section excavation because of their smaller cross-sections;
4. trench excavation: the most time-consuming phase affecting the total work time; operation performed using an excavator;
5. ditch positioning: displacement of the prefabricated elements using a mechanical machine;
6. earth sampling for laboratory analysis;
7. earth transportation to landfill, since the soil performance is not acceptable for this application;
8. subbase compaction: performed with a suitable roller compactor to stiffen the pavement support;
9. slope vegetation, to contrast the erosion due to rainwater.

Time duration calculation

To properly perform the various operations, it is fundamental to evaluate a reliable estimate of costs and times required for each single operation. Obviously, time and cost are not independent, but correlated through roughly complex relationships, related to the available resources (manpower, equipment, materials). The time evaluation can be performed only after the design completion, when each activity is completely defined. The duration of a single phase depends on a special parameter, called "production", related to the composition of a workers' team and to the adopted machines. As an example, considering the excavation operation, the work production can be evaluated through Eq. (25) (Caterpillar, 2004).

$$P = V \cdot \frac{r}{s} \cdot \frac{3600}{T_c} \cdot f \cdot \alpha \cdot \beta \cdot \gamma \quad [m^3/h] \tag{25}$$

Where V is the bucket volume, r is the bucket filling factor, s is the swelling soil factor, T_c is the cycle time, f represents the efficiency of the site construction, α is a coefficient that harmonizes a tower rotation different from 90°, β is the coefficient of bucket comparison different from the front loader, γ is the coefficient of excavation depth different from the optimal one.

Calculated the production, if Q is the earth material in m^3, it is possible to determine the duration taken to perform the whole activity using Eq. (26).

$$t = \frac{Q}{P} \quad [h] \tag{26}$$

Similar expressions could be adopted for all the other activities involving machineries. However, the quantification of these times is not sufficient to identify the total execution duration. It should be considered that the most proper assembly of the available resources assures a considerable contraction of the overall execution time. A PERT or a Gantt chart are the generally adopted schemes to guarantee the task optimization, balancing realization costs and returns. Table 1 shows the activity summary, the involved quantities and the duration of each operation. The duration is deduced, according to the following rules:

- for activities involving only manpower or with a limited use of machines, the duration is derived from the workers' utilization percentage - generally available if the activity unit cost contains details of all the resources;
- for machine operations, the duration is calculated using the specific production formula, as Eq. (25).

Table 1 Summary of the necessary activities, the quantity of the involved material and the duration.

No.	Activity	Quantity (m^3)	Duration (hour)
1	Site preparation	860	7
2	Deforestation	3000	6.85
3	Ditch excavation	300	2.87
4	Trench excavation	6000	40.31
5	Ditch positioning	400	1.11
6	Earth sampling	10	5
7	Earth transportation	6300	7
8	Subbase compaction	1600	6.85
9	Revegetation slopes	1600	2.87

Although suitable coefficients should be considered for more realistic results, the authors have not corrected the durations to simplify the overall analysis.

The Gantt chart

The organization of the activities requires further analyses. The execution of all the activities in series could limit the number of workers, but it is generally impracticable because the total time would be equal to the sum of the single activity durations. For this reason, designers must properly organize the site, with a proper assessment of number and type of workers and machines, to limit costs and time within acceptable ranges. As a preliminary example, assuming deterministic durations the authors propose a rational Gantt chart (Fig. 1). Probably, it should be possible to optimize the chart, but for the research purpose of this paper it would be insignificant.

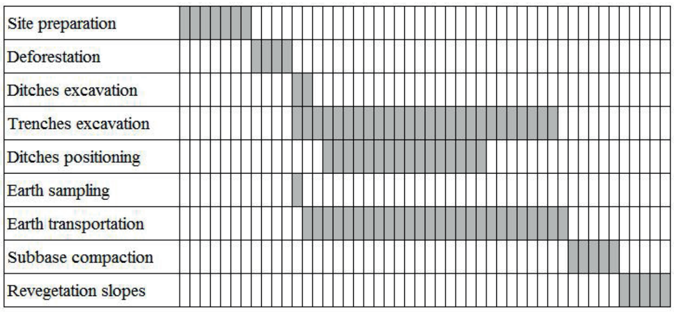

Fig. 1 Gantt chart of the road construction

Handling of the uncertain quantities

Considering the example scenario, the authors individuate some uncertain variables, represented in term of interval rather than deterministic values. The number of uncertain variables is sufficient to appreciate the potential of the method, avoiding at the same time an extreme simplification. In particular, the uncertain quantities are introduced in the following activities:

- deforestation: 1) loader bucket volume V_L; 2) cycle time T_{cL}; 3) efficiency of the construction site f;
- ditch excavation: 1) excavator bucket volume V_{ED}; 2) cycle time T_{cED} , 3) efficiency of the construction site f;
- trench excavation: 1) excavator bucket volume V_{ET}; 2) cycle time T_{cET}; 3) efficiency of the construction site f;
- subbase compaction: 1) roller speed SP; 2) efficiency of the construction site f.

As before mentioned, the authors considered three machines: a loader, an excavator, and a roller compactor. All the related data are listed in Table 2. The variables with the apex I denote quantities expressed in terms of mid-rad, as shown in Eq. (4).

Table 2 Characteristics of the main variables of the involved machines.

		Loader	Excavator Ditches	Excavator Trenches	Compactor
Bucket volume	V^I	<3; 0.3>	<1.7; .17>	<1.7; .17>	-
Speed	SP^I	-	-	-	<3; 0.3>
Filling factor	r	0.8	0.8	0.95	-
Swelling soil factor	s	1.5	1.5	1.1	-
Cycle time	T_c^I	<25; 5>	<20; 5>	<20; 5>	-
Efficiency	f^I	<0.8; .08>	<0.8; .08>	<0.8; .08>	<0.8; 0.08>
Tower rotation	a	-	1	0.88	-
Bucket comparison	b	-	0.8	0.8	-
Escavation depth	g	-	1	1	-
Drum width	L	-	-	-	1.8
No. Passes	n	-	-	-	5

4 Results

As expected, the maximum execution time is relative to the critical path, represented by the five activities arranged as in the following:

1) site preparation → 2) deforestation → 3) ditch excavation → 4) trench excavation → 7) earth transportation → 8) subbase compaction.

The total duration is represented by the sum of the partial durations.

$$D_{TOT} = D_1 + D_2 + D_3 + D_4 + D_7 + D_8 \qquad (27)$$

Only the durations D_2, D_3, D_4 and D_8 present uncertain variables. Since they can be calculated as the ratio between the quantity of the work and the production, it is possible to express the total time D_{TOT} using Eq. (28).

$$D_{TOT} = D_1 + \frac{Q_L}{P_L} + \frac{Q_{ED}}{P_{ED}} + \frac{Q_{ET}}{P_{ET}} + D_7 + \frac{Q_C}{P_C} \qquad (28)$$

Where D_1 and D_7 are the deterministic durations of activities 1 and 7 (Table 1), while Q_L, Q_{ED}, and Q_{ET} are the quantities of excavated earth using the loader and the excavators, expressed in m^3 and referred to the phases 2, 3, and 4. Q_C refers to the surface to be compacted and is expressed in m^2. The quantities P_L, P_{ED}, P_{ET}, and P_C are the productions, respectively, of the loader, the excavators and the compactor.

Expression (29) highlights the uncertain quantities in terms of range numbers.

$$D_{TOT} = D_1 + \frac{Q_L}{V^I_L \cdot \frac{r_L}{s_L} \cdot \frac{3600}{T^I_{cL}} \cdot f^I} + \\ + \frac{Q_{ED}}{V^I_{ED} \cdot \frac{r_{ED}}{s_{ED}} \cdot \frac{3600}{T^I_{cED}} \cdot f^I \cdot \alpha \cdot \beta \cdot \gamma} + \\ + \frac{Q_{ET}}{V^I_{ET} \cdot \frac{r_{ET}}{s_{ET}} \cdot \frac{3600}{T^I_{cET}} \cdot f^I \cdot \alpha \cdot \beta \cdot \gamma} + \\ + D_7 + \frac{Q_C}{\frac{L \cdot sp^I \cdot f^I \cdot 1000}{np}} \quad (29)$$

The term 1/TC introduces small analytical difficulties in the calculation. Considering the general form $1/x$, the function can be replaced by an approximating line, minimizing the related error. The best approximate affine form derived from (22) is presented in Eq. (30).

$$\alpha \hat{x} + \zeta = \alpha \left(x_0 + x_1 \varepsilon_1 + \ldots + x_n \varepsilon_n \right) + \zeta \quad (30)$$

Assuming an interval as [a_b], α (Eq. 31) represents the slope of the line $r(x)$ linking the two extreme points (a, $1/a$) and (b, $1/b$). The point indicated with u, where the function 1/x presents a slope equal to α, can be obtained through Eq. (32), while the optimal value of ζ is calculated using Eq. (33).

$$\alpha = \frac{1/b - 1/a}{b - a} \quad (31)$$

$$u = \frac{1}{4\alpha^2} = \frac{a + b + 2 \cdot (1/a \cdot 1/b)}{4} \quad (32)$$

$$\zeta = \frac{f(u) + r(u)}{2} - \alpha u \quad (33)$$

The max error between the line and the function is in correspondence of the two extremes and in the center points (Eq. 34). Therefore, the best affine form is represented by Eq. (35).

$$\delta = \frac{f(u) - r(u)}{2} \quad (34)$$

$$z_0 + z_1 \varepsilon_1 + \ldots + z_n \varepsilon_n + z_k \varepsilon_k \quad (35)$$

Where:

ε_k is a new noise variable.

$$\begin{aligned} z_0 &= \alpha x_0 + \zeta \\ z_i &= \alpha x_i \quad (with\ i = 1, \ldots, n) \\ z_k &= \delta \end{aligned} \quad (36)$$

In this case, the cycle time Tc has different limits for the loader and the excavators. The results obtained from the above equations can be summarized in Table 3. Figures 2 and 3 illustrate the approximation of the new affine form.

Table 3 Approximation in affine form of the $1/T_c$ function.

	Loader	Excavator
T_c	20_30	15_25
a	-0.001667	-0.002667
U	25	20
z	0.0825	0.105
d	0.00083	0.0016667
x_1	1.5	1.5
x_2	3.5	3.5
x_3	0	0
z_0	0.0408333	0.0516667
z_1	-0.0025	-0.004
z_2	-0.005833	-0.009333
z_3	0	0
z_k	0.0008333	0.0016667

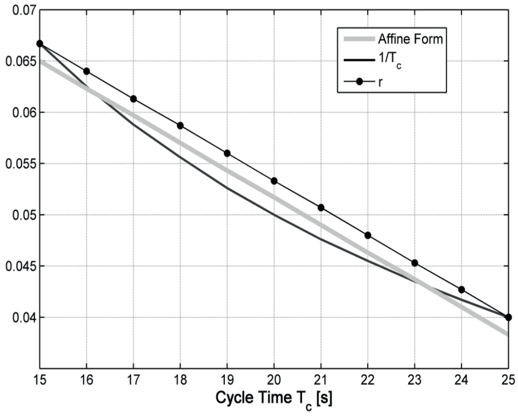

Fig. 2 Affine form of the $1/T_c$ function for the loader

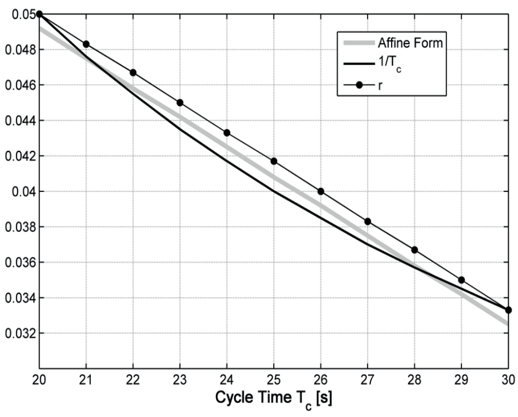

Fig. 3 Affine form of the $1/T_c$ function for the excavators

Chosen the best affine form for $1/T_C$, the resolution of Eq. (29) can be performed through the multiplications in Eq. (22) and (23). In this regard, it is necessary to declare all the variable dependencies. This represents the actual advantage of AA compared to the traditional IA. In this case, three components (ε_1, ε_2, ε_3) of the deviation of an uncertain quantity are identified. They are related to the soil nature, the site morphology, and the workers' ability. The uncertain quantities (V, T_c, Sp and f) have dependencies explicated by the common noise variables and reported in Tables 4 and 5:

Table 4 Common noise variables relative to the different quantities inside the expression useful to calculate the production of the loader and the excavators.

	ε_1	ε_2	ε_3
V	●		●
T_c	●	●	
f	●	●	●

Table 5 Approximation in affine form of the $1/T_c$ function.

	ε_1	ε_2	ε_3
Sp	●	●	●
f	●	●	●

Because of length limitations, in this paper the authors provide the results of the final calculations only. The contribution of the various activities in terms of production and execution time are separated. For more clarity, in order to evaluate the convenience of the proposed numerical technique, the authors reported the calculation of the duration in three distinct situations:

1. Only deterministic variables (Table 6).
2. Variables characterized by the IA technique (Table 7).
3. Variables characterized by the AA technique (Table 8).

Table 6 Results about productions [m^3/h] and durations [h] of all the activities assuming all the quantities as deterministic values. Only the production of the compactor has to be considered in m^2/h.

	PRODUCTION				DURATION			
ACTIVITY	Mid	Rad	Inf	Sup	Mid	Rad	Inf	Sup
ACT1	860	-	860	860	7	-	7	7
ACT2	175.1	-	175.1	175.1	6.85	-	6.85	6.85
ACT3	104.5	-	104.5	104.5	2.87	-	2.87	2.87
ACT4	148.8	-	148.8	148.8	40.31	-	40.31	40.31
ACT8	1440	-	1440	1440	1.11	-	1.11	1.11
ACT9	1600	-	1600	1600	5	-	5	5

Table 7 Results about productions [m^3/h] and durations [h] of all the activities with Interval Analysis. Only the production of the compactor has to be considered in m^2/h.

	PRODUCTION				DURATION			
ACTIVITY	Mid	Rad	Inf	Sup	Mid	Rad	Inf	Sup
ACT1	860	-	860	860	7	-	7	7
ACT2	191.52	73.32	118.2	264.84	7.34	2.81	4.53	10.15
ACT3	118.1	50.41	67.69	168.51	3.11	1.33	1.78	4.43
ACT4	168.28	71.84	96.44	240.12	43.6	18.61	24.99	62.21
ACT8	1454	288	1166	1742	1.14	0.23	0.91	1.37
ACT9	1600	-	1600	1600	5	-	5	5

Table 8 Results about productions [m^3/h] and durations [h] of all the activities with Affine Arithmetic. Only the production of the compactor has to be considered in m^2/h.

	PRODUCTION				DURATION			
ACTIVITY	Mid	Rad	Inf	Sup	Mid	Rad	Inf	Sup
ACT1	860	-	860	860	7	-	7	7
ACT2	182.4	53.2	129.2	235.6	6.58	1.92	4.66	8.5
ACT3	111.41	28.75	82.66	140.17	2.69	0.69	2	3.39
ACT4	158.76	42.73	116.04	201.49	37.79	10.17	27.62	47.96
ACT8	1440	322.2	1117.8	1762.2	1.11	0.25	0.86	1.36
ACT9	1600	-	1600	1600	5	-	5	5

Finally, the final project durations of the three analytical methods are compared in Table 9, considering only by the mid-rad representations.

Table 9 Results about productions [m^3/h] and durations [h] of all the activities assuming all the quantities as deterministic values. Only the production of the compactor has to be considered in m^2/h.

	DURATION					
	Deterministic		Affine Arithmetic		Interval Analysis	
ACTIVITY	Mid	Rad	Mid	Rad	Mid	Rad
ACT1	7	-	7	-	7	-
ACT2	6.85	-	6.58	1.92	7.34	2.81
ACT3	2.87	-	2.69	0.69	3.11	1.33
ACT4	40.31	-	37.79	10.17	43.6	18.61
ACT8	1.11	-	1.11	0.25	1.14	0.23
ACT9	5	-	5	-	5	-

5 Discussion

Despite the simplicity of the numerical example, it permits to quickly identify weaknesses and qualities of the procedure. Obviously, this methodology can be easily applied to a larger number of work activities. A civil engineering construction is more hardly reproducible and controllable than a standard working and thus presents uncertainties that are impractical to configure through traditional methods. For this reason, the probabilistic techniques are not acceptable and produce false mathematical determination that are completely unrealistic. Rather, the AA method assures a proper characterization of the uncertainties in a form very usual for workers in civil engineering. The analytical form of AA can adapt without complications to duration changes during the execution to better evaluate the results.

The advantages of AA in comparison to IA are numerous:

- AA permits to identify a priori the dependencies among variables. In this regard, the authors showed the relationship between the quantities involved in Tables 4 and 5. In particular, cycle time, bucket volume, efficiency of the construction side, and compactor speed are assumed as dependent on soil nature, terrain morphology, and workers' capability. Generally, this study should be more rigorous, but in this context the related results sufficiently highlight the considerable potential of AA. IA, as well as the probabilistic analysis, is computationally convenient only if there is a perfect independence of the uncertain variables. If this hypothesis is not confirmed, the results are affected by obvious errors and, at best, are characterized by unnecessarily large and not reliable intervals.

- the comparison between Table 6 and 7 underlines some inaccuracies concerning the central values of the results. It would be expected to obtain a similar central value in all the three cases, but different interval ranges. Excluding some not remarkable truncation errors, the only approximation source in AA is the determination of the approximate affine form 1/Tc. In this case, this form has been replaced by a line producing a maximum error □. The error magnitude depends strongly on the range of the variable. If this is modest (around 10%), this error is minimal; otherwise, as in this example (the range is about 25 %), the error should be considered by the analyst. However, this approximation is not very significant and produces an error of about 5%. It determines a total error of about 3 unit of time, since the function 1/Tc appears three times in the calculation (1 hour in the loader activity, 1 hour in the activities of ditch and trench excavation).
- the results reported in Tables 7 and 8 show the better performance of the AA technique than the IA method. The IA results are highly questionable: the central values are slightly higher than those obtained with AA. Although they can be considered as precautionary, the real weak point of the IA procedure consists in its too large uncertainty range. The Radius is very large and almost twice that of AA (22.98 against 13.03). This proves a certain error explosion, although the analytic expression of Eq. (29) is sufficiently simple. Moreover, the ranges of the input variables were almost of 10%. It is well known from the literature that, when this percentage grows, the AA maintains its robustness and reliability, while the results obtained through IA quickly degenerate.

6 Conclusions

Generally, a civil engineering construction presents two important indicators: duration and cost. They are usually directly related to each other and, in order to increase the working efficiency, they are monitored during all the work operations. At this regard, some of the existing procedures applied with success in the military and industrial field have been introduced in the civil engineering. However, numerous problems have not been solved yet. For instance, the approximations in Gantt and PERT and the deterministic values assigned to purely stochastic values produce significant imprecisions, For these reason, several researchers tried to combine various procedures- also very complex from an analytical point of view -to solve two major issues:

- Handling the variable dependences in the probabilistic analysis;
- Considering a large number of variables, without causing great computational problems and unreliability of the final result.

The adoption of proper probability density functions makes impractical the procedure. As an alternative, range numbers can be easily implemented in the analysis and can be rapidly evaluated. Generally, expert operators (engineers, managers, workers, executors) are familiar with the superior and inferior limits of a specific quantity even if they do not know how to properly characterize them in probabilistic terms.

As proved by the example presented in the paper, AA brilliantly solves the dependence problem, overpassing the major limitation of the traditional IA. Even in a simple application, IA seems to be unreliable, especially whit variable dependencies, wide ranges, and complex analytic structures.

Theoretical implications and practical applications could be vast. For example, it could be interesting to deepen the trends of the most frequent analytic functions, suggesting an improvement in their approximation, even in non-linear terms.

On a practical level, this tool could be very productive during the design phase to assess the duration of complex works in a realistic way. Moreover, during the work execution, its construction, the executor can highlight discrepancies with the design about the duration of the individual phases of work. Since this could result in a diseconomy, he can differently assemble individual activities with the advantage of knowing more fully the involved resources and variables, getting a result suffering from minor uncertainties.

References

[1] Azaron, A., Fatemi Ghomi, S. M. T. "Lower bound for the mean project completion time in dynamic PERT networks". *European Journal of Operational Research*. 186 (1), pp. 120-127. 2008. DOI: 10.1016/j.ejor.2007.01.015

[2] Azaron, A., Perkgoz, C., Sakawa, M. "A genetic algorithm approach for the time-cost trade-off in PERT networks". *Applied Mathematics and Computation*. 168 (2), pp. 1317-1339. 2005. DOI: 10.1016/j.amc.2004.10.021

[3] Castro, J., Gomez, D., Tejada, J. "A polynomial rule for the problem of sharing delay costs in PERT networks". *Computers & Operations Research*. 35 (7), pp. 2376-2387. 2008. DOI: 10.1016/j.cor.2006.11.003

[4] Castro, J., Gomez, D., Tejada, J. "A rule for slack allocation proportional to the duration in a PERT network". *European Journal of Operational Research*. 187 (2), pp. 556-570. 2008. DOI: 10.1016/j.ejor.2007.03.039

[5] "Caterpillar Performance Handbook (35th ed.)." Caterpillar Inc., Peoria, Illinois. 2004.

[6] Chreitienne, P., Sourd, F. "PERT scheduling with convex cost functions". *Theoretical Computer Science*. 292 (1), pp. 145-164. 2003. DOI: 10.1016/S0304-3975(01)00220-1

[7] Comba, J. L., Stolfi, J. "Affine arithmetic and its applications to computer graphics". In: Anais do VI Simposio Brasileiro de Computaao Grafica e Processamento de Imagens (SIBGRAPI'93", Recife, Brazil, Oct. 9-18, 1993.

[8] Dawson, R. J., Dawson, C. W. "Practical proposals for managing uncertainty and risk in project planning". *International Journal of Project Management*. 16 (5), pp. 299-310. 1998. DOI: 10.1016/S0263-7863(97)00059-8

[9] Dennis, D., Kreinovich, V., Rump, S. M. "Interval and origins of calculus". *Reliable Computing*. 4 (2), pp. 191-197. 1998. DOI: 10.1023/A:1009989211143

[10] Faghihi, V., Reinschmidt, K. F., Kang, J. H. "Construction scheduling using Genetic Algorithm based on Building Information Model". *Expert Systems with Applications*. 41 (16), pp. 7565-7578. 2014. DOI: 10.1016/j.eswa.2014.05.047

[11] Fatemi Ghomi, S. M. T., Rabbani, M. "A new structural mechanism for reducibility of stochastic PERT networks". *European Journal of Operational Research*. 145 (2), pp. 394-402. 2003. DOI: 10.1016/S0377-2217(02)00155-8

[12] Fatemi Ghomi, S. M. T., Teimouri, E. "Path critical index and activity critical index in PERT networks". *European Journal of Operational Research*. 141 (1), pp. 147-152. 2002. DOI: 10.1016/S0377-2217(01)00268-5

[13] Hansen, E. R. "A generalized interval arithmetic." *Lecture Notes in Computer Science*. 29, pp. 7-18. 1975. DOI: 10.1007/3-540-07170-9_2

[14] Hardie, N. "The prediction and control of project duration: a recursive model". *International Journal of Project Management*. 19 (7), pp. 401-409. 2001. DOI: 10.1016/S0263-7863(00)00027-2

[15] Pleguezuelo, R. H., Perez, J. G., Rambaud, S. C. "A note on the reasonableness of PERT hypotheses". *Operations Research Letters*. 31 (1), pp. 60-62. 2003. DOI: 10.1016/S0167-6377(02)00151-7

[16] Madadi, M., Iranmanesh, H. (2012). "A management oriented approach to reduce a project duration and its risk (variability)". *European Journal of Operational Research*. 219 (3), pp. 751-761. DOI: 10.1016/j.ejor.2012.01.006

[17] Maylor, H. "Beyond the Gantt chart: Project management moving on". *European Management Journal*. 19 (1), pp. 92-100. 2001. DOI: 10.1016/S0263-2373(00)00074-8

[18] Moens, D., Vandepitte, D. "Interval sensitivity theory and its applications to frequency response envelope analysis of uncertain structures". *Computer Methods in Applied Mechanics and Engineering*. 196 (21-24), pp. 2486-2496. 2007. DOI: 10.1016/j.cma.2007.01.006

[19] Moore, R. E., Kearfott, R. B., Cloud, M. J. "Introduction to Interval Analysis." 223 p. SIAM, Philadelphia. 2009. DOI: 10.1137/1.9780898717716

[20] Mouhoub, N. E., Benhocine, A., Belouadah, H. "A new method for constructing a minimal PERT network". *Applied Mathematical Modelling*. 35 (9), pp. 4575-4588. 2011. DOI: 10.1016/j.apm.2011.03.031

[21] Muhanna, R. L., Mullen, R. L. "Uncertainty in mechanics: Problems-Interval-Based approach". *Journal of Engineering Mechanics*. 127 (6), pp. 557-566. 2001. DOI: 10.1061/(ASCE)0733-9399(2001)127:6(557)

[22] Mummolo, G. "Measuring uncertainty and criticality in network planning by PERT-path technique". *International Journal of Project Management*. 15 (6), pp. 377-387. 1997. DOI: 10.1016/S0263-7863(96)00088-9

[23] Impollonia, N., Muscolino, G. "Interval analysis of structures with uncertain-but-bounded axial stiffness". *Computer Methods in Applied Mechanics and Engineering*. 200 (21-22), pp. 1945-1962. 2011. DOI: 10.1016/j.cma.2010.07.019

[24] Muscolino, G., Sofi, A. "Stochastic analysis of structures with uncertain-but-bounded parameters via improved interval analysis". *Probabilistic Engineering Mechanics*. 28, pp. 152-163. 2012. DOI: 10.1016/j.probengmech.2011.08.011

[25] Nedialkov, N. S., Kreinovich, V., Starks, S. A. "Interval arithmetic, affine arithmetic, Taylor series methods: why, what next?" *Numerical Algorithms*. 37 (1), pp. 325-336. 2004. DOI: 10.1023/B:NUMA.0000049478.42605.cf

[26] Neumaier, A. "*Introduction to Numerical Analysis*." 366 p. Cambridge University Press, Cambridge. 2001. DOI: 10.1017/cbo9780511612916

[27] Pontrandolfo, P. "Project duration in stochastic networks by the PERT-path technique". *International Journal of Project Management.* 18 (3), pp. 215-222. 2000. DOI: 10.1016/S0263-7863(99)00015-0

[28] Stolfi, J., de Figueiredo, L. H. "An introduction to affine arithmetic". *Trends in Applied and Computational Mathematics*. 4 (3), pp. 297-312. 2003. DOI: 10.5540/tema.2003.04.03.0297

[29] Trietsch, D., Baker, K. R. "PERT 21: Fitting PERT/CPM for use in the 21st century". *International Journal of Project Management*. 30 (4), pp. 490-502. 2012. DOI: 10.1016/j.ijproman.2011.09.004

[30] White, D., Fortune, J. "Current practice in project management – an empirical study". *International Journal of Project Management*. 20 (1), pp. 1-11. 2002. DOI: 10.1016/S0263-7863(00)00029-6

[31] Wilson, J. M. "Gantt charts: A centenary appreciation." *European Journal of Operational Research*. 149 (2), pp. 430-437. 2003. DOI: 10.1016/S0377-2217(02)00769-5

[32] Woodbury, M. A. "*Inverting Modified Matrices*". Princeton University, Statistical Research Group, Princeton, NJ. 1950.