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RESEARCH ARTICLE

New Development to Measure Mode I. Fracture Toughness in Rock

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Abstract

Tensile fracture toughness is one of the dominant characteristics of rocks that play an important role in fracture mechanics of rock structures. In spite of the substantial amount of work that has been conducted as suggested methods to determine the fracture toughness of rock in Mode I., research on new methods are still demanded. The compact tension (CT) specimen is widely used to determine the fracture toughness of metals and is a standard method in accordance with ASTM standard. To conduct similar direct tests in rock, preparation of rock specimen a difficult task and needs some special tools to create notched sample. To address these issues, experimental techniques based direct tensile apparatus have been developed. Developed experimental procedure involves the use of the specimens having a central hole and two notches or cracks at both inner side of proposed specimen. The stress intensity factor formula at the crack tip of a CT specimen can be availed through the literature. However a new expression of stress intensity factor is needed for the developed test in order to account geometry and loading configuration. To develop a new stress intensity factor expression, the numerical models of suggested method were worked out. The focus of the current work is on development of the experimental technique, which involves determining the optimum sample size and shape, loading procedure, fracture toughness for different rock types, and verification of suggested method based on CT test which has been carefully considered in this investigations.

Keywords

Mode I. fracture toughness, CT test, stress intensity factor

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1 Introduction

There have been introduction of various strength parameters based failure criteria, in rock mechanics, used to predict the failure mechanism of rock structures. However, in many cases, process of rock failure cannot merely be described by the conventional failure criteria. For brittle materials such as rock, once a crack and/or discontinuity is developed, the stress state has been significantly changed in the vicinity of the crack tip. Hence the conventional failure criteria such as Mohr-Coulomb and Hoke-Brown cannot explain the fracture process during the crack initiation and propagation in rock materials. To this end, the fundamental of fracture mechanics was introduced to rock mechanics and developed the rock fracture mechanics [1]. Parameters of fracture toughness and strength play important role in controlling the fracture characteristics of rocks. The fracture toughness is a quantitative expression of a material resistance with respect to crack initiation and propagation. This parameter can be applied in rock mechanics as; a parameter for the rock classification, a fragmentation index in tunnel boring and blasting and a material constant in the modelling of rock cutting, hydraulic fracturing, and the stability analysis of rock structures [2].

Three basic fracture modes have been introduced in fracture mechanics based on loading configurations for a crack. The corresponding modes of crack surface displacement are; opening or tensile mode (Mode I.), in-plane shearing mode (Mode II.) and out-of-plane shearing mode (Mode III.) as shown in Fig. 1. The stress intensity factors corresponding to the three basic fracture modes are K_I, K_{II} and K_{III} respectively. The critical value of these stress intensity factors are known as fracture toughness denoted by K_{IC}, K_{IIC}, and K_{IIIC} corresponding to the three basic cracking modes. Since the tensile strength of rocks is lower than compressive and shear strength. Therefore, Mode I. of fracture toughness appears to be the most important parameter which controls the fracture initiation in rocks. In general, most of joints and discontinuities in rock media are related to the tensile mode (Mode I.). To this end, there have been done most of work on the analysis of tensile mode (Mode I.), particularly pertaining to fracture mechanics of rocks.

Hence, a new direct method to measure the Mode I. fracture toughness of rocks is proposed through this paper.

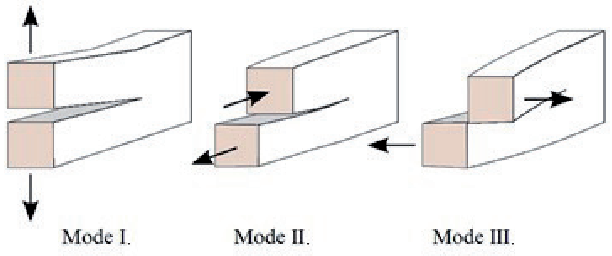


Fig. 1 Three basic modes of fracture [1].

2 Principle of fracture mechanics

An important subject of fracture mechanics is the analysis of stress concentration at the vicinity of crack tip. During compression or tension, existence of a crack in the material changes the state of distribution of stresses in the area near to crack tip and as a result failure can occur due to propagation of the crack. To describe the phenomenon, the theoretical, numerical and experimental methods are used to examine the stresses at the crack tip.

Koloszov [3], Inglis [4], and Muszhelishvili [5] have concentrated on the complex stress formulae to describe the stress states in the area of circular and oval holes in a linear elastic disk. For certain special cases, Westergaardin 1939 [6] proposed a complex stress-dependent solution for stresses condition. Rice and Sih in 1965 [7] introduced a solution for generalized two-dimensional problems for first time. A three dimensional analytical solution has been also developed by Sneddon and Lowengrub in 1969 [8]. Griffith in 1920 [9] proposed a new approach based on the principle of energy near the crack tip. According to Griffith theory, the stress state in the crack tip is singular (Fig. 2).This theory initiated a new direction in the development of fracture mechanics.

According to linear elasticity, the stress and displacement field in the crack tip can be determined by following equations [10].

$$\sigma_{ij}\left(r,\theta\right)=\frac{1}{\sqrt{2\pi R}}\left\{K_I f_{ij}^I\left(\theta\right)+K_{II} f_{ij}^{II}\left(\theta\right)+K_{III} f_{ij}^{III}\left(\theta\right)\right\},$$
$$i,j\Rightarrow x,y,z$$
$$u_i\left(r,\theta\right)=\frac{1}{2\pi}\sqrt{\frac{R}{2\pi}}\left\{K_I g_i^I\left(\theta\right)+K_{II} g_i^{II}\left(\theta\right)+K_{III} g_i^{III}\left(\theta\right)\right\},$$
$$i\Rightarrow x,y,z \tag{1}$$

3 Measurement of tensile mode fracture toughness

A wide variety of direct and indirect experimental methods have been developed to measure the Mode I. fracture toughness of materials [1, 2, 11]. A typical experimental arrangements widely adopted in determining the K_{Ic} of different materials, especially metals, is demonstrated in Fig. 3. The International Society for Rock Mechanics [2, 11] recommended three methods for measuring tensile fracture toughness of rocks using core based specimens. These are Short Rod (SR), Chevron Bend (CB) and Cracked Chevron Notched Brazilian Disc (CCNBD), as shown in Figs. 4, 5 and 6 respectively. The CB and CCNBD tests are most popular methods in rock fracture mechanics.

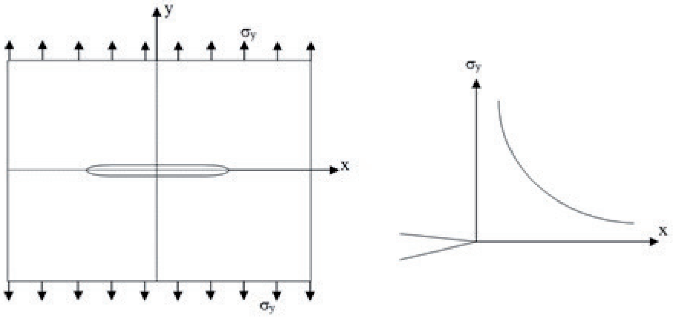


Fig. 2 Crack model and stress concentration according to Griffith [1].

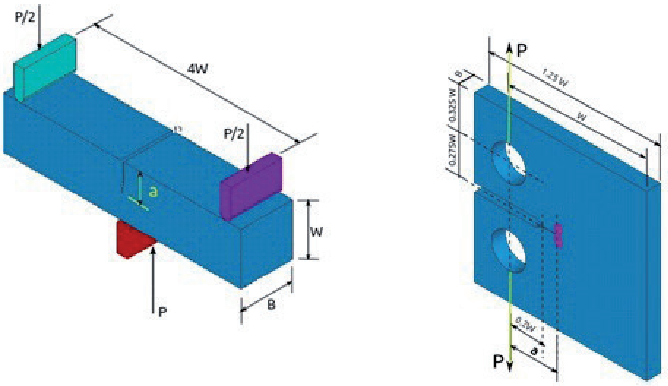


Fig. 3 Typical experimental set up for tensile fracture Toughness tests;
(a) Three-point bended notched specimen,
(b) Compact tension (CT) specimen [12].

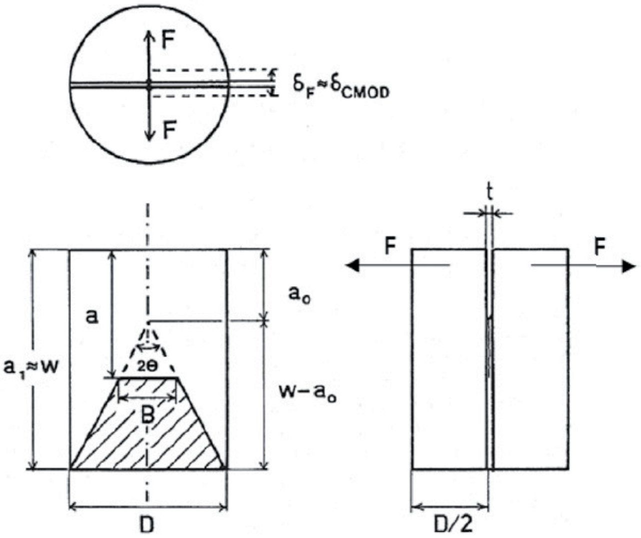


Fig. 4 Short Rod (RS) Specimen with "negative V" notch geometry and loading configuration [13].

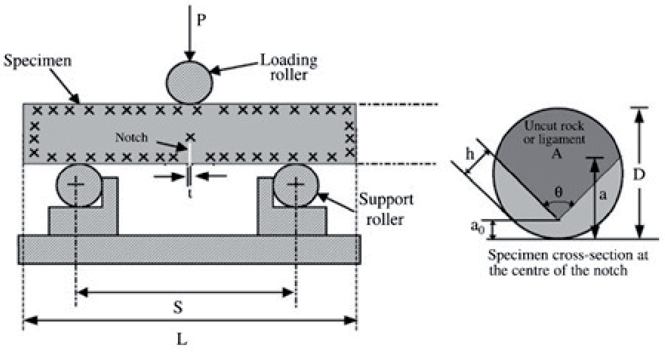


Fig. 5 Experimental arrangements and sample geometry of Chevron Bend (CB) test [13].

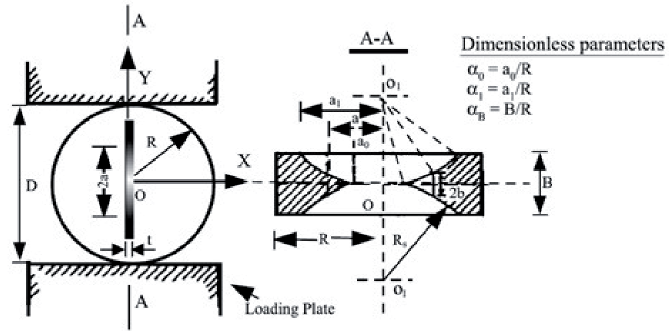


Fig. 6 Experimental arrangements and sample geometry of Cracked Chevron Notched Brazilian Disc (CCNBD) test [13].

As it can be established from rock fracture standard tests in Mode I., most of methods are indirect due to difficulty in apparatus arrangements and sample preparation. However, the values of K_{Ic} measured by indirect methods are always greater than obtained results from direct methods. This can increase uncertainty in designing of the rock structures based on fracture approach. Hence, the lack of a simple direct method in determining the tensile fracture of rocks is evident.

4 Adoption of CT test for rocks to propose a new direct tensile fracture toughness method

Compact tension specimen is a notched sample as shown in Fig. 3 and is a standard specimen in accordance with ASTM standards [12]. CT specimens are used extensively in the area of fracture mechanics, in order to establish Mode I. fracture toughness values for metals. The stress intensity factor at the crack tip of a compact tension specimen is calculated by following equation:

$$K_1 = \frac{P}{B}\sqrt{\frac{\pi}{W}}\left[\begin{array}{l} 16.7\left(\frac{a}{W}\right)^{1/2} - 104.7\left(\frac{a}{W}\right)^{3/2} + 369.9\left(\frac{a}{W}\right)^{5/2} \\ -573.8\left(\frac{a}{W}\right)^{7/2} + 360.5\left(\frac{a}{W}\right)^{9/2} \end{array}\right] \quad (2)$$

where P is the applied load, B is the thickness of the specimen, a is the crack length, and W is the width of the specimen.

The CT specimens have been prepared adopting three rock types in this investigation. These specimens are selected from granite as strength rock, sandstone as mid-strength rock and artificial gypsum-cement as low-strength rock. The geometry of CT sample and test equipment used in this paper is illustrated in Fig. 7. There have been used three similar specimens from each type of rocks to determine the Mode I. fracture toughness. The critical load and fracture toughness for three different rock types are presented in Table 1.

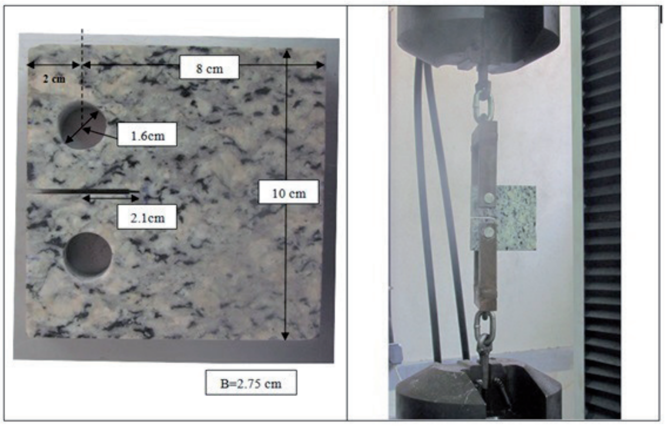
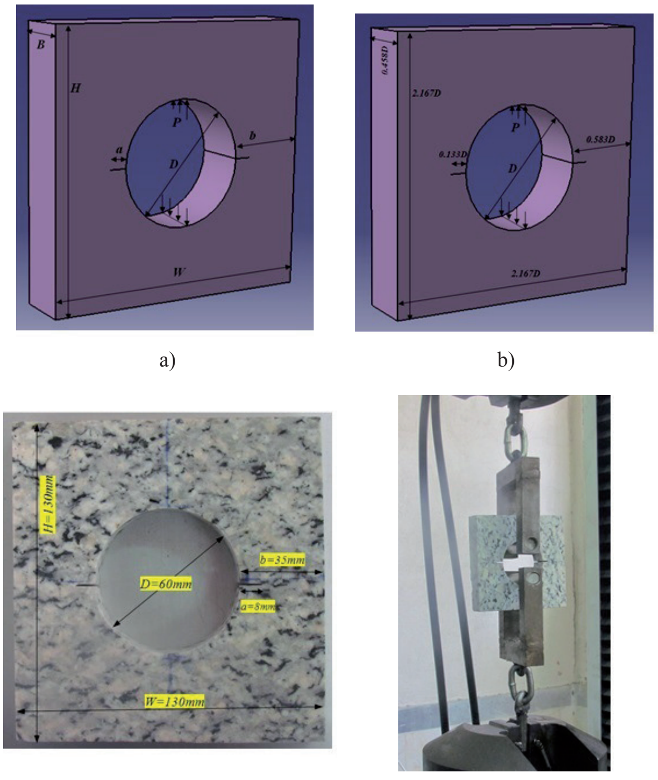


Fig. 7 The geometry of CT specimen and loading configuration.

Table 1 Critical loads and fracture toughness of different rock types by CT test.

Rock Type	Critical Load (kN)	Fracture Toughness $\left(MPa\sqrt{m}\right)$
Granite	2.024	1.427
Sandstone	0.872	0.615
Gypsum-cement	0.263	0.185

As can be seen, drilling of two holes in the CT specimen in rock is a difficult task. During the drilling of holes, the sample is broken due to vicinity of holes to each other. Hence, the more precision and time is needed to prepare the specimen. In spite of this effort, other drawback of the method is that the load required to initiate fracture is relatively low and thereby less precision will be in measurements. To overcome this problem, a new direct method is proposed to measure the fracture toughness of rock in Mode I. The geometry of proposed specimens is illustrated in Fig. 8. All dimensions is defined in term of central hole diameter, D (Fig. 8b). The central hole of sample with diameter, D was drilled by NX core barrel (D=6 cm) and the notch with length of a(a=8mm) was created using diamond wire saw of 0.3 mm thickness. The height, width and thickness of sample used in this study were 13 cm, 13 cm and 2.75 cm respectively.



a) b)

c) d)

Fig. 8 Proposed specimen geometry and test set up loading configuration.

To determine the fracture toughness by the proposed method, a new expression of stress intensity factor at the crack tip of proposed specimen is needed. For this purpose, a new stress intensity factor formula was developed using the finite element model. Figure 9 illustrates two dimensional finite element model of proposed sample presented in this paper. The applied load P was 100 kN and crack length to width ratios (*a/b*) were considered from 0.064 to 0.533 in these numerical models. The nodal stress intensity factors obtained from the FEM analysis for different ratios of *a/b* are presented in Table 2. The nodal stress intensity factors were normalized with regard to crack length (*a*) and far field nominal stress (σ). The normalized stress intensity factors, given as $f(a/b)$, is determined as follows:

$$f\left(a/b\right)=\frac{K_I^i}{\sigma\sqrt{\pi a}} \tag{3}$$

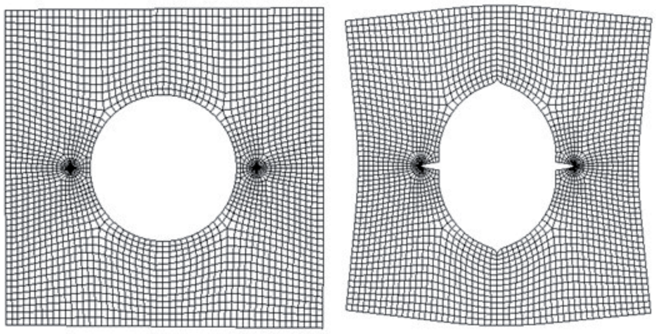
where, is the nodal stress intensity factor, and σ is the nominal stress (=P/2bB).

The function of $f(a/b)$ is determined by curve fitting of the normalized stress intensity factors with respect to ratio of *a/b*. The fourth order of polynomial was used to fit the normalized stress intensity factors in this study. Figure 10 demonstrates the variation of the function $f(a/b)$ with respect to *a/b*. Deduced expression to evaluate stress intensity factor for the proposed specimen is presented as:

$$K_I = f\left(a/b\right)\frac{P}{2bB}\sqrt{\pi a} \tag{4}$$

where

$$f\left(a/b\right)=596.1\left(a/b\right)^4-785.9\left(a/b\right)^3+372.3\left(a/b\right)^2$$
$$-69.30\left(a/b\right)+8.271$$



a) b)

Fig. 9 Finite element model for the proposed specimen:
a) original mesh; b) deformed mesh.

Table 2 Ratios of a/b, nodal stress intensity factor $\left(K_I^i\right)$, and dimensionless shape function $f(a/b)$.

a/b	K_I^i $\left(MPa\sqrt{m}\right)$	$\sigma\sqrt{\pi a}$ $\left(MPa\sqrt{m}\right)$	$f(a/b)$
0.06	0.336	0.063	5.301
0.07	0.358	0.072	4.964
0.08	0.389	0.083	4.656
0.10	0.435	0.099	4.393
0.12	0.512	0.122	4.197
0.16	0.653	0.159	4.120
0.23	0.977	0.226	4.312
0.40	2.175	0.396	5.489
0.53	3.565	0.528	6.747

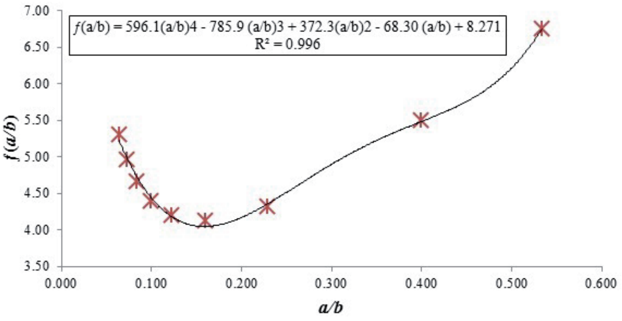


Fig. 10 Variation of the function $f(a/b)$ with respect to *a/b*.

The dimension of proposed specimen, critical load and fracture toughness for three different rock types obtained by new proposed method are presented in Table 3.

Table 3 Critical loads and fracture toughness of different rock types by proposed test.

Rock Type	Critical Load (kN)	Fracture Toughness $\left(MPa\sqrt{m}\right)$
Granite	4.058	1.475
Sandstone	1.806	0.657
Gypsum-cement	0.537	0.192

5 Conclusions

A new experimental method for determining Mode I. fracture toughness of rocks was introduced in this paper. The method is proposed as a simple direct method in rock fracture mechanics to measure tensile fracture toughness. This could approximately eliminate many of drawbacks existing in CT rock tests. The geometry of proposed specimen reveals that the preparation of sample in rock is easier than CT sample. Obtained results of fracture toughness from proposed method illustrated that the values of Mode I. fracture toughness for different rock types are close to CT's values. This implies that the developed stress intensity factor formula by FEM has correctly worked for proposed method. Also, critical loads experienced in proposed method are approximately twice the loads obtained from CT test. Hence, precision in measurement is improved in the new method.

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