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Case Study

Practical Fire Safety Assessment of Steel-beam Floors Made According to the Old Technologies – an Exemplary Case Study. Influence of the Initial Assumptions on the Final Results of Analyses

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Abstract
The purpose of this article is to present procedures and methods for assessing fire resistance of steel-beam floors with the joists hidden within the thickness of the slab. These technologies are currently experiencing their renaissance, both in contemporarily designed buildings and the existing ones, subjected to comprehensive redevelopment, refurbishment or modernization. Due to their simplicity and ease of execution, these floors are just perfect as technology ideal for repairs or alterations of buildings under use or in the case of need of complete replacement of existing floors with new ones. These arguments justify the need to raise the subject of proper safety assessment of these floors in relation to the regulations and requirements of laws applicable in the EU and pursuant to provisions of the latest codes for structural design. A significant part of the study consists of a suggestive computational example, which is a sort of guide, in which the author, by making detailed step-by-step calculations produces a finished pattern of procedure, intended for multiple use. The suggested method of procedure can be successfully used in the assessment of the fire resistance of floor structures with similar technical features. The computational example presented in the study shows that contrary to a popular belief, the use of standard fire model does not always lead to conservative estimates. In the article summary, the author formulates a number of practical applications and conclusions.

Keywords
fire, fire safety, structural element, steel joist, steel-beam floor, standard fire scenario, parametric fire scenario

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1 Introduction

New trends in the structural design and requirements arising from the content of legislation require participants in the construction process to face a difficult task of ensuring that the building and its systems are designed taking into account the danger of fire actions and that in the event of fire, the so-called basic requirements are satisfied, one of which concerns an appropriate structural resistance for a period of time specified in the technical and building regulations. This article has been developed in response to the needs of the building industry including the procedure for the assessment of fire safety of steel-beam floors in relation to the said renaissance and the growing popularity of this type of solutions - used equally in the newly designed projects as well as for the purpose of reconstruction, renovation or modernization of existing buildings. Even in the case of the existing structures, in the investment process there are very often situations, in which it is legally required to demonstrate that the adopted construction method and materials are safe and meet the requirements of currently applicable building regulations. Such situations can for example include a change in function of rooms, use of a building, load changes or a need to replace degraded structural members of the floor.

This article is to help people professionally associated with the construction industry, in particular designers, surveyors, fire protection experts, state authority building inspectors, personnel of building and architectural administration as well as monuments protection services, who in their practice face the need to assess structural safety taking into account fire impact. It is also aimed at pointing to difficulties and explaining any uncertainties that the assessors may encounter during their activities.

2 Procedures for assessing structural resistance under fire

2.1 Determining the actions

When checking the limit state of damage or excessive deformation of the cross-section, structural member or connection it must be demonstrated that in any of the anticipated design situations the design values of effects of actions shall not exceed the corresponding design resistance, which can be expressed in a simplified way by the formula:

$$E_d \le R_d \tag{1}$$

The same applies to the accidental design situation of fire, when the formula (1) may be modified to the following form:

$$E_{fi,d} \le R_{fi,d,t} \tag{1a}$$

where:

$E_{fi,d}$ – the design effect of actions for the fire situation, determined in accordance with [1],

$R_{fi,d,t}$ – the corresponding design resistance of the steel member in the fire design situation, at time *t*.

According to the code [2], in the case of persistent and transient design situations, the design effects of actions on structures should be established on the basis of the so-called fundamental combination, expressed by the following relationship:

$$\sum_{j\ge1}\gamma_{G,j}G_{k,j}\text{"+"}\gamma_P P\text{"+"}\gamma_{Q,1}Q_{k,1}\text{"+"}\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \tag{2}$$

where the sign „+" means, in general terms, that a given component implies "to be combined with".

As the application of the Eq. (2) generally leads to slightly higher estimates, which in turn results in higher consumption of materials, the national annexes to the code [2] sometimes recommend that in the permanent and transient design situation a so-called alternative combination should be adopted as authoritative, defined as a less favourable expression of the two below, described by the Eq. (3a) or (3b):

$$\sum_{j\ge1}\gamma_{G,j}G_{k,j}\text{"+"}\gamma_P P\text{"+"}\gamma_{Q,1}\psi_{0,1}Q_{k,1}\text{"+"}\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \tag{3a}$$

$$\sum_{j\ge1}\xi_j\gamma_{G,j}G_{k,j}\text{"+"}\gamma_P P\text{"+"}\gamma_{Q,1}Q_{k,1}\text{"+"}\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \tag{3b}$$

In the case of accidental design situations, which include fire, the combination of effects of actions for the ultimate limit states takes the form of:

$$\sum_{j\ge1}\xi_j\gamma_{G,j}G_{k,j}\text{"+"}\gamma_P P\text{"+"}\gamma_{Q,1}Q_{k,1}\text{"+"}\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \tag{4}$$

It is recommended that in the case of fire, regardless of the effect of temperature on the material properties, the A_d value should express the design value of indirect actions caused by fire, as determined individually for each design situation. In practical applications, in the case of steel structures, the A_d component is usually not taken into account, since the actual values of any additional axial forces resulting from the thermal elongation of the member are difficult to determine due to the lack of knowledge of the actual rigidity of supporting nodes. In addition, transverse deformations resulting from a fairly rapid decline of the Young modulus in the increased fire temperatures reduce the influence of longitudinal forces arising from the elongation.

The code also recommends that the pre-stressing force *P* should be considered as a permanent action caused by controlled forces or controlled forced structural strain. It ought to be pointed out to the need of distinguishing this type of pre-stressing from other types, such as pre-stressing with tendons or initially forced strains. As it is difficult to talk about the controlled structural pre-stressing in the case of fire, also the component P, which takes into account the effects of pre-stressing forces, is not practicable, therefore the Eq. (4) is simplified to the following form:

$$E_d = \sum_{j\ge1}G_{k,j}\text{"+"}\left(\psi_{1,1}\text{ or }\psi_{2,1}\right)Q_{k,1}\text{"+"}\sum_{i>1}\psi_{2,i}Q_{k,i} \tag{5}$$

As in the majority of fire situations we do not have to deal with more than one important component of variable actions, the above formula in practical applications is usually even further simplified, taking the form of:

$$E_d = \sum_{j\ge1}G_{k,j}\text{"+"}\left(\psi_{1,1}\text{ or }\psi_{2,1}\right)Q_{k,1} \tag{6}$$

The representative value of the variable action may be considered as the frequent value $\psi_{1,1}Q_{k,1}$ or, as an alternative – the quasi-permanent value $\psi_{2,1}Q_{k,1}$.

The use of quasi-permanent value $\psi_{2,1}Q_{k,1}$ or the frequent value $\psi_{1,1}Q_{k,1}$ may be specified in the national annexes to the code EN 1991-1-2 [1]. Generally the use of $\psi_{2,1}Q_{k,1}$ is recommended:

$$E_d = \sum_{j\ge1}G_{k,j}\text{"+"}\left(\psi_{1,1}\text{ or }\psi_{2,1}\right)Q_{k,1} \tag{7}$$

Characteristic combination of actions to be used to assess the irreversible serviceability limit states is expressed by the following formula:

$$\sum_{j\ge1}G_{k,j}\text{"+"}P\text{"+"}Q_{k,1}\text{"+"}\sum_{i>1}\psi_{0,i}Q_{k,i} \tag{8}$$

where the symbols that appear in Eq. (2) – (8) represent respectively:

- $G_{k,j}$ - characteristic value of permanent action j,
- P - relevant representative value of a pre-stressing action,
- A_d - design value of an accidental action,
- $Q_{k,1}$ - characteristic value of the leading variable action 1,
- $Q_{k,i}$ - characteristic value of the accompanying variable action i,
- $\psi_{0,1}$ - factor for combination value of the leading variable action 1,
- $\psi_{0,i}$ - factor for combination value of accompanying variable action i,
- $\psi_{1,1}$ - combination factor for frequent value of the leading variable action,
- $\psi_{2,1}$ - combination factor for quasi-permanent value of the leading variable action,
- $\psi_{2,i}$ - combination factor for quasi-permanent value of the accompanying variable action i,
- $\gamma_{G,j}$ - partial safety factor for permanent action j,
- $\gamma_{Q,1}$ - partial safety factor for leading variable action 1,
- $\gamma_{Q,i}$ - partial safety factor for accompanying variable action *i*,
- γ_P - partial safety factor for pre-stressing actions,
- ξ_j - reduction factor for permanent actions j.

2.2 Determining the critical temperature

The easiest way to assess resistance of a carbon-steel structure not exposed to instability phenomena, assuming a uniform temperature distribution at the section height and along the member length is the design in the time domain, which consists - in the simplest meaning - in determining the time during which the member heats up to the level of the so-called critical temperature. The critical temperature is understood as the structure temperature identified with the moment of total loss of load-bearing capacity of the member with a given stress-strain level as referred to normal conditions. A direct comparison of the time in which the heated member reaches the critical temperature in accordance with the requirements laid down by technical and construction regulations provides and answer to the question whether the member has sufficient resistance within the meaning of the fire safety requirements.

The value of the critical temperature may be determined with some approximation based on the Eq. (9). The relationship described by the formula below has only been provided in the form of a function of one variable – the cross-section degree of resistance utilization factor μ_0 in time $t = 0$, i.e. at the outbreak of fire,

$$\theta_{a,cr} = 39.19 \ln\left[\frac{1}{0.9674\mu_0^{3.833}} - 1\right] + 482 \tag{9}$$

where μ_0 must not be taken less than 0.013.

The Eq. (9) adopted in Eurocode 3 [3] was derived by inverting the formulae approximating the relationship describing the reduction factor of effective yield point $k_{y,\theta}$ as the function of the temperature of the steel member, obtained experimentally, in which the reduction factor was replaced by the degree of utilization μ_0. Due to such a simplification, the Eq. (9) is valid and can be directly applied only for the cases, where the loadbearing capacity in fire design situation is directly proportional to the effective yield strength, i.e. when no stability loss may occur.

Determination of the critical temperature in case of any stability loss can be made only by an iterative procedure, as shown e.g. in [4] or alternatively by incremental procedure, by successive verification in the load domain. The incremental procedure is convenient for computer implementation, whereas the iterative one is more suitable for traditional, hand calculations.

2.3 Fire gas temperature assessment (structural integrity assessment in the temperature domain)

<u>Assessment based on the standard temperature-time curve</u>

The standard temperature-time curve (also known as ISO 834 curve) is a conventional function used in scientific research to evaluate fire resistance of structural members and separate subsystems. Its course is to simulate conditions of a fully developed fire in the premises. The adopted model is simplified, since the gas temperature here is a function of only one variable - time, totally non-dependent on other important parameters that determine the actual course of fire, such as for example the type and distribution of accumulated combustible materials, size of fire compartment or ventilation conditions. The temperature of the fire gases described by the standard curve increases monotonically and does not take into account the cooling phase, which is incompatible with the nature of a real fire. This curve is of historical significance - used for several years to assess the behaviour of structures subjected to strong thermal actions, it has been adopted as a reference for fire resistance parameters (in particular the load-carrying resistance criterion "R"), quoted today in the technical and building regulations [5]. Despite some shortcomings of this fire model, it is still an essential tool for the analysis of the fire safety of structural members of buildings. There is a widespread belief that the estimation of structural fire safety based on the standard description of fire leads to conservative solutions, not reasonable in economic terms, which is not always true.

The standard temperature-time curve is described by the formula:

$$\theta_g = 20 + 345\log_{10}\left(8t + 1\right) \tag{10}$$

where:

- θ_g - gas temperature in the fire compartment, or near the member, [°C];
- t - time (understood as the duration of the fire since its flashover), [minutes].

<u>Assessment based on the parametric temperature-time curve</u>

Fire model described by the parametric temperature-time curve, characterized in more detail in Annex A to the standard [1] constitutes a departure from the aforementioned simplifications. The function describing the course of parametric fire is still a function of one variable - time, however in this case, the time function is dependent on three important physical parameters, such as: fire load density, thermal absorptivity of enclosures separating a given fire compartment, and the size of the ventilation openings in walls. The parametric curves are used in the case of fire compartments of the floor area not exceeding 500m^2, with no openings in the horizontal enclosures and a maximum height of the compartment of 4.0 m. In many situations, especially these first two constraints can be a major obstacle to the use of parametric description for the fire safety analysis of structural members, e.g. in the case of large-surface facilities.

Parametric curve is composed of two fragments, one of which comprises a heating phase and the other one – a cooling phase.

The temperature-time curve in the heating phase is given by:

$$\theta_g = 20 + 1325\left(1 - 0.324e^{-0.2t^*} - 0.204e^{-1.7t^*} - 0.472e^{-19t^*}\right) \tag{11}$$

and in the cooling phase is given by:

(a) in the case of fire controlled by ventilation:

$$\theta_g = \theta_{max} - 625\left(t^* - t^*_{max}\right) \text{ for } t^*_{max} \leq 0,5 \quad (12a)$$

$$\theta_g = \theta_{max} - 250\left(3 - t^*_{max}\right)\left(t^* - t^*_{max}\right) \text{ for } 0.5 < t^*_{max} < 2 \quad (12b)$$

$$\theta_g = \theta_{max} - 250\left(t^* - t^*_{max}\right) \text{ for } t^*_{max} \geq 2 \quad (12c)$$

(b) in the case of fire controlled by fuel supply:

$$\theta_g = \theta_{max} - 625\left(t^* - \Gamma t_{lim}\right) \text{ for } t^*_{max} \leq 0,5 \quad (13a)$$

$$\theta_g = \theta_{max} - 250\left(3 - t^*_{max}\right)\left(t^* - \Gamma t_{lim}\right) \text{ for } 0.5 < t^*_{max} < 2 \quad (13b)$$

$$\theta_g = \theta_{max} - 250\left(t^* - \Gamma t_{lim}\right) \text{ for } t^*_{max} \geq 2 \quad (13c)$$

Figures given in the above formulas mean, respectively:

θ_g - gas temperature in the fire compartment, or near the member, [°C];

t - time (duration of the fire), [h];

$t^* =$ $t \cdot \Gamma$, [h];

$\Gamma =$ $\left(\dfrac{O/0.04}{b/1160}\right)^2$, [-];

b - thermal absorptivity of the total enclosure: $b = \sqrt{(bc\lambda)}$, with the following limits: $100 \leq b \leq 2200$, [J/(m^{2}s$^{1/2}$K)];

ρ - density of the boundary of enclosure, [kg/m^3];

c - specific heat of the boundary of enclosure, [J/kgK];

λ - thermal conductivity of the boundary of enclosure, [W/mK];

O - opening factor of the fire compartment: $O = A_V\sqrt{(h_{eq})} / A_t$, with the following limits: $0.02 \leq O \leq 0.20$, [m$^{1/2}$];

A_V - total area of vertical openings on all walls, [m^2];

h_{eq} - weighted average of window heights on all walls, [m];

A_t - total area of enclosure (walls, ceiling and floor, including openings), [m^2]

$t^*_{max} = (0.0002 \cdot q_{t,d} / O)\cdot\Gamma$, [h];

$q_{t,d}$ - the design value of the fire load density related to the total surface area of the enclosure A_t, whereby: $q_{t,d} = q_{f,d} \cdot A_f / A_t$, respecting the following limits: $50 \leq q_{t,d} \leq 1000$, [MJ/m^2];

$q_{f,d}$ - the design value of the fire load density A_f, [MJ/m^2];

Detailed rules for determining the values of the parameters b, $q_{f,d}$ and t_{lim} are provided in the text of the Annex A to the code [1].

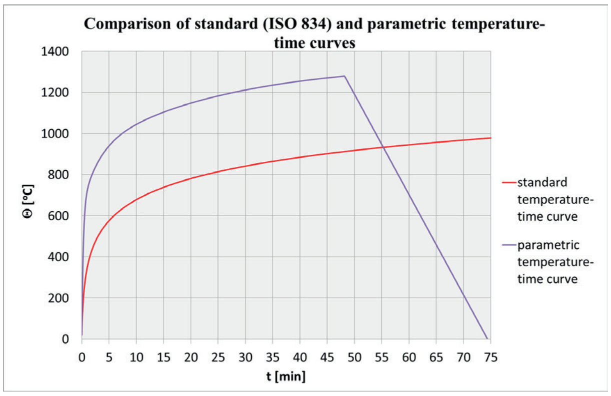

Fig. 1 Comparison of standard temperature-time and parametric temperature-time curves

The Fig. 1 shows a comparison of two curves describing the temperature-time relationship (standard and parametric) determined for the same specific conditions of the fire compartment, adopted in the computational example. Analysis of the functions drawn is contrary to the popular belief, quite commonly propagated in a number of literature references, that the standard fire model is in every case a more conservative approach, resulting in excessively safe estimates of fire safety design. The specific configuration of the fire compartment, resulting from a large supply of fuel, with at the same time favourable ventilation capacity, may in certain circumstances result in much worse conditions within the meaning of the environmental impacts than resulting from the description using the ISO 834 nominal curve.

Special attention should be paid to this fact and one should be extremely self-restraint and humble with regard to the assessment of potential fire environment conditions.

2.4 Determination of the section exposure factor

Formulas enabling determination of computational values of the section factor A_m/V of some unprotected steel members are provided in the Table 4.2 of the code [3]. Analogically, similar formulas but relating to some steel members insulated by fire protection material allowing the determination of design values of the section factor A_p/V are given in the Table 4.3 of the same code [3].

2.5 Calculation of the steel member temperature

Calculation of the temperature of a steel structural member subjected to heating under fire conditions may be carried out using incremental procedures that differ slightly from each other, depending on whether they relate to the unprotected members or members insulated by any fire protection material.

Unprotected internal steelwork

For an equivalent uniform temperature distribution within the cross-section, the increase of temperature $\Delta\theta_{a,t}$ in an unprotected steel member during a time interval Δt should be determined from:

$$\Delta\theta_{a,t} = k_{sh}\frac{A_m/V}{c_a\rho_a}\dot{h}_{net,d}\Delta t \quad (14)$$

where:

k_{sh} - correction factor for the shadow effect;

A_m/V - the section factor for unprotected steel members, [1/m];

A_m - the surface area of the member per unit length, [m²/m];

V - the volume of the member per unit length, [m³/m];

c_a - temperature-dependent specific heat of steel, [J/kgK];

$h_{net,d}$ - the design value of the net heat flux per unit area, [W/m²];

Δt - the time interval, [seconds];

ρ_a - unit mass (density) of steel, [kg/m³].

So as to reach the required level of the calculation accuracy, the time interval Δt cannot be greater than 5 seconds.

The section factor A_m/V means the ratio of the fire exposed area (heated area) to the unit volume of the heated section, which in turn brings down to the ratio of the circumference of the section of a heated member to its cross-sectional area.

The value expressing the total capacity of thermal stresses on the member surfaces exposed to fire is determined by the design value of the net heat flux per unit area $h_{net,d}$. Its size should be determined taking into account the heat flow by convection and by radiation, according to the equation:

$$\dot{h}_{net,d} = \dot{h}_{net,c} + \dot{h}_{net,r} \quad (15)$$

where:

- $\dot{h}_{net,c}$ the net convective heat flux component, and
- $\dot{h}_{net,r}$ the net radiative heat flux component.

The net convective heat flux component should be determined by:

$$\dot{h}_{net,c} = \alpha_c \cdot \left(\theta_g - \theta_m\right) \quad (16)$$

where:

α_c - the coefficient of heat transfer by convection, [W/m²K];

θ_g - the gas temperature in the vicinity of the fire exposed member, (due to assumed fire scenario), [°C];

θ_m - the surface temperature of the member, [°C].

The net radiative heat flux component per unit surface area is determined by:

$$\dot{h}_{net,r} = \Phi \cdot \varepsilon_m \cdot \varepsilon_f \cdot \sigma \cdot \left[\left(\theta_r + 273\right)^4 - \left(\theta_m + 273\right)^4\right] \quad (17)$$

where:

Φ - the configuration factor, usually taken as equal to 1.0;

ε_m - the surface emissivity of the member;

ε_f - the emissivity of the fire;

σ - the Stephan Boltzmann constant (5.67·10-8 W/m2K4)

θ_r - the effective radiation temperature of the fire environment, (for practical purposes it can be assumed that $\theta_r = \theta_g$), [°C];

θ_m - the surface temperature of the member, [°C].

Internal steelwork insulated by fire protection material

For a uniform temperature distribution in a cross-section, the temperature increase $\Delta\theta_{a,t}$ of an insulated steel member during the time interval Δt should be obtained from:

$$\Delta\theta_{a,t} = \frac{\lambda_p A_p/V}{d_p c_a \rho_a}\frac{\left(\theta_{g,t} - \theta_{a,t}\right)}{\left(1+\varphi/3\right)}\Delta t - \left(e^{\varphi/10} - 1\right)\Delta\theta_{g,t} \quad (18)$$

(but $\Delta\theta_{a,t} \geq 0$ when $\Delta\theta_{g,t} \geq 0$),

with:

$$\varphi = \frac{c_p\rho_p}{c_a\rho_a} d_p A_p/V$$

where:

A_p/V - the section factor for steel members insulated by fire protection material;

A_p – the appropriate area of fire protection material per unit length of the member, [m²/m];

V – the volume of the member per unit length, [m³/m];

c_a – the temperature dependent specific heat of steel, [J/kgK], described by the following formulas:

$c_a = 425 + 7.73\cdot10^{-1}\cdot\theta_a - 1.69\cdot10^{-3}\cdot\theta_a^2 + 2.22\cdot10^{-6}\cdot\theta_a^3$, for $20^oC \leq \theta_a \leq 600^oC$ (19a)

$$c_a = 666 + \frac{13022}{738 - \theta_a}\text{, for } 600^oC \leq \theta_a \leq 735^oC \quad (19b)$$

$$c_a = 545 + \frac{17820}{\theta_a - 731}\text{, for } 735^oC \leq \theta_a \leq 900^oC \quad (19c)$$

$c_a = 650$, for $900^oC \leq \theta_a \leq 1200^oC$ (19d)

c_p – the temperature independent specific heat of the fire protection material, [J/kgK];

d_p – the thickness of the fire protection material, [m];

Δt – the time interval, [seconds];

$\theta_{a,t}$ – the steel temperature at time t, [°C];

$\theta_{g,t}$ – the ambient gas temperature at time t, [°C];

$\Delta\theta_{g,t}$ – the increase of the ambient time temperature during the time interval θ_t, [K];

λ_p – the thermal conductivity of the fire protection system, [W/mK];

ρ_a – the unit mass of steel, [kg/m³];

ρ_p – the unit mass of the fire protection material, [kg/m³]

So as to reach the required level of the calculation accuracy, in the case of steel members insulated with fire protection material, the time interval Δt cannot be greater than 30 seconds. Such a considerable difference in the value of the time interval between insulated and unprotected members is due to a greater thermal inertia of the latter. In the calculation example developed for the purpose of this article the same time interval of 5 seconds has been applied in both cases.

Simplified procedure for calculating the temperature of the steel member subjected to thermal actions of fire

In the literature, for example [6], one also may find simplified formulas, allowing estimation of the relationship between the temperature of the steel member analysed (expressed in °C), the time of exposure to fire (expressed in minutes) and the properties of the fire protection coating, if any. These formulas, quoted as in the paper [7], allow inter alia determination of the time required to heat a steel structural member to a predetermined temperature. The time of heating to a temperature θ_a of a steel element exposed to fire, protected with a coating of light insulating material with a thickness of d_p, is provided by the equation:

$$t = 40\left(\theta_a - 140\right)\left[\frac{d_p}{\lambda_p}\cdot\frac{V_m}{A_m}\right]^{0.77} \tag{20}$$

This time is longer than the time of heating to a temperature θ_a of an unprotected steel element exposed to fire, which can be estimated using the following equation:

$$t = 0.54\left(\theta_a - 50\right)\left[\frac{A_m}{V_m}\right]^{-0.60} \tag{21}$$

By transforming these equations in respect of the temperature, in order to keep the same convention as the one adopted in the code [1], we obtain respectively

- for insulated steelwork:

$$\theta_a = 140 + \frac{t}{40\left[\frac{d_p}{\lambda_p}\frac{V_m}{A_m}\right]^{0.77}} \tag{22}$$

- for unprotected steelwork:

$$\theta_a = 50 + \frac{t}{0{,}54\left[\frac{A_m}{V_m}\right]^{-0.60}} \tag{23}$$

Unfortunately, the author of the monograph [6] did not provide, following the original source, the restrictions on the use of these equations, which reduces the possibility of their practical application, especially that they do not provide sufficiently precise estimates for the entire possible range of fire temperatures, covered by the regulations of the codes [1] and [3]. He also did not specify for which type of fire model the equations provided above would estimate the response of the steel structure to the effect of the temperature field with the greatest accuracy.

For comparison, the lines showing the course of functions described by the Eq. (22) and Eq. (23) are shown on Fig. 5 and Fig. 6. Analysis of the drawings confirms small accuracy of the suggested approach, particularly with respect to the curves of heating members subjected to parametric fire actions.

3 Calculation example dedicated to fire resistance assessment of ceramic steel-beam floors with middleweight slab

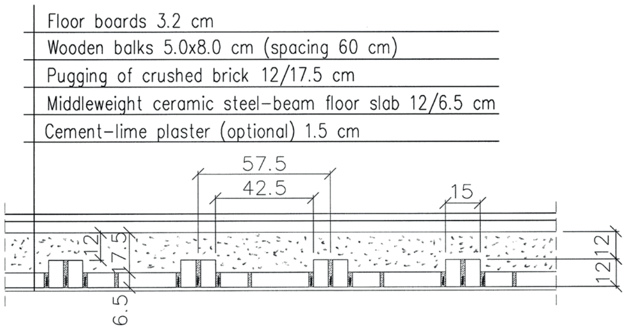

Fig. 2 Cross section through the ceramic slab of steel-beam floor with middleweight slab

Summary of mechanical action per 1 m² of the floor cross section:

Table 1 Permanent actions

No.	Description of action	Charact. value [kN/m²]
1	Floor boards 3.2 cm thick: 0.032·5.5 = 0.180kN/m²	0.18
2	Wooden balks 5 × 8 cm spacing approx. 60 cm: 0.05·0.08·5.5/0.60 = 0.040 kN/m²	0.04
3	Pugging of crushed brick: (0.15·0.12 + (0.575–0.15)·0.175)·18.0/0.575 = 2.890 kN/m²	2.89
4	Ceramic steel-beam floor slab (middleweight): (0.15·0.12 + (0.575–0.15)·0.065)·18.0/0.575 = 1.430 kN/m²	1.43
5	Rabitz-type wire mesh on beam flanges (omitted)	0.00
6	Cement-lime plaster 1.5 cm thick (adopted with a margin instead of the weight of fireproofing plaster): 0.015·19.0 = 0.280 kN/m2	0.28
	∑:	**4.82**

Table 2 Variable actions

No.	Description of action	Charact. value [kN/m²]
1	Uniformly distributed imposed load - area of category A (areas for domestic and residential activities) – Floors [2.000 kN/m²]	2.00
	∑:	**2.00**

Table 3 Permanent actions per single floor beam (with beam spacing of 1.20m)

No.	Description of action	Charact. value [kN/m²]
1	Permanent load of the floor slab: 4.82·1.20 = 5.780 kN/m	5.78
2	Dead weight of the floor beam IPN240: 36.2·9.81/1000 = 0.360 kN/m	0.36
3	Weight of the concrete encasing of the beam section upper part (omitted) [0.000 kN/m]	0.00
	∑:	**6.14**

Table 4 Variable actions per single floor beam (with beam spacing of 1.20 m)

No	Description of action	Charact. value [kN/m²]
1	Uniformly distributed imposed load - area of category A (areas for domestic and residential activities) – Floors of 120 cm wide slab: 2.000·1.20 = 2.400 kN/m	2.40
	∑:	**2.40**

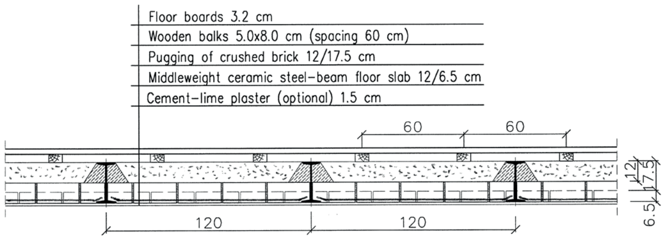

Fig. 3 Cross section through the floor, perpendicular to the directions of steal beams

The following basic data have been adopted:

- Steel grade: S235
- Steel yield point: f_y=235 N/mm²
- Steel density: ρ_a = 7850 kg/m³
- Characteristic value of permanent loads: g = 6.14 kN/m
- Characteristic value of variable loads: q = 2.40 kN/m
- Partial safety factor value for permanent loads: γ_G = 1.35
- Partial safety factor value for variable loads: γ_Q = 1.50
- Combination coefficient value for leading variable action: $\psi_{0,1}$ = 0.7
- Reducing coefficient value for permanent actions: ξ = 0.85
- Combination coefficient value for quasi-permanent value of the leading variable action in an accidental design situation: $\psi_{2,1}$ = 0.3 (as in the residential areas).

Mechanical actions at ambient temperature:

- characteristic value (to check serviceability limit states)

$p_k = g_k + q_k$ = 6.14 + 2.40 = 8.54 kN/m

- design value (to check Ultimate Limit States - ULS in normal conditions) determined according to general rules on the basis of the Eq. (2):

$p_d = \gamma_G g_k + \gamma_Q q_k$ = 1.35 · 6.14 + 1.50 · 2.40 = 11.89 kN/m

- design values (to check ULS in normal conditions) determined according to recommendations of the national annex on the basis of the Eq. (3a) and Eq. (3b):

$p_d = \gamma_G g_k + \gamma_Q \psi_{0,1} q_k$ = 1.35 · 6.14 + 1.50 · 0.7 · 2.40 = 10.81 kN/m

$p_d = \xi \gamma_G g_k + \gamma_Q \psi_{0,1} q_k$ = 0.85 · 1.35 · 6.14 + 1.50 · 2.40 = 10.65 kN/m

Following the recommendations of the national annex to the code [2], the less favourable of the two values calculated above has been adopted for further calculations, namely: p_d= 10.81 kN/m.

To be even more conservative, the value determined according to general rules could be adopted, which is at the same time the maximum of the three optionally determined design combination values of loads: p_d= 11.89 kN/m. The final decision in this regard is left to the designer.

Mechanical actions under fire (design value):

$p_{fi} = g_k + \psi_{2,1} q_k$ = 6.14 + 0.3 · 2.40 = 6.46 kN/m

Please note, that applying the frequent value of the variable action, as recommended e.g. in Polish national annex $\psi_{1,1} q_k$ = 0.5 · 2.40, one would get more conservative results.

Fig. 4 shows a configuration of the rooms on the repeatable storey plan of a sample residential building. Calculations have been performed for the room limited with structural axes 1–3 and B–C, considering them to be representative and robust, both in the so-called standard design case as well as in the case of an accidental design situation under fire.

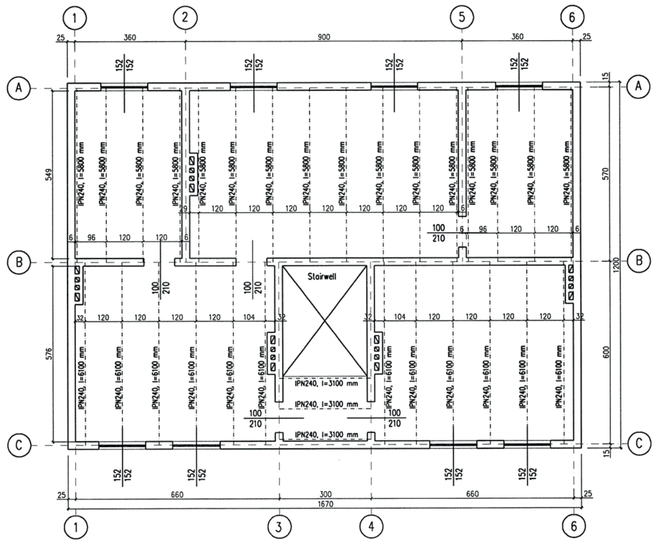

Fig. 4 An example of a repeatable storey of a residential building

Joist design length:

$l_o = l_s + c$, for $c \le 15 + \dfrac{h}{3}$,

where:

l_s – the clear span of joists (between walls),

h – the height of the joist section.

Thus, in our present case:

$l_o = l_s + c = 576 + \left(15 + \dfrac{24}{3}\right) = 576 + 23 = 599\ \text{cm} \rightarrow$ adopted l_o = 6.0 m.

IPN240 beam section has been adopted with the following characteristic parameters:

IPN 240 section dimensions:

h=240.0mm, t_w=8.7mm, b_f=106.0mm, t_f=13.1mm, r=8.7mm.

Geometrical characteristics of the section:

J_y = 4250.0cm⁴, J_z = 221.0cm⁴, A = 46.10cm², i_y = 9.590cm, i_z = 2.200cm, W_y = 354.0cm³, W_z = 41.70cm³, $W_{pl,y}$ = 412.0cm³, $W_{pl,z}$ = 70.00cm³.

3.1 Checking the member load-bearing capacity at normal (ambient) temperature

The maximum design value of the bending moment:

$$M_{Ed} = \frac{p_d \cdot l_o^2}{8} = \frac{10.81 \cdot 6.0^2}{8} = 48.65\,\text{kNm}$$

The maximum design value of the shear force:

$$V_{Ed} = \frac{p_d \cdot l_o}{2} = \frac{10.81 \cdot 6.0}{2} = 32.43\,\text{kN}$$

Checking the class of the cross-section:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{235}} = 1,0$$

Flange:

$$\frac{c}{t} = \frac{(106 - 8.7 - 2 \cdot 8.7)}{2 \cdot 13.1} = 3.05 < 9\varepsilon = 9 \rightarrow$$

the cross-section of Class 1

Web:

$$\frac{c}{t} = \frac{240 - 2 \cdot 13.1 - 2 \cdot 8.7}{8.7} = 22.57 < 72\varepsilon = 72 \rightarrow$$

the cross-section of Class 1

Therefore, the entire cross-section meets the requirements of class 1 section.

It has been assumed that due to the fact of concrete encasing of the upper portions of the steel joist section, it is protected against lateral torsional buckling through continuous lateral bracing of compression flange. Therefore, checking the member resistance is reduced to the issue of verifying the pure resistance of the cross-section.

Bending moment resistance of the cross-section:

$$M_{pl,Rd} = \frac{W_{pl,y} \cdot f_y}{\gamma_{M0}} = \frac{412 \cdot 23.5}{1.0} = 9682\text{kNcm} = 96.82\text{kNm}$$

$$\frac{M_{Ed}}{M_{pl,Rd}} = \frac{48.65}{96.82} = 0.50 < 1.0 \rightarrow \text{resistance condition is met}$$

Shear resistance of the cross-section:

$$A_{V,z} = A - 2bt_f + (t_w + 2r) \cdot t_f =$$
$$46.1 - 2 \cdot 10.6 \cdot 1.31 + (0.87 + 2 \cdot 0.87) \cdot 1.31 = 21.75\text{cm}^2$$

however not less than:

$$\eta h_w t_w = 1.0 \cdot (24 - 2 \cdot 1.31 - 2 \cdot 0.87) \cdot 0.87 = 17.09\text{cm}^2$$

$$V_{pl,Rd} = \frac{A_{V,z} \cdot f_y}{\sqrt{3} \cdot \gamma_{M0}} = \frac{21.75 \cdot 23.5}{\sqrt{3} \cdot 1.0} = 295.10\text{kN}$$

$$\frac{V_{Ed}}{V_{pl,Rd}} = \frac{32.43}{295.10} = 0.11 < 1.0 \rightarrow \text{resistance condition is met}$$

Serviceability Limit State - SLS condition:

$$u = \frac{5}{384} \frac{p_k l_o^4}{EI_y} = \frac{5}{384} \frac{8.54 \cdot 6.0^4}{210 \cdot 10^6 \cdot 4250 \cdot 10^{-8}} = 0.016\text{m} = 1.6\text{cm}$$

$$u_{allow} = \frac{l_o}{250} = \frac{600}{250} = 2.4\text{cm}$$

$$\frac{u}{u_{allow}} = \frac{1.6}{2.4} = 0.67 < 1.0 \rightarrow \text{SLS condition is met.}$$

3.2 Checking the cross-section resistance of the member under fire

The maximum design value of the bending moment: kNm

$$M_{fi,Ed} = \frac{p_{fi} \cdot l_o^2}{8} = \frac{6.86 \cdot 6.0^2}{8} = 30.87\text{kNm}$$

The maximum design value of the shear force: kN

$$V_{fi,Ed} = \frac{p_{fi} \cdot l_o}{2} = \frac{6.86 \cdot 6.0}{2} = 20.58\text{kN}$$

Checking the class of the cross-section under fire:

$$\varepsilon = 0.85\sqrt{\frac{235}{f_y}} = 0.85\sqrt{\frac{235}{235}} = 0.85$$

Flange:

$$\frac{c}{t} = \frac{(106 - 8.7 - 2 \cdot 8.7)}{2 \cdot 13.1} = 3.05 < 9\varepsilon = 9 \cdot 0.85 = 7.65 \rightarrow$$

the cross-section of Class 1

Web:

$$\frac{c}{t} = \frac{240 - 2 \cdot 13.1 - 2 \cdot 8.7}{8.7} = 22.57 < 72\varepsilon = 72 \cdot 0.85 = 61.2 \rightarrow$$

the cross-section of Class 1

Therefore, the entire cross-section meets the requirements of class 1 section.

Determination of the value of the cross-section degree of resistance utilisation factor at the $t = 0$ time of the fire duration:

$$\mu_0 = \frac{E_{fi,d}}{R_{fi,d,0}} = \frac{M_{fi,Ed}}{M_{fi,\theta,Rd(t=0)}}$$

$$M_{fi,\theta,Rd(t=0)} = k_{y,\theta}\left(\frac{\gamma_{M,0}}{\gamma_{M,fi}}\right) \cdot M_{Rd} = k_{y,\theta}\left(\frac{\gamma_{M,0}}{\gamma_{M,fi}}\right) \cdot W_{y,pl} \cdot f_y$$

where:

$\gamma_{M,0}$ – partial safety factor relating to the material properties at ambient temperature; $\gamma_{M,0} = 1.0$,

$\gamma_{M,fi}$ – partial safety factor relating to the material properties at increased temperature; $\gamma_{M,fi} = 1.0$,

$k_{y,\theta}$ - reduction factor of effective yield point

$$M_{fi,\theta,Rd(t=0)} = k_{y,\theta}\left(\frac{\gamma_{M,0}}{\gamma_{M,fi}}\right)\cdot W_{pl,y}\cdot f_y = 1.0\left(\frac{1.0}{1.0}\right)\cdot 412\cdot 23.5 =$$
$$= 9682\text{ kNcm} = 96.82\text{kNm}$$

$$\mu_0 = \frac{M_{fi,Ed}}{M_{fi,\theta,Rd(t=0)}} = \frac{30.87}{96.82} = 0.319$$

Thus:

$$\theta_{a,cr} = 39.19\ln\left[\frac{1}{0.9674\mu_0^{3.833}} - 1\right] + 482 =$$
$$= 39.19\ln\left[\frac{1}{0.9674\cdot 0.319^{3.833}} - 1\right] + 482 = 654.45°\text{C}$$

<u>Checking the cross-section resistance at the estimated critical temperature:</u>

The calculations were performed assuming $\theta_{a,cr}$ = 655 °C

The value of reduction factor of effective yield point at the temperature of 655°C equals:

$$k_{y,\theta} = 0.230 + \frac{0.470 - 0.230}{100}(700 - 655) = 0.338$$

Thus:

$$M_{fi,\theta,Rd(\theta=655°C)} = k_{y,\theta}\left(\frac{\gamma_{M,0}}{\gamma_{M,fi}}\right)\cdot W_{pl,y}\cdot f_y =$$
$$= 0.338\left(\frac{1.0}{1.0}\right)\cdot 412\cdot 23.5 = 3273\text{kNcm} = 32.73\text{kNm}$$

and the resistance condition:

$$\frac{M_{fi,Ed}}{M_{fi,\theta,Rd(\theta=655°C)}} = \frac{30.87}{32.73} = 0.943 < 1.0$$

This shows that the critical temperature value estimated based on the standard Eq. (8) was calculated with a certain approximation and the direct checking of the resistance condition still showed a margin of nearly 6%.

The critical temperature value can be a bit more precisely determined by an iterative method, by determining the value of the reduction factor of effective yield point for successive approximations of the member temperature values, carrying out the calculations by the time when the cross-section utilisation rate $\frac{M_{fi,Ed}}{M_{fi,\theta,Rd}}$ reaches the value as close as possible to 1.0.

Using the iterative method, the critical temperature has been specified at 663°C, for which the value of reduction factor of effective yield point is:

$$k_{y,\theta} = 0.230 + \frac{0.470 - 0.230}{100}(700 - 663) = 0.319$$

Therefore:

$$M_{fi,\theta,Rd(\theta=663°C)} = k_{y,\theta}\left(\frac{\gamma_{M,0}}{\gamma_{M,fi}}\right)\cdot W_{pl,y}\cdot f_y =$$
$$= 0.319\left(\frac{1.0}{1.0}\right)\cdot 412\cdot 23.5 = 3089\text{kNcm} = 30.89\text{kNm}$$

and the resistance condition:

$$\frac{M_{fi,Ed}}{M_{fi,\theta,Rd(\theta=663°C)}} = \frac{30.87}{30.89} = 0.999 \approx 1.0$$

In fact, there is no reasonable need for calculations with such a great accuracy because the procedure for determining the value of reduction factor of effective yield point $k_{y,\theta}$ itself contains an error of approximation. Hence the example presented above should be considered only as illustrative, since the difference in estimating the value of the critical temperature of 8°C does not have, from a technical point of view, a greater importance for the assessment of structural resistance to fire factors in the time aspect.

In this computational example, the following values of the individual characteristic parameters mentioned earlier in the theoretical and descriptive section have been adopted:

αc = 25.0 W/m²K - for the calculation of the standard temperature-time curve (based on [1], §3.2.1(2))

αc = 35.0 W/m²K - for the calculation of the parametric temperature-time curve (based on [1], §3.3.1.1(3))

ε_m = 0.7 (based on [3], §2.2(2))

ε_f = 1.0 (based on [1], §3.1(6))

Φ = 1.0 (based on [1], §3.1(7))

σ = 5.67·10-8 W/m²K⁴ (based on [1], §3.1(6))

k_{sh} = 1.0

ρ_a = 7850 kg/m³

Properties of the protection material (spray application of cement mortar with vermiculite aggregate has been adopted):

ρ_p = 550 kg/m³

c_p = 1100 J/kgK

d_p = 0.008 m = 8 mm

λ_p = 0.12 W/mK

3.3 Fire Gas temperature assessment (structural integrity assessment in the temperature domain)

This example uses an alternative approach, adopting for the sake of comparison the description of fire gas temperature according to:

a. standard temperature-time curve (ISO 864)
b. parametric fire curve.

In each case, calculations have been made for two variants:

a. assuming that a steel beam is not protected against heating by means of fireproof mortars (this is in fact a situation we face in the case of unplastered floors, i.e. for example floors of basements or rooms in industrial or farm buildings)
b. assuming that the beam flange (i.e. the bottom surface of the floor) is plastered in a sealed manner using light fireproof cement mortar with the addition of vermiculite aggregate and the plaster coating adheres well to the substrate.

3.4 Determination of section exposure factor

In the situation analysed in this calculation example there may be, depending on the quality and method of the floor construction, one of the four cases that we are going to consider independently and adopt, in a conservative manner, the least favourable value for further calculations. Each time, when determining the section factor, it ought to be remembered that, in general, it is the ratio of the heated area to the cross sectional area of the heated part of the member.

Case 1

The floor is unplastered and the lower beam flange is flushed out with the underside plane of the slab so that it is exposed to fire temperatures only from the bottom.

$$\frac{A_m}{V} = \frac{b}{bt_f} = \frac{1}{t_f} = \frac{1}{13.1} = 0.076\frac{1}{\text{mm}} = 76.34\frac{1}{\text{m}}$$

Case 2

The floor is unplastered and the lower beam flange protrudes entirely below the underside plane of the slab so that it is exposed to fire temperature on three sides - from the sides and the bottom.

$$\frac{A_m}{V} = \frac{b+2t_f}{bt_f} = \frac{106+2\cdot 13.1}{106\cdot 13.1} = \frac{132.2}{1388.6} = 0.095\frac{1}{\text{mm}} = 95.20\frac{1}{\text{m}}$$

Case 3

The ceiling is plastered with the use of fireproof plaster coating and the lower beam flange is flushed out with the underside plane of the slab so that it is exposed to fire temperatures only from the bottom.

$$\frac{A_p}{V} = \frac{b}{bt_f} = \frac{1}{t_f} = \frac{1}{13.1} = 0.076\frac{1}{\text{mm}} = 76.34\frac{1}{\text{m}}$$

Case 4

The ceiling is plastered with the use of fireproof plaster coating and the lower beam flange protrudes entirely below the underside plane of the slab so that it is exposed to fire temperatures on three sides - from the sides and the bottom.

$$\frac{A_p}{V} = \frac{b+2t_f}{bt_f} = \frac{106+2\cdot 13.1}{106\cdot 13.1} = \frac{132.2}{1388.6} = 0.095\frac{1}{\text{mm}} = 95.20\frac{1}{\text{m}}$$

Due to the fact that the greater the section factor, the smaller (in terms of time) the fire resistance of the structure, a less favourable value has been conservatively preferred for further calculations, which for both protected and unprotected members equals $\frac{A_m}{V} = \frac{A_p}{V} = 95.20\text{m}^{-1}$.

3.5 Fire resistance time of the section subjected to standard fire conditions

Due to the incremental nature of the procedure for determining the temperature of the steel structural members subjected to heating in fire conditions, the calculation in this respect, both for unprotected and insulated members, has been performed using a typical spreadsheet for this purpose.

The results of the calculations have been presented in Table 5 and shown graphically in Fig. 5.

In the case of an unprotected member, the beam reaches a critical temperature, previously determined at 663°C already in the 20th minute after the outbreak of fire. In the light of existing legislation, this corresponds only to fire resistance R15, therefore unplastered floor does not meet requirements laid down in [5].

Fireproof coating insulated member does not reach a critical temperature within the first 60 minutes of the fire flashover, therefore it meets at least the requirements corresponding to fire resistance R60.

Table 5 Selected computational results of the temperature for unprotected steel beam, subjected to standard fire conditions

Time (duration) of fire exposure			t	Θ_g	$\dot{h}_{net.c}$	$\dot{h}_{net.r}$	$\dot{h}_{net.d}$	c_a	$\Delta\Theta_{a.t}$ [°C]	$\Theta_{a.t}$ [°C]
[min]	[sec]	[sec]	[min]	[°C]	[W/m²]	[W/m²]	[W/m²]	[J/kg°C]	-	20.0
	0	1140	19.0000	773.7	3125.3	18997.8	22123.0	811.61	1.7	650.4
	5	1145	19.0833	774.4	3100.2	18910.5	22010.7	814.36	1.6	652.0
	10	1150	19.1667	775.0	3075.5	18823.5	21899.0	817.19	1.6	653.6
	15	1155	19.2500	775.7	3051.0	18736.9	21787.9	820.10	1.6	655.2
	20	1160	19.3333	776.3	3026.8	18650.7	21677.5	823.10	1.6	656.8
	25	1165	19.4167	776.9	3002.9	18565.0	21567.9	826.19	1.6	658.4
	30	1170	19.5000	777.6	2979.2	18479.7	21459.0	829.37	1.6	660.0
19	35	1175	19.5833	778.2	2955.9	18394.9	21350.8	832.66	1.6	661.5
	40	1180	19.6667	778.9	2932.8	18310.7	21243.5	836.05	1.5	663.1
	45	1185	19.7500	779.5	2910.0	18226.9	21137.0	839.55	1.5	664.6
	50	1190	19.8333	780.1	2887.5	18143.8	21031.3	843.16	1.5	666.1
	55	1195	19.9167	780.7	2865.3	18061.2	20926.6	846.88	1.5	667.6

The dotted lines shown in Fig. 5 correspond to the previously described simplified approach for calculating the temperature of steel members exposed to fire thermal actions, as proposed in [6] and [7].

In the present case, they can be considered as a relatively acceptable approximation only in respect of the first 25 minutes of the developed fire duration - in the case of unprotected members – and as a safe approximation in the whole scope covered by these calculations – in the case of isolated structural members.

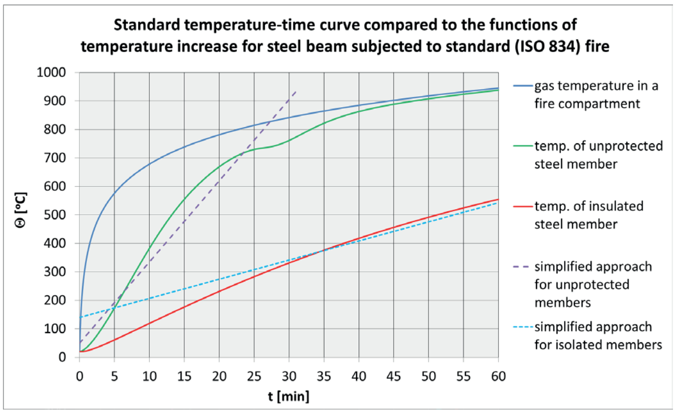

Fig. 5 Standard temperature-time curve compared to the functions of temperature increase for steel beam subjected to standard fire conditions

3.6 Fire resistance time of the section subjected to parametric fire conditions

In the case of residential buildings, a fire compartment usually consists of an entire building or, in exceptional cases, of a single storey. In multi-apartment buildings (in specific design situations), a single apartment may also be a separate fire compartment. For the purpose of this article, it has been established, in order to facilitate the understanding of the computational procedures used, that a single room is a separate fire compartment, separated by the axes 1–3 and B–C, shown in Fig. 4.

It has also been assumed that the walls and ceiling of the room are made of solid bricks and the floor of high-density wood on the concrete underlay of medium density. The walls include two window openings with dimensions b × h = 1.52 × 1.52 m each and two doors with dimensions b × h = 1.00 × 2.10 m. The free height of the storey is h = 3.00 m, whereas the horizontal internal dimensions of the room are: a × b = 5.76 × 6.48 m.

3.7 Determination of basic data

Calculation of the coefficient of thermal absorptivity for the walls and ceiling of solid bricks:

Density $\rho = 1600$ kg/m^3

Specific heat: $c = 840$ J/kgK

Thermal conductivity: $\lambda = 0.7$ W/mK

Coefficient of thermal absorptivity for the walls and ceiling equals:

$$b = \sqrt{\rho c \lambda} = 969.95 \approx 970\,\mathrm{J}/\left(\mathrm{m^2 s^{1/2} K}\right)$$

The calculated value fits well the range of , [J/(m^{2}s$^{1/2}$K)], resulting from the limitations set out by the code.

Calculation of the coefficient of thermal absorptivity for the floor of high-density wood:

Density $\rho = 720$ kg/m^3

Specific heat: $c = 1880$ J/kgK

Thermal conductivity: $\lambda = 0.2$ W/mK

Coefficient of thermal absorptivity for the wooden floor layer equals:

$$b = \sqrt{\rho c \lambda} = 520.31 \approx 520\,\mathrm{J}/\left(\mathrm{m^2 s^{1/2} K}\right)$$

Calculation of the coefficient of thermal absorptivity for the concrete floor underlay of medium density.

Density $\rho = 1800$ kg/m^3

Specific heat: $c = 1000$ J/kgK

Thermal conductivity: $\lambda = 1.15$ W/mK

Coefficient of thermal absorptivity for the concrete floor underlay of medium density equals:

$$b = \sqrt{\rho c \lambda} = 1438.75 \approx 1439\,\mathrm{J}/\left(\mathrm{m^2 s^{1/2} K}\right)$$

In accordance with Annex A, note (5) to the code [1], where the limiting surface is composed of several layers and the coefficient b computed for the further layer in relation to the fire compartment (in this case - the concrete underlay) is greater than value of the same coefficient determined for the closer layer (in this case - the wooden floor), then the deeper layer shall be omitted in further calculations. On this basis, the coefficient of thermal absorptivity for the floor has been adopted as: $b = \sqrt{\rho c \lambda} = 520.31 \approx 520\,\mathrm{J}/\left(\mathrm{m^2 s^{1/2} K}\right)$.

The calculated value fits well the range of $100 \le b \le 2200$, [J/(m^{2}s$^{1/2}$K)], resulting from the limitations set out by the code.

Determination of the fire load density:

In the case of residential buildings, the characteristic fire load density it relation to the unit area (considering fractile of 80%) is given in Table E.4 of the code [1] and is as follows: $q_{f,k} = 948$ MJ/m^2.

The floor area is: $A_f = a \cdot b = 5.76 \cdot 6.48 = 37.32$ m^2

The factor taking into account the fire activation risk due to the size of the compartment has been drawn from Table E.1 [1], using linear interpolation:

$\delta_{q1} = 1.10 + (1.5 - 1.1) \cdot (37.32 - 25) / (250 - 25) = 1.12$

The factor taking into account the fire activation risk due to the type of occupancy equals: $\delta_{q2} = 1.0$

The factor taking into account various active fire protection measures: $\delta_n = 1.0 \div 1.5 \rightarrow$ adopted: $\delta_n = 1.5$, with the assumption that no active firefighting measures have been provided for (Annex E, note (4) [1]).

Hence, the design value of the fire load density per unit floor area, determined by the following relationship, equals:

$$q_{f,d} = q_{f,k}\delta_{q1}\delta_{q2}\delta_n = 948 \cdot 1.12 \cdot 1.0 \cdot 1.5 = 1592.64 \ \mathrm{MJ/m^2}$$

Determination of the thermal properties of the fire compartment.

The total area of the enclosure equals:

$$A_t = 2A_f + 2(a+b)h = 2 \cdot 37.32 + 2 \cdot (5.76 + 6.48) \cdot 3.0 = \\ = 148.08 \ \mathrm{m^2}$$

The total area of the vertical openings on all walls equals:

$$A_V = \sum nh_{op}b_{op} = 2 \cdot 1.52 \cdot 1.52 + 3 \cdot 1.0 \cdot 2.10 = 10.92 \ \mathrm{m^2}$$

The total thermal absorptivity of enclosures separating fire compartment equals:

$$b = \frac{\sum_i (b_i \cdot A_i)}{A_t - A_V} = \frac{1 \cdot 520 \cdot 37.32}{148.08 - 10.92} + \frac{1 \cdot 970 \cdot 37.32}{148.08 - 10.92} + \\ \frac{[2 \cdot (5.76 + 6.48) \cdot 3.0 - 10.92] \cdot 970}{148.08 - 10.92} = 847.56 \frac{J}{m^2 s^{1/2} K}$$

Fire compartment ventilation capacity

Weighted average height of openings in the vertical partitions equals:

$$h_{eq} = \frac{(2 \cdot 1.52 + 3 \cdot 2.10)}{5} = 1.868 \mathrm{m}$$

Therefore the opening factor equals:

$$O = \frac{A_V \cdot \sqrt{h_{eq}}}{A_t} = \frac{10.92 \cdot \sqrt{1.868}}{148.08} = 0.10 \mathrm{m^{1/2}}$$

The value calculated above fits well the range of $0.02 \le b \le 0.20$, [$\mathrm{m^{1/2}}$], resulting from the limitations set out by the code.

Time factor function of the opening factor O and the thermal absorptivity b:

$$\Gamma = \left(\frac{O/0.04}{b/1160}\right)^2 = \left(\frac{0.10/0.04}{847.56/1160}\right)^2 = 3.422^2 = 11.707$$

The fire load density related to the total area of the separating surfaces At:

$$q_{t,d} = \frac{q_{f,d} \cdot A_f}{A_t} = \frac{1592.64 \cdot 37.32}{148.08} = 401.39 \, \mathrm{MJ/m^2}$$

3.8 Assessment of the time required to reach maximum temperature and determination of the maximum gas temperature reached

Medium fire growth rate is expected, for which t_{min} = 20min = 0.333h.

The time t_{max} to reach the maximum gas temperature is determined as:

$$t_{\max} = \max\{(0.0002 \cdot q_{t,d})/O; t_{\lim}\} = \\ = \max\{(0.0002 \cdot 401.39)/0.10; 0.333\} = \max\{0.803; 0.333\} = 0.803\mathrm{h}$$

Due to the fact that t_{max} is determined based on the first component of the above relationship equal to $0.0002 \cdot q_{t,d}/O$, then according to the content of the code [1] the fire may be classified as a ventilation controlled.

The time t^*_{max} required to reach the maximum temperature in the heating phase, taking into account the openings and the thermal absorptivity of the enclosures, is determined as:

$$t^*_{\max} = t_{\max} \cdot \Gamma = 0.803 \cdot 11.707 = 9.401 \ \mathrm{h}$$

The maximum temperature of the fire gases:

$$\theta_{\max} = 20 + 1325 \cdot \begin{pmatrix} 1 - 0.324 \cdot e^{-0.2 \cdot 9.401} - 0.204 \cdot e^{-1.7 \cdot 9.401} + \\ -0.472 \cdot e^{-19 \cdot 9.401} \end{pmatrix} = \\ = 1279.51 °\mathrm{C}$$

The curve in the heating phase

The temperature of the fire gases in the heating phase is determined by the following relationship:

$$\theta_{g,t} = 20 + 1325 \cdot \left(1 - 0.324 \cdot e^{-0.2 \cdot t^*} - 0.204 \cdot e^{-1.7 \cdot t^*} - 0.472 \cdot e^{-19 \cdot t^*}\right)$$

where time t^* is determined as: $t^* = t \cdot \Gamma = 11.707 \cdot t$

The curve in the cooling phase

Where $t^*_{max} \ge 2$ h, the temperature of the gases in the cooling phase in the fire controlled by ventilation is given by the following relationship:

$$\theta_{g,t} = \theta_{\max} - 250(t^* - t^*_{\max}) = 1279.51 - 250(t^* - 9.401) = \\ = 3629.76 - 250 \cdot t^*$$

The resulting curve is presented in Fig. 6 and selected results of the calculations are included in Table 6 for an unprotected member and in Table 7 for a member insulated with fireproof mortar.

In the case of an unprotected member, the beam subjected to parametric fire conditions reaches the critical temperature set at 663°C already in the 7th minute of the developed fire, whereas the isolated beam - after 49 minutes of exposure to similar thermal conditions.

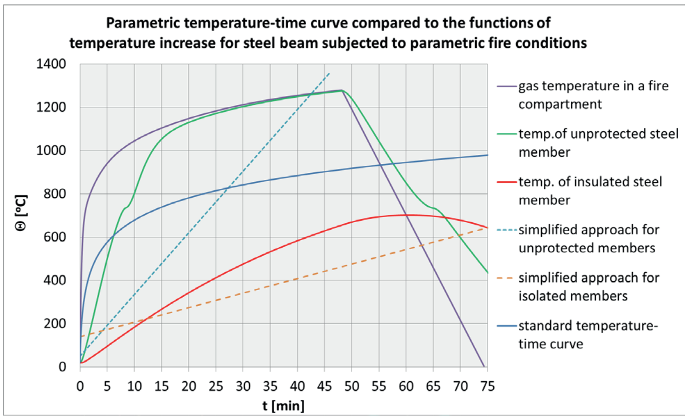

Fig. 6 Parametric temperature-time curve compared to the functions of temperature increase for steel beam subjected to parametric fire conditions

Table 6 Selected computational results of the temperature for unprotected steel beam, subjected to parametric fire conditions

Time (duration) of fire exposure		t	t	t	t*	$\Theta_{g.t}$	$\dot{h}_{net.c}$	$\dot{h}_{net.r}$	$\dot{h}_{net.d}$	c_a	$\Delta\Theta_{a.t}$ [°C]	$\Theta_{a.t}$ [°C]
[min]	[sec]	[sec]	[min]	[h]	[h]	[°C]	[W/m²]	[W/m²]	[W/m²]	[J/kg°C]	-	20.0
	0	360	6.0000	0.1000	1.1707	968.4	13476.0	72908.3	86384.4	741.52	7.1	590.4
	…	…	…	…	…	…	…	…	…	…	…	…
	30	390	6.5000	0.1083	1.2683	980.6	12464.9	72269.3	84734.1	780.50	6.6	631.0
	35	395	6.5833	0.1097	1.2845	982.5	12302.2	72112.0	84414.2	787.55	6.5	637.5
	40	400	6.6667	0.1111	1.3008	984.4	12141.5	71942.2	84083.7	795.41	6.4	643.9
6	45	405	6.7500	0.1125	1.3170	986.3	11983.0	71761.2	83744.2	804.23	6.3	650.3
	50	410	6.8333	0.1139	1.3333	988.2	11827.0	71570.2	83397.2	814.17	6.2	656.5
	55	415	6.9167	0.1153	1.3496	990.0	11673.7	71370.9	83044.5	825.46	6.1	662.6

Table 7 Selected computational results of the temperature for a steel beam insulated by fire protection material, subjected to parametric fire conditions

Time (duration) of fire exposure		t	t	t	t*	$\Theta_{g.t}$	c_a	ϕ	$\Delta\Theta_{g.t}$ [°C]	$\Delta\Theta_{a.t}$ [°C]	$\Theta_{a.t}$ [°C]
[min]	[sec]	[sec]	[min]	[h]	[h]	[°C]	[J/kg°C]	-	-	-	20.0
	0	2940	49.0000	0.8167	9.5607	1239.6	836.4	0.07	-4.1	0.6	662.2
	5	2945	49.0833	0.8181	9.5770	1235.5	837.9	0.07	-4.1	0.6	662.9
	10	2950	49.1667	0.8194	9.5932	1231.5	839.3	0.07	-4.1	0.6	663.5
	15	2955	49.2500	0.8208	9.6095	1227.4	840.8	0.07	-4.1	0.6	664.1
	20	2960	49.3333	0.8222	9.6258	1223.3	842.3	0.07	-4.1	0.6	664.7
	25	2965	49.4167	0.8236	9.6420	1219.3	843.8	0.07	-4.1	0.6	665.4
	30	2970	49.5000	0.8250	9.6583	1215.2	845.3	0.07	-4.1	0.6	666.0
49	…	…	…	…	…	…	…	…	…	…	…
	45	2985	49.7500	0.8292	9.7071	1203.0	849.8	0.07	-4.1	0.6	667.8
	50	2990	49.8333	0.8306	9.7233	1198.9	851.4	0.07	-4.1	0.6	668.3
	55	2995	49.9167	0.8319	9.7396	1194.9	852.9	0.07	-4.1	0.6	668.9

3.9 Comparison of the effectiveness of various mortars

In order to compare the effectiveness of different types of popular mortars used in construction industry, the supplementary calculations was carried out, assuming the alternative application of traditional cement, cement-lime, lime and gypsum roughcast, or even a thin layer of plain concrete, instead of special fire protection mortar with vermiculite aggregate.

During the calculations the following properties of protective materials were adopted:

- for plain concrete:
  ρ_p = 2400 kg/m³
  c_p = 840 J/kgK
  λ_p = 1.70 W/mK
- for cement mortar:
  ρ_p = 2000 kg/m³
  c_p = 840 J/kgK
  λ_p = 1.00 W/mK
- for cement-lime mortar:
  ρ_p = 1850 kg/m³
  c_p = 840 J/kgK
  λ_p = 0.82 W/mK
- for lime mortar:
  ρ_p = 1700 kg/m³
  c_p = 840 J/kgK
  λ_p = 0.70 W/mK
- for gypsum mortar:
  ρ_p = 1300 kg/m³
  c_p = 840 J/kgK
  λ_p = 0.52 W/mK

Analyses were carried out separately for element subjected to standard fire conditions and parametric fire conditions, taking the thickness of the protective layer, respectively equal to 8 mm (similarly as in the case of fire protection mortar with vermiculate aggregate), and the usually adopted thickness of traditional plasterworks, of 15 mm.

The results of the calculations are presented in Table 8.

Table 8 Selected computational results of the time period of fire exposure required to achieve the critical temperature by the steel beam, unprotected and protected with mortars of various types.

Type of fire protecting material	Time period of fire exposure required to achieve the critical temperature of the beam							
	Standard(ISO) fire conditions				Parametric fire conditions			
	Thickness of the protecting layer				Thickness of the protecting layer			
[-]	d = 8.0 mm		d = 15.0 mm		d = 8.0 mm		d = 15.0 mm	
Unprotected steel beam /no protecting material/	19 min 40 sec				6 min 55 sec			
Plain concrete	16 min	05 sec	23 min	25 sec	6 min	25 sec	11 min	15 sec
Cement mortar/roughcast	20 min	50 sec	32 min	00 sec	9 min	25 sec	16 min	35 sec
Cement-lime mortar/roughcast	23 min	15 sec	36 min	05 sec	10 min	55 sec	19 min	10 sec
Lime mortar/roughcast	25 min	25 sec	39 min	45 sec	12 min	20 sec	21 min	30 sec
Gypsum mortar/roughcast	30 min	25 sec	47 min	35 sec	15 min	20 sec	26 min	35 sec
Fire-protecting mortar with vermiculate aggregate	over 60 min		over 60 min		49 min	05 sec	never reached	

It could be observed that the results for 8 millimeters thick layer of plain concrete seem to defy logic. This is due to the mismatch of Eq. (18) in the case of protection materials with low thickness, high density and relatively high thermal conductivity. Such a result should be interpreted as a lack of positive impact (not as the negative impact) of the protection system to the fire resistance of the element.

3.10 Verification of the joist resistance in the strength domain

The fire resistance has been checked in two variants:

a. of an unprotected joist subjected to thermal fire actions described by the standard temperature-time curve. The purpose of the check is to confirm the joist resistance corresponding to the fire resistance class R15.
b. of a joist insulated with sprayed cement mortar with vermiculite aggregate subjected to thermal fire actions described by the parametric temperature-time curve.

The purpose of the check is to confirm the required joist resistance within 45 minutes of the developed fire (note that the author deliberately does not use the concept of fire resistance represented by the symbol R45, since it is assumed that it is reserved for the fire described by means of the standard curve only).

In the case of members with sections of class 1 or 2 and non-uniform temperature distribution both along the length of the beam and at the height of the section, the design bending moment resistance $M_{fi,t,Rd}$ at the time t of the fire duration can be determined using the formula:

$$M_{fi,t,Rd} = \frac{M_{fi,\theta,Rd}}{\kappa_1\kappa_2} = \frac{k_{y,\theta}\cdot M_{Rd}}{\kappa_1\kappa_2}\frac{\gamma_{M,0}}{\gamma_{M,fi}} = \frac{1}{\kappa_1\kappa_2}\frac{k_{y,\theta}\cdot W_{pl,y}\cdot f_y}{\gamma_{M,fi}} \tag{24}$$

where:

κ_1 – the adaptation factor, taking into account the non-uniform temperature distribution at the height of the section, on the basis of [3] point 4.2.3.3.(7)

κ_2 – the adaptation factor, taking into account the non-uniform temperature distribution along the beam length, on the basis of [3] point 4.2.3.3.(8)

<u>Unprotected beam, R15 requirement, according to the standard temperature-time curve.</u>

The cross-section temperature of an unprotected beam after full 15 minutes of developed fire equals (in accordance with the results of calculations carried out) approx. 583°C.

The reduction factor for effective yield strength $k_{y,\theta}$, corresponding to this temperature is:

$$k_{y,\theta} = 0.470 + \frac{0.780-0.470}{100}\left(600-583\right) = 0.523$$

The adaptation factor κ_1 = 0.70 takes into account the fire action on a member unprotected from three sides.

The adaptation factor κ_2 = 1.0 takes into account the static scheme of a simply supported beam.

Design bending moment resistance of the cross-section at the temperature θ_a = 583°C is:

$$M_{fi,t,Rd} = \frac{1}{\kappa_1\kappa_2}\frac{k_{y,\theta}\cdot W_{pl,y}\cdot f_y}{\gamma_{M,fi}} = \frac{1}{0.7\cdot 1.0}\frac{0.523\cdot 412\cdot 23.5}{1.0} = 7233.84\ \text{kNcm} = 72.34\ \text{kNm} > M_{fi,Ed} = 30.87\ \text{kNm}$$

Design shear resistance of the cross-section:

$$V_{fi,t,Rd} = k_{y,\theta}\frac{A_{V,z}\cdot f_y}{\sqrt{3}\cdot\gamma_{M,fi}} = 0.523\cdot\frac{21.75\cdot 23.5}{\sqrt{3}\cdot 1.0} = 154.34\ \text{kN} > V_{fi,Ed} = 20.58\ \text{kN}$$

Resistance conditions in the event of fire for the adopted criterion R15 are met.

<u>Joist insulated with sprayed cement mortar with vermiculite aggregate, requirement of 45 minutes, according to the parametric temperature-time curve.</u>

The maximum temperature in the cross-section of an insulated joist, reached over the entire period of 45 minutes of developed fire did not exceed (according to the results of the calculations made) 638°C.

The value of the reduction factor of effective yield point $k_{y,\theta}$, corresponding to this temperature is:

$$k_{y,\theta} = 0.230 + \frac{0.470 - 0.230}{100}\left(700 - 638\right) = 0.379$$

The adaptation factor κ_1 = 0.85 takes into account the fire action on a member insulated from three sides.

The adaptation factor κ_2 = 1.0 takes into account the static scheme of a simply supported beam.

Design bending moment resistance of the cross-section at the temperature is:

$$M_{fi,t,Rd} = \frac{1}{\kappa_1 \kappa_2} \frac{k_{y,\theta} \cdot W_{pl,y} \cdot f_y}{\gamma_{M,fi}} =$$
$$= \frac{1}{0.85 \cdot 1.0} \frac{0.379 \cdot 412 \cdot 23.5}{1.0} =$$
$$= 4317.03 \text{ kNcm} = 43.17 \text{ kNm} > M_{fi,Ed} = 30.87 \text{ kNm}$$

Design shear resistance of the cross-section:

$$V_{fi,t,Rd} = k_{y,\theta} \frac{A_{V,z} \cdot f_y}{\sqrt{3} \cdot \gamma_{M,fi}} = 0.379 \cdot \frac{21.75 \cdot 23.5}{\sqrt{3} \cdot 1.0} =$$
$$= 111.84 \text{ kN} > V_{fi,Ed} = 20.58 \text{ kN}$$

Resistance conditions in the event of fire for the required 45 minutes of the joist fire resistance are met.

4 Summary and final conclusions

The paper presents a procedure for the assessment of fire resistance of selected type of steel-beam floors with specific technical solution of joists hidden within the thickness of the slab, illustrated by a suggestive computational example.

Calculations and analyses conducted for the purpose of this paper allow formulation of the following conclusions and general comments:

1. Assessment of the fire resistance of structural members of steel-beams floors with joists hidden within the thickness of the slab can be made on the basis of available standard procedures in the thermal, time or strength domain.
2. The procedures presented can be successfully used both to assess the resistance of beam floors with ceramic slab and related solutions: segmental brick vaults, floors filled with prefabricated reinforced concrete slabs, monolithic reinforced concrete slab cast-in-situ on the lower flanges of steel joists, and other structural solutions with joists hidden within the depth of the floor.
3. As there is no composite action between the joist and the floor slab, application of procedures outlined in the standards devoted to the design of composite steel and concrete structures, particularly in EN 1994-1-2 [8], is not substantially justified in the case of this type of technical solutions. The author proposes to use procedures taken from EN 1993-1-2 [3], to determine the fire resistance of this type of floors, while pointing out to the problem of significant discrepancies in final results, depending on the chosen path/variant of procedure and assumptions made.
4. The simplified procedures for calculating the temperature of a steel member subjected to fire thermal actions, suggested in the papers [6], [7], are of a rather limited accuracy and should be avoided.
5. In the case of specific configurations of physical and chemical parameters of separated fire compartments, the use of a standard fire model may not be sufficient and lead to risky, hazardous estimates. The computational example presented showed that contrary to the popular belief, the standard fire model does not always lead to the most conservative results.
6. Continuous development of computational techniques creates opportunities for using in the analysis of building structures of modern methods and computer tools, based on the achievements of the Computational Fluid Dynamics (CFD). This method allows an accurate assessment of the temperature rise of the structural members while taking into account realistic environmental conditions of the fire compartment, adapted to the specific nature of a given category of the structure, its fittings, and the like. At the current stage, the use of computational techniques of this type requires, in addition to skills of using complex, commercial computer tools, also advanced theoretical knowledge, which severely limits the possibility of using these methods in daily engineering practice. In view of these limitations, these methods, in the case of simple, statically determinable structural systems, have so far not become widely used.
7. The use of passive fire protection measures of steel members effectively decreases the rate of temperature growth of load-carrying structures. This increases the likelihood of effectiveness of the rescue operation before the irreversible deformation of structural steel members, thereby reducing the risk of a major accident.
8. As it was proved by the presented calculation of the steel beam, it is possible to somehow increase the current fire resistance of the structure not only by the use of dedicated fire protection products but also by the use of conventional solutions based on traditional materials, of suitable thickness. It can be crucial especially for historical buildings that remain under the supervision of the conservator.
9. Steel is a construction material sensitive to the heating rate and the speed of loading or deformation. Taking into account the observations drawn from the results of research published in the available literature, especially those that have a direct impact on the structural design and assessment process, including fire actions, should be emphasised. It was found that, inter alia, there is a significant effect of the structure heating rate on the value of the critical temperature [9],

the size of the thermal elongation (or deformation in general meaning) [10], and the size of structural creep strains [6], [11], [12]. In the case of structural steel - with increasing heating rate, the value of the critical temperature increases, while the total deformation of the structure, including creep strains, decreases. As the technical and building regulations in general specify correspondingly higher requirements for fire resistance in respect of the main load-carrying members, in practical design this usually translates into a better (more efficient) protection of this class of members against the effects of fire. That results in their slower heating, which may lead to greater strain of these elements caused by creep. This justifies the need to pay special attention to this problem in the process of fire design of these members.

The assessment of the technical condition of structures taking into account fire effects or of the fire affected structures, due to the complex nature of the phenomenon and the ambiguity arising from the ways of its modelling, should be entrusted to a person of recognised professional experience in this field and holding appropriate licenses.

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