Effects of Damping Uncertainties on Damping Reduction Factors

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Abstract

It is apparent that the dynamic response of a building depends on its energy dissipation capacity, hence damping ratio. The damping value experienced by a building during an earthquake differs significantly from the value specified in the design step. This introduces uncertainties in the design process of the building. It would be desirable to consider not only the effects of uncertainties in loading but also the uncertainties in the structural parameters.

In this paper, the effects of uncertainties in the estimation of damping ratio ξ, on the use of Damping Reduction Factors (DRF) for the evaluation of high damping response spectra, are examined. Damping uncertainties are described by a lognormal probability distribution, and the Monte Carlo technique is used to generate the random values of damping. The average of the distribution is the deterministic value of damping (taken equal to 5%, 7.5%, 10%, 20%, 30% and 40%) while three different values of coefficient of variation are considered (i.e. 10%, 20% and 40%, respectively).

All the DRF formulations found in the literature are not able to take into consideration damping uncertainties, leading to significant discrepancies in the high damping response spectra. Based on the results of this study, a new DRF formulation, able to account for uncertainties in damping estimation, is tentatively proposed.

Keywords

Damping ratio, damping reduction factors, High damping response spectra, Uncertainties, Monte Carlo technique

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1 Introduction

In most seismic codes worldwide, the response spectrum is given for a damping ratio ξ = 5%. Civil structures, however, may feature many different values of damping. As a consequence, the 5%-damping response spectrum must be adjusted to other damping levels. In this case, a correction factor is used to evaluate the spectral response for any damping value. Different definitions are used in the literature to identify this correction factor. In this study, we adopt the term Damping Reduction Factor (DRF) [1].

DRF has been studied by many researchers and different expressions of DRF, as function of damping ratio [2–5], damping ratio and period [6–11], damping ratio, period and other earthquake characteristics (e.g. duration, soil conditions, epicentral distance, magnitude) [7,12–16], have been proposed.

One of the first formulation of the DRF is proposed by Newmark and Hall (1973) [17], their results inspired many seismic codes and standards. The proposed expressions are expressed as:

$$DRF = \begin{cases} 1.514 - 0.321 \times \ln(\xi) \\ 1.400 - 0.248 \times \ln(\xi) \\ 1.309 - 0.194 \times \ln(\xi) \end{cases} \quad (1)$$

The studies carried out by Wu and Hanson (1989) [9], presented a set of DRF from a statistical study of inelastic response spectra with high damping ratios. Ten earthquake records were used as ground motions for elastoplastic SDOF systems with damping ratios between 10 and 50%. Outcomes from Wu and Hanson's investigation have been adopted in UBC (ICBO 1994) and NEHRP (1994)

The damping reduction factor proposed by Ashour (1987) [5], can be expressed as:

$$DRF = \sqrt{\frac{0.05 \times (1 - e^{-\alpha \times \xi})}{\xi \times (1 - e^{-0.05 \times \alpha})}} \quad (2)$$

Where α is a coefficient ranging from 18 to 65, according to the earthquake characteristics.

Cardone et al. [1] examined the accuracy of the main DRF formulations included in the actual design codes, Lin et al. [8] carried a series of studies on DRF. These studies were focused

on differentiating the damping effect on displacement and acceleration responses, and the effect of site condition on DRFs was investigated as well.

Among the parameters which the DRF is dependent on, DRF is mainly affected by the damping ratio, therefore, the error in the damping estimation may lead to a wrong value of DRF and subsequently to significant inaccuracies in the dynamic response estimation. This is the main motivation for searching a new DFR relationship, able to account for damping uncertainties in the DRF calculation.

Therefore, the aim of this work is to study the effects of uncertainties in the estimation of the damping ratio ξ on the DRF value. The damping uncertainties are represented by a lognormal distribution of probability, and the Monte Carlo technique is used to generate the random values of damping. The average of this distribution is the deterministic value of damping ratio (taken equal to 5%, 7.5%, 10%, 20% and 30%, respectively) while three different values of coefficient of variation have been considered (i.e. 10%, 20% and 40%, respectively), based on the studies by [18].

Haviland [18] assembled the information provided by complete empirical experiments, and provided a range of data for different levels of response amplitudes and large classes of structural systems and building sizes. The study by Haviland proved that the lognormal and Gamma distributions provide the best fit to damping changes, with a coefficient of variation (COV) ranging from 42% to 87%. Davenport and Carroll [19], after reviewing the database of Haviland, suggested to modify the interval of coefficient of variation (COV), from 33% to 78%, assuming an average value of 40%.

In this study, the Monte Carlo technique is used to simulate a large number of damping values. The most suitable number of samples (200 for each response spectrum) has been found after a statistical study, which is not presented here due to the page number limitation of the paper.

First, three sets of natural records, compatible (on average) with Eurocode 8 [20] response spectra for soil type A, B and C, have been selected. Afterward, those records have been used to derive exact response spectra with uncertain damping through Nonlinear Response History Analysis. Finally, the exact spectra have been compared to the approximate spectra derived using different DRFs from the literature.

Herein, a new formulation of DRF is tentatively proposed, to take into consideration the uncertainties of the damping ratio in the DRF evaluation. The proposed formulation has been found using the software Eureqa [20], which is a tool for searching hidden formulations in a given database. Some conclusions and future research needs are discussed at the end of the paper.

2 Ground motion selection

In this study the Rexel tool [22] has been used to select three sets of spectrum-compatible ground motion records. Rexel is a tool that allows the user to select sets of strong ground motion records that are representative of design response spectra.

The user specifies the target response spectrum and the desired characteristics of the earthquake ground motions in terms of earthquake magnitude, source-to-site distance and other general characteristics. Rexel then selects acceleration time series from the internal database of acceleration time series that satisfy the user-specified selection criteria and provide good fits to the target response spectrum.

Like many codes worldwide, Eurocode 8 (EC8) allows the use of real ground-motion records for the seismic analysis of structures. The main condition to be satisfied is that the average elastic spectrum does not underestimate the code spectrum, with a 10% tolerance, in a broad range of periods, depending on the structure's dynamic properties.

The average spectrum deviation (δ) gives a quantitative measure of how much the spectrum of a record deviates from the spectrum of the code. The definition of (δ) is given by Eq (3):

$$\delta = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(\frac{Sa_0(T_i) - Sa_s(T_i)}{Sa_s(T_i)} \right)^2} \qquad (3)$$

where represents the pseudo-acceleration ordinate of the single record corresponding to the period Ti while is the value of the spectral ordinate of the code spectrum at the same period, and N is the number of values within the considered range of periods. Selecting a record set with low value of δ means to get an average spectrum, which tends to be very close to the code spectrum. Controlling this parameter may allow choosing combinations characterized by records having the individual spectra relatively close to the reference spectrum, and therefore being narrowly distributed around it.

In this work, three sets of seven records, which are very close ($\delta < 10\%$) to the Eurocode design spectra with 475 years return period (peak ground acceleration $a_g = 0.35$ on stiff soil), have been selected. The list of all the records selected for each soil class, with their main characteristics, is presented in the Table 1, 2 and 3. The seismic characteristics presented are the magnitude of the earthquake (Mw) which is in the range 6.3-7.3. The average of the shear wave velocity in the upper 30 meters of the soil (Vs30) which describe the soil type and the epicentral distance

According to EC8, soil type A corresponds to rock, soil type B to deposits of very dense sand, gravel, or very stiff clay and soil type C to deep deposits of dense or medium-dense sand, gravel or clay.

The response spectrums of the 7 records with their averages for soils types A, B and C are presented in the figures 1,2 and 3 respectively. The target response spectrums of the EC8 which are used in Rexel are also presented in this figures for each soil.

Table 1 Record data returned by REXEL for soil A

Earthquake ID	Station ID	Earthquake Name	Date	Mw	Vs30 (m/s)	Epicentral Distance (km)
34	ST20	Friuli	06/05/1976	6,5	1021	23
87	ST54	Tabas	16/09/1978	7,3	826	12
146	ST96	Campano Lucano	23/11/1980	6,9	1100	32
146	ST96	Campano Lucano	23/11/1980	6,9	1100	32
1635	ST2486	South Iceland	17/06/2000	6,5	-	5
41	ST_106	South Iceland	17/06/2000	6,5	-	5,25
101	ST_101	Olfus	29/05/2008	6,3	-	7,97

Table 2 Record data returned by REXEL for soil B

Earthquake ID	Station ID	Earthquake Name	Date	Mw	Vs30 (m/s)	Epicentral Distance (km)
250	ST205	Erzincan	33676	6,6	421	13
1635	ST2482	South Iceland	17/06/2000	6,5	-	15
2142	ST2484	South Iceland (aftershock)	36698	6,4	-	12
41	ST_105	South Iceland	17/06/2000	6,5	-	14,56
64	AQV	L'Aquila mainshock	06/04/2009	6,3	474	4,87
94	LGPC	Loma Prieta	18/10/1989	6,9	477,7	18,75
94	ST_58065	Loma Prieta	18/10/1989	6,9	371	27,59

Table 3 Record data returned by REXEL for soil C

| Earthquake ID	Station ID	Earthquake Name	Date	Mw	Vs30 (m/s)	Epicentral Distance (km)
77	AI_137_DIN	Dinar	1995/10/01	6,4	198,1	0,47
89	EC04	Imperial Valley	1979/10/15	6,5	208,9	27,03
89	EC05	Imperial Valley	1979/10/15	6,5	205,6	27,68
89	EC05	Imperial Valley	1979/10/15	6,5	205,6	27,68
89	EC06	Imperial Valley	1979/10/15	6,5	203,2	27,35
94	ST_47380	Loma Prieta	1989/10/18	6,9	271	29,66
78	ERZ	Erzincan	1992/03/13	6,6	274,5	8,97
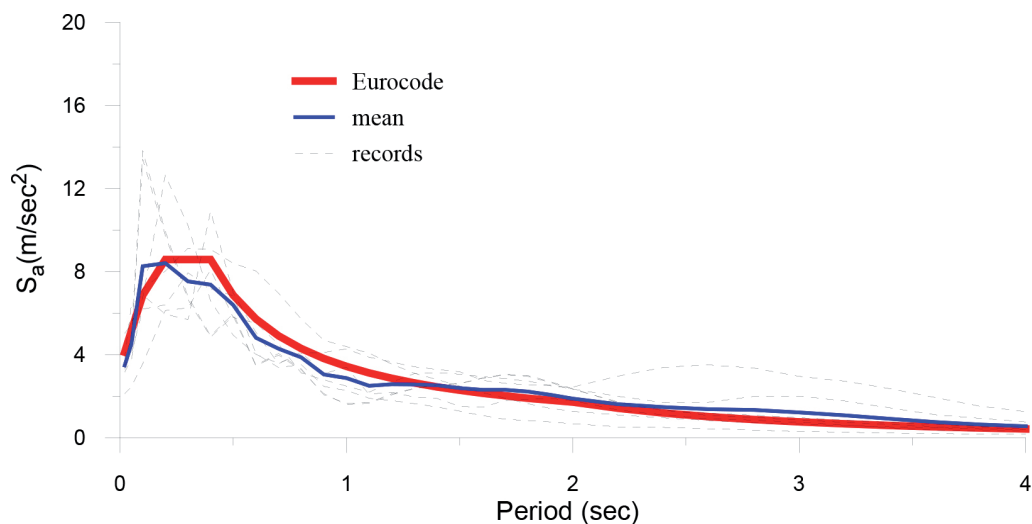
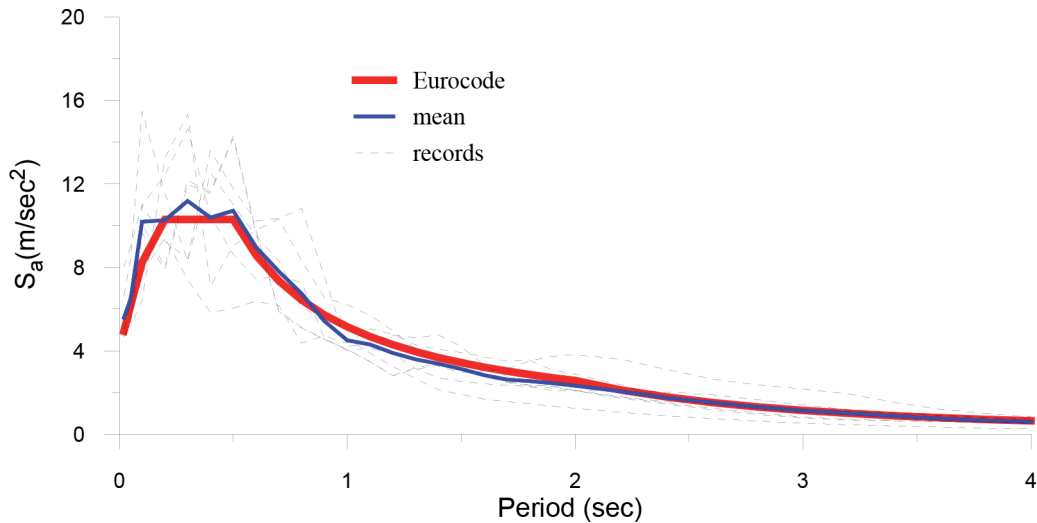Fig. 1 Response spectra for records returned by REXEL for soil A

Fig. 2 Response spectra for records returned by REXEL for soil B
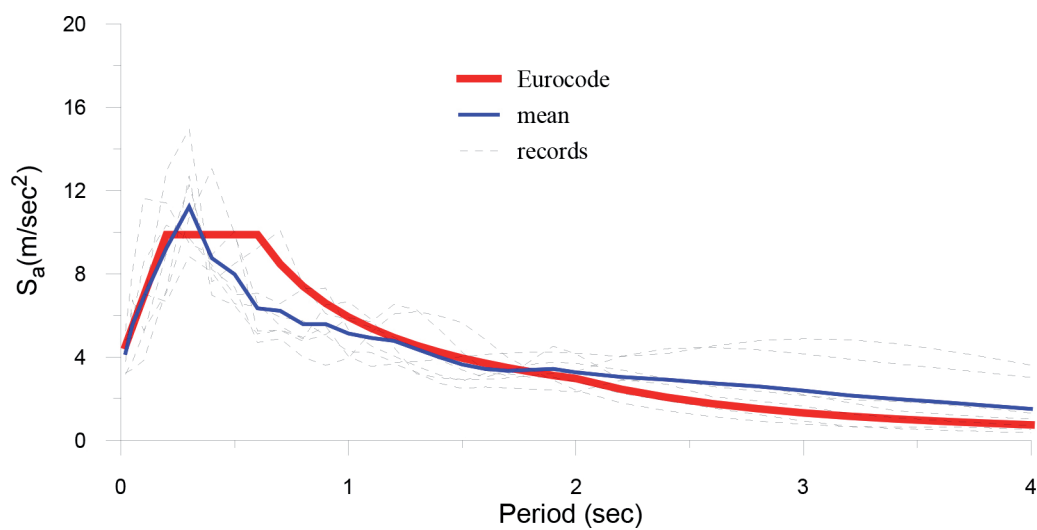

Fig. 3 Response spectra for records returned by REXEL for soil C

3 Monte Carlo simulation

3.1 Simulation technique

The Monte Carlo method is used to generate the distribution of the damping ratio values. In our case, we choose to make them follow the lognormal distribution. Under this assumption, the random values of the sample PSA of size n (i.e. i =1 to N) of corresponding responses are also independent and identically distributed and moreover, by virtue of the law of large numbers, the characteristics of the random sample approach even more statistical characteristics of the population as the sample size n increases.

The steps involved in these calculations can be summarized as follows:

1. Define the problem in terms of all the random variables.
2. Quantify the probabilistic characteristics of all random variables.
3. Generate values for these variables.
4. Assess the problem in a deterministic manner for each series of realization of all random variables;
5. Extracting information for N variables.
6. Determine the effectiveness and the accuracy of the method.

3.2 Number of samples required

An estimate of the number of simulations required for a given physical problem constitutes a very important point of debate in the random computations.

In this work, the Monte Carlo method is used to estimate the values of DRF (ξ, T) associated to a structure of fundamental period T and uncertain damping factor ξ. We assume that the values DRF (i = 1 to N) of the random variable input ξ are independent, and in our case, we considered them following the Log-normal distribution. Under this assumptions, the random values of the sample DRF (ξ, T) of size n (i. e. i = 1, 2, …, N) are also independent and moreover, by virtue of the law of large numbers, the characteristics of the random sample approach even more statistical characteristics of the population that the sample size N increases.

The aim here is to find the minimal number of simulation that gives a representatives results with a minimum cost in term of time. The number of response spectra calculated for each value of ξ simulated is $7 \times 32 \times 5 \times 4$ where 7 is the records number for each soil, 32 is the number of periods, 5 is the number of ξ considered here and 4 is the number of variation coefficients, that means that the increasing the number of simulation from N to $N + 1$ lead to multiplying the number of response spectrums must be calculated by 4480. This justifies the problem of increasing the number of simulations to a very high value of N.

For each period T, we have a number of simulation N which is the number of ξ simulated, that means that we have the same number N for DRF values obtained after the calculation of DRF values using the method represented below.

That means that when N increase the histogram of the DRF values (see figure 4) for a given value T tends to be unchanged and have the limit form (law of large numbers), and practically the same value of average and standard deviation which is the values uses here in this study.
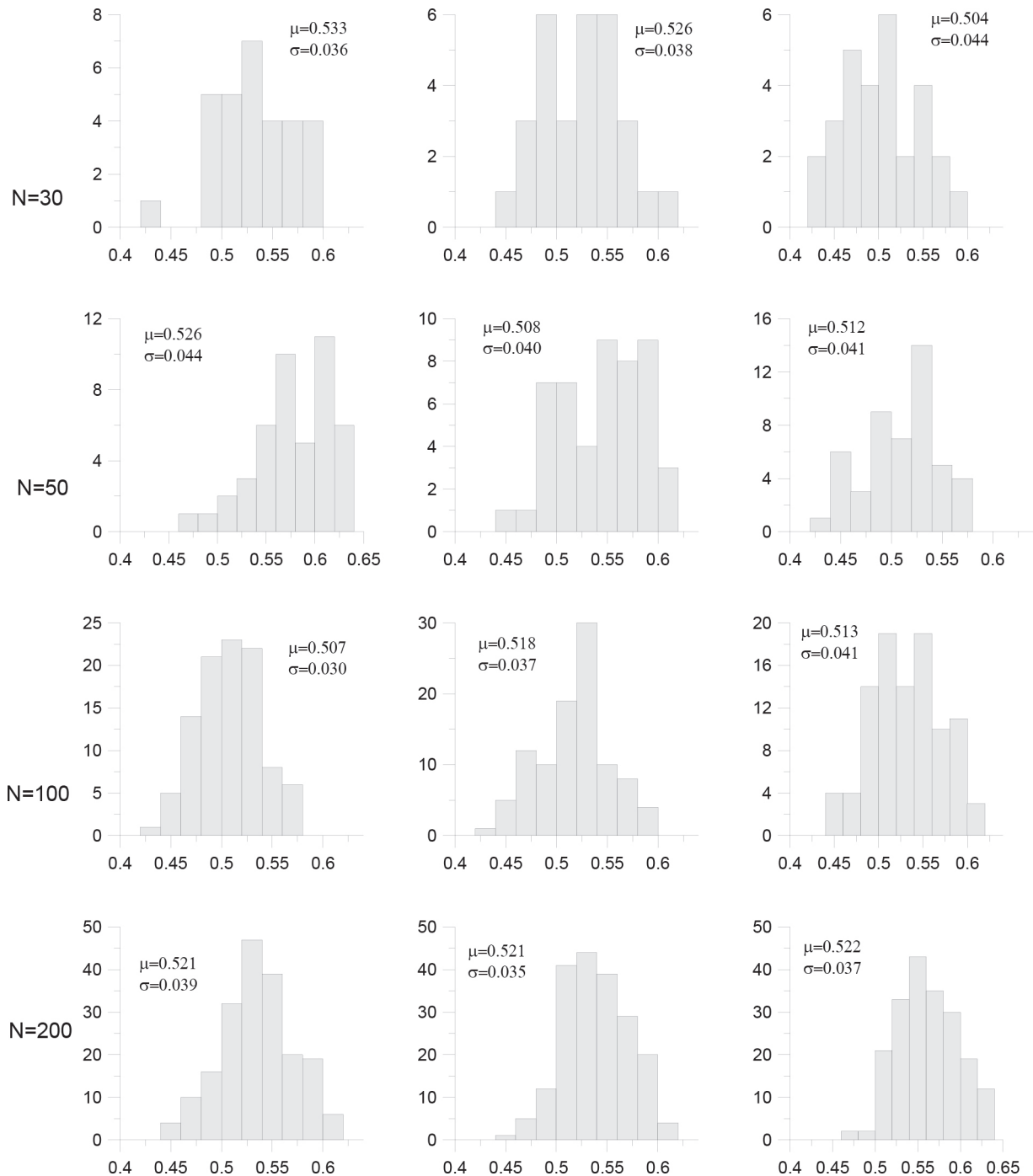

Fig. 4 histograms of DRF values for $T = 2$s for different simulations number (N).

Figure 4 shows the results obtained for the values limits used in this study ($\xi = 0.30$ and COV = 0.40), this presents the maximum of perturbation of results around the mean values of ξ. (if the number of simulation is sufficient here automatically is sufficient for lesser values of ξ and lesser values of COV).

The results are presented for a period value equal 2s and 4 values of N (30, 50,100 and 200) for 3 estimations among 100 estimations carried during the study.

It is clear from the figure 4 that increasing the number of simulation from 30 to 200 have a significant influence in the histogram of DRF values. For $N = 30$, the results average changes from $\mu = 0.504$ to 0.533 and its standard deviation σ changes from $f\sigma = 0.036$ to 0.044, the relative error between the values of μ reach the 5.75%. It is remarkable that is the difference between the cases decrease when N increases.

For $N = 200$, the results average changes from $\mu = 0.522$ to 0.521 and its standard deviation σ changes from $f\sigma = 0.035$ to 0.039, the relative error between the values of μ is practically zero. Which mean that $N = 200$ is a sufficient number of simulation for this case of study and gives reliable results.

4 Results
4.1 Comparison between different COV values and soil conditions

In this section, the results found considering different COV values (0.10, 0.20 and 0.40) are presented.

DRF are obtained with the following equation:

$$DRF = \frac{SD(\xi,T)}{SD(5\%,T)} \qquad (4)$$

where $SD(\xi,T)$ and $SD(5\%,T)$ are the elastic spectral displacements for $\xi > 5\%$ and $\xi = 5\%$, respectively.

In Figure 5, four curves are drawn for each soil type, namely: (i) a deterministic DRF curve, derived considering the deterministic damping value ($\xi = 20\%$, in the case under consideration), (ii) three mean plus sigma ($\mu + \sigma$) DRF curves each one of this curves is derived considering the average response spectra associated with 200 independent values of damping obtained from Monte Carlo random sampling.

For instance, we take the case of COV = 0.1, 200 values of damping are generated according to the log normal distribution, this damping values are used to estimate the DRF values using the equation 4. Then, the estimation of the average and the standard deviation of this DRF values allows us to draw the mean plus sigma ($\mu + \sigma$) curve for each soil.

It is worth noting that all the curves shown in Fig. 5 have the same tendency, regardless the value of COV considered. Obviously, the differences between deterministic curve and ($\mu + \sigma$) curves increase while increasing the COV value.
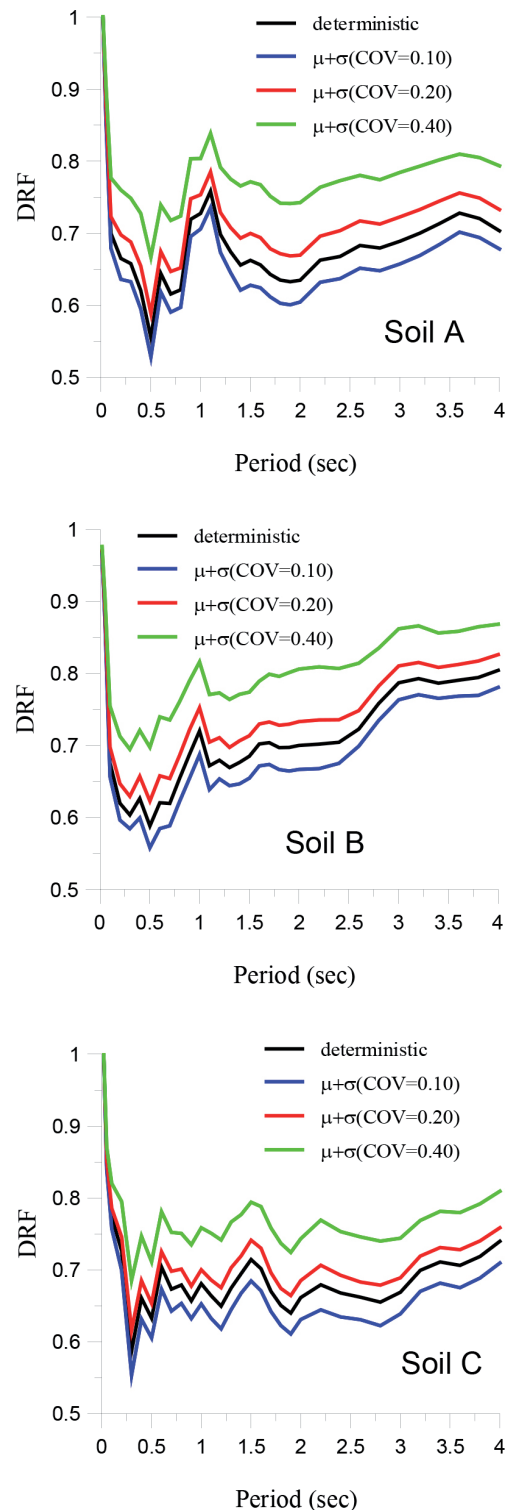

Fig. 5 Comparison between different soil conditions ($\xi = 20\%$).

For instance, for $T = 2$ sec, the deterministic value of DRF is equal to 0.60 while it results equal to 0.74 for COV = 0.4, with a percent error around 23%. Similar results are obtained considering different soil types and period ranges. As a consequence, a damping uncertainty with COV = 40% leads to errors in the DRF estimation greater than 20%.

The values of DRF considering uncertainties turn out be greater than the corresponding deterministic values, in other words, if the uncertainties in damping are not considered, a lower spectral ordinates i.e. (lower design base shear or design displacement) are obtained, which is on the unsafe side (not conservative) of design.

4.2 Comparison with literature formulation

In this section, a comparison between different approximate formulations of DRF from the literature and the exact DRF values derived in this study considering uncertain damping (COV = 0.20) is presented, for different values of ξ (10, 15, 20 and 30 %, respectively).

The approximate formulations of DRF herein considered are those proposed by: (i) Bommer et al (2000), (ii) Lin and Chang (2003), (iii) Hatzigeorgiou (2010) and (iv) Wu and Hanson (1989).

The DRF proposed by Bommer et al. (2000) is expressed by the following formula:

$$DRF = \sqrt{\frac{10}{5+\xi}} \geq 0.7 \tag{5}$$

It has been adopted in the European seismic code (EC8 EN 2004).

Hatzigeorgiou (2010) has proposed a new method for evaluating DRF taking into account the influence of soil conditions and ground motion type (use of natural or artificial accelerograms, near- or far-fault earthquakes), besides viscous damping ratio and period of vibration:

$$DRF(\xi,T) = 1 + (\xi - 5) \cdot \left[1 + c_1 \cdot \ln(\xi) + c_2 \cdot (\ln(\xi))^2 \right]$$
$$\cdot \left[c_3 + c_4 \cdot \ln(T) + c_5 \cdot (\ln(T))^2 \right] \tag{6}$$

The values of the coefficients c_i are given in (Hatzigeorgiou 2010) as a function of soil type and the type of seismic ground motion.
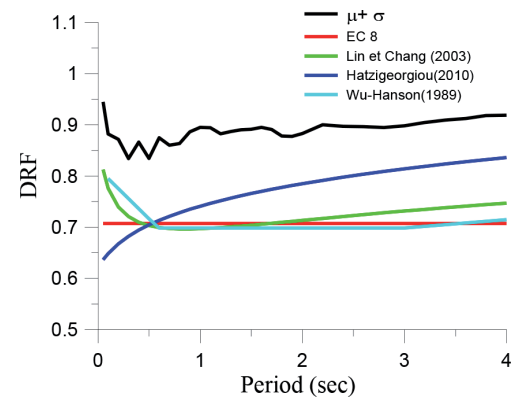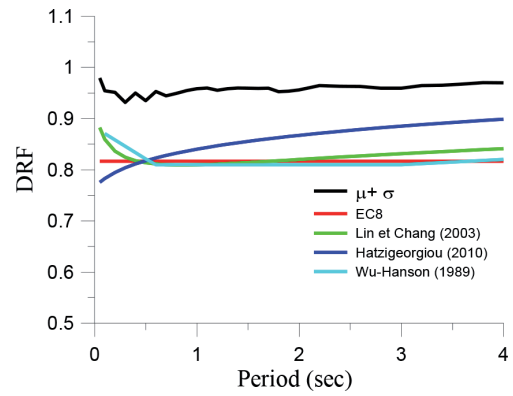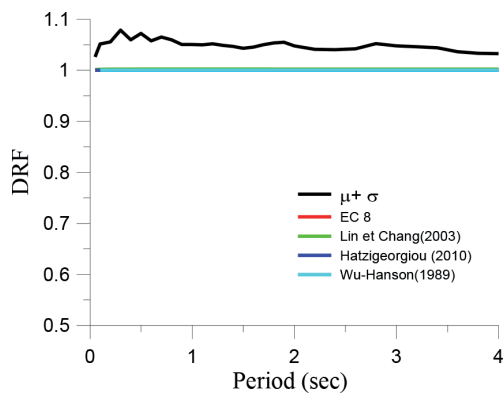

Fig. 6 Comparison between different approximate formulations of DRF and the exact values of DRF ($\mu + \sigma$) considering uncertain damping

The studies carried out by Wu and Hanson resulted in the following expression of DRF.

$$DRF = \frac{\lambda(\xi,T)}{\lambda(5\%,T)} \tag{7}$$

in which is represented by a set of logarithmic relations. Lin and Chang (2003) proposed the following period dependent formulation of DRF:

$$DRF = 1 - \frac{aT^{0.80}}{(T+1)^{0.65}} \tag{8}$$

where $a = 1.303 + 0.436 \ln(\xi)$.

Fig. 6 compares different approximate formulations of DRF and the exact values of DRF ($\mu + \sigma$) derived considering uncertain damping.

From this figure the following conclusions can be drawn: (i) The differences between literature DRF formulations and the results of this study, considering uncertain damping, increase while increasing the damping ratio; (ii) All the literature formulations underestimate the exact DRF values, considering uncertain damping, in some period ranges while they overestimate the exact DRF values in other period ranges.

4.3 Soil effects

To quantify the variability of the uncertain DRF curves for different soils, the DRF curves for each soil and their average (DRF_{all}) are calculated and presented in fgure 7.

The DRF_{all} value can be considered as a damping reduction factor value that neglects site effects. It is clear from the Figure that the influence of soil type on damping reduction factors increases while the damping ratio increases. The maximum error between DRF_{all} and DRF estimated for each site is less than 8.38% for a damping ratio $\xi = 20\%$, and 11.47 % for $\xi = 30\%$. For $\xi = 20\%$, the maximum error is 7.53% for soil A, 8.38% for soil B and 7.18% for soil C. Based on the results shown in this figure, it can be concluded that there is a weak dependency between the DRF values found and the soil type that can be neglected.
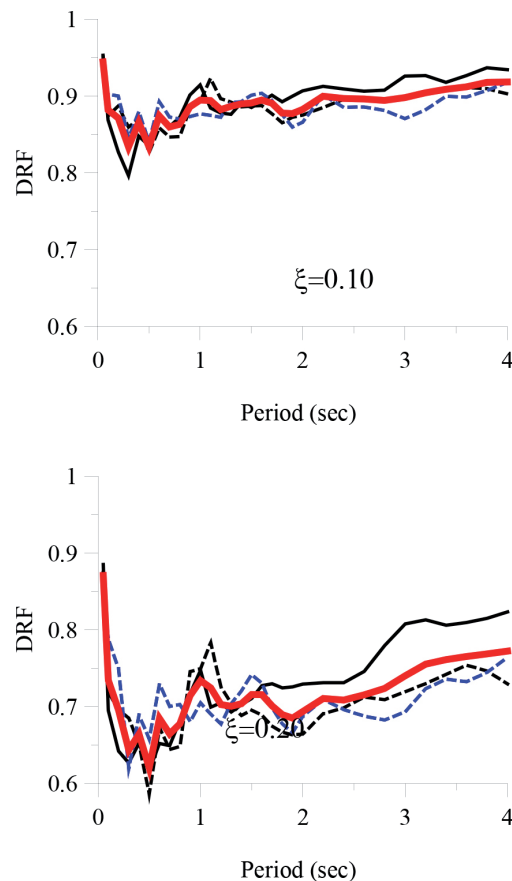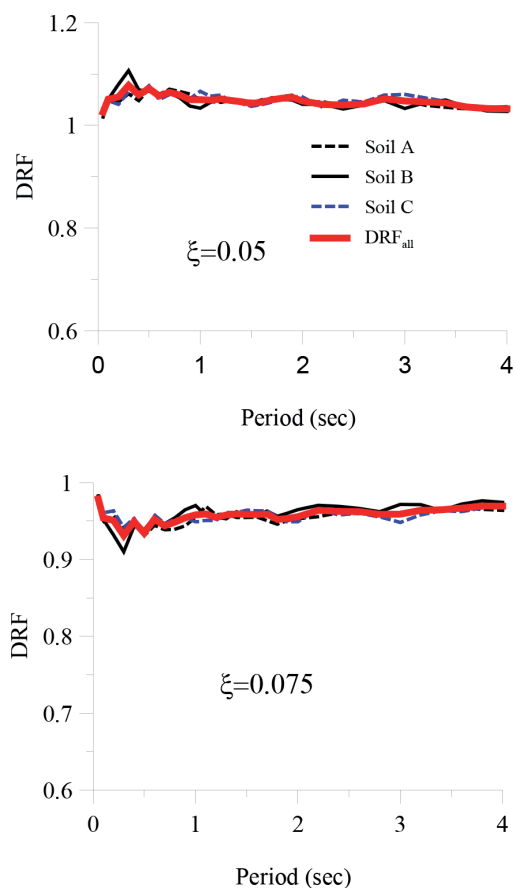

Fig. 7 DRF curves for different soils and their average (DRF_{all}).

4.4 Proposed DRF formulation

As seen before, currently available DRF formulations are not able to take into consideration damping uncertainties in the DRF estimation, so it is necessary to develop a new formulation to fill this gap.

The Eureqa software, sometimes called the robot scientist [21], developed at the Computational Synthesis Lab of the Cornell University by [23] has been used in this study to derive a new DRF formulation accounting for damping uncertainties. The Eureka software uses symbolic regression for detecting equations and hidden mathematical relationships in raw data.

In this case, the input of Eureqa is represented by three vectors providing uncertain values of DRF and corresponding values of damping ratio and period of vibration, respectively.

The model finally selected has been chosen as that featuring the highest R-Squared value and the minimum value of Mean Absolute Error (MAE), also taking into account the simplicity and numerical stability of the model [20].

Herein, the following relationship is tentatively proposed:

$$DRF = 0.941 + \frac{0.009}{\xi} + 0.028 \times \frac{\xi}{T} - 1.335 \times \xi \qquad (9)$$

Figure 8 compares the proposed DRF model and exact values of DRF considering uncertain damping. A good correlation between proposed model and obtained results is observed. This is also confirmed by the relative error between model

predictions and exact results reported in Figure 9. As can be seen, indeed, although the relative error increases while increasing the reference damping ratio, it remains always less than 15 % (for ξ=20%). Focusing the attention on negative errors only (i.e. real results lower than model results), errors less than 7.2 % are found.
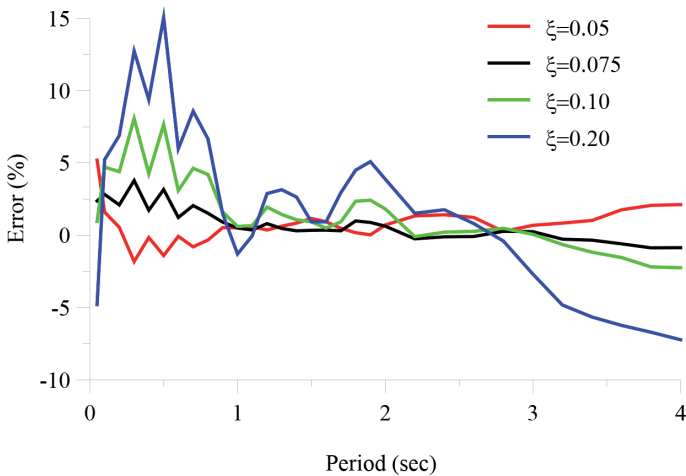

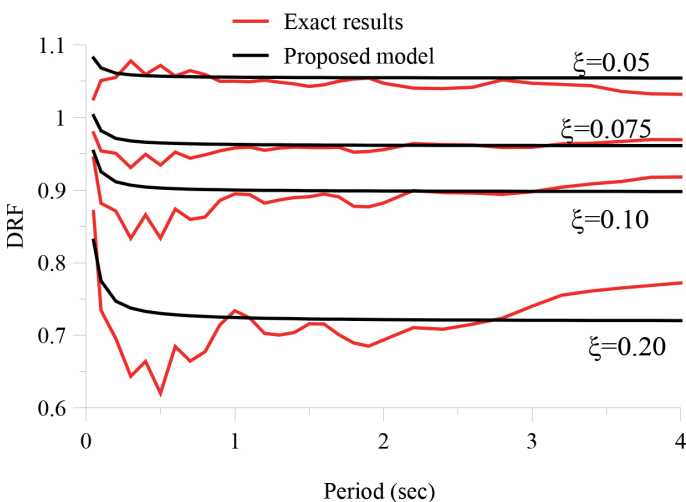**Fig. 8** Comparison between proposed DRF model and exact values of DRF considering uncertain damping.

Fig. 9 Error between proposed DRF model and exact values of DRF considering uncertain damping.

5 Observations and Concluding Remarks

In this paper, the effect of damping uncertainties on the damping reduction factor (DRF) has been examined. All the DRF formulations found in the literature are not able to take into consideration such uncertainties. Based on the results of this study, a new DRF formulation, able to account for uncertainties in damping estimation, has been tentatively proposed. The proposed relationship depends on damping ratio and period only. The error between model predictions and exact results does not exceed 15% for ξ=0.20.

Further studies are needed to take in consideration the effects of period uncertainties in addition to damping uncertainties on damping reduction factor.

Another work is in progress to propose a formulation to determine a direct relation between the damping uncertainty and the uncertainty in the DRF values.

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