

# Modified Dolphin Monitoring Operator for Weight Optimization of Frame Structures

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RESEARCH ARTICLE

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## Abstract

In this article, a modified dolphin monitoring (MDM) operator is introduced and used to improve the performance of the colliding bodies optimization (CBO) algorithm for optimal design of steel structures (CBO-MDM). The performance of the CBO, enhanced colliding bodies optimization (ECBO) and CBO-MDM are compared through three well-established structural benchmarks. The optimized designs obtained by these algorithms are compared, and the results show that the performance of CBO-MDM is superior to those of the other two algorithms. The MDM is found to be a suitable tool to enhance the performance of the CBO algorithm.

## Keywords

modified dolphin monitoring, metaheuristic algorithms, CBO algorithm, ECBO algorithm; optimal design of frame structures

## 1 Introduction

One of the most important applications of optimization in knowledge engineering is optimal design that leads to correct use of what is limited in each engineering problems [1]. Structural optimization leads economical design requiring less material, computational time and human effort [2]. Meta-heuristic algorithms are widely used as robust tools for structural optimization. Some of these can be listed as: Genetic Algorithms (GA) [3], Particle Swarm Optimization (PSO) [4], Charged System Search algorithm (CSS) [5], Krill-herd algorithm (KA) [6], Ray Optimization (RO) [7,8], Dolphin Echolocation Optimization (DEO) [2,9], Colliding Bodies Optimization (CBO) [10], Enhanced Colliding Bodies Optimization algorithm (ECBO) [11], Natural Forest Regeneration algorithm (NFR) [12], Water Evaporation Optimization (WEO) [13], Tug of War Optimization (TWO) [14], Gray Wolf Optimizer (GWO) [15], Ant Lion Optimizer (ALO) [16], Simplified Dolphin Echolocation algorithm (SDEA) [17].

In this study, weight optimization of frame structures is examined. Sections are selected from a standard set of steel sections such as American Institute of Steel Construction (AISC) wide-flange W-shapes. Benchmarks problems of this paper are optimized by the CBO and the ECBO algorithm. In this paper, Modified Dolphin Monitoring (MDM) is introduced and it is shown how it can be improve the performance of the CBO algorithm. MDM is a modified version of the Dolphin Monitoring (DM), in this modified version some new procedures are used to augment the DM ability to reach the global search and prevent entrapment in local optima. Results show that the performance of CBO-MDM algorithm is improved in comparison to the CBO and ECBO algorithms.

This paper is organized as follow: After this introductory section, a brief explanation of the CBO and ECBO algorithms is provided in section 2. Modified dolphin monitoring is presented in section 3. In section 4, the formulations of the strength constraints of AISC load and resistance factor design specifications and displacement constraints are provided. Section 5 includes three well-known benchmark problems. Concluding remarks are presented in the final section.

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## 2 CBO and ECBO algorithms

The main steps of colliding bodies optimization algorithm are presented as follows [8]:

1. Initialization: The initial positions of all the CBs are selected randomly in the search space. It is mimicked as position of some bodies.
2. Search: The value of mass for each CB is assessed, the magnitude of each mass depends on the quality of each CB's cost function. CBs are sorted according to their mass in decreasing order in a way that better CBs and worse CBs are assigned as stationary body and moving body, respectively. Stationary bodies and moving bodies are equally divided into two groups in CBO algorithm. Moving bodies move to stationary bodies and their velocity depend on the distance between them and stationary bodies, velocity of stationary bodies are considered as zero before collision. New position of each CBs after collision are calculated by collision laws.
3. Termination condition check: After the predefined parameter iteration number, the optimization process is terminated.

Additionally in this paper an enhanced form of the CBO (ECBO) is applied to frame structures. ECBO algorithm uses a memory to save better results. It should be added that this algorithm employs a random vector to improve the results.

## 3 Modified dolphin monitoring

The correct adjusting of parameters of each algorithm based on the type of problem is one of the important aspects that must be adhered regarding the meta-heuristic algorithms. This issue mostly depends on designer's personal experience and is therefore considered a disadvantage for meta-heuristic algorithms. Incorrect adjusting of algorithm parameters may result in an algorithm to be trapped in local optima solution or solutions with a penalty. Also, some algorithms are only able to appropriately solve particular problems. One of the conclusions obtained by reviewing convergence factor at any loop is that the probability of early trapped algorithm in the local optima can be reduced with a population having a proper dispersion in the entire search space. Additionally, the ability to achieve global optima algorithm is increased.

The dolphin monitoring method was recently proposed to control the convergence factor, Kaveh and Farhodi [18]. This method is modified in this paper and is called the modified dolphin monitoring. The purpose of this modification is to efficiently control the population dispersion to strengthen the ability of algorithms. This version also does not cause a change in the structure of algorithms, but adds features such as modified dolphin monitoring which must be applied at the end of each loop range is defined to the average locations for each variable for a specified radius factor from standard deviation and this coefficient is equal to 15% in this study, In other word the range for each variable is equal to average  $\pm$  (15%) standard deviation. In each loop, the percent of the population for each variable in the range (available

population dispersion index) must be equal to the amount specified by Eq. (1).

$$MP_i = 10 + 60 \left[ \frac{i-1}{LoopNumber-1} \right] \quad (1)$$

In the above equation,  $MP_i$  is mandatory population dispersion for  $i$ th loop and the present dispersion of population index should reach this amount in each loop. LoopNumber is the total number of loops. It should be mentioned that the value of MP never reaches to 1 and its maximum value is 0.7. This action makes the algorithm more flexible and when the MP reaches to 1, the total population must be necessarily placed within the mentioned range. However, if the MP reaches to a value of 0.7, there will still remain a maximum chance for search.

### 3.1 Modified dolphin monitoring method

- 1.) Calculating the MP in each loop using Eq. (1).
- 2.) Calculating the population within the mentioned range for each variable in each loop and calling it as available population dispersion index.

3.a) If the available population dispersion index is greater than the mandatory population dispersion, the algorithm is moving faster than what is expected to the optimal area (the same range) and the available population dispersion index should be decreased. To do this, the modified dolphin monitoring has considered two mechanisms:

Replacing the variable of interest from population which is in the range with:

- 3.a.1) The variable of interest from available population which are out of the range
- 3.a.2) Values that are randomly generated within the permitted range for each variable.

The modified dolphin monitoring uses both mechanisms at the same time with a probability of 50 percent.

3.b) If the available population dispersion index is smaller than the mandatory population dispersion, algorithm is converging slower than what is expected to an optimal area. In this case, the available population dispersion index should be increased. To do this, the modified dolphin monitoring has considered two mechanisms same as before:

Replacing the variable of interest from population which is out of the range with:

- 3.b.1) The best available optimal variable to the stage.
- 3.b.2) Values that are in the desired range.

Both mechanisms are used with a probability of 50 percent, as the former case.

The first mechanism holds the algorithm not to move away from the best solution and makes the range optimal up to that point and also continues to seek achieved optimum solutions up to that point.

For further clarity, the pseudo-code of the third step (containing 3.a and 3.b) is presented in the following:

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for j = 1:number of variables
  while available population dispersion index(j) ~ mandatory population dispersion(j)
    if available population dispersion index(j) > mandatory population dispersion(j)
      if rand < 0.5
        variable of interest from population which are in the range = variable of interest from available population which are out of the range;
      else
        variable of interest from population which are in the range = values that are randomly generated within the permitted range for jth variable;
      end
    elseif available population dispersion index(j) < mandatory population dispersion(j)
      if rand < 0.5
        variable of interest from population which are out of the range = the best available optimal variable to the stage;
      else
        variable of interest from population which are out of the range = values that are in the desired range;
      end
    end
  end
end
end
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#### 4 Formulation of the optimization problem

In this formulation, the aim is to minimize the weight of frame structures besides satisfying certain design constraints. Design constraints include strength and displacements constraints according to LRFD-AISC specification [19]. The mathematical formulation can be expressed as follow:

Find  $\{x\} = [x_1, x_2, \dots, x_{ng}]$   $x_i \in S_i$

To minimize  $W(\{x\}) = \sum_{i=1}^{nm} \rho_i A_i L_i$  (2)

where  $\{x\}$  is a set of design variables containing the cross sectional area of W-sections;  $ng$  is the numbers of member groups (number of design variables);  $W(\{x\})$  is the weight of the structure;  $nm$  is the number of elements of the structure;  $\rho_i$  is the material density of the member  $i$ ;  $A_i$  and  $L_i$  denote the cross-sectional area and the length of the member  $i$ , respectively. Here,  $x_i$  is the number of a W-section and  $A_i$  is the cross-sectional area of the  $i$ th group.

In this study, discrete optimization is considered, and the  $i$ th variable is selected from  $S_i$  as follow:

$$S_i = (s_{i,1}, s_{i,2}, \dots, s_{i,r(i)}) \quad (3)$$

Thus the problem can be solved as discrete optimization problem,  $r(i)$  is the last available discrete value. To control the constraints, penalty approach is used as follow:

$$fitness(x) = (1 + \varepsilon_1 \nu)^{\varepsilon_2} \times w\{x\}, \nu = \sum_{j=1}^{nc} \max(0, v_j) \quad (4)$$

where  $fitness(x)$  and  $\nu$  are the fitness function and sum of the violations of the design. In this study,  $\varepsilon_1$  and  $\varepsilon_2$  are set to 0.3 and 1, respectively, and  $nc$  is the total number of constraints for each individual design.

According to AISC-ASD [19] constraints are as follow:

(a) Maximum lateral displacement

$$\frac{|\Delta_T|}{H} - R \leq 0 \quad (5)$$

where  $\Delta_T$  is the maximum lateral displacement of the roof;  $H$  is the height of the frame structure;  $R$  is the maximum drift index (in this study it is equal to 1/300).

(b) The inter story drift constraints

$$\frac{|d_i|}{h_i} - R_i \leq 0; \quad i = 1, 2, \dots, ns \quad (6)$$

where  $d_i$  is the inter story drift;  $h_i$  is the story height of the  $i$ th floor;  $ns$  is the total number of stories;  $R_i$  denotes the inter story drift index and its limitation is like  $R$  index.

(c) Strength constraints

$$\frac{P_u}{2\phi_c P_n} + \left[ \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right] - 1 \leq 0; \quad \text{for } \frac{P_u}{\phi_c P_n} < 0.2 \quad (7)$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left[ \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right] - 1 \leq 0; \quad \text{for } \frac{P_u}{\phi_c P_n} \geq 0.2$$

where  $P_u$  is the required strength (tension or compression);  $P_n$  is the nominal axial strength (tension or compression);  $\phi_c$  is the resistance factor ( $\phi_c = 0.9$  for tension and  $\phi_c = 0.85$  for compression);  $M_u$  (containing  $M_{ux}$  and  $M_{uy}$ ) is the required flexural strengths;  $M_n$  (containing  $M_{nx}$  and  $M_{ny}$ ) is the nominal flexural strengths (for two-dimensional frames  $M_{uy} = 0$  and  $M_{ny} = 0$ ); and  $\phi_b$  presents the flexural resistance reduction factor ( $\phi_b = 0.90$ ). The nominal tensile strength for yielding in the gross section is evaluated as follow:

$$P_n = A_g \cdot F_y \quad (8)$$

and the nominal compressive strength of a member is calculated by:

$$P_n = A_g \cdot F_{cr} \quad (9)$$

$$F_{cr} = (0.658^{\lambda_c^2}) F_y; \quad \text{for } \lambda_c \leq 1.5$$

$$F_{cr} = \left( \frac{0.877}{\lambda_c^2} \right) F_y; \quad \text{for } \lambda_c > 1.5 \quad (10)$$

$$\lambda_c = \frac{kl}{r\pi} \sqrt{\frac{F_y}{E}} \quad (11)$$

where  $A_g$  is the cross-sectional area of a member and  $k$  is the effective length factor that is calculated by the approximate formula for unbraced frames based on the study of Dumonteil [20].

$$k = \sqrt{\frac{1.6G_A G_B + 4.0(G_A + G_B) + 7.5}{G_A + G_B + 7.5}} \quad (12)$$

where  $G_A$  and  $G_B$  are stiffness ratios of columns and girders at the two end joints, A and B, of the column section, respectively.

### 5 Optimum design of steel frame using CBO, ECBO and CBO-MDM

In this section, three benchmark frame structures are considered to investigate the performance of the CBO-MDM in comparison with CBO and ECBO algorithms. Minimizing the weight of three frame structures is the aim of this study. These frames are:

- A 1-bay 10-story frame
- A 3-bay 15-story frame
- A 3-bay 24-story frame

In this study, a population of  $n = 60$  is used for all the algorithms (CBO, ECBO and CBO-MDM). One thousand iterations are performed as the maximum number of iterations. For more precise study, each problem was solved 100 times independently. All the considered algorithms can only select discrete values from the permissible cross section for each problem.

All the problems and algorithms are coded in MATLAB and the frame structures are analyzed using a direct stiffness method. It should be noted that the purpose of optimal solution is to find the best answer among answers for each algorithm.

#### 5.1. A 1-bay 10-story frame

Figure 1 illustrates the topology, applied loads and numbering of the member groups for a one-bay 10-story frame. The element grouping results in four beam sections and five column sections for a total of nine design variables. Beam element groups are selected from 267 W-sections, and column groups are chosen from only W14 and W12 sections.

The modulus of elasticity is equal to  $E = 200\text{GPa}$  (29 000 ksi) and the yield stress is  $248.2\text{MPa}$  (36 ksi). The effective length factors of the members are calculated as  $K_x \geq 1.0$  for a sway-permitted frame, and the out-of-plane effective length factor is specified as  $K_y = 1.0$ . Each column is considered as non-braced along its length, and the non-braced length for each beam member is specified as 1/5 of the span length. The frame is designed according to the LRFD specifications [19] and consideration of Section 4.

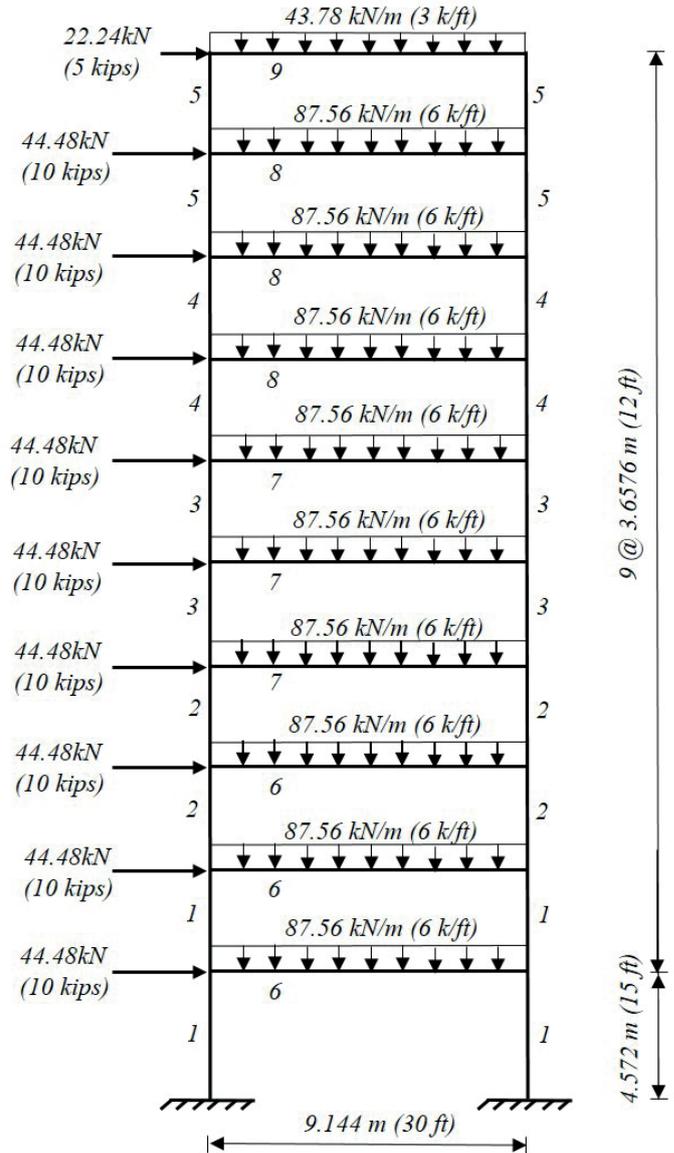


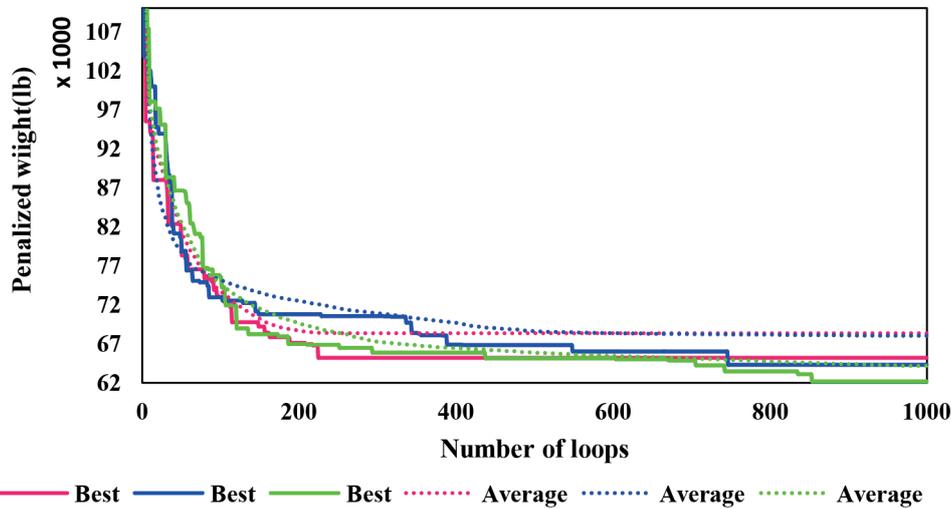
Fig. 1 Schematic of the 1-bay 10-story planar frame.

Table 1 provides a comparison of the optimal designs obtained by CBO, ECBO and CBO-MDM algorithms. This table contains best solution vector and the best, worst and mean weights for all the considered algorithms after 100 individual run.

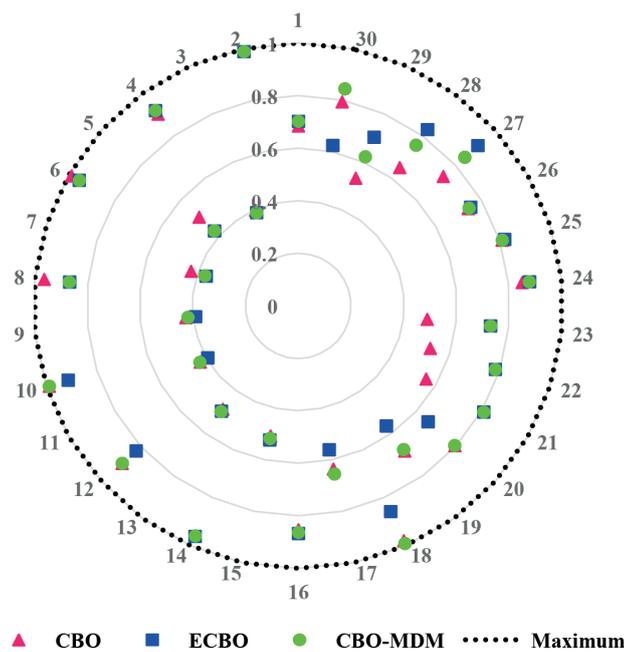
**Table 1** Optimal designs of the CBO, ECBO and CBO-MDM for the 1-bay 10-story frame

Element group	Optimal W-shaped sections		
	CBO	ECBO	CBO-MDM
1	W14x211	W14x211	W14x211
2	W14x159	W14x176	W14x176
3	W14x132	W14x145	W14x132
4	W14x99	W12x106	W14x9
5	W14x61	W12x79	W14x61
6	W40x149	W33x118	W33x118
7	W30x99	W30x99	W30x99
8	W27x94	W24x84	W27x84
9	W18x50	W21x55	W21x44
Best weight (kN)	290.69	286.8	277.28
Worst weight (kN)	320.89	323.55	298.4
Mean weight (kN)	304	302.6	285.16

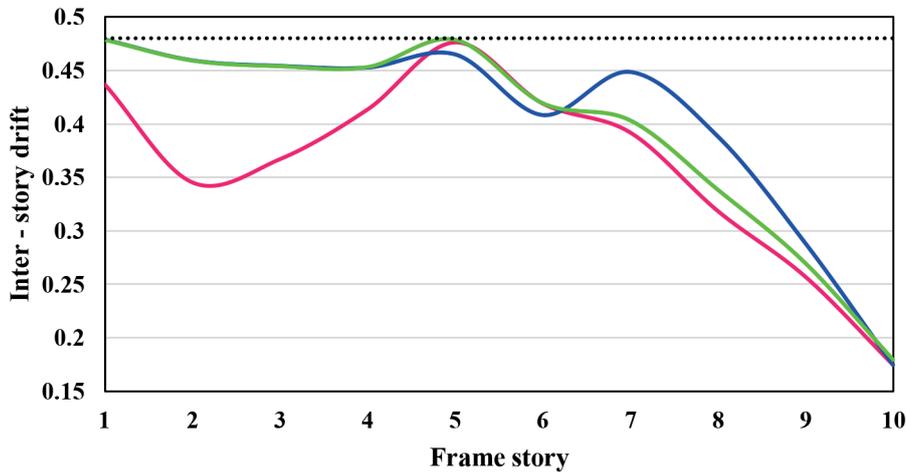
It can be seen that the CBO-MDM has reached the lightest result compared to the CBO and ECBO algorithms. Also the CBO-MDM obtained better mean weight among other algorithms. The best weight of the CBO, ECBO and CBO-MDM are 290.69 kN, 286.8 kN and 277.28 kN, respectively. The CBO, ECBO and CBO-MDM algorithms obtained the optimal design after 225, 747 and 854 iterations respectively. Additionally the CBO-MDM reached the optimal design of CBO and ECBO after approximately 294 and 705 iterations. Fig. 2 illustrates the convergence histories of the CBO, ECBO and CBO-MDM for the best and average designs of the 1-bay 10-story frame. It is obvious that the CBO-MDM obtained better results compared to the CBO and ECBO. Figure 3 and Fig. 4 show the existing stress ratios and inter-story drift for the optimal design of the CBO, ECBO and CBO-MDM algorithms.



**Fig. 2** Convergence curves of the 1-bay 10-story frame.



**Fig. 3** Stress ratios for optimal design of the 1-bay 10-story frame.



— CBO — ECBO — CBO-MDM ..... Maximum

Fig. 4 Inter-story drifts for optimal design of the 1-bay 10-story frame.

### 5.2 A 3-bay 15-story frame

The second design problem is a 3-bay 15-story frame containing 64 joints and 105 members. This is a famous benchmark problem and Fig. 5 illustrates the topology, applied loads and the numbering of member groups for the frame. One hundred and five members are classified into eleven groups (9 column groups and one beam group). The material density is  $E = 200\text{GPa}$  (29 000 ksi) and the yield stress is  $248.2\text{MPa}$  (36 ksi) for all the members.

The effective length factors of the members are considered as  $k_x \geq 0$  for a sway-permitted frame and the out-of-plane effective length factor is indicated as  $k_y = 1.0$ . Each column is considered as non-braced along its length, and the non-braced length for each beam member is determined as one-fifth of the span length. The frame structure is designed according to the LRFD specifications and uses an inter-story drift displacement constraint [19].

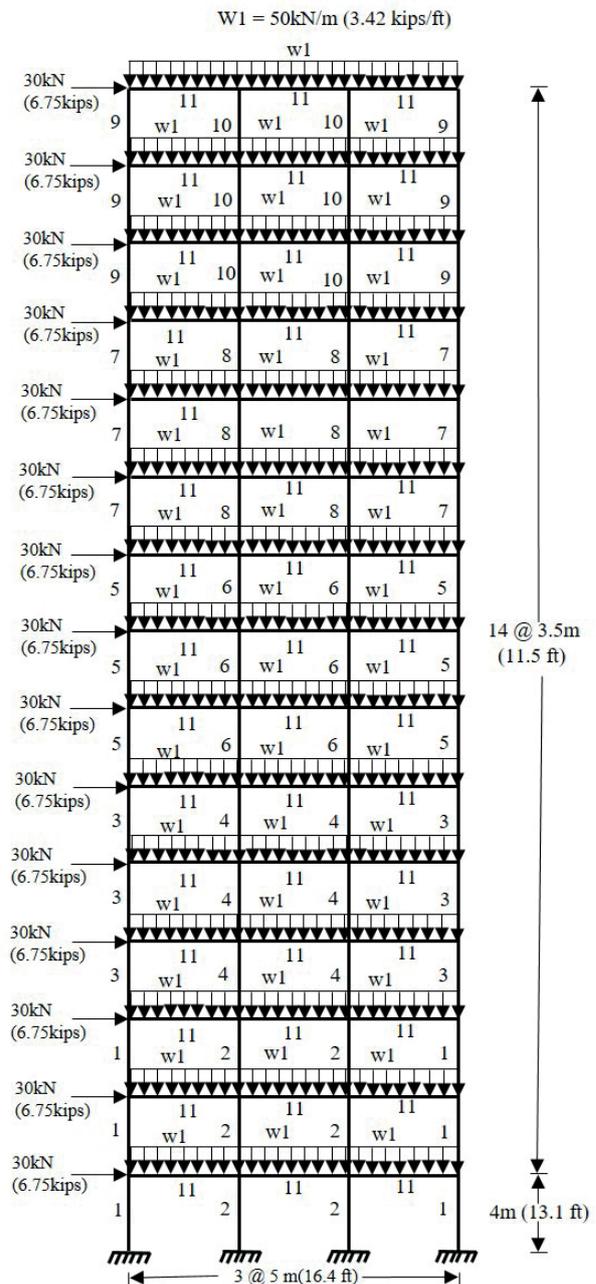


Fig. 5 Schematic of the 3-bay 15-story planar frame.

Table 2 presents the optimal values of the eleven variables achieved by CBO, ECBO and CBO-MDM algorithms; also, this table contains worst and means weights for all the algorithms.

**Table 2** Optimal designs of the CBO, ECBO and CBO-MDM for the 3-bay 15-story frame

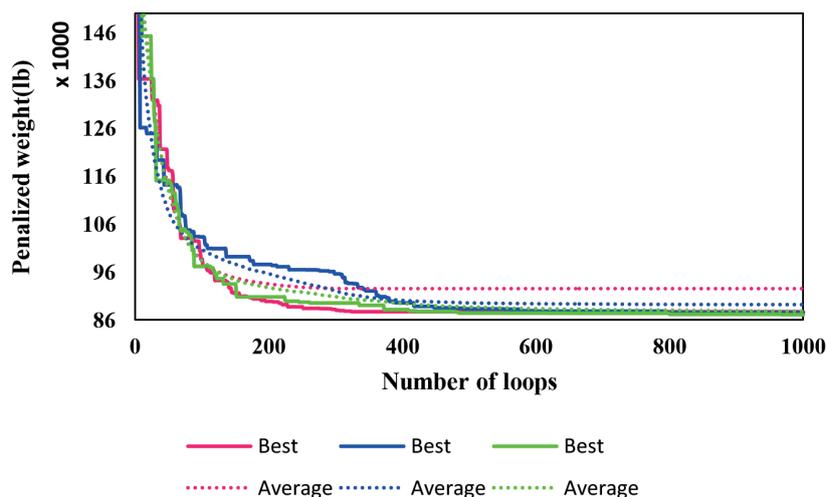
Element group	Optimal W-shaped sections		
	CBO	ECBO	CBO-MDM
1	W24x104	W21x111	W14x99
2	W36x160	W27x146	W27x161
3	W24x84	W27x84	W14x82
4	W24x104	W24x104	W24x104
5	W21x68	W12x65	W16x67
6	W18x86	W18x86	W18x86
7	W18x50	W21x55	W21x48
8	W12x65	W14x61	W14x61
9	W8x28	W14x38	W8x28
10	W10x39	W8x35	W10x39
11	W21x44	W21x44	W21x44
Best weight (kN)	391.15	390.40	387.45
Worst weight (kN)	430.55	414.94	424.20
Mean weight (kN)	411.72	397.07	390.90

CBO, ECBO and CBO-MDM reach 391.15 kN, 390.40 kN and 387.45 kN, respectively; also CBO-MDM obtain better mean weight compared to CBO and ECBO. Additionally CBO, ECBO and CBO-MDM reach the optimal design after 323, 794 and 969 iterations correspondingly, CBO-MDM reaches the optimal design of CBO and ECBO after 480 and 617 iterations, respectively.

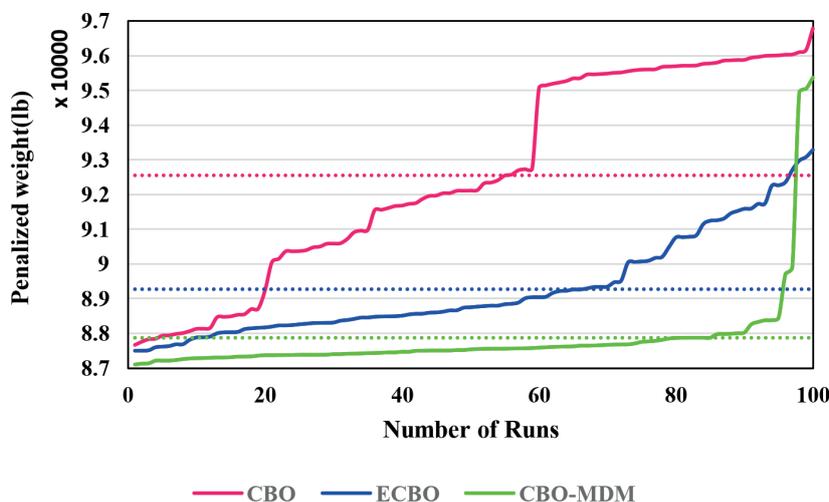
Figure 6 illustrates the best and mean of one hundred runs convergence history for the expected algorithms. Figure 7 displays the final weights obtained by CBO, ECBO and CBO-MDM algorithms in one hundred independent runs. It is obvious that the CBO-MDM has better performance than the other counterparts.

### 5.3 A 3-bay 24-story frame

A 3-bay 24 story frame is shown in Fig. 8 as the last design problem. This structure consists of 168 members that are collected in 20 groups (16 column groups and 4 beam groups). The beam and column element groups are selected from all 267 W-shape and W-14 sections. The material has a modulus of elasticity equal to  $E = 205\text{GPa}$  (29,732 ksi) and a yield stress of  $f_y =$



**Fig. 6** Convergence curves of the 3-bay 15-story frame



**Fig. 7** Final weights of one hundred independent runs for the 3-bay 15-story frame (sorted).



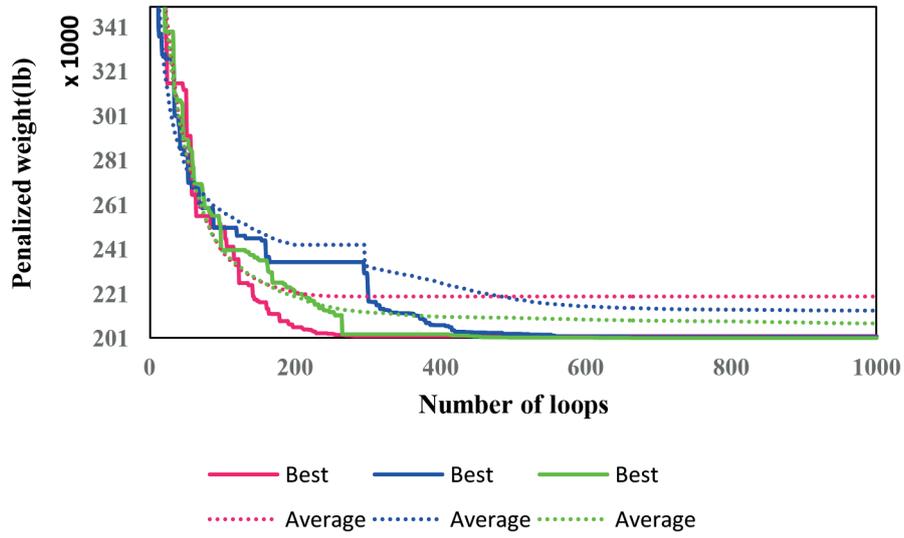


Fig. 9 Convergence curves of the 3-bay 24-story frame.

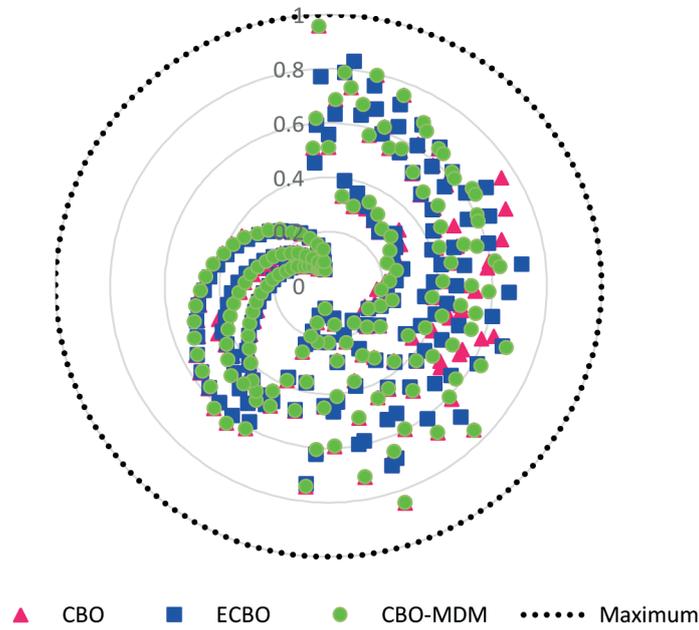


Fig. 10 Stress ratios for optimal design of the 3-bay 24-story frame.

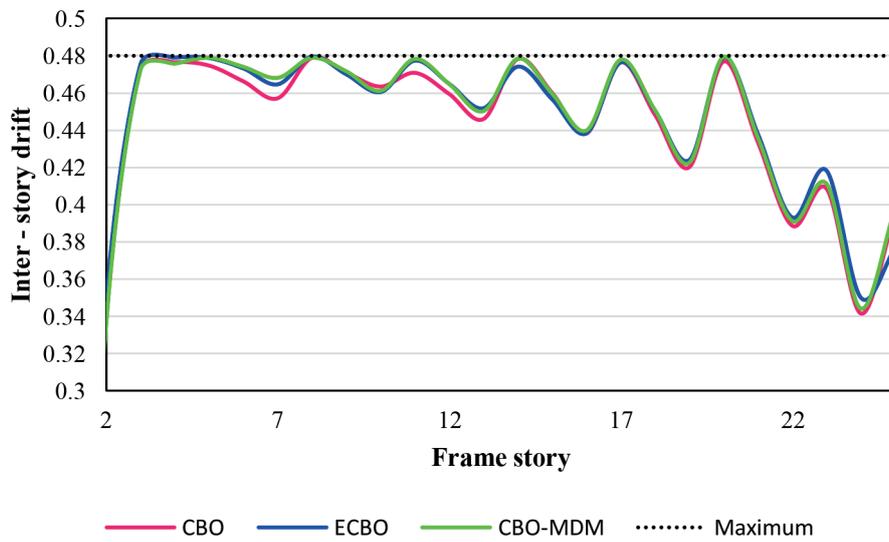


Fig. 11 Inter-story drifts for optimal design of the 1-bay 10-story frame.

## 6 Conclusions

This study presents the Modified Dolphin Monitoring (MDM) operator with its performance when coupled with CBO algorithm. It should be noted that the MDM does not cause any change in the main steps of the optimization algorithm and only performs as an operator on the final solution of each loop, improving the behavior of the main algorithm. To evaluate the performance, the CBO-MDM is compared with that of the CBO and its enhanced version ECBO. Optimal design of three frame structures known in the optimization literature is aimed at minimizing weight according to the existing constraints.

Results of all the considered benchmark problems show that the CBO-MDM is capable to achieve a better solution in comparison to the standard CBO and ECBO algorithms, and found a better mean solution. Apart from one case, its worst solution was also better than the other two methods. On the other hand, according to the figures provided, the method improves the speed of the convergence for the algorithms.

Finally, it seems that using this method promotes the ability the algorithms in achieving the optimum solutions, and the use of this method for other algorithms can be recommended.

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