

## Abstract

Borehole collapse is predicted by common rock failure criteria despite they give conservative results due to some simplification of rock behavior. It is the source of many difficulties in implementation of oil well, underground structures and foundations. Utilizing damage theory could improve the conservancy of rock failure criterion. Damage theory considers loss of function rather than loss of strength. Unlike criteria which were derived from plasticity that assume yielding point as the limit of functionality of rock, damage theory uses a state between failure and yielding in stress-strain diagram as the limit of the rock applicability. In this study, a new model is proposed to predict minimum borehole pressure based on rock damage theory. The model is verified by actual wellbore data. The proposed model predicts minimum borehole pressure close to actual data and more accurate than Mohr-coulomb and Mogi-Coulomb criteria.

## Keywords

collapse pressure, wellbore stability, rock damage, rock micro-mechanics

## 1 Introduction

One of the most challenging issues in geomechanics is wellbore stability. In recent decades many new hydrocarbon reservoirs are found in very deep formations and drilling in those conditions need more accurate calculations and mitigations. Stability related problems cost drilling industry more than Hundreds of million dollars per year worldwide, and possibly as much as one billion dollars annually [1]. Among reservoir formations, Shale is the source of many instability problems and after stress and pressure control to prevent blowouts, it is the second largest geomechanics issue in deep drilling [2].

Instability mechanisms can be grouped into three categories in shale formations [3]: 1) Tensile failure due to excessive wellbore pressure. 2) Compressive failure due to excessively low wellbore pressure 3) Hole size reductions due to swelling of shale, which results in repeated reaming, or in extreme conditions, stuck drill pipe. Many researches have focused on wellbore stability in shale formation considering mechanical, hydrological, chemical and thermal effects which can be found in literature [4–7].

The research show that mechanical effects and wellbore pressure are the main sources of wellbore collapse. Mechanical instability of borehole can be controlled by wellbore pressure and mud density. Mud weight window (MWW) is defined for safe drilling like Fig 1. MWW has upper and lower limits. Excessive wellbore pressure causes fracture on wellbore periphery resulting in mud loss and high drilling cost. Also insufficient mud weight and wellbore pressure causes breakout and even collapse (Fig. 1).

To determine wellbore pressure accurately, proper rock constitutive law should be used. Most of the rock constitutive laws are based on the plastic theory and are too conservative since they assume yielding point of stress-strain diagram as the limit for rock functionality [8]. According to these kinds of criteria, materials which pass yield point are not strong enough to remain functional while some materials such as rock in experiments showed that they can sustain notable plastic strain. Also some rock strength predicting models use peak strength as the final bearing capacity limit which is not applicable in rock

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materials. So realistic rock strength at which the rock material remains functional yet must be between yield and failure limits. In addition, the strength and deformation of rock masses mainly depend on their discontinuity properties [9] and damaged degree.

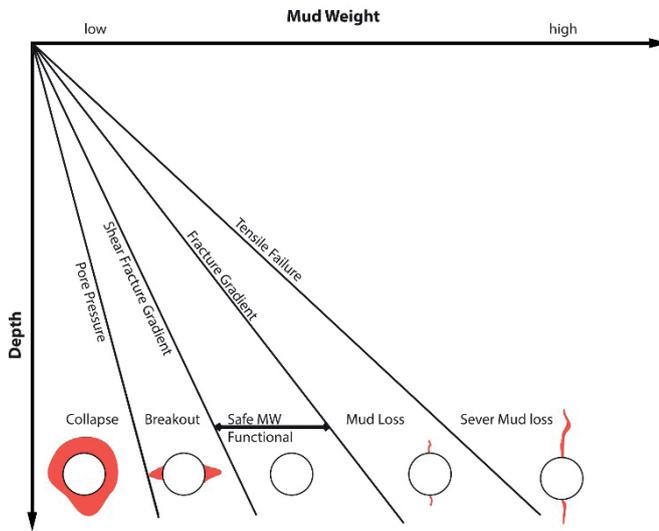


Fig. 1 Mud weight window

Rock failure phenomena starts from micro cracking and it is reasonable to describe the failure process by micromechanics. Damage theory uses “loss of functionality” idea rather than “loss of strength” to describe material failure process [2]. It means, however a material loses its strength after yield point, it is may be usable as a structure. This idea can be utilized to define loss of functionality by reducing the strength of material. In micromechanics point of view, after yielding, microcracks start to develop and by connecting the microcracks together, rock properties alter and provide possibility that rock may no longer remain functional. The stress level at which the microcracks connect is called “crack damage stress” [10].

In this study failure is treated differently from yielding and these two concepts are distinguished to predict rock damage precisely. The damage theory concept is used to link micromechanics to rock failure criterion. The proposed model is utilized to determine wellbore pressure by considering bearing capacity of rock more realistically. The lower limit of MWW is predicted for a real wellbore drilled in Wanaea oil field, in the northwest shelf of Australia. The results of this method is compared and validated by actual case study data which shows a very close agreement.

### Nomenclature

- a,b,c: Material constant
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- $D_0^{el}$ : Initial (undamaged) elasticity matrix
- $\dot{\tilde{\epsilon}}_t^{pl}$ : Equivalent plastic strain rates in tensile loading
- $\tilde{\epsilon}_t^{pl}$ : Equivalent plastic strain in tensile loading
- $\tilde{\epsilon}^p$ : Plastic strain rate

- $\theta$ : Temperature
- $f_i$ : Predefined field variables
- $d$ : Damage parameter
- $d_t$ : Tensile damage variable
- $d_c$ : Compressive damage variable
- $E_0$ : Initial elastic stiffness of the material
- $\bar{E}$ : Stiffness of the material after starting damage
- $\bar{\sigma}_c$ : Effective compressive cohesion stresses
- $\bar{\sigma}_t$ : Effective tensile cohesion stresses
- $\sigma_c$ : Uniaxial compressive strength
- $\sigma_{cd}$ : Crack damage Stress
- $\sigma_y$ : Yielding Stress
- $s$ : Material constant, for intact rock  $s=1$
- $f$ : Plastic potential
- $\lambda$ : Hardening parameter
- $P_0$ : Pore Pressure

## 2 Rock material failure

A rock type material strains under load elastically at first and by increasing magnitude of the load, it may strain inelastically (Fig. 2). After yielding, rock may behave in the form of strain softening, perfect plastic, or work hardening depends on rock type and finally results in failure. Rock failure is brittle or ductile or even something in between. Weak rocks, such as shale formations, under high stress condition, tends to behave ductile. study showed that the higher the confining pressure, the greater the ductility is observed [9] such as conditions observed in deep drilling projects.

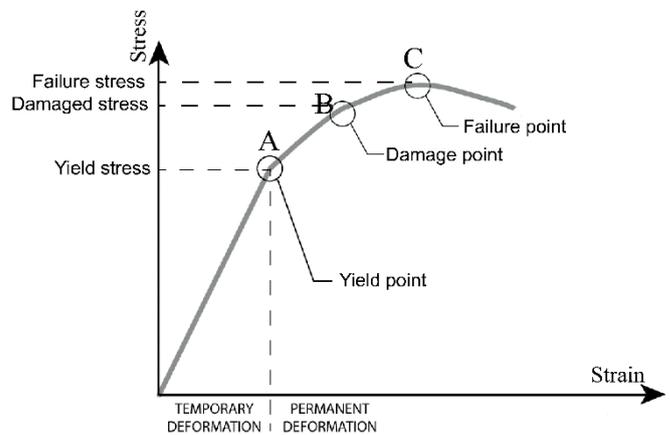


Fig. 2 Stress-strain diagram

The stress state corresponding yield strain is called “yield stress”. Ductile or semi ductile materials can sustain some plastic strain before failure. Corresponding stress at failure is called “failure stress”. The rock damage starts from yielding point to the failure point where the rock is completely damaged. In a point between yielding (A) and failure (C), rock is partially damaged and additive plastic ( $\Delta\epsilon$ ) strain can be estimated linearly as. So for a point like B, it can be written as:

$$\Delta\epsilon = (\sigma_{cd} - \sigma_y) / E.$$

Flow rule theory is used to describe plastic behavior which can lead to an evaluation of material functionality. In flow plasticity theories, total strain in a material is divided into an elastic and a plastic part. Elastic part can be determined from elasticity rules such as Hook law. Determination of the plastic part requires a flow rule and a hardening model. If the yield condition is given by  $f(\sigma) = 0$ , then the normal flow rule takes the following form [11]:

$$\varepsilon^{.p} = \lambda \frac{\partial f}{\partial \sigma} \quad (1)$$

According to rock plasticity, the direction of the plastic strain rate component vector must be normal to the yield surface. The magnitude of these strain rates is not determined by the constitutive laws, reflecting the fact that plastic deformations are rate-independent. So constitutive laws are not able to move with growing plastic strain. To cover this problem some models, such as Cap model, critical state model or double shear model are proposed. Increasing stress deviatory causes rock failure. So stress tensor can be written as:

$$\sigma_{ij} = \sigma_{ij}^e + \sigma_{ij}^p \quad (2)$$

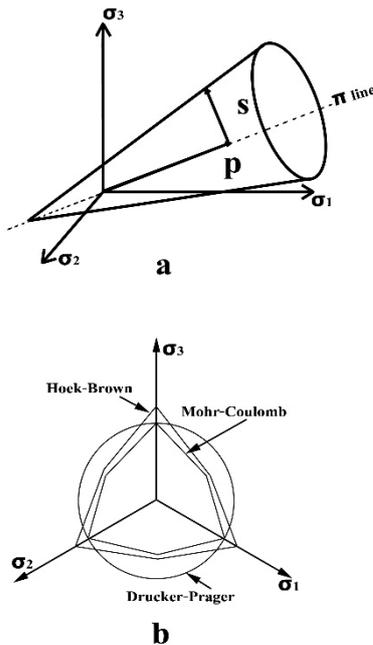


Fig. 3 a) typical yield surface,  
b) rock failure criterion view from hydrostatic line

Therefore rock failure criterion (R) consists of a yielding criterion and a hardening function[12]. Just like the yield condition, the failure condition ( $\sigma_{ij} = 0$ ) can be interpreted as a failure surface in the six-dimensional stress space or alternatively in the three-dimensional space of principal stresses [13]. For example, Mohr-coulomb yielding criterion can be developed to describe plastic behavior of a rock type materials as:

$$f = \sigma_1 - (C + q\sigma_3) = 0 ; \text{ at yielding} \quad (3)$$

$$R = \sigma_1 - (C + q\sigma_3 + h(\sigma) \cdot d) = 0 ; \text{ at failure} \quad (4)$$

$$q = \frac{1 + \sin \phi}{1 - \sin \phi}$$

Yielding points at various stress states make a yielding surface in three dimensional stress state and Cartesian axes (Fig. 3). One of the well-known yielding criteria is Mohr-Coulomb (MC) which assumes rock behavior as elastic-perfectly plastic. Yielding surface of Mohr-Coulomb criterion is illustrated in Fig. 3-b from  $\pi$  line view. In multi axial load conditions, a line with same value of  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  and path through base point is called as  $\pi$  line. Hydrostatic pressure (P) increases along with  $\pi$  line and shear forces (S) is produced by deviatory stress. It is shown that Hoek- Brown (HB) criterion yielding surface is bigger than MC because MC assumes yielding point as the ultimate bearing capacity of rock while HB hypothesis a point above yielding. The difference proves that there some level of rock strength from yielding to failure. Considering rock failure phenomena these levels can be described.

After yielding, crack initiation starts at crack initiation stress level. Existing cracks start to propagate and coalesce to form a dominant shear band. This stress level is defined as “crack damage stress” when cracks connect together. The study of previous researchers’ [10], and) showed that the stress level of cracking can be formulized as below:

$$\sigma_1 - A\sigma_3 = B\sigma_c \quad (5)$$

A and B are material coefficients that vary for each stress level and are determined experimentally for every loading stage. For example, Martin proposed  $A = 1$ ,  $B = 0.4$  for the crack initiation stress level for Lac Du Bonnet granite. By comparing the equation 1 to MC and HB criterion, the coefficients A and B can be drawn for this criterion too [14]. The criterion classical form is:

$$\text{MC: } \sigma_1 - \frac{1 + \sin \phi}{1 - \sin \phi} \sigma_3 = \sigma_c \quad (6)$$

$$\text{HB: } \sigma_1 - \sigma_3 = \left( m \frac{\sigma_3}{\sigma_c} + s \right)^{0.5} \sigma_c \quad (7)$$

Hoek proposed a table to determine m and S values as rock material coefficients based on GSI and damage parameter. He proposed the value of 0.7 for damage parameter defining the loss of functionality for a tunnel excavated with blasting method [14]. According to this criterion for intact rock  $S=1$ . The parameter m can be written in the form of uniaxial tensile and compressive strengths [15].

$$m = \frac{\sigma_c^2 - \sigma_t^2}{\sigma_t} \quad (8)$$

So for a shale which can be assumed as an intact rock the parameter B is:

$$B = \left( \left( \frac{\sigma_c^2 - \sigma_t^2}{\sigma_t} \right) \frac{\sigma_3}{\sigma_c} + 1 \right)^{0.5} \quad (9)$$

### 3 Stress around a wellbore

After drilling, stress distribution changes around the well. These stresses can be written as:

$$\begin{aligned} \sigma_r &= P_w \\ \sigma_\theta &= \sigma_H + \sigma_h - 2(\sigma_H - \sigma_h)\cos 2\theta - P_w \\ \sigma_z &= \sigma_v - 2\nu(\sigma_H - \sigma_h)\cos 2\theta \\ \sigma_{r\theta} &= \sigma_{rz} = \sigma_{\theta z} = 0 \end{aligned} \quad (10)$$

Well stability is controlled by internal pore pressure and mud weight during drilling time. If mud weight is high,  $P_w$  increases in the above formulation and results in decrease of radial stress ( $\sigma_r$ ) and increase of circumferential stress ( $\sigma_\theta$ ). By decreasing mud weight stress state will change oppositely. It means that two limits for mud weight can control stress concentration around a wellbore, lower ( $P_{wb}$ ) and higher mud weight ( $P_{wh}$ ). Lower mud weight limit causes wellbore collapse in the  $\theta = \pm\pi/2$  location considering above equations.

$$\begin{aligned} \sigma_r &= P_w \\ \sigma_\theta &= 3\sigma_H - \sigma_h - P_w \\ \sigma_z &= \sigma_v - 2\nu(\sigma_H - \sigma_h) \end{aligned} \quad (11)$$

Three stress components around well ( ) make 6 stress states considering their magnitudes:

- 1:  $\sigma_\theta \geq \sigma_r \geq \sigma_z$
- 2:  $\sigma_\theta \geq \sigma_z \geq \sigma_r$
- 3:  $\sigma_z \geq \sigma_\theta \geq \sigma_r$
- 4:  $\sigma_r \geq \sigma_\theta \geq \sigma_z$
- 5:  $\sigma_z \geq \sigma_r \geq \sigma_\theta$
- 6:  $\sigma_r \geq \sigma_z \geq \sigma_\theta$

As discussed before, wellbore collapse happens in case of high circumferential stress state ( ). So there are three stress states to determine lower limit of mud weight (states 1, 2 and 3).

### 4 Excavation damage zone

Stress around a wellbore changes while drilling and some parts of the surrounding rock is damaged. Damage parameter is equal to 1 when failed rocks are completely detached from the wall. The degree of damage, also, varies in surrounding rocks and depends on the damage parameter value. In this regard, some regions can be detected. The first region which is usually close to wellbore and loses its function is called “excavation damage zone, EDZ”. In this region the rock become weaker and mechanical and hydraulic properties change significantly [16]. Most of the researchers believe that more than one damaged region can be distinguished around a borehole when it is subjected to stress concentration. Martino defined damaged zones around a borehole as shown in Fig. 4. Three major regions around an opening underground are [17] :

1. Undisturbed zone (UDZ): This region is not subjected to high loads and in-situ stresses and is not altered much.
2. Excavation Altered (distributed) zone (DZ): This part of the rock is an area with disturbed stress conditions without any significant mechanical or hydraulically parameters changed.
3. Excavation damaged zone (EDZ): This part experiences sever stress changes and in some areas, stress concentration induce failure. This zone can also be divided into three sub groups. The area with failed zone, the area with sharp property changes and the area with smooth changes in properties. The location of each part and its extent depends on deviatoric stress values and mean stress values as in Fig. 5.

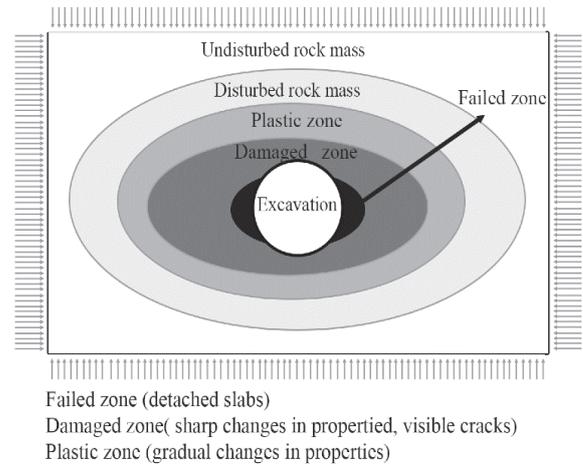


Fig. 4 Excavation damage zone[17]

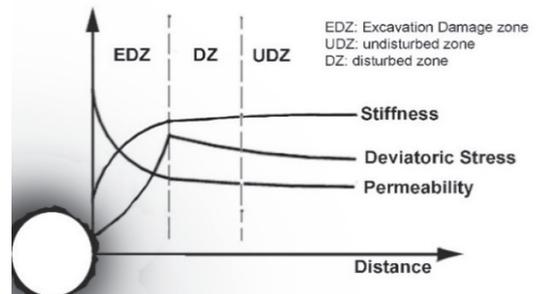


Fig. 5 Hydro mechanical changes around an excavation

Damage parameter ( $d$ ) in undisturbed zone is zero while it could be as high as 1 in EDZ. Research on rock damage showed that 6 damage thresholds could be distinguished in rock behavior [18]. For each stage a relation is proposed in the form of exponential function as below [19].

$$d = 1 - \frac{E'}{E_0} = a + b \left( \frac{\sigma_3}{\sigma_c} \right)^c \quad (12)$$

Considering boundary conditions ( $d = 0$  at yielding and  $d = 1$  at failure),  $a = 0$  and  $b$  and  $c$  should determine through experiment. The damage parameter begins to increase from yielding to failure. Rock plays the role of a structure element in underground cavities as long as it remains functional. In

stability problems, rock cannot be used beyond critical damage parameter value  $d_{cr}$ . Critical damage parameter value is the value of damage parameter at which cracks connect together and cause an EDZ area around excavation. The stress magnitude that causes cracking is crack damage stress threshold. The threshold value is affected by the importance of the structure. Where the structure cannot tolerate any rock damage, this parameter must be low. It means, we should not allow rock to bear much load. In wellbore stability problems, damage parameter is 0.6–0.8. This threshold is between failure and yielding state of the rock.

### 5 Damage model to predict minimum mud weight

Damage theory provides an accurate way of describing failure mechanism. Micro-scale damage gives details about onset of micro cracks and their growth. Generally there are two major categories to study rock failure: a) phenomenological models which study rock failure macroscopically. The followers of this approach use thermodynamics irreversible process to relate stress and strain which are macroscopic variables to tensorial variables. b) micromechanical models that use microcracks initiation and propagation theories to obtain global response of material with respect to damage accumulation [20]. Rock failure process is based on “Damage Mechanics” and “Fracture Mechanics”. Fracture mechanics is concerned with the study of cracks propagation in macro scale which can be seen without equipment aids while damage mechanics focuses on the micro cracks behavior in materials. The domain of application for these theories is presented in Fig. 6. Failure of materials, therefore, is a process of nucleation of micro cavities or micro cracks due to the breakage of atomic bonds from the microscopic view point. From this view point, however, it is a process of crack extension brought about by coalescence of these micro cavities [21].

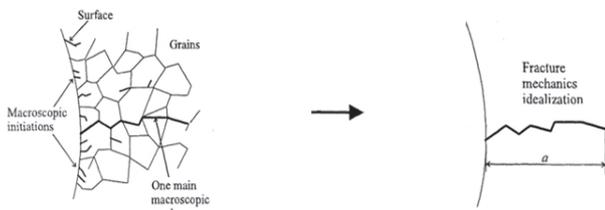
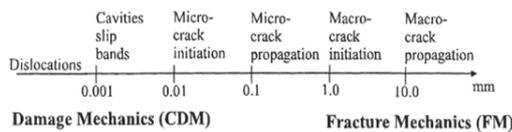


Fig. 6 The area of fracture and damage mechanics study [22]

The most acceptable definition for damage is from Murakami [21]. According to him, the development of cavities in the microscopic, mesoscopic and the macroscopic processes of fracture in materials together with the resulting deterioration

in their mechanical properties are called damage. Therefore the damage parameter is defined as below:

$$d = \frac{A_D}{A} \quad (13)$$

$A_D$  is total surface of defects such as cavities and microcracks and  $A$  is the total area of the sample in a cross section as explained in Fig. 7. By loading a sample, some defects are created and load carrying area decreases which result in magnifying the effect of induced stress by the external force ( $F$ ).

$$\bar{\sigma} = \frac{F}{A - A_D} = \frac{\sigma}{1 - D} \quad ; \bar{\epsilon} = \epsilon,$$

It means both of the below relations have same meaning.

$$\sigma = \bar{E} \bar{\epsilon} \quad \sigma = E \bar{\epsilon}$$

So, damage parameter has below relation with changing the stiffness.

$$d = 1 - \frac{\bar{E}}{E} \quad (14)$$

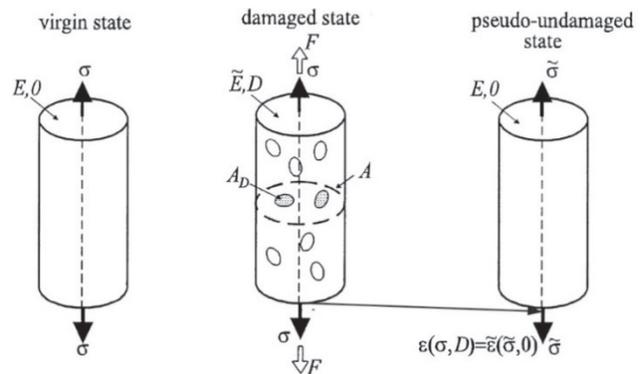


Fig. 7 a sample under damage [22]

Rock behaves differently under compression and tension. Usually under uniaxial tension and before tensile failure, rock stress-strain curve shows a linear elastic form. Under tension, rock cannot sustain inelastic strain so at the failure stress, micro-cracking occurs. Beyond the failure stress level, micro-cracks join together and strain localization is observed.

Under uniaxial compression, the response is linear until initial yield value which corresponds to the point A in Fig. 2. Until this point, cracks inside the rock close and specimen volume reduces. After point A, micro-cracks grow and the specimen volume increases gently. When micro-cracks coincide, the specimen damage begins. This phenomena corresponds to the point B in the same figure. In the plastic regime and after point A, the response is typically characterized by stress hardening followed by a strain softening stage. This representation, although somewhat simplified, captures the main features of rock response. It is assumed that the uniaxial stress-strain curves can be converted into stress-plastic strain curves by below equations:

$$\sigma_t = \sigma_t \left( \tilde{\varepsilon}_t^{pl}, \dot{\varepsilon}_t^{pl}, \theta, f_i \right) \quad (15)$$

$$\sigma_c = \sigma_c \left( \tilde{\varepsilon}_c^{pl}, \dot{\varepsilon}_c^{pl}, \theta, f_i \right)$$

In damage theory, the degradation of the elastic stiffness is characterized by damage variables which are assumed to be functions of plastic strain and other variables. This can be written for tension and compression loading as:

and effective tensile and compressional stresses can be defined as:

$$\bar{\sigma}_t = \frac{\sigma_t}{(1-d_t)} = E_0 (\varepsilon_t - \tilde{\varepsilon}_t^{pl}) \quad (18)$$

$$\bar{\sigma}_c = \frac{\sigma_c}{(1-d_c)} = E_0 (\varepsilon_c - \tilde{\varepsilon}_c^{pl})$$

In the Multi-axial state:

$$\sigma = (1-d) D_0^{el} : (\varepsilon - \varepsilon^{pl}) \quad (19)$$

The last equation shows that stress concentration is occurred after starting the damage process. By evolution damage in a specimen, stress concentration inside rock increase and linear elastic behavior is diverged by nonlinear behavior. When the material starts to damage and considering the continuum form of the equation 19, the criterion can be written as:

$$\frac{\sigma_1}{1-d} - \frac{q\sigma_3}{1-d} = \sigma_c \quad (20)$$

By comparison of equation 4 with equation 7 it can be written as

$$h(\sigma) = (B-1)\sigma_c + (1-q)\sigma_3 \quad (21)$$

then:

$$\sigma_1 - q\sigma_3 = \sigma_c (1-d) + d \cdot [(B-1)\sigma_c + (1-q)\sigma_3] \quad (22)$$

The proposed model can describe stress threshold according to rock damage and EZD idea. Damage parameter helps to describe rock failure precisely. When yielding occurs, damage parameter starts and begins to increase by stress increase. Before yielding,  $d$  is equal to zero and the criterion is the simple Mohr-coulomb criterion. The critical damage parameter value in wellbore stability can be determined by considering the stress state at which the cracks joint together and pore pressure increase suddenly.

## 6 Validation of the proposed model

Since the most accurate results for the determination of primary stresses are provided by in-site investigations[23], here an actual field data was used as an example to control the accuracy of the new proposed model. For this, a vertical borehole drilled through a shale formation at a depth of about 2142 m at Wanaea field in the northwest shelf of Australia was selected. Mohr-Coulomb (MC), Mogi-Coulomb and Mohr-Coulomb Damage model (MCD) were used for the comparison. In this field, at the mentioned depth, pore pressure is equal to 11.1 kPa/m. The shale has cohesion of 3 MPa and friction angle of 31 degrees. The material parameters is summarized in Table 1.

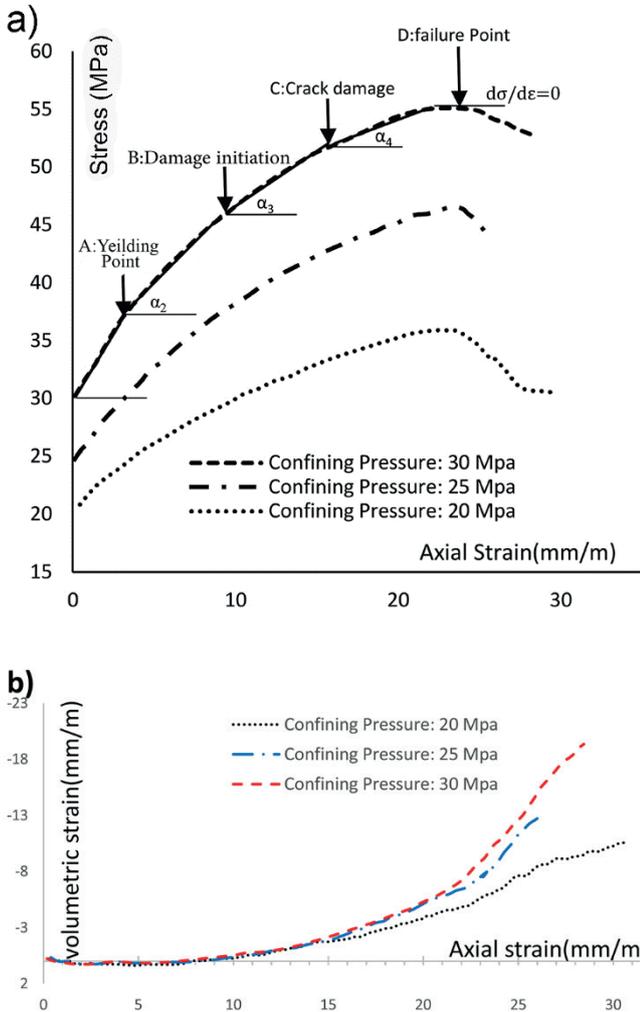


Fig. 8 (a) Stress-strain curve for shale under different confining stress, (b) Volumetric strain – axial strain curve

$$d_t = d_t(\tilde{\varepsilon}_t^{pl}, \theta, f_i); \quad 0 \leq d_t \leq 1, \quad (16)$$

$$d_c = d_c(\tilde{\varepsilon}_c^{pl}, \theta, f_i); \quad 0 \leq d_c \leq 1$$

Damage variable varies from 0 to 1, for undamaged material and fully damaged material respectively. Consequently stress-strain relation is change to:

$$\sigma_t = (1-d_t) E_0 (\varepsilon_t - \tilde{\varepsilon}_t^{pl}) \quad (17)$$

$$\sigma_c = (1-d_c) E_0 (\varepsilon_c - \tilde{\varepsilon}_c^{pl})$$

**Table 1** Material Properties for Wanaea Project [24]

Parameter	Cohesion	Friction angle	$\sigma_H$
Unit	MPa	Degrees	KPa/m
value	3	21	20.8
Parameter	$\sigma_h$	$\sigma_v$	Poisson's ratio
Unit	KPa/m	KPa/m	
value	16.3	20.8	0.25

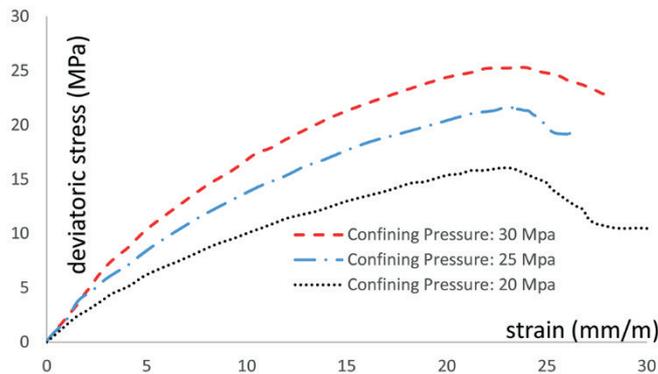
Stress-strain relation for the above mentioned shale is illustrated in figure 8-a. It shows that upon reaching the yielding point, Young's modulus begins to decrease gradually. Between yield and failure points where, at least two distinguished points B and C can be observed. The stiffness of the material at every stage is different and decrease by increasing strain.  $E_0$  is the stiffness of the material at its elastic range while it reaches zero when it fails.

By plotting volumetric strain against axial strain, these distinguished points are very obvious (Fig. 8-b). Three major jump of this graph are:

Yield point is associated with kick off in the volumetric change which occurs at around 5 mm/m of axial strain.

The second jump is called crack initiation point and the final point is crack damage. This jump happens at around 22 mm/m strain. From this point onwards, the slope of volumetric change curve is very sharp due to presence of fully developed cracks and creation of new surfaces in the sample.

However there is another point between crack initiation and crack damage points that is associated to crack coalescence shown by a rapid increase in volumetric strain. When cracks start to coalesce, rock loses its major strength. So this point is selected to be representative of the rock bearing capacity. To obtain this value and the Modulus degradation data, some regression analysis is done on the triaxial laboratory tests at different confining pressures as shown in Fig. 9. Using the stress-strain diagrams in Fig. 8, plot of deviatoric stress versus confining pressure for each level of damage can be presented as in Fig. 10.



**Fig. 9** deviatoric stress –strain curve

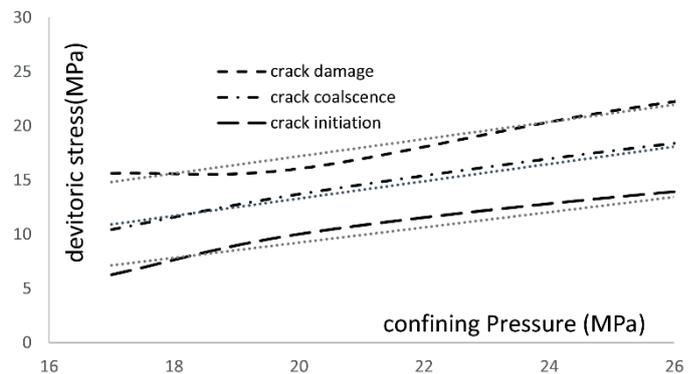
From the above graphs, degradation of rock modulus was also determined. For each stage of damage, the following relations are obtained for the Wanaea shale. Similar studies can be performed for other material and related relationships can be determined.

$$\frac{E'}{E_0} = 0.88 \left( \frac{\sigma_1 - \sigma_3}{\sigma_c} \right)^{0.366} \quad R^2 = 0.998 \text{ crack initiation}$$

$$\frac{E'}{E_0} = 0.62 \left( \frac{\sigma_1 - \sigma_3}{\sigma_c} \right)^{0.313} \quad R^2 = 1 \text{ crack coalescence}$$

$$\frac{E'}{E_0} = 0.46 \left( \frac{\sigma_1 - \sigma_3}{\sigma_c} \right)^{0.294} \quad R^2 = 0.987 \text{ crack damage}$$

By incorporating the last equation with the proposed model, minimum safe mud weight can be estimated. The stress regime in Wanaea field is on the boundary between normal and strike-slip faulting. Drilling was utilized with mud density of 1.42 g/cm<sup>3</sup>. The minimum overbalance pressure versus depth is plotted for both of the failure criteria in Fig. 11. It is apparent that the mud pressure predicted with damage theory is in closer relation with actually-used mud pressure. Applying the Mohr–Coulomb criterion in this field will give significantly conservative collapse pressures.



**Fig. 10** Interpolating three damage levels in the shale sample

Insufficient mud weight may result in shear failure and collapse of the well. The results show that proposed model predicts minimum mud weight more precisely. The field experience showed some minor breakout in the well. The proposed model allows the well to have breakout till some extent. It means the well can sustain some shear failure and still remain stable. This would help drilling engineers especially in underbalance drilling method.

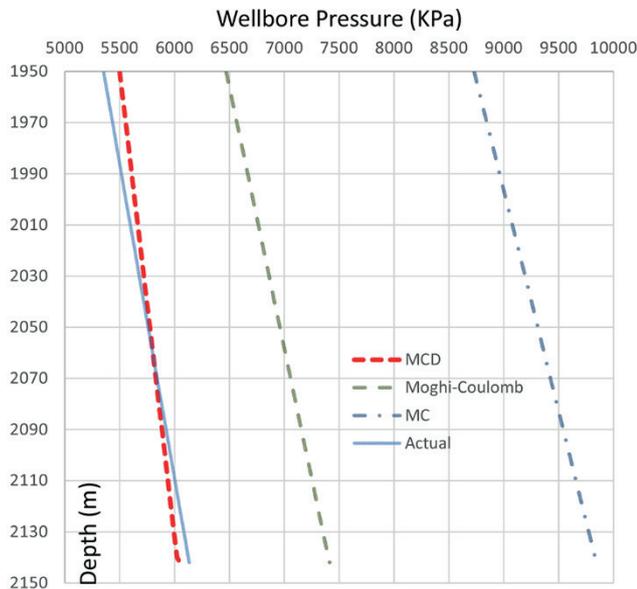


Fig. 11 Minimum mud weight prediction according to damage theory and comparison with other criteria

## 7 Conclusions

In this study, a damage model is proposed to determine minimum mud weight and prevent shear failure around boreholes based on micromechanics concept. Some important conclusions can be taken as below:

1. Damage theory can describe the failure phenomena of rocks more precisely. Rock failure criteria based on yield point such as Mohr-Coulomb and even Mogi-Coulomb failure criteria overestimate rock strength under polyaxial stress conditions.
2. Since rock failure starts from micro cracking, micromechanics can describe rock strength loss and loss of functionality appropriately.
3. Rock failure criterion based on proposed model can evaluate rock applicability better than other conventional criteria such as Mohr-Coulomb.
4. Rock damage is more related to deviatoric stresses rather than amount of stress by itself. It means by hydrostatic stress, contraction and grain movement is dominated to cracking and existing new surfaces.
5. Proposed model based on damage theory, can predict minimum mud weight precisely. By applying the model to a real wellbore drilling data, excavation zone around wellbore is estimated and minimum wellbore pressure to stabilize it is determined. A narrow mud weight window makes so many difficulties in drilling design. The cost and time of wellbore drilling rise when mud weight window is designed very narrowly. The proposed model makes mud weight windows wider.

## References

- [1] Kok, M. V., Uyar, T. T., "A Geomechanical Wellbore Stability Assessment for Different Formations in Petroleum Fields." *Petroleum Science and Technology*. 32 (19), pp. 2355-2364. 2014. DOI: [10.1080/10916466.2013.829856](https://doi.org/10.1080/10916466.2013.829856)
- [2] Dusseault, M. B. "Geomechanical challenges in petroleum reservoir exploitation." *KSCE Journal of Civil Engineering*. 15 (4), pp. 669-678. 2011. DOI: [10.1007/s12205-011-0007-5](https://doi.org/10.1007/s12205-011-0007-5)
- [3] Zhang, J. "The impact of shale properties on wellbore stability." The University of Texas at Austin. 2005.
- [4] Rafieepour, S., Ghotbi, C., Pishvaie, M. R. "The Effects of Various Parameters on Wellbore Stability During Drilling Through Shale Formations." *Petroleum Science and Technology*. 33 (12), pp. 1275-1285. 2015. DOI: [10.1080/10916466.2011.606253](https://doi.org/10.1080/10916466.2011.606253)
- [5] Ma, J., Zhao, G., Khalili, N. "A fully coupled flow deformation model for elasto-plastic damage analysis in saturated fractured porous media." *International Journal of Plasticity*. 76, pp. 29-50. 2016. DOI: [10.1016/j.ijplas.2015.07.011](https://doi.org/10.1016/j.ijplas.2015.07.011)
- [6] Wang, Q. Y., Zhu, W. C., Xu, T., Niu, L. L., Wei, J. "Numerical Simulation of Rock Creep Behavior with a Damage-Based Constitutive Law." *International Journal of Geomechanics*. 17 (1), 2017. DOI: [10.1061/\(ASCE\)GM.1943-5622.0000707](https://doi.org/10.1061/(ASCE)GM.1943-5622.0000707)
- [7] Mansourizadeh, M., Jamshidian, M., Bazargan, P., Mohammadzadeh, O. "Wellbore stability analysis and breakout pressure prediction in vertical and deviated boreholes using failure criteria – A case study." *Journal of Petroleum Science and Engineering*. 145, pp. 482-492. 2016. DOI: [10.1016/j.petrol.2016.06.024](https://doi.org/10.1016/j.petrol.2016.06.024)
- [8] Sofianos, A. I., Nomikos, P. P. "Equivalent Mohr-Coulomb and generalized Hoek-Brown strength parameters for supported axisymmetric tunnels in plastic or brittle rock." *International Journal of Rock Mechanics and Mining Sciences*. 43 (5), pp. 683-704. 2006. DOI: [10.1016/j.ijrmms.2005.11.006](https://doi.org/10.1016/j.ijrmms.2005.11.006)
- [9] Schwartzkopff, A. K., Melkounian, N. S., Woithe, S. D. "Design improvements for a true triaxial cell to monitor initiation and propagation of damage and body cracks during testing." In: *Mine Planning and Equipment Selection*. (Drebenstedt, C., Singhal, R. (Eds)), pp. 551-560. Springer International Publishing, Switzerland. 2014. DOI: [10.1007/978-3-319-02678-7\\_53](https://doi.org/10.1007/978-3-319-02678-7_53)
- [10] Cai, M., Kaiser, P. K., Tasaka, Y., Maejima, T., Morioka, H., Minami M. "Generalized crack initiation and crack damage stress thresholds of brittle rock masses near underground excavations." *International Journal of Rock Mechanics and Mining Sciences*. 41 (5), pp. 833-847. 2004. DOI: [10.1016/j.ijrmms.2004.02.001](https://doi.org/10.1016/j.ijrmms.2004.02.001)
- [11] Malek, J., Prusa, V., Rajagopal, K. R. "Reviews in geomechanics." 172 p. Jindrich Necas Center for Mathematical Modeling, Lecture Notes, Vol. 3. Matfyzpress, Praha. 2007. URL: <http://www.karlin.mff.cuni.cz/~prusa/ncmm/notes/download/workshopOnGeomaterials.pdf>
- [12] Hadei, M. R., Kemeny, J., Ghazvinian, A., Rezaiepoor, A., Sarfarazi, V. "New Development to Measure Mode I Fracture Toughness in Rock." *Periodica Polytechnica Civil Engineering*. 61 (1), pp. 51-55. 2017. DOI: [10.3311/PPci.9264](https://doi.org/10.3311/PPci.9264)
- [13] Gross, D., Seelig, T. "Classical fracture and failure hypotheses." In: *Fracture Mechanics*. pp. 39-50. Springer, Berlin, Heidelberg. 2011. DOI: [10.1007/978-3-642-19240-1\\_2](https://doi.org/10.1007/978-3-642-19240-1_2)
- [14] Hoek, E., Carranza-Torres, C., Corkum, B. "Hoek-Brown failure criterion-2002 edition." In: *Proceedings of NARMS-Tac, Toronto, 2002*. 1, pp. 267-273. URL: <https://roscience.com/documents/hoek/references/H2002.pdf>

- [15] Pariseau, W. G. "Design analysis in rock mechanics." 698 p. CRC Press. 2011. URL: <https://www.crcpress.com/Design-Analysis-in-Rock-Mechanics-Second-Edition/Pariseau/p/book/9780415893398>
- [16] Saiang, D. "Damaged rock zone around excavation boundaries and its interaction with shotcrete." Luleå Tekniska Universitet. 2004.
- [17] Martino, J. "The excavation damage zone in recent studies at the URL." In: Proceedings of the 2002 Int. EDZ Workshop—The Excavation Damage Zone—Causes and Effects. AECL report. 2003.
- [18] Eberhardt, E. "Brittle rock fracture and progressive damage in uniaxial compression." University of Saskatchewan Saskatoon. 1998.
- [19] Chang, S. H., Lee, C. I., Lee, Y. K. "An experimental damage model and its application to the evaluation of the excavation damage zone." *Rock Mechanics and Rock Engineering*. 40 (3), pp. 245-285. 2007. DOI: 10.1007/s00603-006-0113-8
- [20] Li, X., Cao, W. G., Su, Y. H. "A statistical damage constitutive model for softening behavior of rocks." *Engineering Geology*. 143–144, pp. 1-17. 2012. DOI: 10.1016/j.enggeo.2012.05.005
- [21] Murakami, S. "Mechanical modeling of material damage." *Journal of Applied Mechanics*. 55 (2), pp. 280-286. 1988. DOI: 10.1115/1.3173673
- [22] Skrzypek, J. J., Ganczarski, A. "Modeling of material damage and failure of structures: theory and applications." 326 p. Springer, Berlin, Heidelberg. 1999. DOI: 10.1007/978-3-540-69637-7
- [23] Kálmán, E. "In-situ measurements in Overconsolidated Clay: Earth Pressure at rest." *Periodica Polytechnica Civil Engineering*. 56 (2), pp. 239-244. 2012. DOI: 10.3311/pp.ci.2012-2.10
- [24] Al-Ajmi, A. "Wellbore stability analysis based on a new true-triaxial failure criterion." 138 p. PhD thesis, KTH. 2006. URL: [http://rymd.lwr.kth.se/Publikationer/PDF\\_Files/LWR\\_PHD\\_1026.pdf](http://rymd.lwr.kth.se/Publikationer/PDF_Files/LWR_PHD_1026.pdf)