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# A NEW APPROACH TO THE CALIBRATION PROBLEMS OF THREE-DIMENSIONAL LASER SCANNERS

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## Abstract

Synchronized laser scanners are the most popular 3D image capture systems for industrial applications. The accuracy of the scanned picture is a key factor of the complete system. In this paper, a new mathematical description of synchronized laser scanners will be presented, which is necessary to the developed calibration method. The mathematical model is based on the geometrical design of the triangulation and it can make the application of scanners easier. The calibration method is working by tests on reference planes, which should be scanned and some reference points should be chosen on it. Due to the combined relations among the geometrical and system parameters, it is better – as shown – if the system parameters are estimated step-by-step by a linear error correcting method from the measured data and from the coordinates of the points on the reference planes. The new mathematical model allows simulating the function and the errors of the triangulation system easy. The error analysis of the system can help us to obtain important data from the model to design synchronized scanners.

*Keywords:* laser scanner, robot vision, camera calibration.

## 1. Introduction

3D imaging is one of the most interesting challenges of industrial picture processing. There are a lot of different methods for 3D image capturing (binocular stereo, monocular systems, different kinds of structured lighting, etc.). One of the most popular solutions is the use of laser scanners. This equipment uses a laser light source, some kind of deflection units (typically galvano-mirrors), electrooptical position-sensitive transducer and evaluation electronics for triangulation. A well-known optical arrangement developed by RIOUX [1] is the synchronized line scanner, which makes a good compromise between depth resolution and viewing field possible. At the Department of Control Engineering and Information Technology, in cooperation with the University of Karlsruhe we extended this method for field scanning, and a new system was developed for industrial applications with high resolution and large viewing field [2].

The accuracy of the scanner depends on the amount of uncertainty in the parameter description. On the other hand, the electro-optical system is very complex.

Two basic questions arise: a.) How are the different parameters affecting the final accuracy, and b.) What is the optimal parameter setup for a given application?

To find a solution for this problem a method is necessary for parameter estimation and for calculation of the expected quality of the captured image. In the following we describe a mathematical model and parameter estimation procedure for laser scanners. The result of simulations will be also presented.

## 2. Mathematical Model

The mathematical model is based on the geometrical construction of the synchronized triangulation scanner, shown in *Fig. 1*. The light source is a HeNe laser. Its beam falls on the $x$ mirror rotated by the $x$ scanner. The mirror reflects the beam to the fixed mirror ($r$), then the beam falls onto the surface of the object. The illuminated point of the object is projected onto the sensor by the lens after reflection on the fixed mirror ($l$) and on the opposite side of the $x$ mirror. The synchronization is realized by double reflection on the $x$ mirror.

The center of the coordinate system of our model is on the axis of the $x$ scanner. The $z$ axis is identical with the incident beam of the laser source, the $x$ axis is perpendicular to it. The angles of the mirrors and the beam are shown in *Fig. 1*. Two beams are indicated in the projection part only. The beam, indexed with an asterisk ($*$), is called the central beam, which goes through the object point (**P**) and the center of the lens. The object point is projected to the point on the sensor, where the central beam reaches the surface of the sensor. The point **E** is an object point, the image of which is in the center of the sensor, in case of the initial deflecting angle of the $x$ scanner. This means, that the central beam of this point is on the axis of the lens between the sensor and the $x$ mirror, and it is identical with the virtual axis of the lens before the $x$ mirror. Parameters of this beam are indexed with a letter $T$. The system parameters are defined by the geometrical construction. They are the position parameters $t_1$, $t_2$, $\vartheta_1$, $\vartheta_2$ of the fixed mirrors and the initial deflecting angle $\omega_{x0}$ of the $x$ mirror. The maximum value of the deflection angle $\omega_x$ is defined by the applications. Relations can be found among the angles of the deflected beam and the angles of the mirrors.

$$\alpha_x = 2\omega_x - 90^\circ, \tag{1}$$

$$\theta = 2(\vartheta_1 - \omega_x) + 90^\circ, \tag{2}$$

$$\gamma^* = \varphi^* - 2(\omega_x - \vartheta_2), \tag{3}$$

$$\gamma^T = 90^\circ - 2(\omega_x - \vartheta_2), \tag{4}$$

Each object point corresponds to a sensor point. Relations can be established using coordinate-geometry but this is so complicated that the model would be uncontrollable.

To reduce the complexity of the relations, we project the center of the coordinate system (**O**) through the fixed mirrors ($r$) and ($l$), which results **R** and **L**.

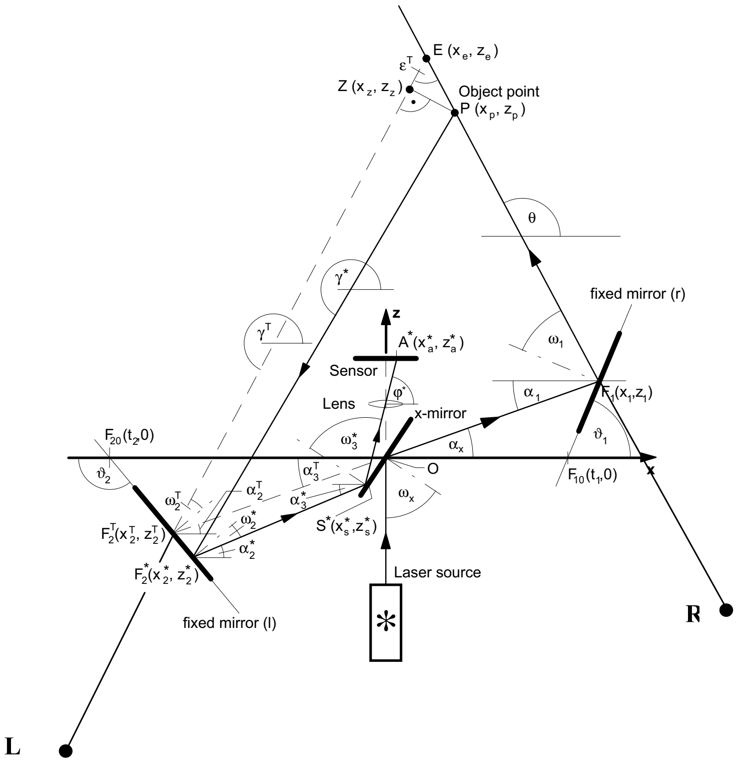


*Fig. 1.* Synchronized triangulation geometry

According to geometry, point **R** is the virtual center of the deflected beams. If the image of an object point falls onto the center of the sensor, the central beam starting from the object point is going virtually to the point **L**. It can be proved that these points are independent of the deflection. In addition, the angle $\varepsilon^T$, which can be found between the deflected beam and the central beam of the projection, is also independent of the deflection.

$$\varepsilon^T = 180^\circ - 2(\vartheta_2 - \vartheta_1). \tag{5}$$

Taking into account the above mentioned fact, a circle can be found containing the virtual points **R** and **L** and the object point **E**, the image of which is the center of the sensor. The center of the circle is indicated as **C** (See *Fig.* 2).

There exist the following *geometrical relations*, which can be proved (they are prominent in the calibration procedure): a) The points **R**, **L** and the point **F** (virtual focus) are on the circle, which we call as reference circle; b) **LF** = **OF** = **RF**; c) **CF** is perpendicular to **LR**; d) **LD** = **RD**, where **D** is the point of intersection of **CF** and **LR**.

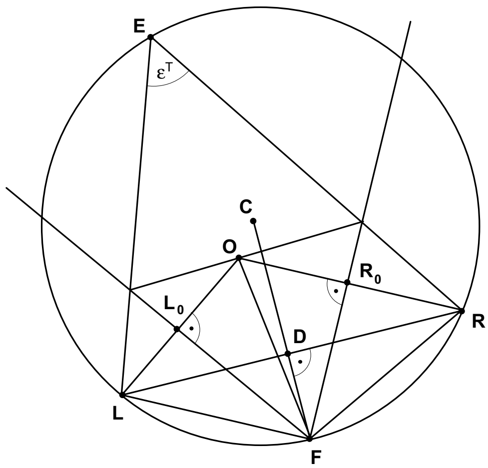


*Fig. 2.* Reference circle and virtual centers

The coordinates of virtual centers $\mathbf{R}(x_R, z_R)$, $\mathbf{L}(x_L, z_L)$ and virtual focus $\mathbf{F}(x_F, z_F)$ can be determined from the system parameters using the following equations:

$$x_R = \frac{2t_1}{1 + \dfrac{1}{\tan^2 \vartheta_1}}, \qquad z_R = \frac{-t_1 \dfrac{2}{\tan \vartheta_1}}{1 + \dfrac{1}{\tan^2 \vartheta_1}}, \tag{6}$$

$$x_L = \frac{2t_2}{1 + \dfrac{1}{\tan^2 \vartheta_2}}, \qquad z_L = \frac{-t_2 \dfrac{2}{\tan \vartheta_2}}{1 + \dfrac{1}{\tan^2 \vartheta_2}}, \tag{7}$$

$$x_F = \frac{t_2 \tan \vartheta_2 - t_1 \tan \vartheta_1}{\tan \vartheta_2 - \tan \vartheta_1}, \qquad z_F = \frac{(t_2 - t_1) \cdot \tan \vartheta_1 \cdot \tan \vartheta_2}{\tan \vartheta_2 - \tan \vartheta_1}. \tag{8}$$

The center point $\mathbf{C}(x_C, z_C)$ and the radius $(r)$ of the reference circle can be determined from the **LFR** triangle.

$$x_C = \frac{K_1(x_L + x_F) - K_2(x_R + x_F) - z_L + z_R}{2(K_1 - K_2)}, \tag{9}$$

$$z_C = \frac{K_1 K_2 (x_L - x_R) + K_1(z_R + z_F) - K_2(z_L + z_F)}{2(K_1 - K_2)}, \tag{10}$$

where

$$K_1 = \frac{x_F - x_L}{z_L - z_F}, \qquad K_2 = \frac{x_F - x_R}{z_R - z_F}, \tag{11}$$

and the radius is given by the following equation:

$$r = \sqrt{(x_F - x_C)^2 + (z_F - z_C)^2}. \tag{12}$$

The importance of these parameters lies in the fact that they are independent of the deflection.

To reduce the complexity of description of the projection, the projection axis can be straightened (See *Fig. 3*). The distance from the lens to the detector is $a$; from the lens to the axis of the $x$ mirror is $b$. The length of the optical axis from point **E** to point **O** is $l$, and from point **O** to point **Z** is $l_z$, respectively. The distance of the object point from the virtual axis is $h$. The image of the object point on the sensor is measured from the center of the detector indicated by $x_a^*$.
It can be proved that a positive $x_a^*$ belongs to a negative $h$ and vice versa. This means that as the sensor detects an image point with a positive coordinate $x_a^*$, the object point is inside the reference circle, otherwise it will be found outside the reference circle.

The relations which are determined according to *Fig. 3* can be simplified, so the parameters of the optical projection can be evaluated from the following equations:

$$l = 2r \cdot \sin(\delta_1 + \delta_2), \tag{13}$$

$$l_z = \frac{l \cdot a \cdot \tan \varepsilon^T - b \cdot x_a^*}{x_a^* + a \cdot \tan \varepsilon^T}, \tag{14}$$

$$h = -\frac{(l + b) \cdot x_a^* \cdot \tan \varepsilon^T}{x_a^* + a \cdot \tan \varepsilon^T}, \tag{15}$$

where

$$\delta_1 = \arctan \frac{z_R - z_L}{x_R - x_L} = \text{constant}, \tag{16}$$

$$\delta_2 = 180^\circ - \theta = 90^\circ - 2(\vartheta_1 - \omega_x). \tag{17}$$

The parameters $l_z$ and $h$ yield the coordinates of the object point in the coordinate system fixed to the virtual axis. The required coordinates can be determined after a coordinate transformation from the coordinate system fixed to the virtual axis to the coordinate system fixed to the non-moving mirrors and to the detector. After the transformation the coordinates of the object point $\mathbf{P}(x_P, z_P)$ are the following:

$$\left.\begin{array}{rcl} x_P &=& h \cdot \sin \gamma^T - l_z \cdot \cos \gamma^T + x_L \\ z_P &=& -h \cdot \cos \gamma^T - l_z \sin \gamma^T + z_L \end{array}\right\}. \tag{18}$$

The importance of the model is that the complicated equations (*Eqs.* (6)–(12)) should be evaluated only once before the measurements and the distance values can be computed easily using simple relations (*Eq.* (4) and *Eqs.* (9)–(14)).

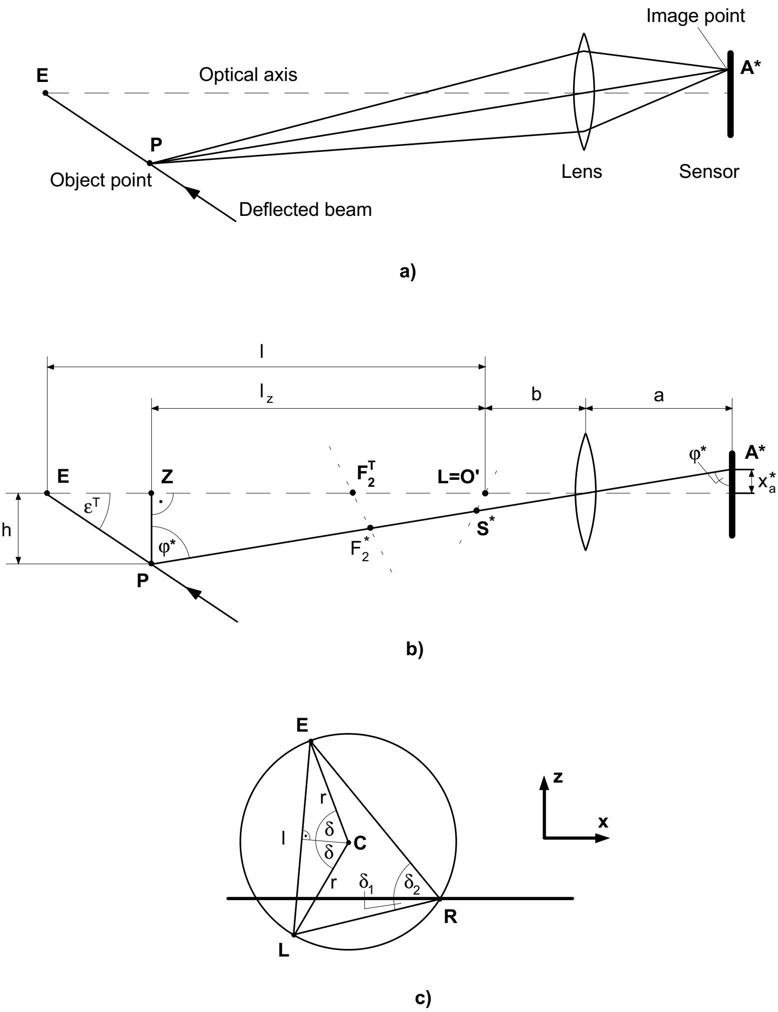


*Fig. 3.* a) Projection axis straightened, b) optical geometry, c) angles in the circle

## 3. Calibration Method

The calibration of the synchronized scanners means the determination of system parameters. It can be seen from the equations that the inverse function of system parameters is not a simple relation. Therefore, another way is chosen to determine the system parameters.

The calibration method is based on measurements on reference planes. The system parameters can be estimated step-by-step by an error correction method. The planes are illuminated at different deflection angles and some reference points $\mathbf{P}_{ij}$ are chosen on it. *Fig. 4* shows the calibration measurements and the reference planes. Though the orientation of the planes is optional, the simplest choice is if they are parallel to each other. It is necessary to find some object points $\mathbf{E}_i$ on the planes, the image of which is in the middle of the detector. The coordinates of all points $\mathbf{P}_{ij}(x_{ij}, z_j)$ and $\mathbf{E}_i$ should be noticed. The number of deflections is $q$, the number of $\mathbf{E}_i$ points is $k$, respectively.

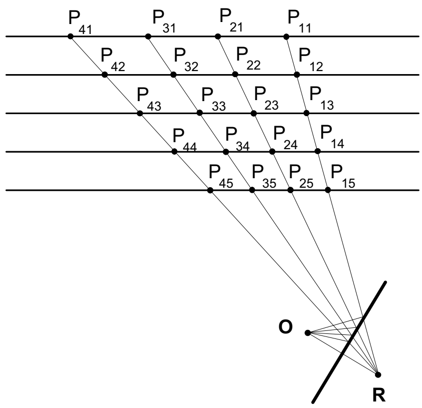


*Fig. 4.* Reference planes for calibration

The first step of the calibration is to determine the virtual deflection center ($\mathbf{R}$), where the deflected beams intersect. The equation of the straight line $i$ is

$$z_j = m_i x_{ij} + b_i. \tag{19}$$

Due to measurement errors, $m_i$ and $b_i$ can be estimated by an error-correction method. The most simple solution is the linear Least-Squares estimation. The linear model is given by

$$\mathbf{y} = \mathbf{X}_i \mathbf{b}_i + \mathbf{e}_i, \tag{20}$$

where

$$\mathbf{y} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}, \quad \mathbf{X}_i = \begin{bmatrix} x_{i1} & 1 \\ x_{i2} & 1 \\ \vdots & \vdots \\ x_{in} & 1 \end{bmatrix}, \quad \mathbf{b}_i = \begin{bmatrix} m_i \\ b_i \end{bmatrix}. \tag{21}$$

The Least Squares estimation of the parameter vector $\mathbf{b}_i$ is

$$\hat{\mathbf{b}}_i = (\mathbf{X}_i^T \mathbf{X}_i)^{-1} \mathbf{X}_i^T \mathbf{y}. \tag{22}$$

The reduced form of the estimated parameters is the following:

$$\hat{\mathbf{b}}_i = \begin{bmatrix} \hat{m}_i \\ \hat{b}_i \end{bmatrix} = \frac{1}{n \cdot \sum_{j=1}^{n} x_{ij}^2 - \left(\sum_{j=1}^{n} x_{ij}\right)^2} \times \begin{bmatrix} n \cdot \sum_{j=1}^{n} x_{ij} z_j - \sum_{j=1}^{n} x_{ij} \cdot \sum_{j=1}^{n} z_j \\ \sum_{j=1}^{n} x_{ij}^2 \cdot \sum_{j=1}^{n} z_j - \sum_{j=1}^{n} x_{ij} \cdot \sum_{j=1}^{n} x_{ij} z_j \end{bmatrix}. \tag{23}$$

The $\hat{\mathbf{b}}_i$ vector contains the estimated parameters of the straight lines. Now the most probable intersection point should be estimated. Each straight line $i$ contains the point $\mathbf{R}$, so the following equation can be written

$$z_R = \hat{m}_i x_R + \hat{b}_i, \tag{24}$$

where $\hat{m}_i$ and $\hat{b}_i$ are estimated in the previous step. The equations can be summarized by the following matrix form:

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e}, \tag{25}$$

where

$$\mathbf{y} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \vdots \\ \hat{b}_q \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} -\hat{m}_1 & 1 \\ -\hat{m}_2 & 1 \\ \vdots & \vdots \\ -\hat{m}_q & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} x_R \\ z_R \end{bmatrix}. \tag{26}$$

The Least-Squares estimation gives the estimated coordinates of the point $\mathbf{R}$:

$$\hat{\mathbf{b}} = \begin{bmatrix} \hat{x}_R \\ \hat{z}_R \end{bmatrix} = \frac{1}{q \cdot \sum_{i=1}^{q} \hat{m}_i^2 - \left(\sum_{i=1}^{q} \hat{m}_i\right)^2} \times \begin{bmatrix} -q \cdot \sum_{i=1}^{q} \hat{m}_i \hat{b}_i + \sum_{i=1}^{q} \hat{m}_i \cdot \sum_{i=1}^{q} \hat{b}_i \\ -\sum_{i=1}^{q} \hat{m}_i \cdot \sum_{i=1}^{q} \hat{m}_i \hat{b}_i + \sum_{i=1}^{q} \hat{b}_i \cdot \sum_{i=1}^{q} \hat{m}_i^2 \end{bmatrix}. \tag{27}$$

The way of estimation of the system parameters was shown above. Only the most important steps of the estimation will be given here without the mathematical details.

The virtual deflection center (**R**) is the image of the center of the coordinate system (**O**) reflected by the fixed mirror ($r$). Therefore, the mirror lies on the bisecting perpendicular straight line of the section **RO**, so the parameters (axis intersection $t_1$, angle $\vartheta_1$) of the mirror ($r$) can be estimated.

The initial deflecting angle ($\omega_{x0}$) can be determined from *Eq.* (2). The angle $\vartheta_1$ of the fixed mirror ($r$) is already known. The step angle $\Delta\omega_{xi}$ of the moving mirror can be set accurately on the scanner. The necessary relations are the following:

$$\omega_{x_i} = \omega_{x_0} + \Delta\omega_{x_i}, \tag{28}$$

$$\left.\begin{array}{ll} \hat{\theta}_i = \arctan \hat{m}_i, & \text{if} \quad \hat{m}_i > 0 \\ \hat{\theta}_i = 180^\circ + \arctan \hat{m}_i, & \text{if} \quad \hat{m}_i < 0 \end{array}\right\}. \tag{29}$$

Using a Least-Squares estimation, the initial deflecting angle can be determined.

The next step is to determine the reference circle, which is required to calculate the center and the radius of the circle. We will use the noted $\mathbf{E}_i$ points ($k$ pieces) and the virtual deflection center ($\mathbf{E}_{k+1} = \mathbf{R}$) because they are on the circle. If $k+1 \geq 3$, the circle is determined.

We have chosen two points from them in every combination. Each section between points $\mathbf{E}_i$ and $\mathbf{E}_j$ is a chord of the circle, and its bisecting perpendicular contains the center of the circle. The center can be determined by Least-Squares estimation of the intersection points of bisecting perpendiculars. The determination of the radius can be performed easily, as the center and some other points of the circle are known.

The intersection of the mirror ($r$) and the circle gives the intersection point of the fixed mirrors (**F**). The point **L** can be determined using the geometrical relations a) and c) on page 273. If we drop a perpendicular from point **R** to the line **CF**, the intersection with the circle gives the point **L**. The point **L** is the image of the center of the coordinate system (**O**) reflected by the fixed mirror ($l$). The mirror ($l$) is the bisecting perpendicular straight line of the section **LO**, so the parameters (axis intersection $t_2$, angle $\vartheta_2$) of the mirror ($l$) can be estimated.

Now we have all system parameters of the synchron scanner in the coordinate system of the fixed mirrors. The position and the orientation of the scanner in real environment can be determined by capturing data from fixed reference object. The transformation from the coordinate system fixed to the scanner to the coordinate system fixed to the reference object is given by the measured data. The inverse of the transformation matrix gives the reverse transformation matrix. We can get the real data from any object using the reverse transformation.

The mathematical model and the calibration method can be applied for both 3D triangulation and light-strip methods. Although the model describes line scanners, it can be used for 3D range finders with double deflection too, because the projection is independent of the parameters of the vertical synchronized scanner. The parameters of the vertical scanner can be estimated by a similar calibration method as described above.

## 4. Examination

The examinations of some optical parameters can help us to design synchronized scanners. The viewing field of the system is shown in *Fig.5*. (The following typical parameters are the used: $t_1 = 150$ mm, $t_2 = -150$ mm, $\vartheta_1 = 47.5°$, $\vartheta_2 = 130°$, so $\varepsilon^T = 15°$. The lens and detector parameters are $a = 40$ mm, $b = 100$ mm, the width of the detector is 12 mm.)

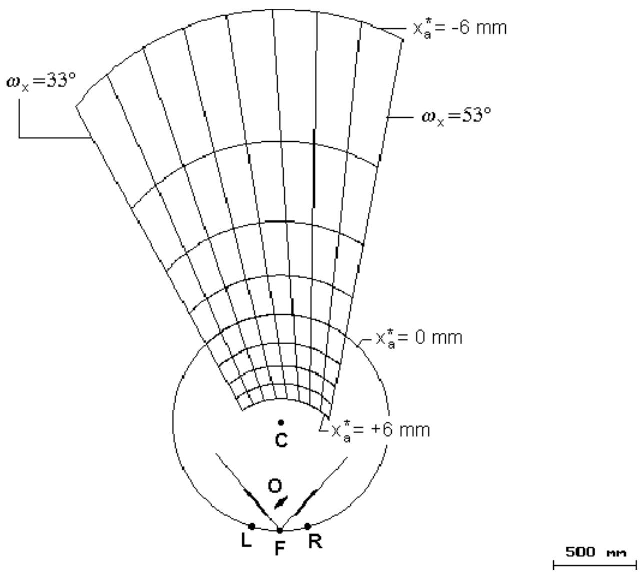


*Fig. 5.* Viewing field ($\varepsilon^T = 15°$)

The straight lines contain the object points related to the deflection in the interval $\omega_x = 33° - 53°$. The object points, the images of which fall onto predefined locations on the detector (+6 mm; +4.5 mm; +3 mm; etc.), are on the arcs.

The resolution is given by the distance between an object point and another object point, the image of which is in the center of the detector at the same deflection (see *Fig. 6*). The tangent of the function is the reciprocal form of the resolution. The non-linearity can be seen on the figure, but the resolution at 'nearer' object points (inside the circle) is approaching linear and independent of the deflection.

The tests of the calibration are shown in *Fig. 7*. The system parameters are known and some calibration points (object point) were determined. An error was added to the coordinates of the object points and the 'new' system parameters were estimated by the calibration method. The figure shows the distance among the original object points and the new calibration points determined by the 'new' system parameters. The test shows that the accuracy of theoretical resolution (determined

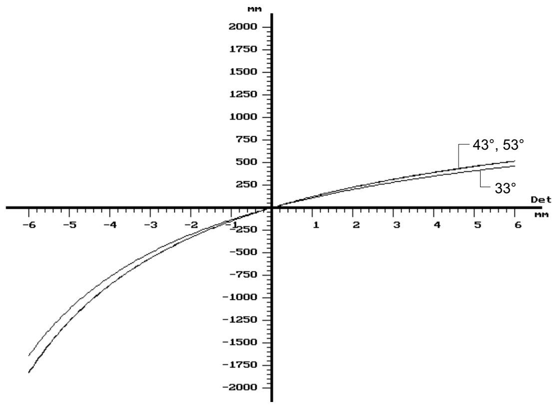


*Fig. 6.* Resolution ($\varepsilon^T = 15°$)

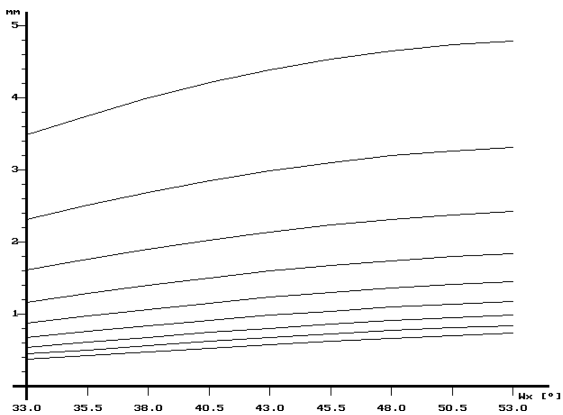


*Fig. 7.* Calibration error analysis

from the pixel number of the detector) can be reached when the system parameters are unknown and should be estimated by calibration. However, it is true for certain system parameters only, so we need a-priori knowledge about the parameters and we have to investigate calibration tests such as shown in *Fig. 7*.

## 5. Summary

A new mathematical model of a synchronized scanner and its application for calibration was presented. The mathematical model is based on the geometrical construction of the triangulation. The system parameters can be determined from the geometrical data by combined relations, but they should be evaluated only once before the measurement series. The distance values can be easily determined from the system parameters and from the measurement values. The calibration method works by test images from reference planes. The system parameters are estimated step-by-step by a linear error correcting method from the measured data and from the coordinates of the points on the reference planes. The error analysis of the system can help us to obtain important data from the model to design synchronized scanners. Both the error analysis of the calibration method and the experimental results show that the error of the calibration method can be kept in the range of the maximum resolution. It means that measurements based on calibration give results as accurate as possible according to the resolution.

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