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ELECTRICAL DIMENSIONING OF INVERTER-INDUCTOR-LOAD SYSTEM IN INDUCTION HEATING OF FERROMAGNETIC PLATES AS LOAD

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Abstract

An important practical field of induction heating is the heating of ferromagnetic plates. A significant part of the heating arrangement is a medium-frequency – so called length-field – heating inductor. It is supplied by a medium-frequency inverter of serial resonance circuit. The paper presents the detailed dimensioning method of the inductor–load system, based on approximating analytical computations, together with the interaction between that and the inverter. It gives the computed results for a concrete situation.

Keywords: induction heating, heating of ferromagnetic plates, dimensioning inductors and inverters.

1. Introduction

An important practical solution of the problem described in the title, the heating of plastic covered steel plates, was dealt with by presentation (KOLLER and TEVAN, 1998). These are manufactured in such a way, that the zincous steel plates whose temperature is increased to 120 . . . 200 °C, are covered by a plastic foil by a production line, through which one or more band shape plates with a depth of 0.5 . . . 1.0 mm and a width of max. 1400 mm go at a speed of 3 . . . 5 m/s. The alternating magnetic field excited by the length-field heating inductor (*Fig. 1*) – parallel to the plane of the band – induces eddy currents which heat the steel band. The computation of the equivalent excitation length, which is necessary for the dimensioning, is detailed in the paper (TEVAN and KOLLER, 1999). In this article the electromagnetic field is determined between the ferromagnetic plate and the flux conductor – inductor system with approximating analytical calculation of 2D-model. Then the power is calculated, penetrated into the ferromagnetic plate and from this the equivalent excitation length can be obtained with the condition that a uniform excitation of equivalent length gives the same power.

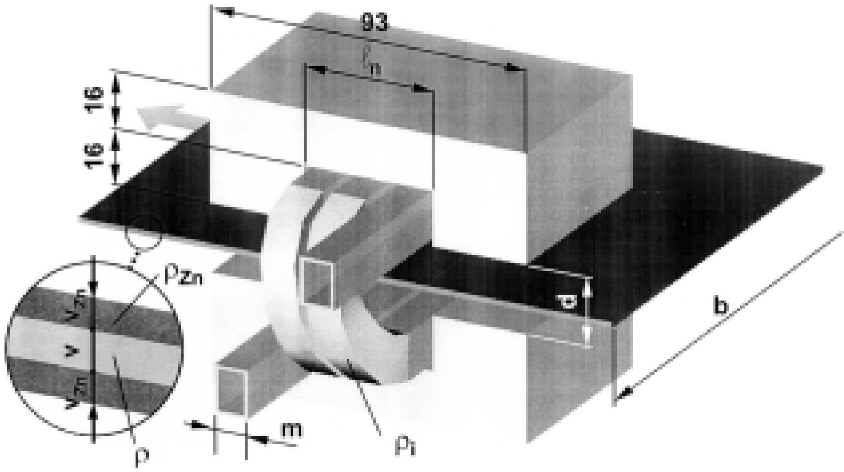

Fig. 1. Arrangement of length-field inductor

2. Dimensioning

During the dimensioning procedure the task is to induce power with the amplitude and frequency defined by the technology in the ferromagnetic and moving band. Nevertheless, this power can be generated only by the interaction of the serial resonance circuit having a capacitance of C and the inductor-load system, therefore not only the non-linear feature of the inductor-load system has to be taken into consideration solving this complex dimensioning problem, but the operation of the inverter, thus the fitting between these two components as well.

In case of the inductor only the distance between the pole surfaces, signed by d and the number of turns, signed by N and in connection with it the width of the conductor m are changeable. The current, flowing from the inverter through the inductor and the capacitor, and the adjustable U_t terminal voltage of the inverter have common zero point, which is set by the control system of the inverter. The equivalent circuit, which is valid for the first harmonic can be seen in *Fig. 2*. Disregarding R_{Zn}, there is the serial equivalent diagram of the inductor-load system between the terminals of the inductor, signed by AB, where R_s and X_s are the AC resistance and reactance of the steel layer(s) of the band(s) to be heated, at circular frequency ω, concerning the output of the inverter,

$$R_i = \rho_i \cdot \frac{l_i \cdot N}{m\sqrt{\dfrac{2 \cdot \rho_i}{\omega \cdot \mu_0}}}$$

symbolise the AC resistance of the coil with a length of l_i,

$$X_0 = \omega \cdot \mu_0 \cdot \frac{b(d - z \cdot v)}{l_s + \dfrac{d}{2}} \cdot N^2$$

is the reactance of the air gap, where z is the number of the plates.

The equivalent circuit does not contain the inner reactance of the coil, because that is much smaller than X_0. Based on the derivation published in the Appendix, the effective AC resistance of the zinc layer(s),

$$R_{Zn} = \rho_{Zn} \cdot \frac{2 \cdot b \cdot z}{l \cdot v_{Zn}} \cdot N^2$$

is connected parallel to the impedance of the steel layer(s).

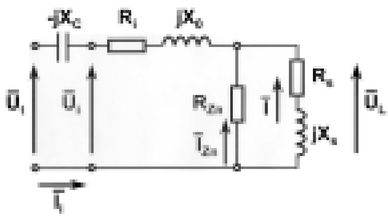

Fig. 2. Equivalent circuit

In the equivalent circuit R_s and X_s are current-dependent, because the plate is ferromagnetic. According to the model (MacLean, 1954) using step-function magnetisation characteristics, in case of sinusoidal excitation and full wave absorption, the first harmonic field impedance can be written as follows (equation (240) in book (Tevan, 1985)):

$$\mathbf{Z}_s = \frac{16}{3\pi}\sqrt{\frac{B_0\omega\rho}{2H_m}}\left(1 + \frac{1}{2}j\right),$$

where B_0 is the 75 percentage of a pre-defined saturation induction (which depends a bit on the excitation), ρ is the resistivity of the plate and H_m is the maximal value of the sinusoidal magnetic field strength on the surface of the ferromagnetic plate, so $H_m = \sqrt{2}H$, here H is real, effective value. The effective value of the first harmonic phazor of the electric field strength is:

$$\mathbf{E} = \mathbf{Z}_s \cdot H = \frac{8\sqrt[4]{2}}{3\pi}\sqrt{B_0\omega\rho H}\left(1 + \frac{1}{2}j\right).$$

Based on the previous equation, the resultant load voltage of z pieces of band with a width of b can be formulated:

$$\mathbf{U}_L = 2 \cdot z \cdot b \cdot \mathbf{E} \cdot N = \frac{16 \cdot \sqrt[4]{2}}{3 \cdot \pi} \cdot \sqrt{\rho \cdot B_0 \cdot \frac{N}{l} \cdot \omega \cdot z \cdot b \cdot N} \cdot \sqrt{I} \cdot \left(1 + \frac{1}{2} \cdot j\right), \quad (1)$$

where I, the load current, flowing in the ferromagnetic part of the band exciting H is also real, while l is the average equivalent excitation length (TEVAN and KOLLER, 1999), thus:

$$H = \frac{N \cdot I}{l}.$$

In case of one plate ($z = 1$) – situating asymmetrically in the air gap – (*Fig.* 3.*a*) the equivalent excitation lengths are different on the two sides of the plate ($\ell_1 < \ell_2$), because the distances between these sides and the opposite pole surface are different ($\Delta_1 < \Delta_2$).

From *Eq.* (1) it can be seen, that voltage $\mathbf{U}_L$ is inversely proportional to the square root of average equivalent length ℓ, therefore in this case that is determined from *Eq.* (2).

$$\frac{1}{\sqrt{l}} = \frac{1}{2} \cdot \left(\frac{1}{\sqrt{l_1}} + \frac{1}{\sqrt{l_2}}\right). \quad (2)$$

The explanation of multiplier 2 in *Eq.* (1) is that there are eddy currents on both sides of the plate, because of the both sides excitation (*Fig.* 3.*b*).

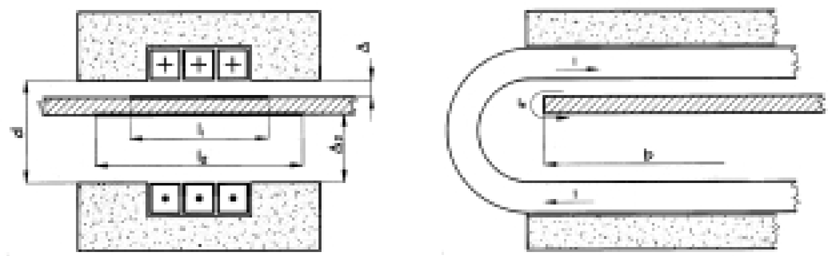

Fig. 3. The equivalent excitation lengths when an asymmetrically positioned plate is heated

In case of two, symmetrically positioned plates ($\Delta_1 = \Delta_2$) $\ell = \ell_1 = \ell_2$ (*Fig.* 4.*a*). In *Fig.* 4.*b* it can be observed that the eddy currents flow in opposite direction close to the opposite plate surfaces.

In case of three or more asymmetrically positioned plates (*Fig.* 5.*a*) the average equivalent distance depends on the distance between the two plates, situating closest to the pole surfaces, so formula (2) can be used in this case as well.

In *Fig.* 5.*b* it can be seen that eddy currents flow in the plate in the middle, because they are induced by the eddy currents flowing in the outside plates. Because

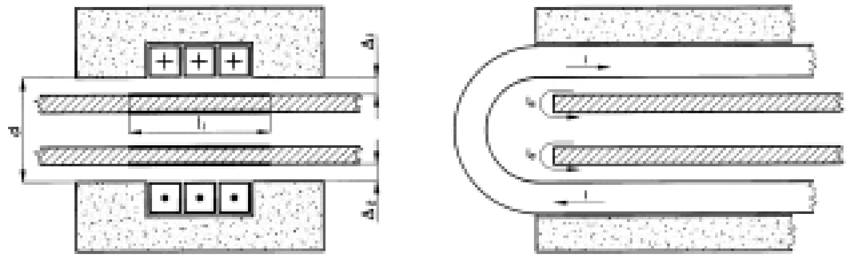

Fig. 4. The equivalent excitation lengths in heating of two, symmetrically positioned plates

there are eddy currents flowing in each plate, therefore in *Eq.* (1) it was necessary to use the number of the plates (z) as a multiplier.

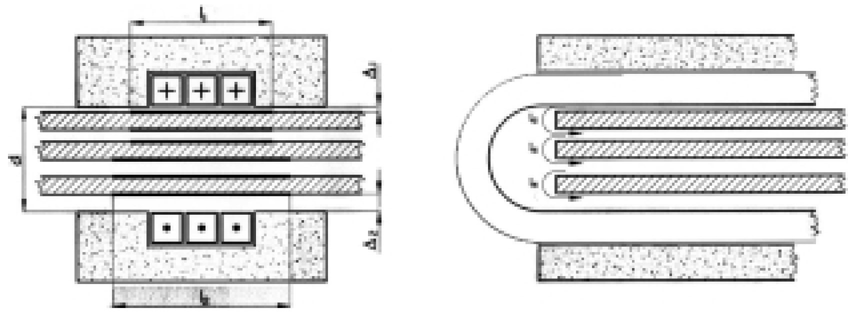

Fig. 5. The equivalent excitation lengths in heating of three, asymmetrically positioned plates

Introducing notation

$$K = \frac{16\sqrt[4]{2}}{3\pi}\sqrt{\rho B_0 \frac{\omega}{l}} N\sqrt{N} z b, \tag{3}$$

the voltage phazor of the load:

$$\mathbf{U}_L = K\sqrt{I}\left(1 + \frac{1}{2}j\right). \tag{4}$$

From *Eqs.* (3) and (4), it can be seen that the impedance of the steel layer is non-linear, its features are different related to the other linear components of the equivalent circuit (including the resistance of the zinc).

On the one hand, the value of the impedance depends on the value of the current, namely it is inversely proportional to the square root of that. On the other hand, the impedance of the steel layer is directly proportional not to the square of the number of turns, but to its power of 3/2.

According to *Fig. 2*, the complex effective value of the inductor current is:

$$\mathbf{I}_i = I + \frac{\mathbf{U}_L}{R_{Zn}} \tag{5}$$

and the terminal voltage of the inverter:

$$\mathbf{U}_t = \mathbf{U}_L + \mathbf{I}_i \cdot (R_i + j \cdot X_0) - j \cdot X_C \cdot \mathbf{I}_i.$$

By the substitution of (2), the terminal voltage can be given by the following formula:

$$\mathbf{U}_t = K\sqrt{I}\left(1 + \frac{1}{2}j\right) + \mathbf{I}_i(R_i + jX_0 - jX_C). \tag{6}$$

Based on the previously described equations and taking the control of the inverter into consideration, the electrical parameters can be calculated.

2.1. *Dimensioning by Assuming Sinusoidal Current*

In case of serial resonance circuit inverter, to a first approximation, the current can be considered to be sinusoidal. The control of the inverter ensures to be the zero points of the current and square voltage of the inverter at the same time. It means that the zero points of the first harmonic of the square voltage will be at the same time like the previous ones. Therefore the first harmonic circular impedance has to be purely effective resistance:

$\mathrm{Im}\left(\frac{\mathbf{U}_t}{\mathbf{I}_i}\right) = 0$, where Im means the imaginary component.

Using formulae (6), (5) and (4)

$$\frac{\mathbf{U}_t}{\mathbf{I}_i} = \frac{K\sqrt{I}\left(1 + \frac{1}{2}j\right)}{I + \dfrac{K\sqrt{I}\left(1 + \frac{1}{2}j\right)}{R_{Zn}}} + R_i + j(X_0 - X_C) \equiv R_i + j(X_0 - X_C) +$$

$$+ \frac{R_{Zn}\left(1 + \frac{1}{2}j\right)}{\dfrac{R_{Zn}}{K}\sqrt{I} + 1 + \frac{1}{2}j} \equiv R_i + j(X_0 - X_C) + \frac{R_{Zn}\left(1 + \frac{1}{2}j\right)\left(1 + \dfrac{R_{Zn}}{K}\sqrt{I} - \frac{1}{2}j\right)}{\left(1 + \dfrac{R_{Zn}}{K}\sqrt{I}\right)^2 + \frac{1}{4}},$$

thus

$$0 = \mathrm{Im}\left(\frac{\mathbf{U}_t}{\mathbf{I}_i}\right) = X_0 - X_C + \frac{R_{Zn}^2}{2K}\frac{\sqrt{I}}{\left(1 + \dfrac{R_{Zn}}{K}\sqrt{I}\right)^2 + \frac{1}{4}}.$$

From that the following quadratic equation can be written for $\sqrt{I}$:

$$(\sqrt{I})^2 - \left[\frac{K}{2(X_C - X_0)} - \frac{2K}{R_{Zn}}\right]\sqrt{I} + \frac{5K^2}{4R_{Zn}^2} = 0; \tag{7}$$

by this I can be calculated. According to (1) K depends on ℓ, the average equivalent excitation length ℓ depends on the excitation, which means that it depends on I, therefore determination of I requires iteration. Knowing I, $\mathbf{U}_L$ can be calculated from (4), $\mathbf{I}_i$ from (5) and $\mathbf{U}_t$ from (6). The other electrical parameters – like power, voltage of the condenser, power coefficient and efficiency – can be determined based on the equivalent circuit.

A computer program was developed for the dimensioning, which calculates these electrical parameters at a given capacitance and by prescribing the inverter voltage (according to the real operation) or prescribing the useful power.

2.2. More Accurate Dimensioning

In reality the current of the serial resonance circuit is not sinusoidal, but – in steady state – it is a series of oscillations damping during a half of a period, whose circular frequency is well known:

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}};$$

here L, R and C are the inductance, resistance and capacitance of the circuit. (Since by inverter of resonance circuit the process has to be periodical, therefore $\frac{1}{LC} > \frac{R^2}{4L^2}$, so $\sqrt{\frac{L}{C}} > \frac{R}{2}$ is valid.) The terminal square voltage of the inverter has the same circular frequency, so its first harmonic has it as well. According to that, the capacitive phase angle is

$$\tan\psi = \frac{\frac{1}{C\omega} - L\omega}{R} = \frac{1 - LC\omega^2}{RC\omega} = \frac{1 - LC\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}{RC\omega} = \frac{R}{4L\omega},$$

thus

$$\tan\psi = \frac{1}{4\tan\varphi}, \tag{8}$$

where φ is the phase angle of the inductor-load system at circular frequency ω. As the computed $\tan\psi$ was positive, it means that the first harmonic of the current is ahead of the first harmonic of the voltage, so it reaches the zero point before the commutation times. The computation is traced back to the $\psi = 0$ situation in such a way that the condenser with capacitance C is supposed to be two condensers connected serially, so

$$\frac{1}{C} = \frac{1}{C_0} + \frac{1}{C_1}, \tag{9}$$

and the capacitance C_0 of the first condenser is chosen so that if it is connected in the circuit only, then $\psi = 0$. Therefore the following can be written:

$$\frac{1}{C_0 \omega} = L\omega. \tag{10}$$

On the other hand,

$$\frac{1}{C\omega} - L\omega = R \tan\psi,$$

based on this and from *Eq.* (8)

$$\frac{1}{C\omega} = L\omega + R\tan\psi = R(\tan\varphi + \tan\psi) = R\left(\tan\varphi + \frac{1}{4\tan\varphi}\right),$$

dividing it by *Eq.* (10), the result is:

$$\frac{C_0}{C} = \frac{R}{L\omega}\left(\tan\varphi + \frac{1}{4\tan\varphi}\right) = 1 + \frac{1}{4\tan^2\varphi}. \tag{11}$$

However, $\tan\varphi$ depends on that ω, which is determined by C_0 and the program part calculating with $\psi = 0$. Therefore the program has to be completed by an additional iteration cycle.

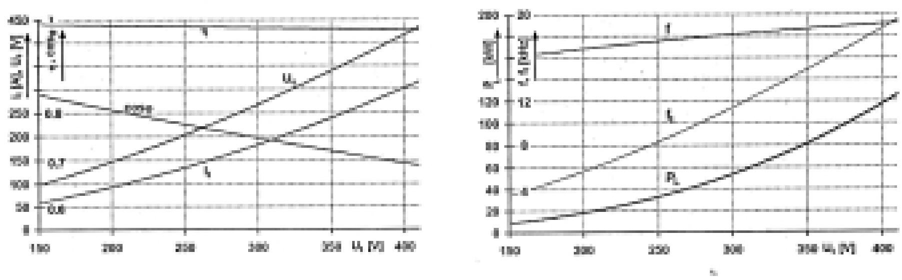

Fig. 6. Dimensioning results ($\psi = 0$)

3. Results

Here the dimensioning can be presented for only one situation ($d = 2$ mm; $N = 3$; $m = 11$ mm; $\rho_i = 2 \cdot 10^{-8}$ Ωm; $v = 0.5$ mm; $\rho = 2 \cdot 10^{-7}$ Ωm, $v_{zn} = 7.1$ μm, $\rho_{Zn} = 1 \cdot 10^{-7}$ Ωm; $b = 1250$ mm, $\xi = 9 - 9$ mm; $z = 2$; $C = 6\mu$F). The parameters were represented on diagrams in *Fig. 6.a* and *6.b* at $\psi = 0$ as a function of $\mathbf{U}_t$ in full wave absorption domain. In *Fig. 6.a* the current I_i, the condenser voltage U_C, the efficiency of the inductor-load system signed by η and the power

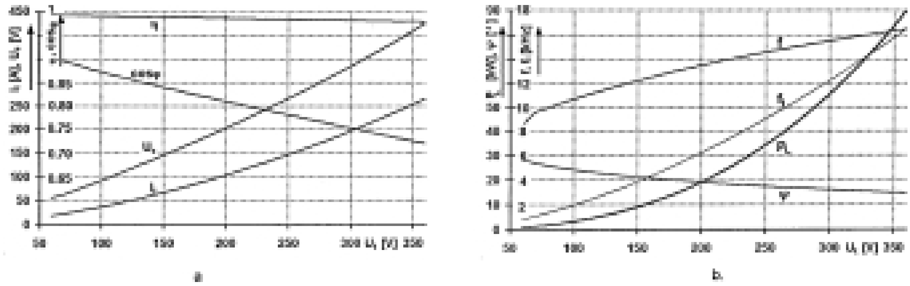

Fig. 7. More accurate dimensioning results

coefficient $\cos\varphi$ can be seen, while in *Fig. 6.b* the power P_L, the actual frequency f, and the frequency belonging to the full wave absorption as a borderline case, f_l can be observed. The ideal heating for the technology is at $\mathbf{U}_t = 405$ V inverter voltage, where $f = f_l = 19.2$ kHz and $P_L = 120$ kW. The same diagrams are presented for $\psi \neq 0$ as well (*Fig. 7.a* and *7.b*). In this case the 'ideal' parameters are as follows: $\mathbf{U}_t = 358$ V, $f = f_l = 16.46$ kHz; $P_L = 90$ kW.

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Appendix

Taking the Zinc Layer on the Steel Plate into Consideration

The zinc layer is very thin on the surface of the plate (on each side). Therefore the phazor of the current density and in connection with it the phazor of the electrical field strength is constant inside this layer (*Fig. F1*). Thus, using the notation of the figure $\mathbf{E}_1 \cong \mathbf{E}_0$ and the current density inside the zinc layer is:

$$\mathbf{J} = \frac{\mathbf{E}_0}{\rho_{Zn}},$$

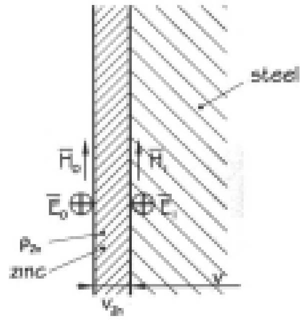

Fig. F1. Field quantities on the surface of a thin zinc layer

where ρ_{Zn} is the resistivity of the zinc. Based on the excitation law

$$\ell_H(\mathbf{H}_0 - \mathbf{H}_1) = \ell_H v_{Zn}\mathbf{J} = \ell_H v_{Zn}\frac{\mathbf{E}_0}{\rho_{Zn}},$$

so

$$\mathbf{H}_0 = \mathbf{H}_1 + \frac{v_{Zn}}{\rho_{Zn}}\mathbf{E}_0.$$

But

$$\mathbf{H}_0 = \frac{N\mathbf{I}_i}{\ell}, \qquad \mathbf{H}_1 = H = \frac{NI}{\ell}; \qquad \mathbf{E}_0 2bzN = \mathbf{U}_C,$$

and substituting these relations into the previous equation, it can be written:

$$\mathbf{I}_i = I\frac{\mathbf{U}_c}{\rho_{Zn}\dfrac{2bz}{\ell v_{Zn}}N^2},$$

so the resistance connecting parallel

$$R_{Zn} = \rho_{Zn}\frac{2bz}{\ell v_{Zn}}N^2.$$