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ANALYSIS OF TRANSIENT TRANSFER PROPERTIES OF CURRENT TRANSMITTERS. COMPUTATIONAL METHODS

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Abstract

In this paper such analytical and numerical methods are presented, which are necessary for the analysis of transient transfer properties of self-integrating and current-integrating type current transmitters operating based on the principle of magnetic voltage measurement. The methods can be used for arbitrary exciter signals. The analytic solution of response signal measurable on the output of the transmitter is also presented, using the analytical method in the case of four given exciter signals (fault far from the generator, fault near to the generator, half-sine wave current, current signal of an inverter).

Keywords: current transmitters, current transformer, transient signal transmission.

1. Introduction

The practically used current transmitters are operating based on the well-known principle of magnetic voltage measurement. This principle was published by CHATTOCK (CHATTOCK, 1887), more than hundred years ago, and later by ROGOWSKI (ROGOWSKI, 1912). Current transmitters consist of a ring-shaped current measuring coil, which surrounds the conductor and an integrating circuit connected to the outlets of the coil. For the operation of the so called passive (self-integrating and current-integrating type) current transmitters no supply voltage is required. Nowadays the interest for current transmitters is strongly increasing, due to the wide spreading of modern electronic protections and measuring equipment. Current transmitter can be magnificently connected to these devices because it provides such a voltage signal on its output which is proportional to the primary current. Using it, the restriction for the impedance applicable in the secondary circuit of traditional current transformers, the long distance signal transport is not necessary. There is no danger of death during the disconnection of the secondary circuit. Separated measuring and relay core are not necessary as it is in the traditional current transformers. The measuring coil contains no iron, therefore the current transmitter has linear transfer characteristic (in a wide range of measurable currents), including the conform transmission of transient currents (containing DC component). The problems of measuring network frequency currents were solved by inventing a new passive, current-integrating current transmitter (KOLLER, STEFÁNYI 1985).

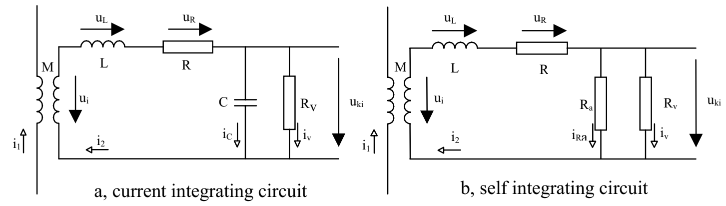


Fig. 1.

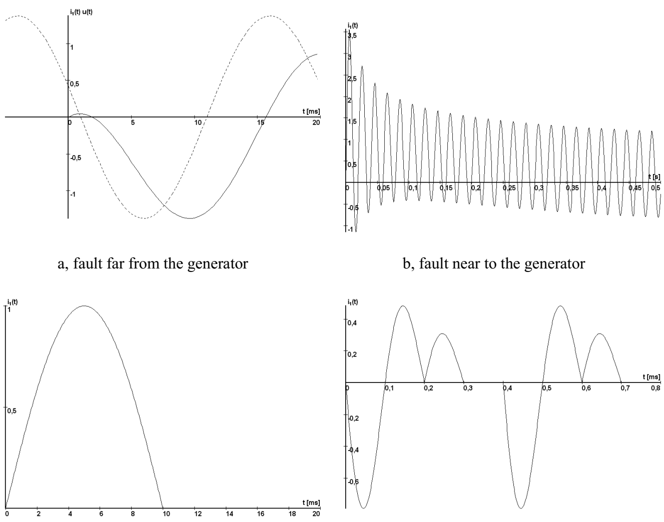
This current transmitter with the other, passive self-integrating type – which can be supposed as a traditional one – can be applied to transmission stationary currents from network frequency to some 10 kHz frequencies (KOLLER, 1994). The computational models and design methods of the transmission of quasi-stationary currents are available. (KOLLER, 1993) shows the results regarding the examination, centrated parts of network are represented in *Fig. 1* (KOLLER and STEFÁNYI, 1985) with the following parameters:

i_1	current to measure (input signal)
i_2	current of measuring circuit
u_i	voltage induced in the measuring coil
M	mutual inductance of the measuring coil and the conductor to measure
L	self inductance of measuring coil
R	effective resistance of measuring coil
R_v	resultant resistance of the loss resistance of the condenser and the input resistance of the measuring instrument (current-integrating circuit)
R_v	input resistance of measuring instrument (self-integrating circuit)
C	capacitance
R_a	output resistance of the measuring circuit
$U_{ki} = U_C$	output voltage signal (current-integrating circuit)
$U_{ki} = U_{Ra}$	output voltage signal (self-integrating circuit)

Based on these circuits, the dimensioning of current transmitters was accurate for quasi-stationary signal transmission – even at some 10 kHz frequency. The accuracy could be justified by comparing the measurement results. So these models are well applicable for examining the transient signal transmission up to some 10 kHz frequency. Reducing frequency (in the case of transients, DC component is possible) the models describe reality more and more accurately.

One of our goals is to calculate the output signals for both of the current transmitters, based on the presented models, for the following input signals – frequent in practice:

1. Current transients at a fault far from the generator (*Fig. 2a*). The effect of the generator can be neglected by most of the short circuits in power networks.
2. Current transients at a fault near to the generator (*Fig. 2b*). Examination of the transmission of transient and subtransient currents flowing in a network, which contains non-linear elements of synchronous generator.
3. Sinusoidal half wave current to test circuit breaker (*Fig. 2c*). This signal is applied by synthetic short circuit tests of circuit breakers.
4. Current of impulse-operating inverter (*Fig. 2d*). Current transmitter is used for supplying control electronics in the main circuit of the resonance circuit inverter applied in induction heating.



a, fault far from the generator

b, fault near to the generator

c, sinusoidal halfwave

d, current of inverter

Fig. 2.

Further goals are to analyse the features of transient transmission and to determine the dimensioning principles used at transient signal transmission.

The comparison of computation and measurement results would be extremely difficult, – because of the complicated measurements – but according to the previously mentioned facts, it is not necessary because the applied computational models describe the operation of current transmitters accurately enough.

For the examination, two kinds of computational methods were used. On the one hand, the equations, which can be formulated for the measuring circuit, were solved analytically, on the other hand, choosing appropriate time division, the output response signals were determined by a numerical method. The accuracy can be evaluated by comparing the results of the two different methods.

2. Computational Methods

The equations necessary for computations are formulated based on the current-integrating circuit (*Fig. 1a*). The input signal, the current to measure itself, flows in the conductor encircled by the measuring coil. Generally it can be a current of optional time function, let's sign it by $i_1(t)$. The voltage induced in the measuring coil, u_i, is proportional to the derivative of the exciter current, while the coefficient is the mutual inductance (M) between the primary conductor and the measuring coil:

$$u_i = M \cdot \frac{\mathrm{d}i_1}{\mathrm{d}t}. \tag{1}$$

The value of the mutual inductance can be supposed to be constant, irrespective of the value of the current, because it is provided by the design of the ideal magnetic voltage measuring coil.

This voltage signal gives the input signal of the measuring circuit. The output signal, in the case of current-integrating circuit, appears as a voltage signal in the outlet of the capacitor (in the case of self-integrating current transmitter, the output signal appears on the output resistor).

The voltage of the condenser can be expressed from a Kirchoff-I equation, based on the current-integrating circuit in *Fig. 1a*:

$$L \cdot \frac{\mathrm{d}i_2}{\mathrm{d}t} + R \cdot i_2 + u_C = u_i. \tag{2}$$

The previous equation is a first order, linear differential equation, which contains a new variable, the current of the measuring circuit (i_2). If the voltage of the condenser is expressed by its current, the equation we get contains only one variable, but it is an *integro-differential equation*:

$$u_i - L \cdot \frac{\mathrm{d}i_2}{\mathrm{d}t} - R \cdot i_2 - U_{C_0} - \frac{1}{C} \cdot \int_0^t i_2(\tau)\,\mathrm{d}\tau = 0. \tag{3}$$

The solution of this depends on the time function of the input signal, according to (1). $u_c(0) = 0$ and $i_2(0) = 0$, when the elements of the circuit have no energy. The situation is more complicated, if the boundary conditions are not equal to zero $-u_c(0) = U_{co}$ and $i_2(0) = I_{2o}$.

In the case of self-integrating current transmitter (*Fig. 1b*) the exciter signal (u_i) can be described also by *Eq.* (1) and from that the output voltage can be expressed also based on a Kirchoff-I equation. The situation is more simple than it was in the previous case, because the circuit contains only one energy storing element, so the differential equation is only a first order one:

$$u_i - L \cdot \frac{\mathrm{d}i_2}{\mathrm{d}t} - R \cdot i_2 - i_2 \cdot R_{ki} = 0, \tag{4}$$

where $R_{ki} = \dfrac{R_a \cdot R_v}{R_a + R_v}$.

If the coil contained some energy at moment $t = 0$ – its current differed from zero –, initial value $i_2(0) = I_{2o}$ has to be taken into consideration.

Initial values can be considered by inserting ideal voltage and current source. Because there will be more than one sources, theorem of superposition is suitable to use.

The solution will be created by two different methods.

- Analytical method
  The input signals presented in the introduction can be divided into simpler elementary functions (e.g. sinusoidal, exponential functions, their sum or product), the time function of the measuring circuit is written for them and after the substitution into the differential equation, the equation is solved by *Laplace-transformation*. Taking the advantage of additive features of the transformation, it is easier to write the response for complex test signals, based on the results. However, the exciter signal is simple, the final formula we get in such a way is extremely complicated.
- Applying a numerical method
  The initial differential equation is solved by computer, in a numerical way. The applied algorithm is the simplest one which fulfils the requirements in terms of accuracy and speed.

2.1. Analytical Method

The first step is to write the initial *Laplace-transformed* form which gives the basis of the *Laplace-transform* of output signals for input signals with arbitrary time function. Here the most common situation is given, when a parallel connected 'loss' resistance is present in the circuit and the storing element of the circuit is not energy-free.

As an effect of input signal $i_1(t)$, induced voltage $u_i(t)$ (its *Laplace-transform* is: $U_i(s)$) appears on the outlet of the coil. The further possible exciter signals are given by the initial conditions.

2.1.1. General Solution

When the energy storing element in the case of current-integrating current transmitter (*Fig. 1a*) is supposed to have some energy, then a

$$U_{Co}(s) = \frac{U_{Co}}{s} \tag{5}$$

ideal voltage source – connected serially to the condenser – and an

$$I_{2o}(s) = \frac{I_{2o}}{s} \tag{6}$$

ideal current source – connected parallel to the inductance – are inserted.

Because there are more than one excitations in the circuit, the theorem of superposition is applied. Let's take exciter signal $U_i(s)$ first and substitute the current source by a disconnection and the voltage source by a short circuit. Using voltage dividing formula, the voltage of the condenser:

$$U_{C_1}(s) = U_i(s) \cdot \frac{\dfrac{1}{L \cdot C}}{s^2 + s \cdot \left(\dfrac{R}{L} + \dfrac{1}{R_v \cdot C}\right) + \dfrac{R_v + R}{R_v \cdot L \cdot C}}.$$

The simplest way to determine the voltage of condenser generated by the current of current source (6) is to use current dividing formula:

$$U_{C_2}(s) = I_{2_o} \cdot L \cdot \frac{\dfrac{1}{L \cdot C}}{s^2 + s \cdot \left(\dfrac{1}{R_v \cdot C} + \dfrac{R}{L}\right) + \dfrac{R + R_v}{L \cdot C \cdot R_v}}.$$

The voltage of condenser created by the voltage source (5) can be written by the formula of voltage division:

$$U_{C_3}(s) = \frac{U_{C_0}}{s} - \frac{U_{C_0}}{s} \cdot \frac{\left(\dfrac{s \cdot L}{R_v} + \dfrac{R_v + R}{R_v}\right) \cdot \dfrac{1}{L \cdot C}}{s^2 + s \cdot \left(\dfrac{1}{R_v \cdot C} + \dfrac{R}{L}\right) + \dfrac{R + R_v}{L \cdot C \cdot R_v}}.$$

According to the theorem of superposition and adding the voltages with correct sign:

$$\begin{aligned} U_C(s) &= U_{C_1}(s) + U_{C_2}(s) + U_{C_3}(s) \\ &= \frac{U_{C_0}}{s} + \left[U_i(s) + I_{2_0} \cdot L - \frac{U_{C_0}}{s} \cdot \left(\frac{s \cdot L}{R_v} + \frac{R_v + R}{R_v}\right)\right] \cdot \frac{\dfrac{1}{L \cdot C}}{(s - s_1) \cdot (s - s_2)}, \end{aligned} \tag{7}$$

$$s_1 = -\frac{1}{2} \cdot \left[\frac{R}{L} + \frac{1}{R_v \cdot C} - \sqrt{\left(\frac{R}{L} + \frac{1}{R_v \cdot C}\right)^2 - \frac{4 \cdot (R + R_v)}{L \cdot C \cdot R_v}}\right],$$

where

$$s_2 = -\frac{1}{2} \cdot \left[\frac{R}{L} + \frac{1}{R_v \cdot C} + \sqrt{\left(\frac{R}{L} + \frac{1}{R_v \cdot C}\right)^2 - \frac{4 \cdot (R + R_v)}{L \cdot C \cdot R_v}}\right]. \tag{8}$$

Let's introduce the integrating time constant of the integrator $T_i = R \cdot C$, the nominal circular frequency $\omega_n^2 = 1/L \cdot C$, the time constant of the parallel, so called ‘loss-circuit’ $T_v = R_v \cdot C$. The previous roots expressed by the introduced quantities are:

$$s_1 = -\frac{1}{2} \cdot \left[T_i \cdot \omega_n^2 + \frac{1}{T_v}\right] - \sqrt{\left(T_i \cdot \omega^2 + \frac{1}{T_v}\right)^2 - 4 \cdot \frac{(R + R_v)}{L \cdot C \cdot R_v}},$$
$$s_2 = -\frac{1}{2} \cdot \left[T_i \cdot \omega_n^2 + \frac{1}{T_v}\right] + \sqrt{\left(T_i \cdot \omega^2 + \frac{1}{T_v}\right)^2 - 4 \cdot \frac{(R + R_v)}{L \cdot C \cdot R_v}}. \tag{9}$$

In *Eq.* (7) the two components can be separated very well. The first part is the response of the system to the exciter signal, while the second one represents the output signals caused by the initial energies. These signals can be examined absolutely independently and finally summarised. The output voltage caused by the initial energies can be obtained by a simple *inverse Laplace-transformation.*

$$u_C(t) = U_{C_0} + \frac{\dfrac{I_{2_0}}{C} - \dfrac{U_{C_0}}{R_v \cdot C}}{s_1 - s_2} \cdot (e^{s_1 \cdot t} - e^{s_2 \cdot t})$$
$$- \frac{U_{C_0} \cdot (R_v + R)}{R_v \cdot L \cdot C} \cdot \left[\frac{1}{s_1 \cdot s_2} + \frac{e^{s_1 \cdot t}}{s_1 \cdot (s_1 - s_2)} + \frac{e^{s_2 \cdot t}}{s_2 \cdot (s_2 - s_1)}\right]. \tag{10}$$

In the case of self-integrating circuit, the *Laplace-transformed* form can be written based on *Eq.* (4). If the coil is not free of energy at switching in, an ideal current source has to be inserted. As with the previous case, the theorem of superposition can be applied. The voltage appearing at the outlet as a result of the input signal is the following:

$$U_{R_{a1}}(s) = U_i(s) \cdot \frac{R_{ki}}{R_{ki} + R + s \cdot L} = U_i(s) \cdot \frac{\dfrac{R_{ki}}{L}}{\dfrac{R_{ki} + R}{L} + s}.$$

The second exciter signal, the current source generates the following voltage at the output:

$$U_{R_{a2}}(s) = I_{2_s} \cdot \frac{R_{ki}}{\dfrac{R_{ki} + R}{L} + s}.$$

Summarising the two components – using notation $s_1 = -\dfrac{R_{ki} + R}{L}$ the output signal can be obtained:

$$U_{R_a}(s) = U_{R_{a1}}(s) + U_{R_{a2}}(s) \left[U_i(s) + L \cdot I_{2_0}\right] \cdot \frac{\dfrac{R_{ki}}{L}}{s - s_1}. \tag{11}$$

If there is no excitation, the output voltage caused by the initial energies is:

$$u_{R_a}(t) = I_{2_0} \cdot R_{ki} \cdot e^{s_1 \cdot t}. \tag{12}$$

To know the total response signal, the response for the excitation has to be known. In the case of complex signals used in practice it would give complicated mathematical deductions, therefore, using the additive property of *Laplace-transformation*, the test signals are divided into elementary functions and after the *inverse Laplace-transformation*, the response signals are provided by the appropriate linear combination of the results (this summarisation is independent of the type of the integrator). These elementary functions are sinusoidal, exponential functions or originating from these products. The results can be found in *Table 1*. A further simplification will be made: we do not take into account that the transmitters are receiving their input signals via inductive connection. It means that not function $i_1(t)$ will be simplified, but $u_i(t)$. With this method a derivation can be saved and, anyway, for us the *Laplace-transformed* form of $u_i(t)$ is necessary in the initial equations.

2.1.2. *Transmission of Exciter Signals*

In this part the response signals caused by the input signals, that we wanted to examine originally, will be determined analytically, using the results we got, like simple panels, which have to fit together in an appropriate way to get the response of more complex signals, which have practical significance.

a. Fault far from the generator (Fig. 2a)

The time function of the current will be the sum of a cosinusoidal and an exponential function.

$$\begin{aligned} i_1(t) &= I_m \cdot \left[\cos(\omega \cdot t + \psi - \varphi) - e^{-\frac{t}{T_h}} \cdot \cos(\psi - \varphi)\right] \\ &= I_m \cdot \left[\cos(\omega \cdot t + \alpha) - e^{-\frac{t}{T_h}} \cdot \cos(\alpha)\right]. \end{aligned} \tag{13}$$

The parameters are: I_m is the stationary peak value of the fault current, T_h is the time constant of the fault circuit, φ is the phase angle of the circuit, ψ is the angle of switching in (it is coincidental, its value depends on the phase angle of the supply voltage at the moment of switching in), ω is the circular frequency.

The response signal can be determined from the previously obtained functions (*Table 1*) in the following way:

$$\begin{aligned} u_{ki}(t) = &-M \cdot I_m \cdot \omega \cdot [L_3(t) \cdot \cos(\alpha) + L_4(t) \cdot \sin(\alpha)] \\ &+ I_m \cdot \frac{M}{T_h} \cdot \cos(\alpha) \cdot L_2(t, T_h). \end{aligned} \tag{14}$$

Table 1.

$u_i(t)$	Current-integrating	Self-integrating
$\varepsilon(t)$	$L_1(t) = \frac{1}{L\cdot C}\cdot\left[\frac{1}{s_1\cdot s_2}+\frac{e^{s_1\cdot t}}{s_1\cdot(s_1-s_2)}-\frac{e^{s_2 t}}{s_2\cdot(s_1-s_2)}\right]$	$L_1(t)=\frac{R_{ki}}{L}\cdot\frac{e^{s_1\cdot t}-1}{s_1}$
$e^{-\frac{t}{T}}$	$L_2(t,T)=\frac{e^{-\frac{t}{T}}}{L\cdot C}\cdot\left[\frac{1}{s_1'\cdot s_2'}+\frac{e^{s_1'\cdot t}}{s_1'\cdot(s_1'-s_2')}-\frac{e^{s_2' t}}{s_2'\cdot(s_1'-s_2')}\right]$	$L_2(t,T)=\frac{R_{ki}}{L}\cdot e^{-\frac{t}{T}}\cdot\frac{e^{s_1'\cdot t}-1}{s_1'}$
$\sin(\omega\cdot t)$	$L_3(t)=\frac{1}{L\cdot C}\cdot\left[A_1\cdot\sin(\omega\cdot t)+A_2\cdot\cos(\omega\cdot t)+A_3\cdot\omega\cdot e^{s_1\cdot t}+A_4\cdot\omega\cdot e^{s_2 t}\right]$	$L_3(t)=\frac{R_{ki}}{L}\cdot\left[-A_1\cdot\sin(\omega t)-A_2\cdot\cos(\omega\cdot t)+A_3\cdot\omega\cdot e^{s_1\cdot t}\right]$
$\cos(\omega\cdot t)$	$L_4(t)=\frac{1}{L\cdot C}\cdot\left[A_1\cdot\cos(\omega\cdot t)-A_2\cdot\sin(\omega\cdot t)+A_3\cdot s_1\cdot e^{s_1\cdot t}+A_4\cdot s_2\cdot e^{s_2\cdot t}\right]$	$L_4(t)=\frac{R_{ki}}{L}\cdot\left[-A_1\cdot\cos(\omega t)+A_2\cdot\sin(\omega\cdot t)+A_3\cdot s_1\cdot e^{s_1\cdot t}\right]$
$e^{-\frac{t}{T}}\sin(\omega\cdot t)$	$L_5(t,T)=\frac{e^{-\frac{t}{T}}}{L\cdot C}\cdot\left[A_1'\cdot\sin(\omega\cdot t)+A_2'\cdot\cos(\omega\cdot t)+A_3'\cdot\omega\cdot e^{s_1'\cdot t}+A_4'\cdot\omega\cdot e^{s_2'\cdot t}\right]$	$L_5(t,T)=\frac{e^{-\frac{t}{T}}\cdot R_{ki}}{L}\cdot\left[-A_1'\cdot\sin(\omega t)-A_2'\cdot\cos(\omega\cdot t)+A_3'\cdot\omega\cdot e^{s_1' t}\right]$
$e^{-\frac{t}{T}}\cos(\omega\cdot t)$	$L_6(t,T)=\frac{e^{-\frac{t}{T}}}{L\cdot C}\cdot\left[A_1'\cdot\cos(\omega\cdot t)-A_2'\cdot\sin(\omega\cdot t)+A_3'\cdot s_1'\cdot e^{s_1'\cdot t}+A_4'\cdot s_2'\cdot e^{s_2'\cdot t}\right]$	$L_6(t,T)=\frac{e^{-\frac{t}{T}}\cdot R_{ki}}{L}\cdot\left[-A_1'\cdot\cos(\omega t)-A_2'\cdot\sin(\omega\cdot t)+A_3'\cdot s_1'\cdot e^{s_1' t}\right]$

$$s_1' = s_1 + \frac{1}{T} \qquad s_2' = s_2 + \frac{1}{T}$$

$$A_1 = \frac{s_1\cdot s_2-\omega^2}{(s_1^2+\omega^2)\cdot(s_2^2+\omega^2)} \qquad A_1' = \frac{s_1'\cdot s_2'-\omega^2}{(s_1'^2+\omega^2)\cdot(s_2'^2+\omega^2)}$$

$$A_2 = \frac{\omega\cdot(s_1+s_2)}{(s_1^2+\omega^2)\cdot(s_2^2+\omega^2)} \qquad A_2' = \frac{\omega\cdot(s_1'+s_2')}{(s_1'^2+\omega^2)\cdot(s_2'^2+\omega^2)}$$

$$A_3 = \frac{1}{(s_1^2+\omega^2)\cdot(s_1-s_2)} \qquad A_3' = \frac{1}{(s_1'^2+\omega^2)\cdot(s_1'-s_2')}$$

$$A_4 = \frac{1}{(s_2^2+\omega^2)\cdot(s_2-s_1)} \qquad A_4' = \frac{1}{(s_2'^2+\omega^2)\cdot(s_2'-s_1')}$$

$$A_1 = \frac{s_1}{s_1^2+\omega^2} \qquad A_1' = \frac{s_1'}{s_1'^2+\omega^2}$$

$$A_2 = \frac{\omega}{s_1^2+\omega^2} \qquad A_2' = \frac{\omega}{s_1'^2+\omega^2}$$

$$A_3 = \frac{1}{s_1^2+\omega^2} \qquad A_3' = \frac{1}{s_1'^2+\omega^2}$$

b. Fault near to the generator (Fig. 2b)

One phase fault current of a generator consists of several, different type signals. It consists of a transient, subtransient, a stationary and a DC component. The function is the following (RETTER, 1992):

$$
\begin{aligned}
i_1(t)=&\left[(I_Z'' - I_Z') \cdot e^{-\frac{t}{T_d''}} + (I_Z' - I_Z) \cdot e^{-\frac{t}{T_d'}} + I_z\right] \cdot \cos(\omega \cdot t + \alpha) \\
&- I_Z'' \cdot \cos(\alpha) \cdot e^{-\frac{t}{T_S'}}, \\
u_{ki}(t)=&M \cdot \Bigg\{(I_Z'' - I_Z') \cdot \left[L_5(t, T_d'') \cdot \left(\frac{\sin(\alpha)}{T} - \omega \cdot \cos(\alpha)\right)\right. \\
&\left.-L_6(t, T_d'') \cdot \left(\frac{\cos(\alpha)}{T} + \omega \cdot \sin(\alpha)\right)\right] \\
&+ (I_Z' - I_Z) \cdot \left[L_5(t, T_d') \cdot \left(\frac{\sin(\alpha)}{T} - \omega \cdot \cos(\alpha)\right)\right. \\
&\left.-L_6(t, T_d') \left(\frac{\cos(\alpha)}{T} + \omega \cdot \sin(\alpha)\right)\right] \\
&-I_Z \cdot \omega \cdot [L_3(t) \cdot \cos(\alpha) + L_4(t) \cdot \sin(\alpha)] + \frac{I_Z''}{T_S'} \cdot \cos(\alpha) \cdot L_2(t, T_S')\Bigg\}.
\end{aligned}
\tag{15}
$$

In this case the given functions can be substituted, but let's disregard this because it would result in a long and complicated formula.

c. Sinusoidal halfwave (Fig. 2c)

This is such a case of simple sinusoidal wave when only one half of the total period is taken into consideration. Because the argument of the *Laplace-integral* is valid in the total positive time domain, the function can be generated as a sum of a $\sin(\omega t)$ function, switched in at $t = 0$ and a $(-\sin(\omega t))$ function switched in at $t = \pi/\omega$. The formula using unit step signal:

$$
\begin{aligned}
i_1(t) &= I_m \cdot [\varepsilon(t) \cdot \sin(\omega \cdot t) - \varepsilon(t - T) \cdot \sin(\omega \cdot t)] \\
&= I_m \cdot [\varepsilon(t) \cdot \sin(\omega \cdot t) + \varepsilon(t - T) \cdot \sin(\omega \cdot (t - T))],
\end{aligned}
$$

where $T = \pi/\omega$,

$$u_{ki}(t) = M \cdot I_m \cdot \omega \cdot [\varepsilon(t) \cdot L_4(t) + \varepsilon(t - T) \cdot L_4(t - T)].$$

d. Current signal of an impulse operating inverter (Fig. 2d)

In steady transient state (disregarding the run up of the inverter) the current signal of the inverter consists of several signals based on (KOLLER and TEVAN 1993) as follows.

$$
\begin{aligned}
i_1(t) &= -C \cdot \omega \cdot U_m \cdot \left(1 + \frac{k^2}{4}\right) \cdot e^{-\frac{k}{2} \cdot \omega \cdot t} \cdot \sin(\omega \cdot t) \\
&= -I_m \cdot e^{-\frac{k}{2} \cdot \omega \cdot t} \cdot \sin(\omega \cdot t) \qquad (0 \le t \le T),
\end{aligned}
$$

where

$$\beta = \frac{T_i}{T}, \qquad k = \frac{\cos\varphi}{\sqrt{1-\cos^2\varphi}}, \qquad \omega = 2\cdot\pi\cdot f_s = \sqrt{\frac{1}{L\cdot C} - \left(\frac{R}{2\cdot L}\right)^2},$$

$$I_m = C\cdot\omega\cdot U_m\cdot\left(1+\frac{k^2}{4}\right),$$

$$i_1(t) = I_m\cdot\left(1-e^{-\frac{k\cdot\pi}{2}}\right)\cdot e^{-\frac{k}{2}\cdot\omega\cdot(t-T)}\cdot\sin(\omega\cdot t) \qquad (T < t \le 3T/2).$$
$$i_1(t) = 0 \qquad (3T/2 < t < T).$$

If only one period is examined, the total function will be the following, using unit step functions:

$$\begin{aligned} i_1(t) = &-\varepsilon(t)\cdot I_m\cdot e^{-\frac{k}{2}\cdot\omega\cdot t}\cdot\sin(\omega\cdot t) + \varepsilon(t-T)\cdot I_m\cdot e^{-\frac{k}{2}\cdot\omega\cdot t}\cdot\sin(\omega\cdot t)\\ &+\varepsilon(t-T)\cdot I_m\cdot\left(1-e^{-\frac{k\cdot\pi}{2}}\right)\cdot e^{-\frac{k}{2}\cdot\omega\cdot(t-T)}\cdot\sin(\omega\cdot t)\\ &-\varepsilon(t-\frac{3}{2}\cdot T)\cdot I_m\cdot\left(1-e^{-\frac{k\cdot\pi}{2}}\right)\cdot e^{-\frac{k}{2}\cdot\omega\cdot(t-T)}\cdot\sin(\omega\cdot t), \end{aligned}$$

$$\begin{aligned} u_{ki}(t) = M\cdot I_m\cdot\Bigg\{&-\varepsilon(t)\cdot\left[\omega\cdot L_6\left(t,\frac{2}{k\cdot\omega}\right) - \frac{1}{T}\cdot L_5\left(t,\frac{2}{k\cdot\omega}\right)\right]\\ &+\varepsilon(t-T)\cdot\left(e^{-\frac{k}{2}\cdot\omega\cdot T}+1-e^{-\frac{k\cdot\pi}{2}}\right)\cdot\left[\omega\cdot L_6\left(t-T,\frac{2}{k\cdot\omega}\right)\right.\\ &\left.-\frac{1}{T}\cdot L_5\left(t-T,\frac{2}{k\cdot\omega}\right)\right] - \varepsilon\left(t-\frac{3T}{2}\right)\cdot\left(1-e^{-\frac{k\cdot\pi}{2}}\right)\cdot e^{-\frac{k}{2}\cdot\omega\cdot\frac{T}{2}}\\ &\cdot\left[\omega\cdot L_6\left(t-\frac{3\cdot T}{2},\frac{2}{k\cdot\omega}\right) - \frac{1}{T}\cdot L_5\left(t-\frac{3\cdot T}{2},\frac{2}{k\cdot\omega}\right)\right]\Bigg\}. \end{aligned}$$

This formula gives the response signal for one period of the current signal of the inverter substituting the appropriate functions.

Because it is difficult to determine the results analytically in the case of complicated signals, it is useful to make the calculations by a numerical method. A further advantage is that by comparing the results obtained from different methods, we can check their correctness. The method can be built on the Kirchoff-I equation of the circuit and the necessary quantities can be calculated without iteration – using numerical integration and derivation.

Appropriate accuracy is required, because the results (output signals) we got in this way have to be compared to the exciter signals and small differences have to be determined.

If the numerical procedure is not accurate enough, the results will be strongly distorted.

E.g. if the current transmitter transmits the given signal with a fault of 1% and the difference between the numerical and the analytical method is also 1%, the distortion of the transmission errors can reach 100%. Therefore the time step has to be kept on a small value. Its only disadvantage is that it increases the computational requirements and in such a way the computing time as well.

The basic idea of the method is that the integrals and derivatives of the variables in the Kirchoff-I equation are generated from the results of the previous time moments.

If the Kirchoff-I equation for the current-integrating circuit remains an *integro-differential equation* (3), we have to integrate and derive numerically. The voltage of the condenser can be expressed from its current in such a way that the mean value of the currents in the previous and current moment is generated (*Fig. 3*). For that it is necessary to compute the rate of the change in the current. This will be the quantity to find, and from that the current of the circuit can be expressed, as well as the voltage of the condenser.

If in the case of current-integrating circuit, the initial equation (2) (which is a *first order differential equation*) is written in a slightly different form – variable $i_2(t)$ is expressed from variable $u_C(t)$ –, we get a second order differential equation.

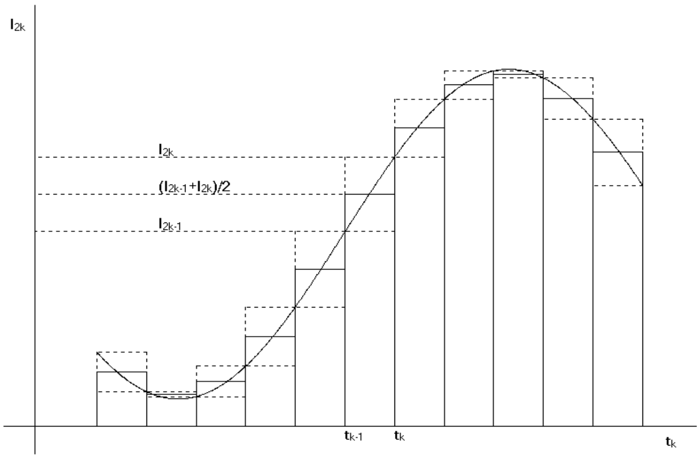


Fig. 3.

This second method gives more accurate results, in the case of the same time division. However, it produces, understandably, a great error at switching in because the second derivative is calculated from the results of two previous moments. The error decreases drastically after some (5–10) time steps.

The variables and their derivatives can be substituted by their numerical versions.

2.2. Comparing the Numerical and the Analytical Methods

By an appropriate computer program, it is possible to compare the numerical and analytical methods. Choosing different exciter signals, the deviance between the output signal calculated by a numerical method and the analytical solution can be examined. Increasing the time step (Δt) the deviance is increasing rapidly (*Fig. 4*). In the case of a fault far from a 50 Hz generator, the following relative deviance (Δh) can be determined:

	Current-integrating	Self-integrating
$\Delta t = \ \ 1\ \mu$s	$2.621\ 10^{-4}$%	$1.046\ 10^{-4}$%
$\Delta t = 10\ \mu$s	$2.621\ 10^{-3}$%	$1.046\ 10^{-3}$%

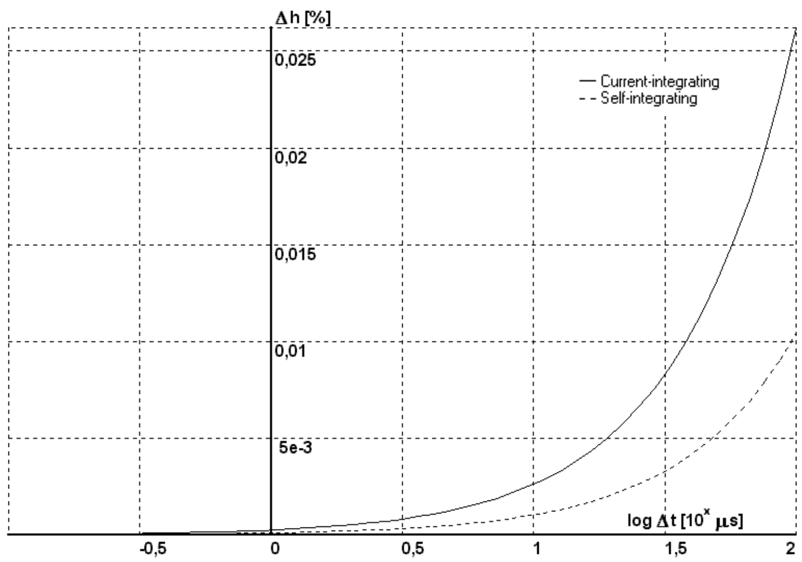


Fig. 4.

2.3. Numerical Method

These deviations are very small related to the errors that we want to examine because the error of the transient signal transmission of practically used current transmitters is 1–2%.

If the time step is chosen to be approximately at least 200 times smaller than the time of period (in the case of 50 Hz it is 10 μs), the error of the numerical method is negligible. Naturally, if the frequency of the excitation is increasing, the time step has to be reduced proportionally to that.

The formulas of the analytic method are complex and complicated, therefore it requires more computational time than the numerical method. Therefore it is more advantageous to use the numerical method for computer-aided analysis.

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