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A NEW APPROACH TO DISCRETE-EVENT DYNAMIC SYSTEM THEORY

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Abstract

The paper presents an original formulation of discrete-event dynamic systems (DEDS) strictly consistent with the Kalman definition of dynamic systems. The paper starts with a clear definition of event as a pair (occurrence time, fact), where the time is a real number and the fact is an element of a set with algebraic properties. The introduction of the concept of event sequences and of suitable operations over their set allows to formulate DEDS as causal operators transforming input event sequences into output event sequences. The definition of a state for such operator allows to give a state representation of the input-output relation. The state representation is a state equation as in the standard continuous or discrete-time systems, and allows to compute the free and the forced responses of the system. The paper terminates by providing the elementary stability definitions and the state equations of linear and time-invariant DEDS.

Keywords: discrete-event dynamic systems, events, stability, state equations.

1. Introduction

Many and different approaches to discrete-event dynamic systems (DEDS) are encountered in the literature. As a tentative, they can be subdivided in two main classes: logical DEDS and timed DEDS.

Logical DEDS are concerned with modelling and control of dynamic systems whose state variables evolve in time with the following properties. It is defined a numerable state set $\mathcal{X}$, usually without any algebraic property, i.e. it is just an alphabet. A specific state value is denoted by x_j, $j = 1, 2, \ldots$. The state may change its values only at a numerable set $\mathcal{T}$ of time instants t_k, called *occurrence times*. When the set $\mathcal{T}$ is pre-defined, it may be substituted by the integer set $\mathcal{Z}$, the time instants t_k may be forgotten and the new state value at times $t > t_k$ may be indicated with $x(k)$. In this case, the state evolution may be written as a sequence of state values $x(k)$ and the corresponding DEDS are called *logical or untimed DEDS*. Typical logical DEDS are Finite State Automata (RAMADGE et al., 1989) and Petri Nets (PETERSON, 1981).

For each state value x_j, the set $\mathcal{X}_j$ of the state values x_h which are reachable at the next occurrence time is finite and is a subset of the set $\mathcal{X}$. The pair (x_j, x_h) defines a possible state variation, called *transition*. Given a state value x_j at time t_k, a specific transition is triggered by the occurrence of an *event* e which is an element of a numerable set $\mathcal{E}$. The same event may trigger more than one transition, also starting from the same state value, in which case uncertain transitions may be considered (ÖZVEREN et al., 1991). The relation between events and state values is usually expressed by a transition function $\delta : \mathcal{E} \times \mathcal{X} \to \mathcal{X}$, whose domain is actually a subset of $\mathcal{E} \times \mathcal{X}$. Given a state value, the subset of the events which might trigger a transition is called feasible. Apparently the concept of event is an intrinsic feature of the model, like a parameter set, and not an external variable, like an input variable. However, input variables (disturbances and control) may act on the events, by modifying their occurrence order, or in other words, their discrete time. For instance the control problems studied by (RAMADGE et al., 1989) aim at designing feedback rules for disabling some events to occur as far as the state passes through certain values.

Timed DEDS, differently from untimed ones, have a mechanism for generating the occurrence times based on the event occurrence. For instance in (CASSANDRAS et al., 1989), each event has a lifetime, which can be random or deterministic, and a new event occurs when the lifetime of an event which is still feasible expires. If applied to Finite State Automata, random lifetime mechanisms may generate dynamic models called Generalised Semi-Markov Processes (GSMP) (HO et al., 1992). Markov chains are a class of such processes when the event lifetime generation is exponential. The state variables of GSMP, unlike those in Markov chains and in logical DEDS, must be defined as pairs (automaton state value, lifetimes of feasible events), the latter time variables being called supplementary variables. Similar mechanisms are used in timed Petri Nets, where a lifetime may be associated to each transition (or to places) (COHEN et al., 1989).

The original theory of discrete-event dynamic system proposed in this paper departs in different aspects from the usual DEDS formulations:

1. A clear distinction is made between the facts which may occur and the event which occurs, the latter being defined as a pair (occurrence time, fact), the occurrence time being a real number. In the literature, as already pointed out, the term event just means the fact which may occur at some time: a simplification which looks reasonable for the untimed models (logical DEDS) where occurrence times are defined as positive integers, and consequently, they are implicit in the event sequences. The concept of event as (occurrence time, fact) is instead clearly mentioned as timed event in the timed models (CASSANDRAS et al., 1989; INAN et al., 1989; ZEIGLER, 1976 and 1989), but, apparently, also such DEDS are not explicitly built over such concept, by exploiting for instance the machinery of event sequences.

2. The sets of the facts may be endowed, if necessary, with algebraic properties, i.e. they are not just alphabets. In this way, operations, like addition, may be introduced over the event set and over the sets of the event sequences.
3. The state of the discrete-event dynamic systems is defined to be a potential event which will occur at a future time to modify the system state, if no external input event (controlled or uncontrolled) will occur meanwhile. This definition allows to formulate discrete-event state equations having forced and free evolutions as in the usual continuous or discrete-time dynamic systems. In the literature, events appear to be used just to describe the state transitions forced by external inputs (RAMADGE et al., 1989), which means that if all the events are disabled or cannot occur, the state remains constant, or otherwise said, the dynamic system has no free response, since events are not able to concatenate themselves.
4. A DEDS formulation like the one proposed in this paper, being strictly consistent with the Kalman definition of dynamic systems, is expected to be suitable for control design and performance evaluation. In the literature some difficulties are perceived for what concerns the use of DEDS in evaluating system performances, difficulties which might have brought to develop specific algebraic methods for untimed (INAN et al., 1989) and for timed models (COHEN et al., 1989) or approaches like perturbation analysis for predicting performances from sample simulation paths (CASSANDRAS et al., 1989; HO et al., 1992; SURI, 1989).

The paper will start with the basic definitions of time, events and event sequences and the introduction of their operations. Based on such elements, the input and output event sequences of discrete-event dynamic systems will be defined and their state equations will be formulated upon definition of the system state. The paper will then provide the basic stability definitions and it will end with the presentation of the linear and time-invariant discrete-event dynamic systems.

2. Time, Events and Event Sequences

2.1. Basic Definitions

Time, fact and event. Given a real variable $t \in \mathcal{T} \subseteq \mathbf{R}$ called *time*, where $\mathbf{R}$ denotes the set of the real numbers, and given a generic set Ξ of elements $\xi \in \Xi$ called *facts*, the Cartesian product $\mathcal{E} = \mathcal{T} \times \Xi$ is defined. Its elements $e = (t, \xi) \in \mathcal{E}$ are called *events.* Any event is therefore described by the pair t = '*the occurrence time instant of the event*', ξ = '*the fact associated to the event*'.

Simultaneous events. Two events $e_1 = (t_1, \xi_1)$, $e_2 = (t_2, \xi_2)$ are said to be *simultaneous* when they have the same occurrence time, i.e. when $t_1 = t_2$ holds.

In a discrete-event system, simultaneous events will be only admitted when it will be necessary to represent in a distinct way elementary facts which can be further combined to represent the facts of more complex events. Under such conditions, simultaneous events do not appear in contrast with the usual assumption that it is practically impossible that two different events occur at the same time. Their introduction should, however, be accompanied by the introduction of a mechanism for combining the facts of simultaneous events into the single fact of a new event. Of course, different facts to be combined together, will have to be at some extent *mutually compatible*, a concept which shall be formally expressed by saying that: *two or more facts are mutually compatible if they are summable and their original set Ξ of facts is closed under the addition.* The combination mechanism will be then a binary operation called *event addition*, applicable to simultaneous events only. As a conclusion, the occurrence of simultaneous events will be admitted if and only if they are summable.

Addition of simultaneous events. The addition $e_3 = e_1 + e_2$ of two simultaneous events $e_1 = (t, \xi_1)$, $e_2 = (t, \xi_2)$ belonging to the same event set $\mathcal{E} = \mathcal{T} \times \Xi$ is defined as a third event $e_3 = (t, \xi_3) \in \mathcal{E}$, such that $\xi_3 = \xi_1 + \xi_2$. The event addition exists if and only if: 1) the addition $\xi_3 = \xi_1 + \xi_2$ is defined in the set of facts Ξ and 2) the set Ξ is closed under addition. The definition of the addition in the set Ξ implies the existence in the same set of the null element θ, such that $\xi = \xi + \theta$ holds for any $\xi \in \Xi$. Then, the event $e = (t, \theta)$ will be called the *null event*, meaning that no fact is occurring at time t.

2.2. Event Sequences and Operations between Sequences

Event sequences. An event sequence σ is a *numerable* set of events belonging to the same event set $\mathcal{E}$, which is *completely ordered* by their occurrence times, i.e.

$$\sigma = \{e_1 = (t_1, \xi_1), \ldots, e_i = (t_i, \xi_i), \ldots\} \quad \text{with the constraint} \quad t_{i+1} > t_i. \quad (1)$$

Therefore, if the event sequence would include simultaneous events, they had to be added into a single event, as explained in the previous section. Therefore an event sequence can be always expressed as a numerable set of non simultaneous events.

By denoting with $\mathcal{T}_\sigma \subset \mathcal{T} \subseteq \mathbf{R}$ the numerable set $\{t_1, \ldots, t_i, \ldots\}$ of the different occurrence times of an event sequence σ, the sequence itself may be defined as a function $\sigma : \mathcal{T}_\sigma \to \Xi$ which is everywhere defined in $\mathcal{T}_\sigma$ and possesses a single value in Ξ. Therefore an event sequence will be indicated indifferently either with $\sigma = \{(t_i, \xi_i)\}$, $i \in [1, n]$, where n may be infinite, or as a function value $\xi(t_i)$, $t_i \in \mathcal{T}_\sigma$. The set of all the (numerable) sequences which can be constructed over the event set $\mathcal{E} = \mathcal{T} \times \Xi$ will be indicated with $\Sigma(\mathcal{E})$.

Over the set $\Sigma(\mathcal{E})$ of the event sequences, the following operations are defined.

Restriction of a sequence. The restriction of an event sequence σ to a time interval $t_i > t$ is defined as the operation $R[\sigma, t_i > t]$ whose result is a sub-sequence $\sigma' \in \Sigma(\mathcal{E})$ of σ such to include only the events e_i occurring at times $t_i > t$. The restriction operation will be indicated with $\sigma' = R[\sigma, t_i > t]$ or simply with $\sigma' = \sigma(t)$.

Addition of sequences belonging to the same set. Consider a pair of sequences $\sigma_1, \sigma_2 \in \Sigma(\mathcal{E})$, $\mathcal{E} = \mathcal{T} \times \Xi$, defined on the same event set $\mathcal{E}$. Their sum $\sigma_3 = \sigma_1 + \sigma_2$ is defined as the union of the events of the two addenda. Accordingly: 1) two event sequences which do not include simultaneous events are always summable 2) instead, two event sequences which include simultaneous events are summable if and only if the addition operation is defined in their set of facts Ξ. The addition of the simultaneous events allows to obtain a sum σ_3 without simultaneous events in agreement with the definition of an event sequence.

3. Discrete-Event Dynamic Systems

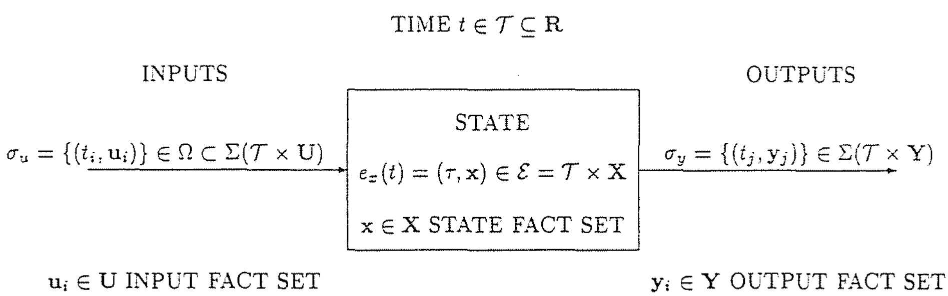

Fig. 1. Input-output diagram of a discrete-event dynamic system

A discrete-event dynamic system is a system whose inputs and outputs are made of asynchronous event sequences (*Fig. 1*). With reference to the basic definitions introduced in the previous paragraph, the following notations are used.

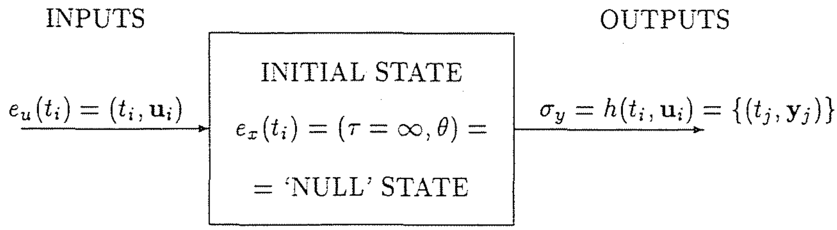

Fig. 2. The output event sequence $h(t_i, \mathbf{u}_i)$ as an effect of the input event $e_u(t_i) = (t_i, \mathbf{u}_i)$ starting from null initial state

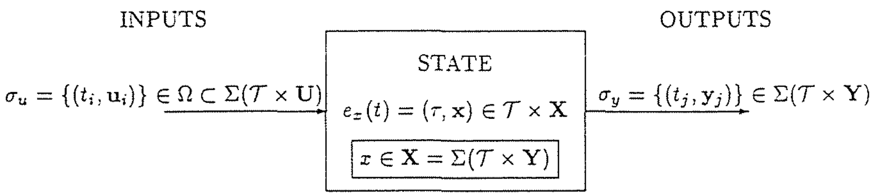

Fig. 3. State equation representation of linear time invariant discrete event dynamic systems

Let $\mathbf{U}$ be the set of facts $\mathbf{u}_i$ associated with the events which make up the sequences $\sigma_u = \{(t_i, \mathbf{u}_i)\} = \{e_u(t_i)\}$ of the input events of the system. $\Omega \subset \Sigma(\mathbf{U})$ denotes the set of the admissible input sequences.

Let $\mathbf{Y}$ be the set of the facts $\mathbf{y}_i$ associated with the events which make up the sequences $\sigma_y = \{(t_j, \mathbf{y}_j)\} \in \Sigma(\mathbf{Y})$ of the output events of the system.

The causality property of the system is assumed, so that the generic output event of the system $e_y(t_j) = (t_j, \mathbf{y}_j)$ turns out to be an effect of previous input events $\sigma_u = \{(t_i, \mathbf{u}_i)\}$ ($t_i < t_j$ in the hypothesis of strict causality, $t_i \leq t_j$ in the hypothesis that an input event can have an immediate effect on the outputs) and of the initial conditions. Such a dependence is expressed through the state of the system.

In the discrete-event dynamic systems introduced here, the state at time $t \in \mathcal{T} \subseteq \mathbf{R}$ is expressed by a potential future event $(\tau, \mathbf{x})$ with $\tau > t$, being $\mathbf{x} \in \mathbf{X}$, where $\mathbf{X}$ is the set of facts associated with the events which express the state of the system. The event $(\tau, \mathbf{x})$ expressing the state at time $t < \tau$, is said to be 'potential', since its actual realization can be taken as certain only in the hypothesis that no input events of the system will occur during the time interval $(t, \tau]$.

The notation $e_x(t)$ is introduced to indicate the event $(\tau, \mathbf{x})$ which expresses the state of the system at time t, being $\tau > t$. Similarly, the notation $(\tau(t), \mathbf{x}(t))$ can be used to show that the state at time t is described by an event $(\tau, \mathbf{x})$ where both the fact $\mathbf{x}$ and its time of occurrence τ are functions of the time t.

In summary, the state $e_x(t)$ is defined at each time $t \in \mathcal{T}$ and it is a function whose value changes only at a numerable set of times; the state variations being called *state transitions*. The occurrence times of the state transitions depend on the times t_j of the input event sequence (forced evolution times) and on the times τ of the future potential events $(\tau, \mathbf{x})$. Note that when a potential event occurs, a new potential event may be generated, giving rise to a concatenated sequence of events, the free evolution, occurring in absence of input events.

The relation input-state-output is described by two different sets of equations, which hold in correspondence of the input events (forced evolution) and of the state events (free evolution) respectively.

Let us consider the instant t_i when the input event $e_u(t_i) = (t_i, \mathbf{u}_i)$ belonging to the sequence $\sigma_u = \{(t_i, \mathbf{u}_i)\}$ occurs and denote with t_{i-} and t_{i+}, respectively, the instants which *immediately* precede and follow the occurrence of the event. The transition function of the state at time t_i is defined as follows

$$e_x(t_{i+}) = F(e_x(t_{i-}), e_u(t_i), t_i) , \tag{2}$$

and it can also be written in the equivalent form

$$(\tau(t_{i+}), \mathbf{x}(t_{i+})) = F((\tau(t_{i-}), \mathbf{x}(t_{i-})), (t_i, \mathbf{u}_i), t_i) . \tag{3}$$

In the hypothesis of a non strictly dynamic system, in correspondence of an input event $e_u(t_i) = (t_i, \mathbf{u}_i)$ an output event occurs, which is expressed by

$$e_y(t_i) = G(e_x(t_{i-}), e_u(t_i), t_i) . \tag{4}$$

In absence of input events the state of the system remains unchanged up to the time τ of occurrence of the event $e_x(t) = (\tau(t), \mathbf{x}(t))$ which describes the state. In other words, by denoting with t_i and t_{i+1} the instants when two subsequent input events occur and by having set $e_x(t_{i+}) = (\tau, \mathbf{x})$, it results

$$e_x(t) = (\tau, \mathbf{x}) \quad \text{for} \quad t_i < t < t_{i+1} \quad \text{and} \quad t < \tau . \tag{5}$$

At time $t = \tau$ the state evolves freely, originating output events according to the relations:

$$e_x(\tau_+) = \Phi(e_x(\tau_-), \tau) , \tag{6}$$

$$e_y(\tau_+) = \Gamma(e_x(\tau_-), \tau) . \tag{7}$$

4. System Stability

The symbol θ denotes the null element of the set of the facts $\mathbf{X}$ associated with the state events of the system. θ is therefore the fact at whose occurrence nothing happens.

The system is said to be in the *zero* state (in physical terms: the system is totally devoid of energy) whenever the system state is expressed by the event $(\tau = \infty, \theta)$. A system in the null state does not originate output events and can be removed from such a state only by applying input events.

The system is said to be in a stable state $(\tau, \mathbf{x})$ whenever, being in free evolution, reaches the null state in a finite time through a finite sequence of state transitions (events). A system state is said to be *asymptotically stable* whenever, starting from such a state, the system, which is evolving freely, reaches the null state through an infinite sequence of state events, or in other words, the system *tends* to the null state when the number of the state events tends to infinity. Under conditions of asymptotic stability the null state will be reached during a time tending to infinity, unless the time interval between subsequent state events tends to zero or the system itself tends to become a continuous system.

Note that the asymptotic stability requires the definition of the limit operation in the set of sequences of state events and therefore the definition of a metric in the set of the state facts.

A system is said to be stable whenever its stability property holds for any admissible state.

5. Linear Time-Invariant Systems

A system is said to be time-invariant whenever a shift of the time origin does not modify the validity of the *Eqs.* (2), (3), (5) and (6) which define the forced and free evolution of the system. A system is said to be linear whenever the superposition of the effects holds.

In a linear time-invariant system the sequence of the output events $\sigma_y = \{(t_j, \mathbf{y}_j)\}$ which corresponds to the forced system response to a given input event sequence $\sigma_u = \{(t_i, \mathbf{u}_i)\} = \{e_u(t_i)\}$ starting from the null initial state can be expressed in the form:

$$\sigma_y = \sum_i h(t_i, \mathbf{u}_i) \, , \tag{8}$$

where $h(t_i, \mathbf{u}_i)$ is the sequence of output events due to the input event $e_u(t_i) = (t_i, \mathbf{u}_i)$ (see *Fig. 2*). Because of the invariance property, the sequence of output events $h(t_i, \mathbf{u}_i)$ can be obtained by applying a time shift t_i to the sequence $h(t = 0, \mathbf{u}_i)$ which is the response to the input event $(t = 0, \mathbf{u}_i)$.

Consequently, a discrete-event system, linear and time-invariant, can be described by the output event sequence $h(t = 0, \mathbf{u}_i)$ expressing the forced system response to the input events $(t = 0, \mathbf{u}_i)$ when $\mathbf{u}_i$ is varying in the set $\mathbf{U}$ of the admissible input facts. This description can be convenient when the set $\mathbf{U}$ is finite and does not have peculiar properties.

Let us consider, for instance, the mathematical model of a factory where $\mathbf{U}$ is the set of manufacturing operations which can be performed in the factory and $\mathbf{Y}$ is the set of the operations drawing/storing raw materials, semifinished or finished products, as a result of the manufacturing operations. The sequences of the output events $h(t = 0, \mathbf{u}_i)$ for $\mathbf{u}_i \in \mathbf{U}$ express the mathematical model of the manufacturing operation $\mathbf{u}_i$. *Eq.* (7) allows to describe the factory performance due to a sequence of input events (commands of manufacturing operations) in the hypothesis of linear and time-invariant system.

The transformation from the input-output model corresponding to *Eq.* (7), to the more general state equations model corresponding to *Eqs.* (2), (3), (4), (5) and (6), as it is well known, never has a unique solution, because of the freedom in selecting the state variables. A simple way for moving from *Eq.* (7) to the state equations model, is the one of assuming the set $\sum(\mathbf{T}\times\mathbf{Y})$ of the possible output event sequences as the set $\mathbf{X}$ of the facts associated with the state events.

The state at time t with $t_i < t < t_{i+1}$ is then expressed by the event $(\tau, \mathbf{x})$ where $\mathbf{x}$ is the event sequence $\sum_{j_x} h(t_j, \mathbf{u}_j)$ (forced response of the system to the input events up to time t) restricted to the output events with occurrence time longer than t. The occurrence time τ of the state event is then the occurrence time of the first event of the sequence $\mathbf{x}$ defined above, representing the fact of the event $(\tau, \mathbf{x})$ which expresses the system state at time t.

The fact $\mathbf{x}$ associated with the event $e_x(t)$ expressing the system state at time t coincides therefore with the output event sequence, which is the effect of the previous input events at time t, or with the free system response due to the initial conditions of the system at time t. By setting $\mathbf{x} = \{(\tau_k, \mathbf{y}_k)\}$, $k \in [1, n]$ it results in $e_x(t) = (\tau_1, \mathbf{x})$, being τ_1 the occurrence time of the first event of the event sequence $\mathbf{x}$. In the hypothesis that the occurrence time t_{i+1} of the first input event be shorter than τ_1, the state transition relation holds

$$e_x(t_{i+1+}) = (\tau', \mathbf{x}') , \quad \mathbf{x}' = \mathbf{x} + h(t_{i+1}, \mathbf{u}_{i+1}) , \tag{9}$$

where τ' is the time of occurrence of the first event of the event sequence $\mathbf{x}'$.

In the hypothesis that $\tau_1 < t_{i+1}$, the transition occurs at time τ_1 and it is a transition of free evolution. It results:

$$e_x(\tau_{1+}) = (\tau_2, R[\mathbf{x}, \tau_i > \tau_1]) , \tag{10}$$

where R denotes the restriction operation of a sequence.

If the system is strictly causal, the output event sequences $h(t=0, \mathbf{u}_i)$, expressing the forced system response to the input events $(t=0, \mathbf{u}_i)$ when $\mathbf{u}_i$ is varying in the set $\mathbf{U}$ of the admissible input facts, never contain events occurring at time $t=0$. Consequently, the output events occur only in correspondence of the free state transitions, i.e. in coincidence with the state event $e_x = (\tau_1, \mathbf{x})$. Having denoted with $\mathbf{x} = \{(\tau_k, y_k)\}$ the event sequence which makes up the fact of the state event, the output event occurring at time τ_1 results in

$$e_y(\tau_1) = (\tau_1, \mathbf{y}_1)\,. \tag{11}$$

Eqs. (8), (9) and (10) describe in terms of state equations the linear, time-invariant, strictly causal system, which is also described by *Eq.* (7) in terms of input-output relations.

6. Conclusion

The preliminary results of a new theoretical and original approach for modelling discrete-event dynamic systems have been presented. The approach, which is consistent with the classical definition of dynamic systems given by Kalman, appears promising and open to further new developments.

Specifically, the definition of the system state as an event allows to clearly formulate the free evolution of the discrete-event dynamic systems as a concatenation of events, which can be interrupted by the forced evolution imposed by the input events. A formulation like this appears quite innovative with respect to the usual formulation of DEDS.

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