UP-TO-DATE TECHNIQUE FOR EASY HIGH-ACCURACY POSITION ACQUISITION WITH SINUSOIDAL INCREMENTAL ENCODERS

Bernhard HÖSCHELER* and László SZÁMEL**

*Institute for Electrical Drives
University of Erlangen
Cauerstr. 9, D–91058 Erlangen, Germany
** Department of Electrical Drives
Technical University of Budapest
H–1521 Budapest, Hungary

Abstract

Currently known methods for high-resolution position acquisition with sinusoidal incremental encoders (i.e. VeCon) always require additional hardware components for counting whole cycles, two A/D-converters for quantifying the position within the current cycle and a routine for synchronising both information. The presented method uses only the information of the two A/D-converters with a relatively simple procedure. The most important fact is that the position may be calculated even at a rotation speed, where the fundamental frequency of the encoder signals exceeds the sampling frequency by a multiple value, so that the sampling theorem may be violated.

The paper discusses the development of this technique, the implementation of an additional error detection method, a practical application as well as experimental results.

Keywords: position measurement, signal processing, error detection.

1. Introduction

Modern servo drives demand permanently increasing accuracy for position and speed measurement. Therefore it is useful to reduce the effort in sensor technology hardware as far as possible by efficient digital signal processing.

Digital (optical) encoders seem to be a good solution, because the position is available in digital form in this case. Improving dynamic behaviour and higher sampling frequencies, however, lead to an increasing problem, that may be clear with a simple example:

If a speed resolution of less than 0.1 rpm at a sampling frequency of 2 kHz is required, a sensor with a minimum number of 300 000 cycles per revolution (c/r) will be necessary (using a quad edge counting method). Apart from the high cost, such an encoder will produce a fundamental frequency of already 15 MHz at a speed of 3000 rpm [4] !

This is the cause, why the digital encoders are more and more replaced by encoders with analog sine/cosine outputs, where the position information

is continuously available. Applying these encoders it is possible to calculate the position within one cycle, so that a smaller number of cycles per revolution (as 1000...5000) will be sufficient. Unfortunately, the effort in hardware and software increases: recent methods (as e.g. VeCon [3], [5]) are proceeding in a parallel structure of two components as seen in *Fig. 1*.
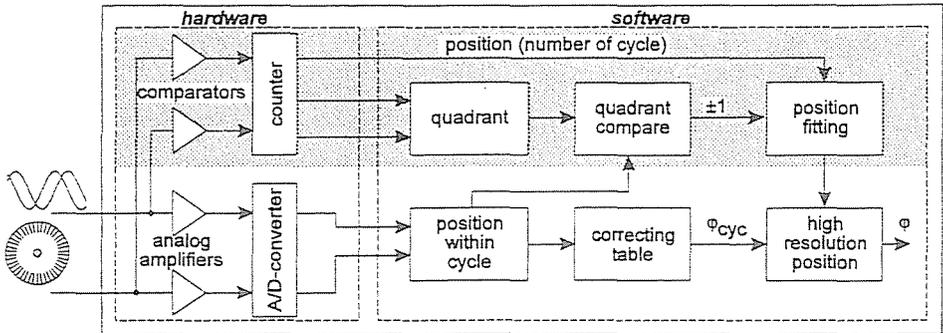

Fig. 1. Schematic diagram of high-accuracy position acquisition with sinusoidal incremental encoders

The calculation of the position is done here by combining different information:

First, the zero crossings of the encoder outputs are detected by comparators, and a hardware counter calculates the number of the current cycle in a 'classical' way. Apart from that, the number of quadrant within the cycle is detected. As a second point, the analog encoder tracks are sampled, using two A/D-converters. A software routine then computes the position within the cycle. x Now there is the problem to combine these data. Even a very small offset in time between counter and A/D-converter may lead to a situation, where both informations belong to different cycles. In this case, calculating the absolute position will lead to a false value. Instead of that an additional block is required where the synchronism is proofed by comparing the two different quadrant values. If they are different, the counter has to be fitted by ± 1. The counter value and the position within the current cycle are then formed to the value of the absolute position.

The presented method manages this in a much more easier way so that all components in *Fig. 1* marked in grey may be omitted. This is done by using a very effective algorithm in the block indicated as 'high resolution position'. But first let us see, how to calculate the position within one cycle.

2. Calculation of the Position within a Cycle

The two analog encoder tracks are named $s(\varphi)$ and $c(\varphi)$ to show the affinity to sine and cosine like signals clear. φ means the absolute position, which is scaled to the number of cycles per revolution.

The object is to calculate the position within one cycle as close as possible to the real value. One possibility is to use an arc sine or arc cosine function for one of the encoder signals. Unfortunately, the encoder tracks show deviations from an ideal sinus, especially offset and amplitude errors are often observed.

Therefore a method is used which eliminates common amplitude errors. This is done by dividing the (absolute) smaller one of the encoder signals by the other and adding an offset. *Table 1* shows the very simple rules of computation [8].

Table 1. Calculating the position within one cycle

position within cycle	$0 \ldots \frac{1}{8}$	$\frac{1}{8} \ldots \frac{3}{8}$	$\frac{3}{8} \ldots \frac{5}{8}$	$\frac{5}{8} \ldots \frac{7}{8}$	$\frac{7}{8} \ldots 1$
rule for calculating position within cycle	$\frac{s(\varphi)}{8c(\varphi)}$	$\frac{1}{4} - \frac{c(\varphi)}{8s(\varphi)}$	$\frac{1}{2} + \frac{s(\varphi)}{8c(\varphi)}$	$\frac{3}{4} - \frac{c(\varphi)}{8s(\varphi)}$	$1 + \frac{s(\varphi)}{c(\varphi)}$

By choosing the right method depending on the position within one cycle, a monotonous and continuous function between the real and calculated position is reached. The decision which rule to take is calculated with the sign of sum and difference of the two encoder tracks. But, the calculated position is just an approximation, ideal sinusoidal encoder tracks will lead to errors of about 1.2% of one cycle. This error may be reduced by processing an arc-tangent/arc-cotangent function, as it is also done in the VeCon Chip by a look-up table. However, having encoder tracks errors (e.g. offset) this correction will fail. So an individual correcting table for each encoder is used here.

It is very useful to correct the calculated position within the cycle (in the following named with ϕ'_{cyc}) with an individual look-up table. For that point, the information of both encoder tracks is needed for a range of one cycle as a function of the true position within the cycle (ϕ_{cyc}).

It is obvious that the reference measurement should be done at a very constant speed. But, having no additional sensors, this was up to now only achieved by adding rotating masses, which is a fatal restriction for practical use. For that reason, a method was developed, which works without any additional sensors or masses and a reference measurement is possible whenever wanted. This technique works as follows:

The main idea is that the fundamental frequency of the encoder signals exceeds the fundamental frequency of the mechanical rotation by the value

of the cycles per revolution, e.g. 2500. Watching the time characteristic within only one encoder cycle, we may assume that the encoder signals will also be influenced very little by the mechanical speed fluctuation and speed may nearly be constant.

If the distortion in time would only be slight, ϕ_{cyc} would increase linearly from 0 to 1 between the two positive zero crossings of the sine track. But regarding the fluctuation, the position has to be calculated more accurately. This is done by estimating a polynomial of 3rd order, which is determined with the four positive zero crossings liming the two neighbouring cycles. The exact time for every crossing is calculated by an interpolation between two sine tracks samples around the crossing. Having calculated the polynomial rule, the real position $\phi_{\mathrm{cyc}}(k)$ and the uncorrected position $\phi'_{\mathrm{cyc}}(k)$ is then calculated for every sample (k) within the cycle. As the wanted look-up table is of the shape $\phi_{\mathrm{cyc}}(\phi'_{\mathrm{cyc}})$, the samples have to be recalculated to equidistant values of ϕ'_{cyc}. This calculation is individually done for a large number of cycles and averaging.

Based on the analysis of simulating 50–100 samplings per cycle are sufficient for interpolation with acceptable accuracy. The error derived from interpolation is max. 0.05% also in the case of 50 sampling periods. In course of the simulation the signals of encoders with different shapes have been tested. The analysis has produced a very interesting result, namely, it will be a negligible small deviation if the signals of the encoder are not sinusoidal, either they are offset or have an amplitude error. The above accuracy is valid too in that extreme case if the offset is 40% and both encoders have a 20% amplitude error.

If at least 50 sampling points are necessary at an encoder of 2500 cycles and 50 kHz sampling frequency, then the speed should be less than 24 rpm. But if the speed is so small, the fluctuation of speed cannot be neglected. A mean speed of 15 rpm has been chosen so that the maximum value of the fluctuating speed as well should be less than 24 rpm. As we have no checked position value, the speed has been generated by a brushless DC motor supplied by a constant voltage phasor, rotating at a speed of 15 rpm. In this case the speed has fluctuated between 9 and 20 rpm.

To fill in a look-up table, a memory more than the minimum is necessary because of the speed oscillations. So the measurement should cover 3 whole periods, 1000 values per signal (4 Bytes on the DSP one by one), i.e. altogether 4 kBytes are used. Consequently, 12 kBytes are necessary for storing the sine and the cosine signals, and the angle of period. Furthermore, the correction table requires 800 Bytes.

At usual operation mode it is not necessary to fill in the correction table once more. It can be loaded from the hard disc of PC or can be read from the EEPROM memory. At the experiments we used a PC for storage.

The program on the DSP was enlarged with that algorithm so that a reference measurement is possible, whenever it is desired.

3. Calculation of the Absolute Position

The next task is to gain the absolute position only from the calculated values of the position within one cycle. This would be very easy if there were always many samples per cycle. Because of the high number of cycles per revolution, encoders, however, reach high fundamental frequencies even at lower speed, so that the sampling theorem may be violated and a reconstruction of the position seems to be impossible.

But we can use one more information:

A mechanical system can change its speed only at a limited acceleration rate. So in a sampled data system, every value of absolute position has to be in a finite interval compared to the last positions [6],[7]. Considering that the algorithm for the position within the cycle produces a value between 0 and 1, it will be clear that a reconstruction of the absolute position will be possible, if acceleration is so low that the current absolute position will be around the last position in an interval with the width of less than one. This will be shown in the following figure (*Fig. 2*).
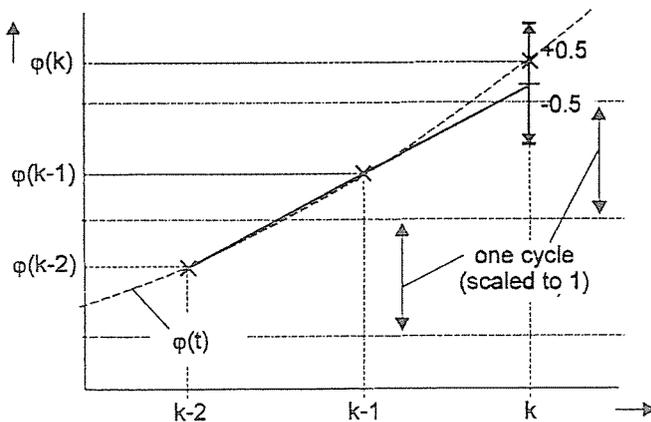

Fig. 2. Possible interval (± 0.5) of the current absolute position $\varphi(k)$ postulating a certain maximum acceleration compared to the past two positions $\varphi(k-1)$ and $\varphi(k-2)$ (dashed: continuous progression of position)

If the speed would be constant (acceleration $= 0$) the current position $\varphi(k)$ may be extrapolated by a line going through $\varphi(k-1)$ and $\varphi(k-2)$. Postulating that the current position may be in an interval around ± 0.5 of this extrapolated value, a reconstruction is able with the following algorithm:

Denominating the difference of two successive position samples with $u(k)$

$$u(k) = \varphi(k) - \varphi(k-1) \tag{1}$$

we can forecast the absolute position $\varphi(k)$ in the following interval:

$$\varphi(k) \in [\varphi(k-1) + u(k-1) - \frac{1}{2}...\varphi(k-1) + u(k-1) + \frac{1}{2}[. \tag{2}$$

Substituting (1), we get:

$$\varphi(k) \in [2\varphi(k-1) - \varphi(k-2) - \frac{1}{2}...2\varphi(k-1) - \varphi(k-2) + \frac{1}{2}[. \tag{3}$$

Scaling the position $\varphi(k)$ to cycles, it may be simply described by a sum of number of cycle (i) and position within cycle (φ_{cyc}):

$$\varphi(k) = i(k) + \varphi_{\mathrm{cyc}}(k) . \tag{4}$$

The possible number of the current cycle $i(k)$ is then:

$$i(k) \in [2\varphi(k-1)-$$

$$-\varphi(k-2) - \varphi_{\mathrm{cyc}}(k) - \frac{1}{2} ... 2\varphi(k-1) - \varphi(k-2) - \varphi_{\mathrm{cyc}}(k) + \frac{1}{2}[. \tag{5}$$

With the abbreviation

$$c(k) = 2\varphi(k-1) - \varphi(k-2) - \varphi_{\mathrm{cyc}}(k) \tag{6}$$

we can say

$$i(k) \in [c(k) - \frac{1}{2} ... c(k) + \frac{1}{2}[. \tag{7}$$

Because $i(k)$ has to be integer there is only one definite value that may be estimated simply by rounding:

$$i(k) = \mathrm{Round}\,((c(k)) . \tag{8}$$

The absolute position may be calculated in the following way:

$$\varphi(k) = \mathrm{Round}\,(2\varphi(k-1) - \varphi(k-2) - \varphi_{\mathrm{cyc}}(k)) + \varphi_{\mathrm{cyc}}(k) . \tag{9}$$

The formula shows that the three values $\varphi(k-1)$, $\varphi(k-2)$ and $\varphi_{\mathrm{cyc}}(k)$ are sufficient to gain $\varphi(k)$ in a recursive algorithm independent of the rotative speed.

This algorithm was implemented on a signal processor DSP32C (50 MHz) from AT&T where calculating the absolute position requires only 780 nsec. The whole program including sampling the encoder signals, calculating and correcting the position between the cycles, calculating the absolute position and counting revolutions is done in less than 8 μsec, so that

a sampling frequency of 25 kHz was chosen [1],[2] and excellent dynamic behaviour is achieved. For testing the algorithm at an extreme point where only little encoder information is available, the sampling frequency was reduced from 25 kHz to 1 kHz. Then, the encoder was driven with a speed of ±2000 rpm. Even at the maximum speed, when the algorithm gets only one sample every 83th cycle, the algorithm worked without any errors.

But there are two points we have to respect: the start-up and the acceleration limit.

Concerning the first problem, the algorithm has to be started at standstill or such a low speed, where the position changes less than half a cycle during a sampling interval. This will in practice lead to no problem, when starting the algorithm at the time the system is put into operation. The incremental encoders are applied in drives where measuring the position is also necessary. On the other hand, when measuring the position, the initialization is carried out before the start of the motor but after switching in the controller.

Another point is the restriction of maximum acceleration. Calculating this leads to a rule in the following form:

$$\frac{d\omega}{dt} \leq \frac{\pi}{\text{cycles per revolution}} f_s^2 \,, \tag{10}$$

where f_s is the sampling frequency of the encoder tracks.

In the implemented case (encoder with 2500 c/r and a sampling frequency of 25 kHz) there can be seen that a maximum angular acceleration of $\dot{\omega}_{\max} \approx 785000\,\text{rad/sec}^2$ is still allowed. Compared to accelerations of highly dynamic servo machines ($\approx 20000\,\text{rad/sec}^2$), the restriction will have little impact in practice. So there are two important consequences: First the method may also be applied in systems with less computing power at lower sampling frequencies. Second, if such a high acceleration does not appear at all, the reserve may be used for realising an error detection. The algorithm therefore is based as follows.

4. Error Detection

Postulating a lower acceleration rate, the interval for the possible position will shrink to less than one cycle. If for instance a value for the possible interval of $\pm\frac{1}{3}$ is chosen, the current number of cycle will be:

$$i(k) \in [c(k) - \frac{1}{3} \ldots c(k) + \frac{1}{3}[\,. \tag{11}$$

At 'normal' operating this interval will, however, still include one Integer which is the current number of cycle. In the other case, when the inter-

val leads to no unique cycle number (e.g. 1.24 ... 1.91), there is an error situation. This can be proved by checking the following rule:

$$| \text{Round}\,[c(k)] - c(k)| < \frac{1}{3}\,. \tag{12}$$

If the absolute value on the left side is greater than $\frac{1}{3}$, an error has occurred, where either the maximum acceleration was exceeded or there was a sampling error. Enlarging the program on the DSP with such an error detection, the calculating time increased only for 800 nsec so that this algorithm is also practical. The maximum permissible acceleration then has to be reduced to 66%.

For testing the error detection in practice, the sampling frequency was reduced to 1 kHz to get a situation where the maximum acceleration will be exceeded by a highly dynamic servo motor.

The measurements confirmed the theory:

Using an acceleration where the possible interval for the position is greater than the allowed value of $\pm\frac{1}{3}$ cycle but less than $\pm\frac{1}{2}$ cycle, the position is still calculated correctly, and error is detected. If acceleration is increased so that an interval of more than $\pm\frac{1}{2}$ cycle will result, an error is still detected, but the position cannot be calculated any more correctly (the algorithm will supply position values as if the speed was lower).

Fundamentally, the fault detection serves for filtering out the rough errors (false connection of signals of the encoder, A/D conversion fault) in the course of putting the drive into operation. But if the drive works, the calculation algorithm is of high safety, as it is difficult to overcome the high acceleration limit. The hardware we developed is in operation without errors at three different drives for more than a year in the laboratory circumstances.

It follows from the recursive feature of the calculation algorithm that an error occurring eventually produces constant disturbance. Applying the method of the traditional hardware counter, the above problem does not occur at the speed measurement but at the position measurement, because this method too is a recursive procedure. But at these applications fault detection frequently cannot be found. The calculation algorithm can be completed at the drives demanding high safety. The up-to-date encoders produce one period of a sine wave per revolution. They make a rough estimation of both the position and the speed possible. If the calculation of two kinds results in a significant difference, the false measurement can be improved as follows:

- Setting up the $(i(k))$ value of the speed range on the base of the estimated speed.

- Setting back the position value at the following zero impulse.

5. Conclusion

The described method for position acquisition is a progress concerning the following aspects:

- Usually required hardware may be omitted by using an additional algorithm which is very easy to implement.
- The algorithm will have no limit concerning a maximum rotational speed. The allowed maximum acceleration is so high, that it will have little impact in practice.
- There is nearly no restriction for the shape of the encoder tracks. Harmonic distortions and offset of the encoder tracks are allowed without producing additional errors, if the proposed algorithm for gaining an individual correcting look-up table is used.
- Calculating the position by dividing the encoder tracks, the algorithm is non-sensitive regarding common amplitude errors of the encoder tracks.
- A very high resolution is available which is primarily influenced by the quantization of the used A/D-converter. Using 12 bit converters, a resolution of a 1/5800 cycle will be reached, if the the quantization error is ± 1 LSB (last significant bit).
- An additional error detection is very easy to implement. With that an exceeding acceleration is detected as well as an error in the incoming cables.

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