GENERAL $Q - U$ CHARACTERISTIC APPROACH OF NON-LINEAR LOADS

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Abstract

Almost all computer simulation programs use the constant impedance (Z type), constant current (I type), and constant power (S type) [ZIP type], for load modeling. For the non-linear loads there is the exponent type of the $Q - U$ characteristics with constant exponent. This paper deals with the estimation techniques of voltage dependent exponent characteristics of non-linear 'equivalent inductive and inductive-capacitive load' ($\gamma(u)$). The analysis of mathematical model, laboratory measurement of parallel connected loads and data of on-site measurement proved the usefulness of the novel method, to calculate voltage dependent non-linear $Q - U$ characteristics. Continuous $\beta_t(u)$ and $\gamma(u)$ functions are produced of voltage dependent $Q - U$ static characteristics for parallel inductive and combined inductive-capacitive non-linear loads. These functions can be implemented in advanced network simulation programs like Electro-Dynamic Simulation (EDS) by different regression orders.

Keywords: non-linear load models, voltage dependent static load characteristics, equivalent models for combined inductive and inductive-capacitive loads.

1. Introduction

The fundamental frequency non-linear loads in the power system include different types of inductive loads, mainly transformers, and rotating electric machines, due to their non-linear voltage-current characteristics. In electric networks, of course, there are large-scale parallel connected capacitive and non-linear inductive loads. Computer programs simulating this type of non-linearity, mostly use the constant exponent type of the $Q - U$ characteristics with proposed range of the constant exponent (e.g. $\alpha = 2 \dots 6$). Generally, the Z type model is employed in several widely-used simulation programs, resulting in a limited accuracy. This paper is based on a former one [16], where a novel approach of getting a more accurate model of the saturation characteristics of single non-linear inductive loads was discussed, *introducing the voltage dependent exponent* $\beta(u)$ and there was given its analytical solution with first and second order regression for the single load $Q - U$ characteristics. By the evaluation of the mathematical model, laboratory measurements and on-site measurements of some measured loads,

$Q-U$ static characteristics of the voltage dependent $\beta_t(u)$ and $\gamma(u)$ are produced as a 'new non-linear load characteristics', for pure inductive and combined inductive-capacitive non-linear load, first by graphical function, later by analytic way.

In this publication the reactive power consumed by the non-linear load is expressed in the next forms:

$$Q(u) = Q_0(U/U_0)^{\alpha}, \tag{1a}$$

or

$$Q(u) = Q_0(U/U_0)^{\beta(u)}, \tag{1b}$$

or

$$Q(u) = Q_0(U/U_0)^{\gamma(u)}, \tag{1c}$$

where: U_0 and Q_0: reference values of voltage and reactive power;
α : constant (old type) exponent for the polynom type model;
$\beta(u)$: new voltage dependent exponent of single inductive load characteristics according to the [16][1];
$\gamma(u)$: new voltage dependent exponent of the combined loads (parallel connected inductive-capacitive loads) according to this publication;
$u = U/U_0$: actual voltage in p.u.

In this publication the verification is given that *the earlier proposed model can be more general and adaptable for most types of inductive and combined loads.* Furthermore, the series of investigations and main steps of the evaluation of $\beta_t(u)$ and $\gamma(u)$ are explained.

Due to the limited volume of this publication the discussion was made only for minimum number of selected cases in order to show the overall results.

2. Goals and Methods

The main objectives of this work are as follows:

- to have non-linear load representation with sufficient flexibility enabling the modeling of most typical non-linear load voltage-current characteristics regarding the voltage dependency,

[1] In the previous publication the terms 'polynom' and 'mononome' were used for the mathematical expressions where the base of the exponential function is voltage as the variable and the exponent is sometimes a constant (X^k form) or sometimes the variable itself. The 'mononome' is the special case of polynom by a single component.

- to show that pure inductive and combined non-linear load characteristics should be represented with variable voltage dependent function,
- to increase the accuracy of the computer simulation models,
- to represent most of the effects of the non-linear load elements,
- to develop load model characteristics that should correspond to the physical loads,
- to have a load model with sufficient flexibility to allow several forms of representation in the computer simulation programs like EDS. In order to have pure and controllable results, based on the objectives listed above, the following types of investigations were developed:

I. Mathematical models of non-linear load simulations (Analytical method).
II. Laboratory measurements.

Both investigations were made with the following variations:

- More parallel inductive components, with different U-I curves.
- Parallel inductive and capacitive loads of different compensation levels.

III. On-site measurements and evaluation of the results.

The variations in the methods (mathematical model, tests in laboratory and on-site measurements) support:

- the use of new and more accurate, advanced models,
- the evaluation of the effect of the selection of U_{ref} on the $\beta_t(u)$ and $\gamma(u)$ values,
- the evaluation and comparison of the accuracy of the different models,
- the 'feed-back' between the physically tested models and the different types of advanced mathematical analysis.

On the basis of the investigations and their conclusions, it is not possible to model such high (and non-linear) voltage dependency over the voltage range of interest surrounding the nominal voltage with the most commonly used constant type models. Therefore, a new proposed approach of power relationship to voltage as 'polynom type' model was produced *with voltage dependent exponent of $\beta(u)$ and $\gamma(u)$ instead of the constant exponent.*

Most of the steps of this work are ready in an algorithm, and the algorithm is realized and tested in computer programs.

The generally used polynom type of the non-linear models by constant exponent has been assumed. The generally used equation *for parallel non-linear inductive load representation* which is considered to be the generally used polynom type of the non-linear models by constant exponent is given below:

$$Q_{L10}\left(\frac{U}{U_0}\right)^{\alpha_1} + Q_{L20}\left(\frac{U}{U_0}\right)^{\alpha_2} \cong Q_{Lt0}\left(\frac{U}{U_0}\right)^{\alpha_t} . \qquad (2)$$

On the left hand side there are the components of the single load. On the right hand side there is the 'expected' polynom (or mononome) type for the equivalent models. In the general case, if $\alpha_1 \neq \alpha_2$, there is no accurate solution because the sum of two polynoms cannot be equal to a third polynom (mononome) except at one operating point.

To solve this contradiction, there is an assumption to find the solution by the help of the 'voltage modulated' or 'voltage dependent' exponent of the polynom:

$$Q_{L1}(u) + Q_{L2}(u) = Q_{Lt0}(U/U_0)^{\beta(u)}. \tag{3}$$

In these equations:
α_1, α_2 : constant exponent of each inductive load ($\alpha_1, \alpha_2 > 2$),
α_t : constant exponent of the combined inductive loads ($\alpha_t > 2$),
Q_{L10}, Q_{L20} : reference reactive power of each inductive load,
Q_{Lt0} : reference reactive power of the total inductive load,
Q_{L1}, Q_{L2} : actual reactive power of each inductive load.

If the assumption that α_1, α_2 are constants is not correct, then there is a general assumption that the components are explained as 'voltage dependent exponents' $\beta_1(u)$ and $\beta_2(u)$, documented in the previous publication [16], and the total load can be expressed in a similar way. This proposed new general form of parallel pure non-linear inductive load representation is given below:

$$Q_{L1}(u, \beta_1(u)) + Q_{L2}(u, \beta_2(u)) = Q_{Lt0}(u, \beta_t(u)), \tag{4}$$

where: $\beta_1(u), \beta_2(u)$: new voltage dependent exponent of inductive load ($\beta_1(u), \beta_2(u) > 2$);
$\beta_t(u)$: new voltage dependent exponent of total inductive load ($\beta_t(u) > 2$).

The generally used equation *for non-linear combined inductive-capacitive load representation* is given below:

$$Q_{L0}\left(\frac{U}{U_0}\right)^{\alpha} - Q_{C0}\left(\frac{U}{U_0}\right)^{2} = Q_{t0}\left(\frac{U}{U_0}\right)^{\gamma}. \tag{5}$$

Here are the same problems. First: if it were right that for the inductive load $\alpha > 2$ and it were constant, then *the subtraction of two polynoms could not be produced* as a third polynom (mononome). Second: in general case for the inductive loads $\alpha \neq$ constant. To solve these contradictions there is an assumption to find the solution, similar to the previous $L_1 + L_2$ case [4], in the proposed new general form:

$$Q_L(u, \beta(u)) - Q_C(u, \alpha = 2) = Q_t(u, \gamma(u)), \tag{6}$$

where: Q_{t0} : reference reactive power of the total combined load,
Q_{L0}, Q_{C0} : reference reactive power of the single inductive or capacitive load,
Q_L, Q_C : actual value of single inductive or capacitive load,
Q_t : actual value of equivalent combined load,
γ : exponent of the combined load.

3. Analysis of the Mathematical Model

3.1. Pure Inductive Loads

Constant exponent for the load components is assumed, also two parallel connected inductive loads were assumed, with different voltage–current or voltage–reactive power characteristics. The evaluation is carried out in two parts as shown in *Table 1*, with running reference voltage (U_{ref}). The data are generated by the analytical equation (2) with voltage variation between 0.8 to 1.20 p.u.

Table 1. Parameters of the mathematical model

Loads	L1	L2
α	6.0	2.0
variant 1–Q_L	50.0	100.0
variant 2–Q_L	100.0	50.0

3.1.1. Smaller Load with Greater Exponent

The first inductive load ($L1$) power exponent α_1 is set to $\alpha_1 = 6$, and the reactive power value is set to $Q_{L1} = 0.5 * Q_{L2}$, the second inductive load has power exponent α_2 set to $\alpha_2 = 2$.

The result of the calculated combined inductive load characteristic $\beta_t(u)$ is shown in *Table 2* and plotted in *Fig. 1*. The $r1 \ldots r3$ parameters are different variations of the reference voltages in *Eq.* (2).

3.1.2. Greater Load with Greater Exponent

The first and second load exponents α_1 and α_2 are kept constant with the same values as in the previous part. The reactive power of the second load is set to $Q_{L2} = 0.5 * Q_{L1}$ (greater load has greater exponent).

Table 2. $\beta_t(u)$ of smaller load with greater exponent

Cases (r_i)	U_{ref} p.u.	$\beta_t(u)$
1	0.80	$2.73 > \beta_t(u) < 3.29$
2	1.00	$2.98 > \beta_t(u) < 3.67$
3	1.20	$3.29 > \beta_t(u) < 3.96$

The results of the calculated combined inductive load characteristics $\beta_t(u)$ are shown in *Table 3* and plotted in *Fig. 2*.

Table 3. $\beta_t(u)$ of greater load with greater exponent

Cases (r_i)	U_{ref} p.u.	$\beta_t(u)$
1	0.80	$3.90 > \beta_t(u) < 4.56$
2	1.00	$4.24 > \beta_t(u) < 4.96$
3	1.20	$4.56 > \beta_t(u) < 5.17$

From the results of the mathematical model of pure inductive load *Figs. 1* and *2*, it is seen that:

- the 'equivalent load' $\beta_t(u)$ exponent is not constant,
- the range of $\beta_t(u)$ depends on the parameters of the components (Q_{Li0} and α_i),
- the change of U_{ref} causes some shifting of $\beta_t(u)$ characteristics,

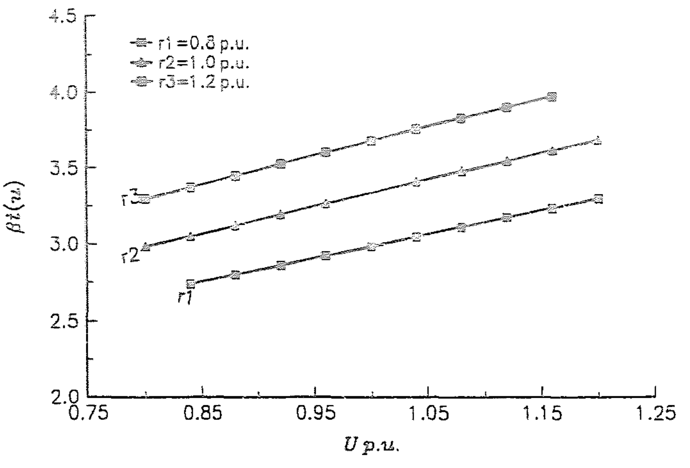

Fig. 1. $\beta_t(u)$ characteristics of smaller load with greater exponent

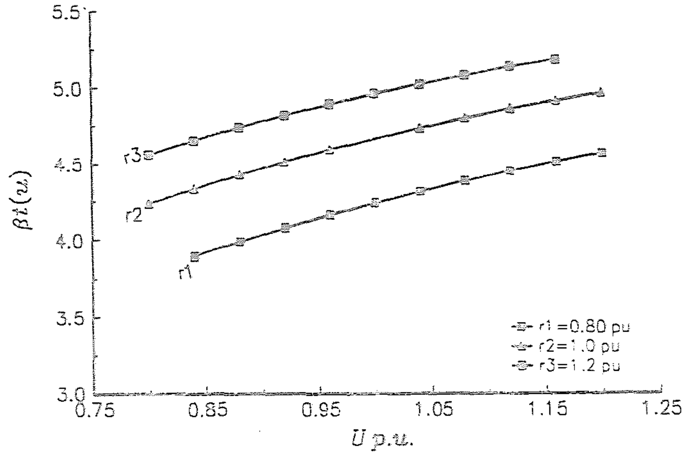

Fig. 2. $\beta_t(u)$ characteristics of greater load with greater exponent

- the position and the form of the $\beta_t(u)$ function depend on the selection of U_{ref};
- the $\beta_t(u)$ is less than the larger α_i exponent and greater than the smaller α_i exponent, and closer to the exponent having larger Q_{Li0} value:
- the pure combined inductive load characteristic can be expressed by means of a continuous, non-linear exponent $\beta_t(u)$ (see *Figs. 1, 2*).

3.2. Combined Inductive-Capacitive Loads

It has been mentioned that the equivalent $Q-U$ static characteristics cannot have constant exponent, due to the subtraction of the inductive and capacitive reactive power components [4]. This was only a remark in this publication that started the analysis of how a general method of the evaluation can be developed for non-linear loads. The combined load data is generated by the analytical equation (5), with assumed reference values of $Q_{L0} = 20$ kVar with constant $\alpha = 6$ for all the cases and $Q_{C0} = 15.0$ kVar for case 1, $Q_{C0} = 25.0$ kVar for case 2.

Due to the role of the compensation level on the calculation, it is needed to introduce compensation ratio (K), which, in general, is a frequency and voltage dependent variable expressed as follows:

$$K(f,u) = \frac{Q_C(f,u)}{Q_L(f,u)}. \tag{7a}$$

Frequency dependent load is not considered in this study, so *Eq.* (7a) can be rewritten as

$$K(u) = \frac{Q_C(u)}{Q_L(u)} \qquad \text{for general cases, like } \alpha > 2 \text{ as well.} \tag{7b}$$

It means that, in general, the compensation ratio (K) is not constant due to different voltage functions of the inductive and capacitive components of the power. In the linear network $\alpha = 2$, disappears the role of the voltage dependent reactive power in the inductive and capacitive components. Only in this case K is constant and it can be expressed by the reference values of power:

$$K_0 = \frac{Q_{C0}}{Q_{L0}}. \tag{7c}$$

Of course, for theoretical reasons, there is the possibility to define this ratio for the non-linear cases, as well. In these cases K_0 represents this ratio *only in the reference point.* In other words it means that for non-linear network itself *the resonance depends on the voltage.* This is a singular point in *Eq.* (5) because the right hand side of this equation is zero, 'actual combined load reactive power is zero', the coefficient 'reference reactive power of the combined load' $Q_{t0} = Q_{L0} - Q_{C0}$ is zero, too, and the searched exponent $\gamma(u)$ is undefined. To find this critical voltage, these are the following steps to calculate it:

In the point of the resonance *Eq.* (5) gives zero and in consequence the two components are equal:

$$Q_{L0}\left(\frac{U}{U_0}\right)^{\alpha} = Q_{C0}\left(\frac{U}{U_0}\right)^{2}, \tag{8}$$

$$\left(\frac{U}{U_0}\right)^{\alpha} = \left(\frac{Q_{C0}}{Q_{L0}}\right) * \left(\frac{U}{U_0}\right)^{2} \Rightarrow \alpha * \mathrm{Log}\left(\frac{U}{U_0}\right) = \mathrm{Log}\left(\frac{Q_{C0}}{Q_{L0}}\right) + 2 * \mathrm{Log}\left(\frac{U}{U_0}\right)$$

$$(\alpha - 2) * \mathrm{Log}\left(\frac{U}{U_0}\right) = \mathrm{Log}\left(\frac{Q_{C0}}{Q_{L0}}\right) \Rightarrow \mathrm{Log}\left(\frac{U}{U_0}\right) = \frac{1}{(\alpha-2)} * \mathrm{Log}\left(\frac{Q_{C0}}{Q_{L0}}\right)$$

$$\frac{U}{U_0} = \sqrt[\alpha-2]{\frac{Q_{C0}}{Q_{L0}}}, \qquad \text{hence} \qquad U = \sqrt[\alpha-2]{\frac{Q_{C0}}{Q_{L0}}} * U_0 = \sqrt[\alpha-2]{K_0} * U_0.$$

For example, if on the nominal voltage there is $K_0 = 0.9$ of the compensation and for the inductive component $\alpha = 6$ then, $U = \sqrt[6-2]{K_0} * U_0 = 0.974U_0$, that is about 3% change in the voltage makes singularity in this model. (On this type of network 2.6% decrease in the voltage makes the case of the resonance!).

The previous facts result in the new approach of the problems, so more attention has to be paid to the resonance and to the behavior of the characteristics before and after the resonance. To avoid the discontinuity at the resonance point, *the calculation of $\gamma(u)$ characteristic must be divided* *into two segments*, the first one which is before the resonance zone and the second one is after the resonance zone. The evaluation of the combined load characteristic $\gamma(u)$ was made for these two cases, $Q_{C0} < Q_{L0}$ and $Q_{C0} > Q_{L0}$.

3.2.1. *Case 1* $Q_{C0} < Q_{L0}$

The compensation level at the nominal voltage in this case was set to $Q_{C0} = 0.75 * Q_{L0}$ ($K_0 = 75\%$), and the voltage at which the resonance occurs is calculated ($U = 0.93U_0$). The resonance itself is not only a function of the Q_{C0}/Q_{L0} ratio, but a function of voltage, according to *Eq.* (7a) or (7b). *Fig. 3* shows the inductive and capacitive components, and the total value of the generated reactive power.

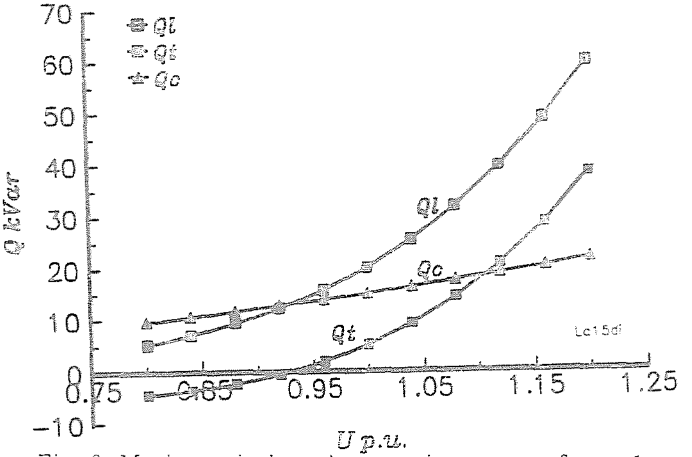

Fig. 3. Mathematical model, reactive power of case 1

From *Fig. 3*, the following remarks are drawn:

a) In the point of resonance the total reactive power $Q_t(u)$ is zero, and the voltage at this point is called resonance voltage,

b) In the case of ($\alpha > 2$ and $U > U_{\text{resonance}}$) the total reactive power $Q_t(u)$ is positive (the case of 'over the resonance voltage'). For the mathematical analysis this is the 'normal' case and the unknown $\gamma(u)$ exponent is expected in the 'normal' range (but not constant).

c) In the cases of ($\alpha > 2$ and $U < U_{\text{resonance}}$) the total reactive power $Q_t(u)$ is negative (the case of 'under the resonance voltage'). For the mathematical analysis this is an 'opposite'/'unusual' case: the coefficients of the left hand side (Q_{L0} and Q_{C0}), 'reference reactive power of inductive and capacitive loads' in *Eq.* (5) are positive, the coefficient of the right hand (Q_{t0}) 'reference reactive power of the combined load' is negative and the unknown $\gamma(u)$ exponent is in an 'unusual' range (see the results later).

The method itself for the identification of the parameters for this type of the simulation needs some measured value of the total reactive $Q_t(u)$ in both voltage ranges ('before' and 'after' the resonance) and two segments of the evaluations. It means that the free selection of the pairs of the reference values (Q_{t0} at U_0 voltage) is done twice: a pair of them in the range 'after the resonance' and another one in the range 'before the resonance'.

The consequence of this method is the duplication of the implementation of the combined load simulation to the computer programs: simulations before and after the resonance. The feature of this new method will be explained by studying and evaluating the next figures, edited for these cases. It is necessary to declare that the feature of this method is due to the use of usual polynome / exponent / power type of the mathematical model for the non-linear loads. Results of the calculated $\gamma(u)$ are shown in *Fig. 4.* In this diagram the $(r_1, \ldots\ldots, r_i)$ parameters are different variations for the declared referent voltages in *Eq.* (5).

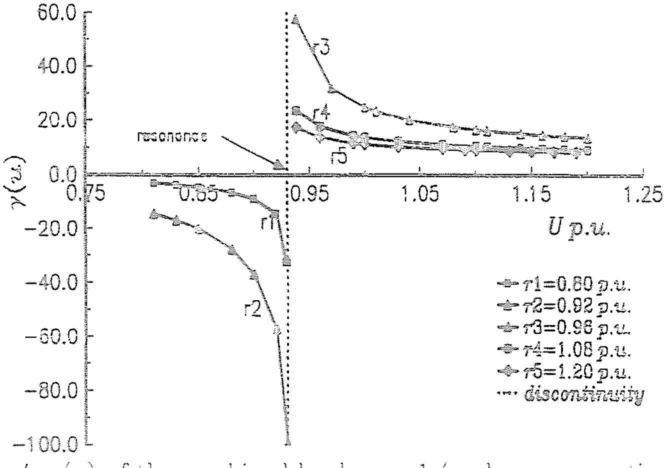

Fig. 4. $\gamma(u)$ of the combined load, case 1 (undercompensation)

In Fig. 4 a new characteristic resulted, the calculated $\gamma(u)$ produced a new characteristic form, compared to the pure inductive loads. The main differences are:

- the characteristics are not continuous, there is a singularity at the point of resonance;
- the range of the exponent is much greater, especially in the lower voltage domain;
- there is a voltage domain where negative exponent is needed.

3.2.2. Case 2 $Q_{C0} > Q_{L0}$

Compensation level at the nominal voltage in this case was set with value of $K_0 = 125\%$, $Q_{C0} = 1.25 * Q_{L0}$, and the voltage at which the resonance resulted is calculated ($U = 1.057U_0$). Results of the calculated $\gamma(u)$ are shown in *Fig. 5*. In *Figs. 4, 5* the range of $\gamma(u)$ in both zones is decreasing, as the voltage increases compared to the previous inductive cases.

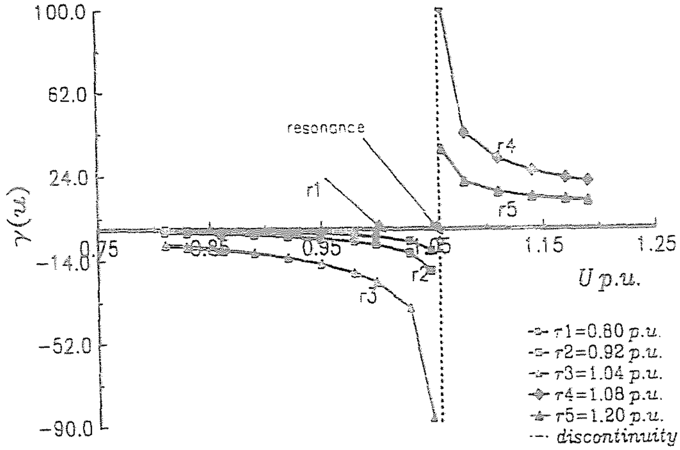

Fig. 5. $\gamma(u)$ of the combined load, case 2 (undercompensation)

From the evaluation of the mathematical model, for combined inductive-capacitive load, it has been documented that:

- the 'equivalent load' $\gamma(u)$ exponent is not constant;
- the position and the form of $\gamma(u)$ function depend on the selection of U_{ref}, the change of U_{ref} causes some shifting of $\gamma(u)$ characteristics;
- the range of $\gamma(u)$ depends on the parameters of the load components;
- in the voltage variation at the resonance voltage there is a singularity in the calculation of $\gamma(u)$ characteristics and there is a change of its sign;
- the $\gamma(u)$ value in both zones has a decreasing tendency, this is due to over- and under-compensation of the combined load characteristics.

3.2.3. Tendency and Conclusions for Combined Inductive-Capacitive Loads

There are very significant common tendencies and conclusions for these cases (combined loads, 3.2.1 and 3.2.2).

In chapter 3.1 the non-linearity, caused by the single or parallel pure inductive loads was studied. For these cases the $\beta_i(u)$ exponents were not constant but they had some positive (monotonous) slope in the range of 2...6.

There are very important differences in the values and sign of the $\gamma(u)$ exponents if they are compared in *the combined (inductive-capacitive) cases and the 'normal cases' (pure inductive loads).*

1) *For the combined loads* there are the following main types of the $\gamma(u)$ exponents:
   a) if the actual voltage is *large* (it is in the range greater than the resonance) and the reference voltage in *Eq.* (5) is also in this *large* range, then the $\gamma(u)$ exponents are *positive* numbers, in the range of 5....25;
   b) if the actual voltage is *small* (it is in the range smaller than the resonance) and the reference voltage is in this *small* range, then the $\gamma(u)$ exponents are *negative* numbers, in the range of $-5.... - 35$;
   c) these large ranges of the voltage dependent $\gamma(u)$ exponent in a) and b) are the consequence of the subtraction of the inductive and capacitive reactive power components in *Eq.* (5). By the pure cases of parallel connected inductive-capacitive components of the mathematical model it can be stated that only a forced value of the exponent in the equivalent load can compensate the deficit in the $Q_{t0} = Q_{L0} - Q_{C0}$ coefficient. It is true for the real combined loads, too. The range of the voltage dependent exponent $\gamma(u)$ depends on the parameters of the components.
2) There is a *singularity* in the point of the resonance voltage and this point divides into *two 'sides' the model*;
3) It is not easy to find a 'common model' *for the hole voltage range* if there is a model by *Eq.* (5) and the right hand side form is decided to use;
4) For advanced computer simulations of the combined LC loads the authors can propose two main types of the model:
   a) *Mathematical models* by $\gamma(u)$ voltage dependent exponents (see *Figs. 4, 5*) and/or by different approximations of $\gamma(u)$ curves in analytical form;
   b) *Component models* by separation of the combined L and C loads to $Q_L(u)$ and $Q_C(u)$ components and to implement these sub-models to computer programs.
   These are significant new results, they do not allow the use of constant range and method for $\alpha = 2...6$, [11].

4. Analysis of Laboratory Measurements

Evaluation of the laboratory measurement was made first for two different parallel connected inductive load components and then for load composed of parallel inductive and capacitive components. The compensation level was changed by increasing the number of capacitors.

4.1. Pure Inductive Loads

Calculated $\beta_t(u)$ characteristics of parallel connected inductive loads for the three cases of the evaluations by different reference voltage values are shown in *Table 4* and plotted in *Fig. 6*.

Table 4. $\beta_t(u)$ of the combined measured loads

Cases (r_i)	U_{ref} p.u.	$\beta_t(u)$
1	0.85	$3.07 > \beta_t(u) < 3.89$
2	1.00	$3.43 > \beta_t(u) < 3.99$
3	1.20	$3.64 > \beta_t(u) < 3.87$

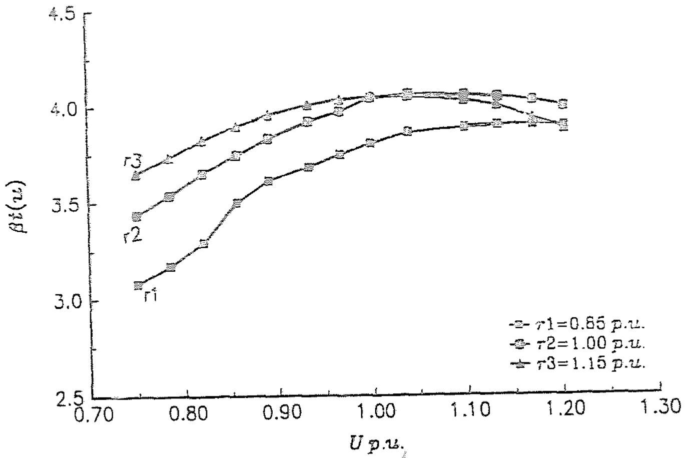

Fig. 6. $\beta_t(u)$ of the measured pure inductive loads

4.2. Combined Inductive-Capacitive Load

4.2.1. Case 1 $Q_{C0} < Q_{L0}$

In this case the compensation level at the nominal voltage was $K_0 = 79.2\%$, $Q_{C0} = 0.792 * Q_{L0}$, and the voltage at which the resonance resulted is

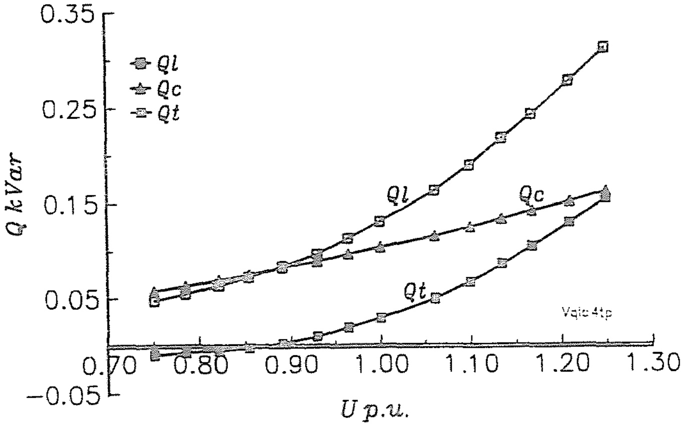

Fig. 7. Measured reactive power

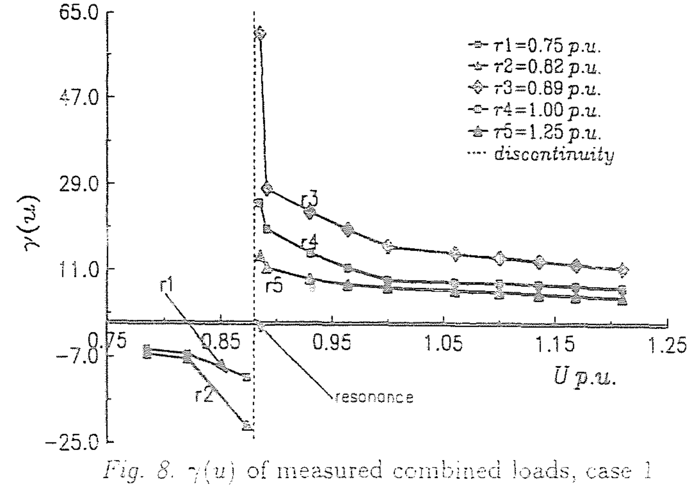

Fig. 8. $\gamma(u)$ of measured combined loads, case 1

$(U = 0.88U_0)$. Results of the absolute value of the measured reactive power are plotted in *Fig. 7* and the calculated $\gamma(u)$ values are shown in *Fig. 8*. The cases shown in *Figs. 7* and *8* have resulted qualitatively in similar characteristics as in 3.2.1 (see *Figs. 3* and *4*).

4.2.2. Case 2 $Q_{C0} > Q_{L0}$

In this case the compensation level at the nominal voltage was $K_0 = 110.7\%$, and the voltage at which the resonance resulted is $(U = 1.05U_0)$. Calculated $\gamma(u)$ values are plotted in *Fig. 9*.

For the measured cases of the combined loads there are the same tendencies as for the mathematical models: singularity at the resonance voltage, large

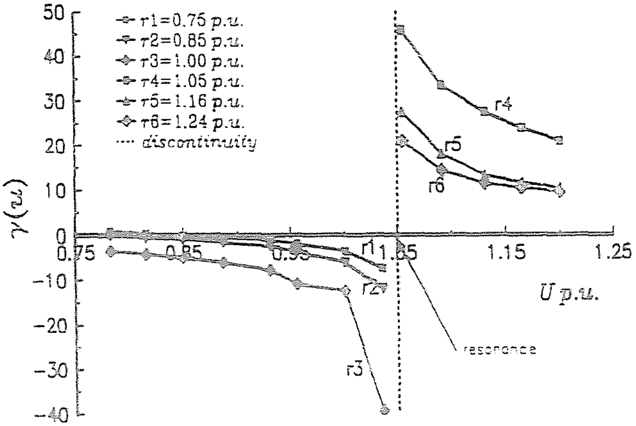

Fig. 9. $\gamma(u)$ of measured combined loads, case 2

negative or positive values of the exponents in the ranges 'before' and 'after' the resonance voltage.

On an actual network in the time there are large changes of inductive and/or capacitive components in the loads. The results of both mathematical and measured cases document that in the cases of the compensated networks (combined non-linear inductive and capacitive loads), it is not right to simulate by constant exponents the reactive part of the reduced power on the nodes and the values of the exponents can exceed the values given in the publications, both in magnitude and in sign, as well. (The negative exponents are valid if there is an overcompensation).

5. Analysis of On-site Measurements

In order to validate and test the novel method, analysis of gathered data from field measurement was made for some measured loads. A voltage–current load characteristic measurement at the Rákoskeresztur substation was performed, with staged transformer tap-change on the 20 kV bus side with step of about $\cong 2\%$. Evaluation of the load characteristics of some loads was made where the reference voltage was selected to be the first point on the tap step. Results of $\gamma(u)$ calculation are shown in *Table 5* and plotted in *Fig.10*.

Fig. 10 shows that the load characteristic values of the selected loads are varying between $1.0 < \gamma(u) < 9.0$.

Table 5. $\gamma(u)$ ranges of the measured loads

Load/tap steps	1	2	3	4
IKARUSZ $\gamma(u)$	3.29	4.63	4.52	4.63
PESTI ut $\gamma(u)$	3.76	4.74	5.23	5.14
Gödöllő $\gamma(u)$	8.92	6.38	5.93	6.23
Cinkota $\gamma(u)$	4.86	4.45	4.20	4.19
Vegyiművek $\gamma(u)$	1.66	1.39	1.75	2.86

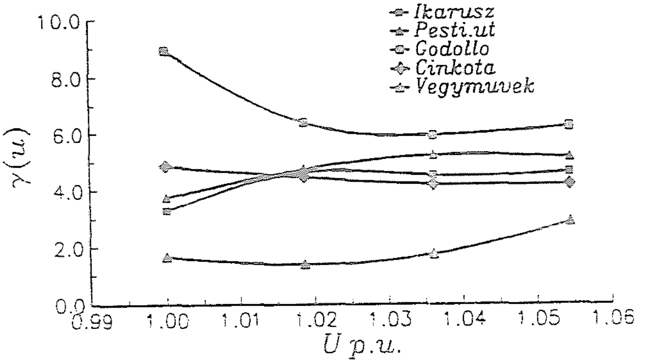

Fig. 10. $\gamma(u)$ ranges of the measured loads

6. Tendencies and Conclusions

From the evaluations of the mathematical model, laboratory measurements and on-site measurements, the following conclusions can be drawn:

- for the 'equivalent load' $\beta_t(u)$ and $\gamma(u)$ exponents are not constant;
- the position and form of the $\beta_t(u)$ and $\gamma(u)$ functions depend on the selection of U_{ref}, the change of U_{ref} causes some shifting of the $\beta_t(u)$ and $\gamma(u)$ characteristics;
- the range of $\beta_t(u)$ depends on the parameters of the components (Q_{Li0} and α_i) and the range of $\gamma(u)$ on the components (Q_{t0} and α_i);
- the $\beta_t(u)$ is less than the larger α_i exponent and greater than the smaller α_i exponent, and it is closer to the exponent having larger Q_{Li0} value;
- the compensation ratio increases to 100%, it results in singularity (resonance point) in the calculation of $\gamma(u)$ parameters and also in changes of its sign;

- the large ranges of the voltage dependent $\gamma(u)$ exponent are the consequence of the subtraction in *Eq.* (5) for a pure case of parallel connected inductive-capacitive components of the mathematical model and laboratory measurements:
- the non-linear inductive and combined load characteristics cannot be expressed as constant exponent but as a continuous, non-linear exponent $\beta_t(u)$ and $\gamma(u)$ functions (see *Figs. 1, 2, 6* and *Figs. 3, 4, 8, 9, 10*);
- the constant exponent for the simulation is not so accurate in all the cases, so the method developed can be more accurate than the method used so far.
- for a longer interval of U, it is not easy to find an analytical function for the exponent of $\beta_t(u)$ and $\gamma(u)$ but as an advanced solution it can be expressed in *a polynomial form* by different order;
- to follow the curve/bend of the $\beta_t(u)$ and $\gamma(u)$ characteristics and to express their convex or concave running, regression of different order can be used.

Of course, all the regressions produce some error but first order or second order analytical expressions can result in more accurate simulation than that of the constant exponent approximation. Evaluation of single non-linear load characteristics has shown that it can be modeled more accurately with the second order regression algorithm [16].

It has been realized how important it is to know the values of Q_{C0}, Q_{L0} and $\beta_t(u)$ from the point of view of resonance of the system, therefore, it is suggested to develop an algorithm to calculate these unknowns based on the calculation of $\gamma(u)$ of the combined load.

Most commercially available programs use only ZIP model which does not reflect most of the load variation in an accurate way, due to voltage variation [11]. So the constant exponent (power) for the simulation is not valid any more. On the other hand, the proposed models will have significant effect on the study of system simulations, in this way the risk of voltage overshoot in the load side and later voltage dip in the bulk power system can be accurately simulated. The application of the proposed algorithms in advanced network simulation programs like EDS can result in much better and more influential model representation which can reflect to the essential behavior of the loads.

7. Summary

This paper introduces a general novel approach for modeling and simulation of non-linear inductive and combined loads based on a former one [16]. The proposed models are based on the generally used polynomial form of $Q-U$ static characteristics, but by the $\beta_t(u)$ and $\gamma(u)$ voltage dependent exponents developed much more accurate load simulation can be obtained. By more on-site measurements of combined loads on selected sites of the network, calculation of voltage dependent load characteristics and the parameters of the regressions can be implemented to a computer program like EDS.

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