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SLIDING MODE BASED FEEDBACK COMPENSATION FOR MOTION CONTROL

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Abstract

The main contribution of this paper is to examine, via experimental investigations of a Type 2 servomechanism, the engineering feasibility of a sliding mode based control design, in which a discontinuous estimator is used in feedback compensation of uncertainties and exogenous disturbances. Experimental results of a transputer controlled single-degree-of-freedom motion control system are presented. The experimental system consists of a conventional DC servo gear motor with encoder feedback and variable inertia load coupled by a relatively rigid shaft.

Keywords: sliding mode, feedback compensation, disturbance observer.

1. Introduction

Sliding mode has been introduced in the late 1970's [1] as a control design approach for the control of robotic manipulator. The main utility of sliding mode in this control design problem is to decouple the normally highly coupled nonlinear dynamics, and to desensitize the robot's tracking performance to payload variations, unknown system parameters, and externally applied forces. In the early 1980's, sliding mode was further introduced for the control of induction motor drives [2]. Its utility in this hybrid discipline, consisting of power electronics and motion control, is to provide direct switching strategy [3] to the power electronics devices such that, in spite of the nonlinear dynamics of the induction motor, the control design is decomposed into a nonlinear control synthesis problem, and a linear control design problem of reduced order. These two early applications of sliding mode indicated the versatility of the underlying control theoretic principles in the design of feedback control systems for motion control, regardless of the origin or the nature of the particular system performance specifications and design goals.

These initial works were followed by a large number of research papers in robotic manipulator control and in motor drive control. References can be found in [4]. In some of these works, experimental results were obtained [5], [6]. However, despite of the theoretical predictions of superb closed loop system performance of sliding mode, some of the experimental works indicated that sliding mode in practice has limitations due to the need of high sampling frequency to reduce the high frequency oscillation phenomenon about the sliding mode manifold – collectively referred to as 'chattering'. In most of the experimental works involving sliding mode, the efforts spent on understanding the theoretical basis of sliding mode control are generally minimized, while a great deal of energy were invested in empirical techniques to reduce chattering. Among these experimental works, a few succeeded to show closed loop system behaviour which were predicted by theory. Those who failed to manage the experimental designs successfully concluded that chattering is a major problem in realizing sliding mode control in practice.

On the theoretical front, the 1980's saw a continued growth of R&D in new extensions of the original theory. The connection of sliding mode control to model reference adaptive control introduced some excitement in the research community. In addition, the design of sliding mode observers [7], [8], [9] provided additional capabilities to a sliding mode based feedback control loop. Experimental results for sliding mode observer were obtained for robotic manipulator control recently [10]. Finally, the issue of discrete-time sliding mode was raised from the theoretical perspective, resulting in a number of different definitions of discrete-time sliding mode [12], [11]. A comparison of classical and discrete-time sliding mode control, via experimental investigations of a single-degree-of-freedom motion control system can be found in [13].

2. Sliding Mode Based Feedback Compensation

The system with external disturbances and uncertain parameters satisfying the so-called DRAZENOVIC condition [16] is written in the regular form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} + \Delta A_{21} & \bar{A}_{22} + \Delta A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{B}_2 + \Delta B_2 \end{bmatrix} u + \begin{bmatrix} 0 \\ E_2 \end{bmatrix} f(t) \ , \tag{1}$$

where $x_1 \in R^{n-m}$, $x_2 \in R^m$, $u \in R^m$, $\bar{A}_{ij}, i, j = 1, 2$, and $\bar{B}_2$ denote the nominal or desired (ideal) system matrices, $\Delta A_{2j}, j = 1, 2$ and ΔB_2 are the respective uncertain perturbations, and $f(t)$ is an unknown, but bounded disturbance with bounded first time derivative with respect to time. According to [14] [15], we design a sliding mode estimator

$$\dot{\hat{x}}_2 = \bar{A}_{21} x_1 + \bar{A}_{22} \hat{x}_2 + \bar{B}_2 (u + \nu) \ , \tag{2}$$

using a discontinuous feedback control ν to reach a desired manifold,

$$\sigma = x_2 - \hat{x}_2 = 0 \ , \quad \sigma \in R^m \ . \tag{3}$$

The condition for existence of sliding mode is

$$\sigma_i \dot{\sigma}_i < 0 \ , \tag{4}$$

where σ_i is the i-th element of vector σ. The simplest control law which can lead to sliding mode is the relay

$$\nu_i = M_i \mathrm{sign}(\sigma_i) \ . \tag{5}$$

If sliding mode exists ($\sigma = 0$ and $\dot{\sigma} = 0$) then there is a continuous control, so-called equivalent control, ν_{eq}, which can hold the system on the sliding manifold, (but it does not guarantee the convergence to the switching manifold in general). The primary goal of this design is to obtain the equivalent control of ν for the motion on this manifold. If the system in sliding mode

$$\dot{\sigma} = \Delta A_{21} x_1 + \Delta A_{22} x_2 + \Delta B_2 u + E_2 f - \bar{B}_2 \nu_{eq} = 0 \ ; \tag{6}$$

$$\bar{B}_2 \nu_{eq} = \Delta A_{21} x_1 + \Delta A_{22} x_2 + \Delta B_2 u + E_2 f \ . \tag{7}$$

Clearly, ν_{eq} contains information on the system's parametric uncertainties and the external disturbance which can be used for feedback compensation. In the practice, there is no way to calculate the equivalent control ν_{eq} precisely, but it can be estimated.

The system response with the control

$$u' = u + \nu_{eq} \tag{8}$$

coincides with the nominal or ideal, undisturbed system response for the control u, as showed in *Fig. 1*.

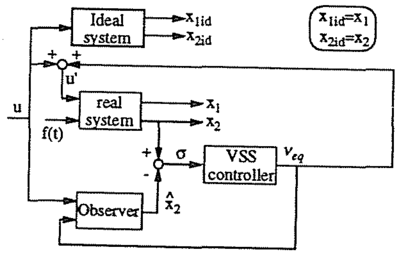

Fig. 1. Sliding mode based feedback compensation

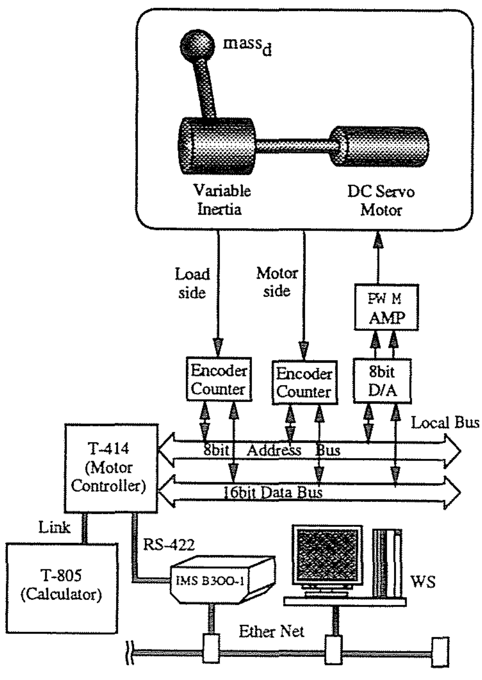

Fig. 2. System configuration

3. Experimental System

3.1 Configuration

The experimental system consists of a conventional DC servo gear motor with encoder feedback and variable inertia load coupled by a relatively

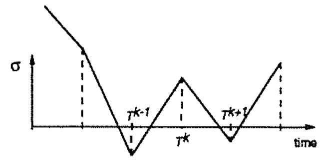

Fig. 3. Discrete-time chattering phenomenon

rigid shaft as shown in *Fig. 2.* The parameters of the motor and load are given in *Table 1.* The controller is implemented using a transputer as the computation engine. The DC motor is supplied by a DC chopper whose switching frequency was more than ten times bigger than the controller sampling frequency.

$$T_{sampling} = 640\ \mu s\ , \quad T_{chopper} = 50\ \mu s$$

3.2 System Equation

In course of control design, a reduced order model is used, the armature inductance and the flexibility of the shaft are ignored. The state variables are the shaft position, θ, and the shaft angular velocity, ω. The effect of mass_d is considered as disturbance. The nominal model is the following

$$\dot{x} = \bar{A}x + \bar{B}u\ , \tag{9}$$

where the control u is the motor voltage and

$$x = [\Theta \quad \omega]^T\ , \tag{10}$$

$$\bar{A} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-D_n}{J_n} + \frac{-K_{en}K_{tn}}{R_{an}J_n} \end{bmatrix}, \tag{11}$$

$$\bar{B} = \begin{bmatrix} 0 & \dfrac{K_{tn}}{R_{an}J_n} \end{bmatrix}^T\ . \tag{12}$$

4. Type 2 Servo Servo Design with Disturbance Compensation

The control with disturbance compensation is given by

$$u' = \underbrace{-k_I \int_0^t (\theta_r(\tau) - \theta(\tau))d\tau - k_\theta\theta - k_\omega\omega}_{u:\ \text{PID control}} \quad \underbrace{-\hat{\nu}_{eq}}_{\text{compensation}}\ , \tag{13}$$

Table 1
Nominal parameters

Motor and Gear Parameters		
torque	τ_n	5.2 [Nm]
output power	P_{outn}	18.5 [W]
angular speed	ω_n	3.6 [r/s]
gear ratio		1/ 88
torque constant	K_{tn}	5.07 [Nm/A]
back e.m.f voltage constant	K_{en}	5.06 [V/r/s]
armature resistance	R_{an}	2.7 [Ω]
armature inductance	L_{an}	1.1 [mH]
inertia of the motor transformed to the load side	J_{mn}	0.07 [kgm^2]
gear damping	D_n	2 [Nms]
shaft stiffness	K_{sn}	10000 [Nm/r]
Load Parameters		
inertia of the cylindrical mas	J_{ln}	0.06 [kgm^2]
load damping	D_n	0.001 [Nms]
diameter		20 [cm]
length		15 [cm]
Disturbance Parameters		
mass$_d$	m_{dn}	1 [kg]
arm length	l_{an}	40 [cm]

where $\hat{\nu}_{eq}$ is an approximation of the equivalent control ν_{eq} and Θ_r is the reference position.

The estimator is constructed in the following way

$$\dot{\hat{\omega}} = k_{\hat{\omega}}\hat{\omega} + k_u(u + \nu) \tag{14}$$

where ν undergoes discontinuities the desired manifold is written in the following form

$$\sigma_\omega = \omega - \hat{\omega} = 0 \ . \tag{15}$$

The simplest control law which might lead to sliding mode is the relay

$$\nu = k_\sigma \text{sign}(\sigma_\omega) \quad (k_\sigma > 0) \ . \tag{16}$$

In case of relay control law, the condition for existence of sliding mode is

$$k_\sigma > \max\{|\nu_{eq}|\} \ . \tag{17}$$

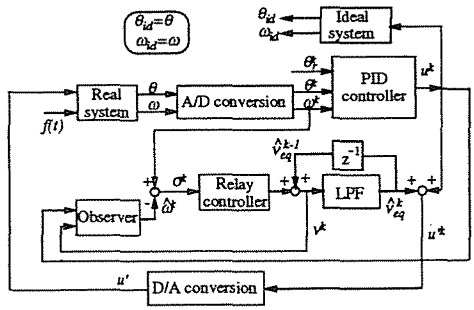

Fig. 4. The overall controller scheme

According to the equivalent control method, the system in sliding mode behaves as if ν is replaced by its equivalent value ν_{eq}. As it is known, ν_{eq} can be considered as the average value of the high frequency switched ν, consequently, an approximation of the equivalent control ν_{eq} is obtained from the discontinuous control ν by low pass filtering in the following way

$$T_c^3\hat{\nu}_{eq}^{(3)} + 3T_c^2\hat{\nu}_{eq}^{(2)} + 3T_c\hat{\nu}_{eq}^{(1)} + \hat{\nu}_{eq} = \nu \ , \tag{18}$$

where $^{(i)}$ denotes ith derivate with respect to time.

4.1 Discrete-Time Implementation

The robustness of continuous-time sliding mode control is obtained by high-frequency switching of high-gain control inputs. To adapt the sliding-mode philosophy for a digital controller, the sampling frequency should be increased compared to other types of control method.

If ν_{eq} is small but $\nu_{eq} \neq 0$ then σ might chatter around the manifold $\sigma = 0$ as shown in *Fig. 3*, where T^k denotes the time of kth sampling. In case of relay control, (16), the control switches from $+\nu$ to $-\nu$ and vice versa resulting $\hat{\nu}_{eq} = 0$. The role of the discontinuous term in the control law is to hide the effect of the uncertain perturbations and bounded disturbance. The more knowledge in the control law is implied, the smaller discontinuous term is necessary. Since ν_{eq} is continuous, only the change of ν_{eq} during the sampling period should be covered by the discontinuous term. The chattering can be reduced by the following discrete-time control law:

$$\nu^k = \hat{\nu}_{eq}^{k-1} + k_\sigma \mathrm{sign}(\sigma_\omega^k) \ , \tag{19}$$

where the superscript k refers to the kth sampling period. The overall controller scheme is shown in *Fig. 4*.

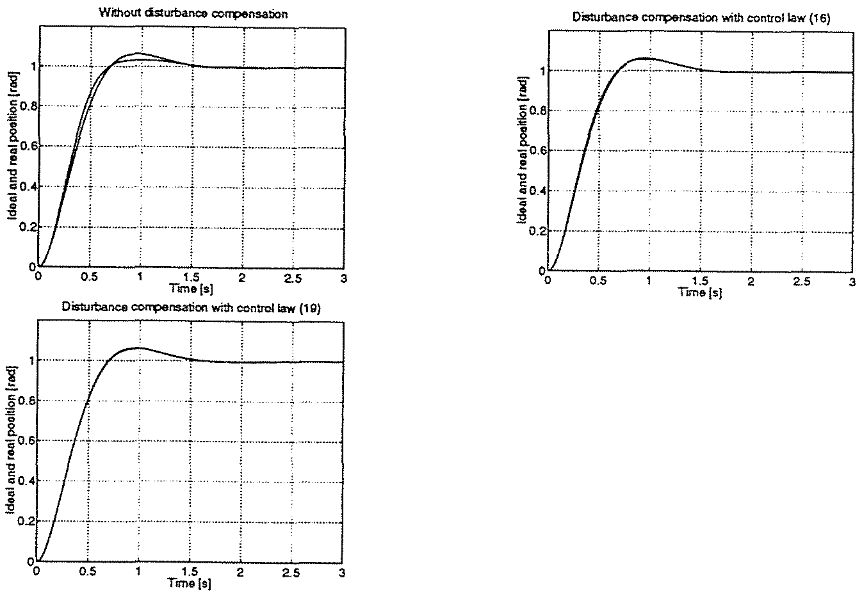

Fig. 5. Ideal and actual positions

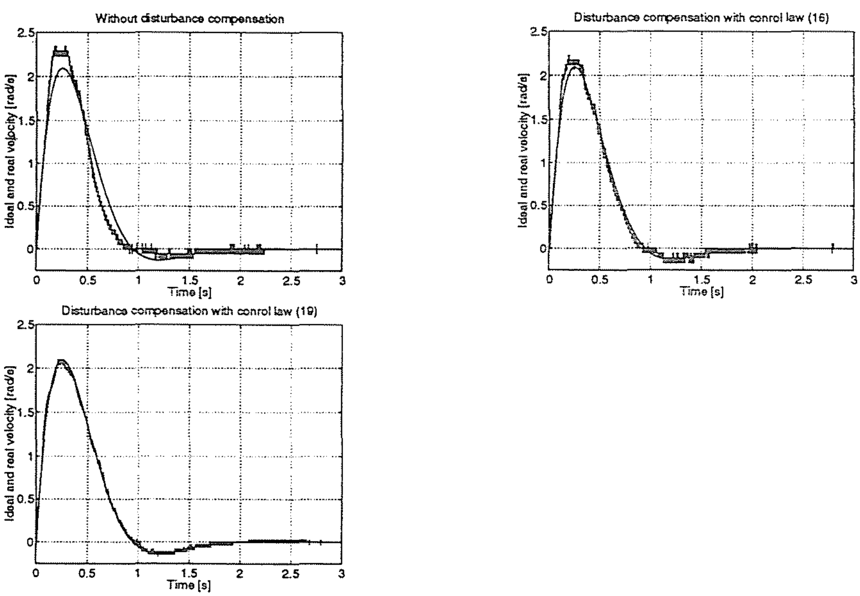

Fig. 6. Ideal and actual angular velocities

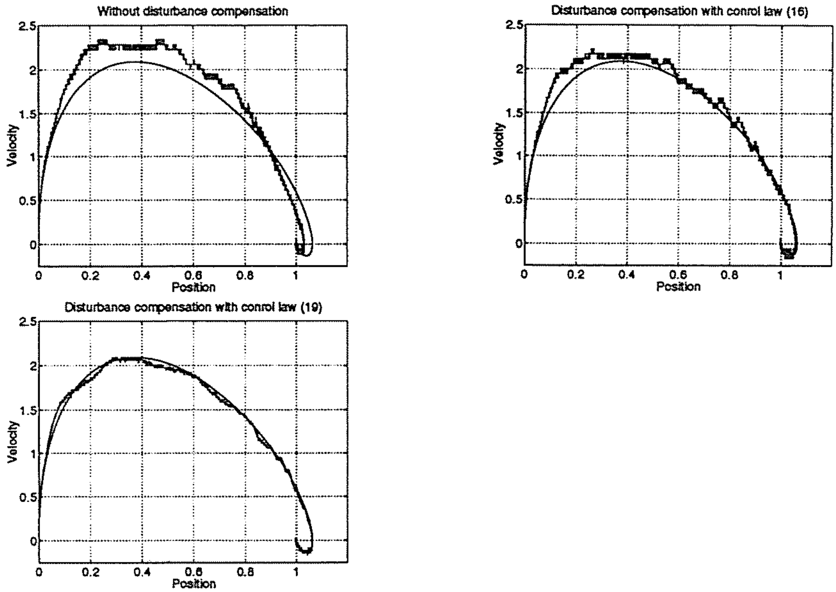

Fig. 7. Ideal and actual phase-trajectories

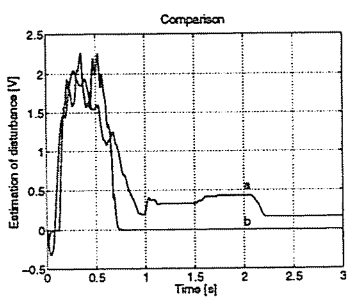

Fig. 8. Estimated disturbance a. Control law (19) b. Control law (16)

5. Simulation and Experimental Results

In all cases, the control parameters are set as follows

$$
\begin{aligned}
k_I &= 500\ [V/rs]\ , \\
k_\Theta &= 150\ [V/r]\ , \\
k_\omega &= 15\ [Vs/r]\ ,
\end{aligned}
\tag{20}
$$

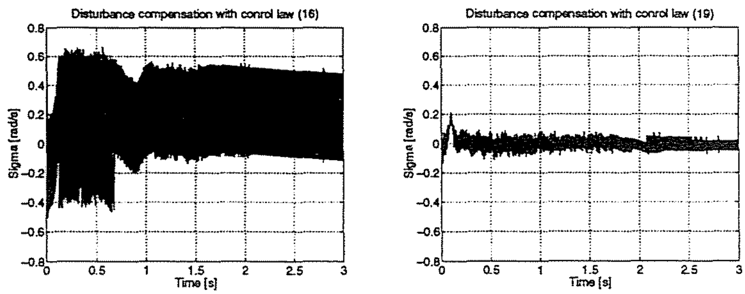

Fig. 9. Chattering phenomenon

$$k_\sigma = \begin{cases} 0 & \text{if compensation is switched off,} \\ 15 & \text{if control law (16) is applied,} \\ 3 & \text{if control law (19) is applied,} \end{cases} \tag{21}$$

$$T_c = 0.007\ s\ . \tag{22}$$

Because of the physical limitation of the experimental system, it was impossible to increase the disturbance mass. Instead of external disturbance the parameter uncertainties were increased, i.e. the desired dynamic behavior of the real system was changed. From the nominal parameters it can be calculated that

$$\begin{aligned} k_{\dot{\omega}n} &= -41.1\ [1/s]\ , \\ k_{un} &= 6.7\ [r/Vs^2]\ . \end{aligned} \tag{23}$$

The system with the desired dynamics has the following parameters:

$$\begin{aligned} \bar{k}_{\dot{\omega}} &= -427.1\ [1/s]\ , \\ \bar{k}_u &= 39.49\ [r/Vs^2]\ . \end{aligned} \tag{24}$$

The reference signal is a step change

$$\Theta_r^k = 1[\text{rad}] \quad (t > 0)\ . \tag{25}$$

The first set of plots in *Fig. 5* shows the time functions of the ideal and the measured positions with the three control laws. The corresponding angular velocities are shown in *Fig. 6*. The phase trajectories of the ideal and measured state variables are shown in *Fig. 7*. It should be emphasized that in this case the sliding 'manifold' is a point in the one dimensional 'phase-space' of σ and *Fig. 7* shows the overall control behavior. Applying the control laws (16) and (19), the estimated disturbances and chattering actions are compared in *Fig. 8* and in *Fig. 9*, respectively.

6. Conclusion

This paper presents an experimental adaptation of a sliding mode based feedback compensation. The experimental results demonstrate that the discontinuous estimator in sliding mode is a promising tool to use in eliminating the effect of a big scale parameter perturbation and bounded external disturbance.

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