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NOVEL METHOD TO SIMULATE SINGLE NON-LINEAR INDUCTIVE LOAD VOLTAGE-REACTIVE POWER CHARACTERISTICS

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Abstract

In the computer programs for the power system simulations there are different methods for the load simulations. Almost all methods use the S and Z type of the load modelling. For the nonlinear inductive loads there is the exponent type of the Q–U static characteristics with constant exponent. After the analysis of the measured characteristics of the synchronous machines and transformers, novel method is suggested to calculate the voltage dependent non-linear Q–U static characteristics. The main steps of the method are: to evaluate the measured characteristics and to produce by mathematical procedures a continuous $\beta(u)$ function of the varying not-constant exponent for the Q–U static characteristics. The $\beta(u)$ function can be expressed by regression of different order and it can be implemented in an advanced network simulation program like the Electro Dynamic Simulation (EDS).

Keywords: nonlinear load models, voltage dependent static load characteristics, variable exponent of reactive power–voltage characteristic.

1. Introduction

The constant power (S type) and constant impedance (Z type) of the load simulations are generally used in the load flow part of the power system simulation studies. The exponent type of the Q–U or P–U characteristics with constant exponent is mostly used, if the program permits the simulations of the non-linearity. In the publications there is a proposed range of this constant exponent, for example for the Q–U characteristics the $\alpha(qu) = (2..6)$ and for the P-U characteristics the $\alpha(pu) = (1..3)$. The data and the method are explained in reference publications in this task (see e.g. [1, 2, 3, 4, 6]). In the publication [5] it was mentioned that the equivalent Q–U static characteristics cannot have a constant exponent, due to the subtraction of the inductive and capacitive reactive power components. This was only a remark in this publication that started the analysis of how a general method of the evaluation can be developed for this type of the loads and how to implement these results to an advanced model of the load characteristics in the power system studies. (The detailed information on this task will be explained in another publication.) The investigations of this topic itself started the analysis for the components of the combined load, for the developing of methods, to do the evaluations of the different measured characteristics for the different type of loads, and to propose an advanced method for the modelling of the non-linear loads.

The main features of this methodology and the main arguments for its using are summarized below:

- By the evaluation of the measured U–I or Q–U static characteristic calculation of the formally not constant exponent $\beta(u)$ can be done as 'new characteristics', named exponent-voltage characteristic; in the first form it is a graphical function, and by this way the steps of the evaluation can be continued;
- By the variations of the reference voltage values (U_{ref} and Q_{ref}) needed for the evaluations, the influence of the reference values on the $\beta(u)$ function can be studied in order to get the best fit of the approximate curve.
- By different types of the regression an analytical function can be produced to express $\beta(u)$. This form is no more a graphical function but it is an analytical one, by a given level of accuracy;
- By the developing of the load simulation method the advanced non-linear load characteristics can be calculated, and the parameters of these characteristics can be applied by the user to the program;
- The outputs of the analysis of the $\beta(u)$ analytical function give the parameters of the non-linear load characteristics;
- By these more accurate load simulation can be done.

Temporarily the most steps of the previous methodology are ready in algorithm, and the algorithms are realized and tested in computer programs.

The next chapters explain the elements of the methods.

In Chapter 2: sum of the main considerations and the equations needed for the method.

In Chapter 3: explanation of how to derive the Q–U characteristics for the synchronous machines from their magnetizing curve.

In Chapter 4: graphical results of the analysis for the $\beta(u)$ exponent-voltage characteristics (for the synchronous machines and for the transformers).

In Chapter 5: explanation of the tendency and the conclusions of the analysis for the $\beta(u)$ exponent-voltage characteristic.

In Chapter 6: types of the regression for the graphical $\beta(u)$ functions are given, to have analytical $\beta(u)$ functions, and the parameters of an advanced non-linear load simulation method.

2. Main Consideration and Base of the Analysis

The main sources of the non-linearities in the power system are the transformers, shunt reactors and the rotating machines due to their non-linear voltage–current characteristics (magnetic curves). There is a wide ranging model to study the non-linearity: if the voltage–current ratio is not constant then the reactive power–voltage characteristics can be written in the next form:

$$Q(u) = Q_0(U/U_0)^{\alpha}\,, \tag{1}$$

where Q_0: the reactive power of the load at the given reference voltage (U_0) and α = constant $\gg 2$, is the exponent (power) of an exponent type function, having more than the second order.

In this publication there is a documentation of a method that has the same base, but after laboratory measurement and theoretical considerations an advanced method is produced for the analysis of the non-linear load. By this an advanced model for the non-linear load simulation can be developed.

Steps of the analysis:

- to produce a chart / tabulation of the reactive power–voltage characteristics of a given load range of the voltage enough to realize the needed range of the non-linearity (minimum 5....8 points);
- to fix U_0 and Q_0 reference values;
- to presume that the model is searched in the from of the *Eq.* (1) and to calculate the exponent α for each 'running step' related to the 'fix' U_0 and Q_0 points by the expression:

$$\alpha = \frac{\log(Q/Q_0)}{\log(U/U_0)}\,; \tag{2}$$

- if it is found that α is constant, the needed parameter for the load simulation is found;
- if α is not constant, it can be declared as a function of the voltage, $\beta(u)$ and drawn in a $\beta - u$ co-ordinate system;
- to repeat the previous evaluation by the change of the 'fix' U_0 and Q_0 reference values, for studying the sensitivity of the $\beta(u)$ characteristics on the selection of the reference values;
- to try to produce an analytic function, for $\beta(u)$ by the regression of the evaluated $\beta(u)$ characteristics;
- by this function the reactive power–voltage characteristics can be expressed in this combined form:

$$Q(u) = Q_0(U/U_0)^{\beta(u/u_0)}\,. \tag{3}$$

The significant difference between the old model or the base of *Eq.* (1) (α = const. > 2) and the new model of *Eq.* (3) that itself the 'exponent' is the function of the voltage.

By this way continuous Q–U characteristics having less error in the analysis than the old model can be produced. If this new type of the load models will be implemented to the network studies (e.g. voltage-reactive power control, dynamic simulation) and the needed parameter will be supported by the measurement done on the network, then more accurate network analyses will be done.

3. Derivation of Q–U Characteristics from a Magnetizing Curve of Synchronous Machines and their Evaluations

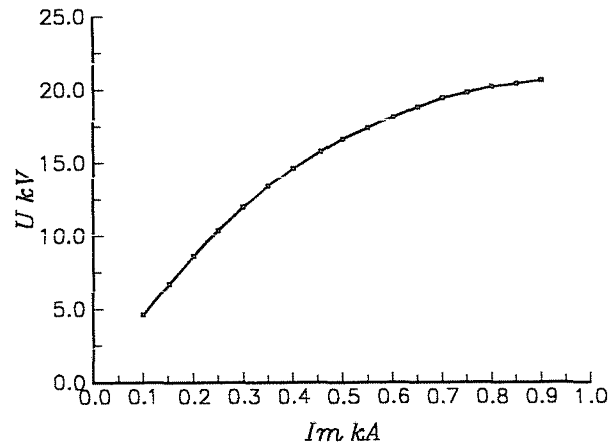

Fig. 1. Saturation curve of Syn.259 MVA

The magnetizing curve of the synchronous machines is measured, generally, by the excitation of DC rotor current and by the measuring of the no loaded terminal voltage. The stator reactive power–voltage characteristic (as a large non-linear inductive load) is needed for the evaluations. Next an overview of the considerations and steps of how to derive the Q–U characteristics from the magnetizing curve will be shown. The method will be explained by the example of a given synchronous machine. The synchronous machine's saturation data are shown in *Table 1* and saturation curve is plotted in *Fig. 1*. For this machine these are the factory given parameters:

$$\epsilon_d = 240\% , \quad U_{t.n} = 15.75 \text{ kV} , \quad S_n = 259 \text{ MVA} .$$

ϵ_d : synchronous reactance at the nominal voltage in %,
$U_{t.n}$: nominal terminal voltage in kV,
S_n : nominal power in MVA.

χ_d is the synchronous reactance at the nominal voltage in ohms:

$$\chi_d = \left(\frac{\chi_d\%}{100}\right)\left(\frac{U_{t.n}^2}{S_n}\right) = \left(\frac{240}{100}\right)\left(\frac{(15.75\ \text{kV})^2}{259.0\ \text{MVA}}\right) = 2.299 \cong 2.3\ \Omega\ .$$

If the magnetizing current is supplied from the stator side, the synchronous machine is equivalent to a large non-linear reactor. On the nominal voltage this reactive power is

$$Q_{t.n} = \frac{U_{t.n}^2}{\chi_d} = \frac{(15.75\ \text{kV})^2}{2.3\ \Omega} = 107.85\ \text{Mvar}\ .$$

For this, the nominal stator magnetizing current is:

$$I_{s.m.n} = \frac{Q_{t.n}}{\sqrt{3}U_{t.n}} = \frac{107.85\ \text{M var}}{\sqrt{3}\cdot 15.75\ \text{kV}} = 3958\ \text{A}\ .$$

From the factory test data it is known that for the field (rotor) side $I_{f.n} =$ 455 A is needed to magnetize to the $U_{t.n} = 15.75$ kV. Physically there is the same flux in the air-gap. By this equivalency it is defined the current reduction coefficient from the rotor to the stator current (involved the reduction for the DC current to the stator AC RMS value):

$$K_I = I_{s.m.n}/I_{f.n} = 3958\text{A}\,/455\text{A} = 8.699\ .$$

- Using this coefficient the next reductions can be done:
- to re-scale the factory produced $I_r - U_t$ magnetizing characteristics to the $I_s - U_t$ curve;
- to express the stator side reactive power by the rotor current:

$$Q_t = \sqrt{3}U_t I_{s,m} = \sqrt{3}U_t K_I I_r\ . \tag{4}$$

If the 'formal magnetizing power' is defined by the multiplication of U_t and I_r (by using the factory measured data)

$$Q_{formal} = U_t I_r = Q_f \tag{5}$$

and to define the coefficient of the power reduction

$$K_Q = Q_t/Q_{formal} = \sqrt{3}U_t I_s/U_t I_r = \sqrt{3}U_t K_I I_r/U_t I_r \tag{6}$$

then (with the given data)

$$K_Q = \sqrt{3}K_I = \sqrt{3}\cdot 8.699 = 15.07\ .$$

By these considerations the reduced magnetizing characteristics to the stator current are produced first, next, by the multiplication of U_t and I_s values, the $Q_f - U_t$ non-linear characteristics are obtained.

It is needed to make remark for these considerations and for these coefficients. The saturation of the machines means that the current needed for magnetizing is not proportional if the induced voltage is increased. By the flux equivalency this does not depend on weather the machine is excited from the rotor or stator side, these currents are proportional. The previous consideration or the reduction is needed because the data produced by the factory for the magnetizing are given for two different sides of the synchronous machine.

By the previous reduction the $Q_t(u)$ non-linear characteristic can be defined according to the *Eq.* (3)

$$Q_t(u) = Q_0(U/U_0)^{\beta(u/u_0)} \tag{3a}$$

and to calculate the exponent by the *Eq.* (2)

$$\beta = \frac{\log(Q_t/Q_0)}{\log(U_t/U_{t_0})} . \tag{2a}$$

Of course in the *Eqs.* (2a) and (3a) the direct result of the formal multiplication $Q_{formal} = U_t I_r$, can be used, because the K_Q coefficient disappears in the *Eqs.* (3a) and (2a)

$$\beta = \frac{\log(Q_t/Q_0)}{\log(U_t/U_0)} = \frac{\log(K_Q Q_f / K_Q Q_{f_0})}{\log(U_t/U_0)} = \frac{\log(Q_f/Q_{f_0})}{\log(U_t/U_0)} . \tag{2b}$$

4. Analytical and Graphical Results of the $\beta(u)$ Characteristics

4.1. Synchronous Machines

In this part, two types of synchronous machines were chosen as a single load type for this investigation. The magnetizing characteristics of large and small synchronous machines are good examples of single inductive load representation. The calculation of $\beta(u)$ values is divided into cases. The cases (C1, C2, ...) are chosen based on the selection of reference voltage value ($U_{\text{ref.}}$), in order to show the significance of $U_{\text{ref.}}$ on the value of $\beta(u)$. The number of cases also depends on the number of measured points. The $\beta(u)$ characteristics are plotted versus the voltage values instead of p.u. values, in order to combine all the cases in single plot for easy comparison of all the cases.

4.1.1. Large Synchronous Machine (Syn.259 MVA) Type

Measured parameters of the saturation curve of 259 MVA synchronous machine and 15.75 kV nominal voltage was taken from the factory test. The parameters are shown in *Table 1* and plotted saturation characteristic is shown in *Fig. 1*.

Table 1. Syn.259 MVA data

U_f kV	I_{rotor} A
4.60	100
6.70	150
8.65	200
10.40	250
12.00	300
13.40	350
14.60	400
15.75	455
16.60	500
17.40	550
18.10	600
18.80	650
19.40	700
19.80	750
20.20	800
20.40	850
20.60	900

Evaluation of Syn.259 MVA machine was divided into five cases (C1, 2, 3, 4, 5), which represent the linear, knee, nominal and the saturation segments. Calculated $\beta(u)$ for every case are shown in *Table 2*, and plotted characteristics of $\beta(u)$ with voltage variation are shown in *Fig. 2*.

Table 2. $\beta(u)$ ranges of Syn.259 MVA for cases (C1, 2, 3, 4, 5)

Case	$U_{ref.}$kV	$U_{ref.}$p.u.	$I_{r\ ref.}$A	$\beta(u)$
1	4.60	0.292	100	$2.0 \leq \beta(u) < 2.5$
2	12.00	0.762	300	$2.1 \leq \beta(u) < 3.1$
3	15.75	1.000	455	$2.2 \leq \beta(u) < 3.5$
4	18.80	1.194	650	$2.3 \leq \beta(u) < 4.6$
5	20.60	1.308	900	$2.4 \leq \beta(u) < 6.3$

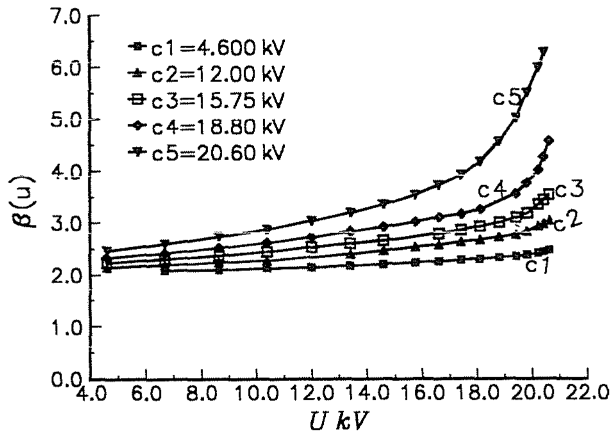

Fig. 2. $\beta(u)$ of Syn.259 MVA for cases (C1, 2, 3, 4, 5)

4.1.2. Small Synchronous Machine (Syn.12 KVA)

Saturation characteristics of small laboratory synchronous machine (Syn.12 KVA) with rated power of 12 kVA and nominal voltage of 220 Volt is shown in *Fig. 3*.

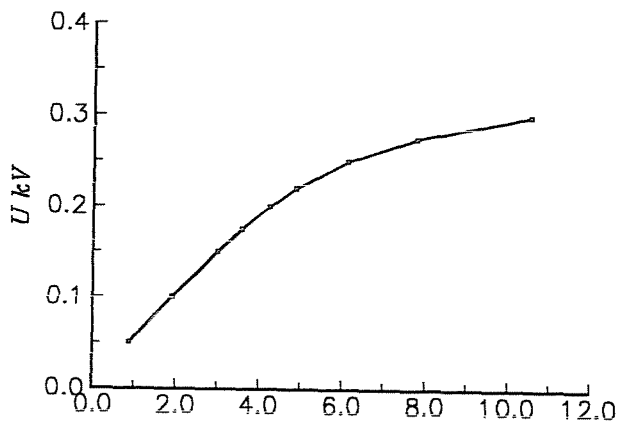

Fig. 3. Saturation curve of Syn.12 KVA machine

The data for the evaluation was taken from the saturation curve after scanning, and the evaluation was divided into four cases only. The calculated $\beta(u)$ for every case are shown in *Table 3* and plotted characteristics of $\beta(u)$ with voltage variation (up and down step) are shown in *Fig. 4*.

Table 3. $\beta(u)$ ranges of Syn.12 KVA for cases (C1, 2, 3, 4)

Case	$U_{ref.}$kV	$U_{ref.}$p.u.	$I_{r\ ref.}$A	$\beta(u)$
1	50.0	0.227	0.867	$2.0 \leq \beta(u) < 2.40$
2	150.0	0.682	3.000	$2.0 \leq \beta(u) < 2.81$
3	220.0	1.000	4.867	$2.1 \leq \beta(u) < 3.50$
4	300.0	1.364	10.500	$2.3 \leq \beta(u) < 4.50$

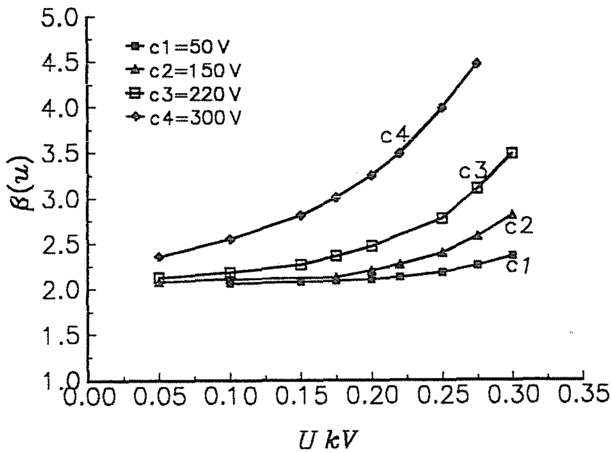

Fig. 4. $\beta(u)$ of Syn.12 kVA for cases (C1, 2, 3, 4)

4.2. Transformers

Four different transformers were evaluated ranging from laboratory, distribution to bulk transmission transformers. The data of these transformers were mostly supplied by Hungarian factories. The evaluations were divided into several cases for every transformer based on the selection of U_{ref}.

4.2.1. Transformer (tr.20 KVA)

Saturation characteristic of small laboratory transformer (tr.20 KVA) with rated power of 20 kVA and nominal primary voltage of 231 Volt is plotted on *Fig. 5.* The cases (C1, 2, 3, 4) for tr.20 KVA were evaluated and the results of the calculation of $\beta(u)$ are shown in *Table 4.* Plotted characteristics of $\beta(u)$ for every case are shown in *Fig. 6.*

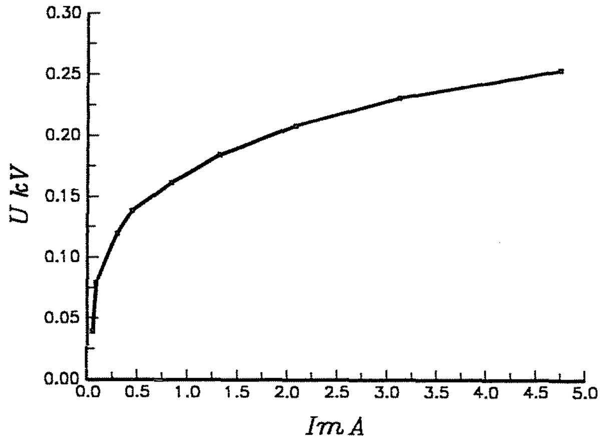

Fig. 5. Saturation characteristic of tr.20 KVA

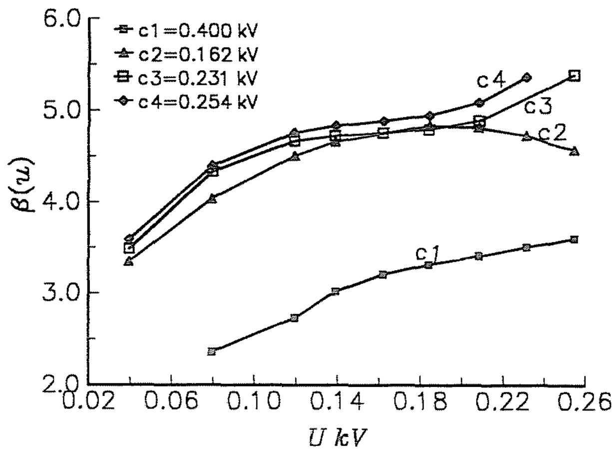

Fig. 6. $\beta(u)$ of tr.20 kVA for cases (C1, 2, 3, 4)

Table 4. $\beta(u)$ ranges of Tr.20 KVA for cases (C1, 2, 3, 4)

Case	$U_{ref.}$kV	$U_{ref.}$p.u.	$I_{r\ ref.}$A	$\beta(u)$
1	39.5	0.171	0.042	$2.35 \leq \beta(u) < 3.59$
2	161.8	0.700	0.841	$3.34 \leq \beta(u) < 4.57$
3	231.0	1.000	3.120	$3.48 \leq \beta(u) < 5.36$
4	254.3	1.100	4.740	$3.59 \leq \beta(u) < 5.37$

4.2.2. Transformer (tr.40 MVA)

Tr.40 MVA is distribution transformer with rated power of 40 MVA and nominal secondary voltage of 22 kV. The saturation characteristic is shown on *Fig. 7.* The cases evaluated and the calculated $\beta(u)$ values are shown in *Table 5*, also the results of $\beta(u)$ calculation with voltage variation are plotted in *Fig. 8.*

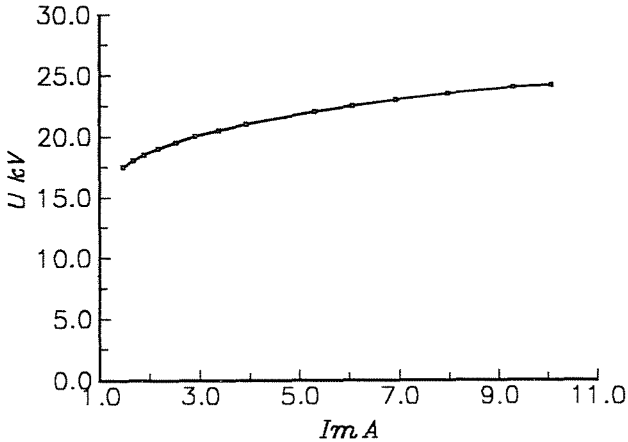

Fig. 7. Saturation characteristic of tr.40 MVA

Table 5. $\beta(u)$ ranges of Tr.40 MVA for cases (C1, 2, 3, 4)

Case	$U_{ref.}$kV	$U_{ref.}$p.u.	$I_{r\ ref.}$A	$\beta(u)$
1	17.5	0.795	1.450	$3.73 \leq \beta(u) < 6.86$
2	19.5	0.886	2.516	$6.35 \leq \beta(u) < 7.20$
3	22.0	1.000	5.300	$6.59 \leq \beta(u) < 7.85$
4	24.0	1.091	9.293	$6.80 \leq \beta(u) < 8.43$

4.2.3. Transformer (tr.200 MVA)

Tr.200 MVA is a bulk transmission transformer with rated power of 200 MVA and nominal tertiary voltage of 18.0 kV. The saturation characteristic is shown in *Fig. 9.* The cases evaluated and the calculated $\beta(u)$ values are shown in *Table 6*, results of $\beta(u)$ calculation with voltage variation are plotted in *Fig. 10.*

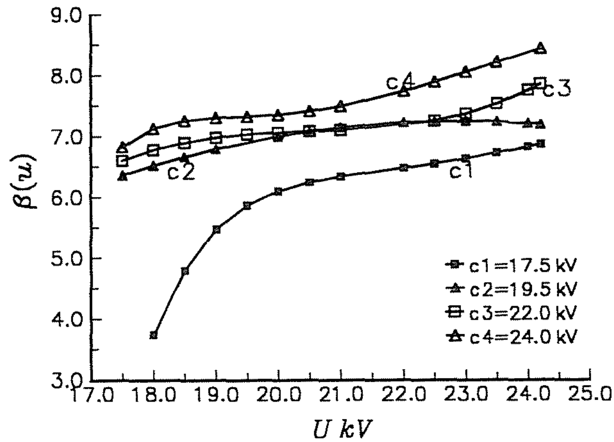

Fig. 8. $\beta(u)$ of tr.40 MVA for cases (C1, 2, 3, 4)

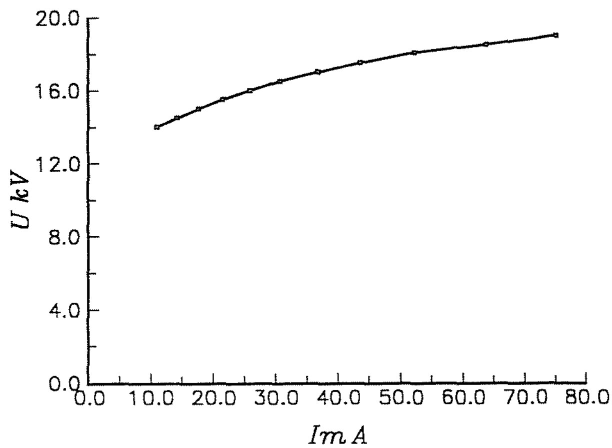

Fig. 9. Saturation characteristic of tr.200 MVA

Table 6. $\beta(u)$ ranges of Tr.200 MVA for cases (C1, 2, 3, 4)

Case	$U_{ref.}$kV	$U_{ref.}$p.u.	$I_{r\ ref.}$A	$\beta(u)$
1	14.00	0.775	10.976	$7.31 \leq \beta(u) < 9.73$
2	16.00	0.886	25.538	$6.42 \leq \beta(u) < 7.57$
3	18.00	1.000	55.200	$6.80 \leq \beta(u) < 7.70$
4	19.00	1.052	75.000	$7.00 \leq \beta(u) < 7.75$

4.2.4. Distribution Transformers Characteristics

Distribution transformers of 630, 1000 and 1600 kVA rating, with nominal primary voltage of 11 and 22 kV were evaluated. These transformers are

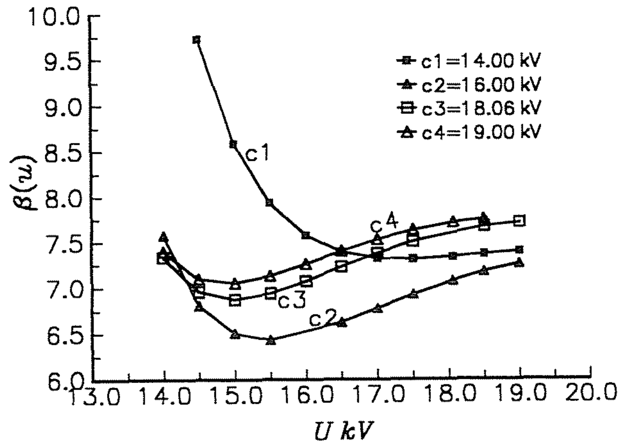

Fig. 10. $\beta(u)$ of tr.200 MVA for cases (C1, 2, 3, 4)

Hungarian manufactured with the same iron core characteristics. Saturation curve *Fig. 11*, is calculated from the manufacture iron core curve and the specification tables. The nominal voltage was chosen at 1.6 Tesla. The general characteristic is valid for different types of the transformers. The printed case is calculated for the 630 kVA and 11 kV.

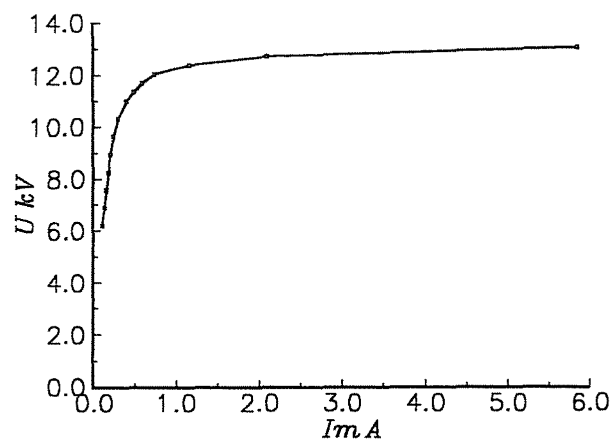

Fig. 11. Distribution transformer(s) saturation characteristics

The $\beta(u)$ calculations of the transformer characteristics are carried out in three cases. *Table. 7* shows these cases and the ranges of $\beta(u)$. Plotted results are shown in *Fig. 12*.

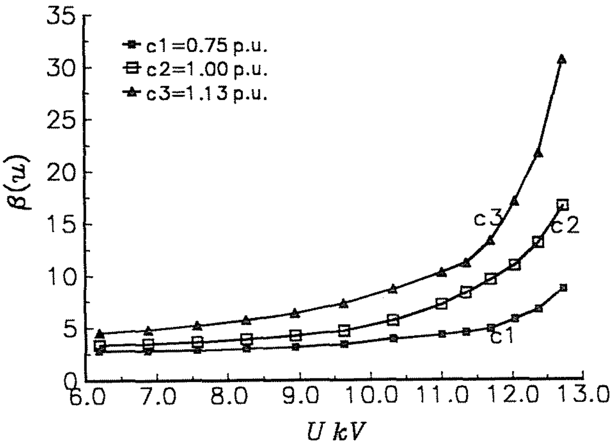

Fig. 12. $\beta(u)$ of distribution transformer for cases (C1, 2, 3)

Table 7. $\beta(u)$ of Distribution transformers cases (C1, 2, 3)

Case	$U_{ref.}$kV	$U_{ref.}$p.u.	$I_{r\ ref.}$A	$\beta(u)$
1	8.250	0.750	0.175	$2.77 \leq \beta(u) < 8.64$
2	11.00	1.000	0.400	$3.32 \leq \beta(u) < 16.62$
3	12.3750	1.125	1.597	$4.50 \leq \beta(u) < 30.60$

5. Tendencies and Conclusions by the Evaluations

The synchronous machines and the transformers studied with their cases produced qualitatively similar results which can be summarized in the following paragraphs:

1. the exponent $\beta(u)$ is not constant (the exponent: means the power of the reactive power–voltage function), but we can analyze it as continuous function of the voltage (see *Figs. 2, 4, 6, 8, 10, 12*);
2. from the Tables and Figures which correspond to the studied cases it is clear that the value of $\beta(u)$ is very sensitive to the selection of U_{ref}. When U_{ref} was selected in the linear segments of the saturation curve, there was small increase in $\beta(u)$ value with respect to the voltage change. It is clear as U_{ref} increased toward the saturation part, $\beta(u)$ values had an increased tendency, too.
3. the proposed method itself shows small change of the $\beta(u)$ value in the linear part, too. In these cases as a simplification, it is allowed to use the constant exponent type of $Q - U$ functions. These are only some cases that the constant impedance (Z type) models are relatively

good. The new proposed model by $\beta(u)$ gives solution for the small and large voltage range, too.

4. from the cases of the U_{ref} in the linear part where $\beta(u)$ is in the range of (2.2...2.5), the simplification of $\beta(u) = 2.0$ causes an error in $Q(u)$ value of up to 20% for the synchronous machine and as high as 90% error for distribution transformers (see *Figs. 14* and *16*).
5. the proposal suggests the use of the exact nominal voltage level as U_{ref} or a value near to it, due to the knee and saturation level which causes large ranges of $\beta(u)$ value, see e.g. *Table 2* (case 3, $2.2 \leq \beta(u) < 3.5$), *Table 3* (case 3, $2.1 \leq \beta(u) < 3.5$), *Table 4* (case 3, $3.48 \leq \beta(u) < 5.36$), *Table 5* (case 3, $6.59 \leq \beta(u) < 7.85$), *Table 6* (case 3, $6.8 \leq \beta(u) < 7.7$) and *Table 7* (case 2, $3.32 \leq \beta(u) < 16.62$).
6. the generally applied simulation programs use only the constant type of the exponent $(0, 1, 2)$, selection of the voltage ranges wanted to be studied, and the choice of appropriate value of a constant power (α) value is needed.
7. significant and important practical results for load modelling simulation has been demonstrated in the preceding section. This new modified model simulates accurately non-linear inductive loads. So it can be concluded that a more general tool for representing and analyzing the non-linear inductive load is developed.
8. it is proposed to implement these advanced reactive power-voltage characteristics $\beta(u)$ to those electric network studies, where the highly changing non-linear inductive loads significantly influence the results (e.g. Voltage and Reactive power control, Electro Dynamic Simulation, etc.). This calculation of $\beta(u)$ is possible and the range of error in the simulation can be minimized to a large extent.
9. the proposed method is much more accurate than the method used so far frequently.

6. Smoothing and Regressions of the $\beta(u)$ Characteristics

6.1. Algorithm

In the previous chapters it was defined the voltage dependent type exponent $\beta(u)$ for the reactive power–voltage characteristics and there were documented some graphical solutions for $\beta(u)$. The next step of the novel method is to implement the result to the computer programs used for the network calculations. It might be more methods to do this such as:

a) to do tabulated form to simulate $\beta(u)$ graphical characteristics;
b) to find analytic functions to simulate $\beta(u)$ graphical characteristics, for example by linear or second order or higher regression of the measured characteristics;

c) to find analytic functions to simulate directly the non-linear Q–U characteristics, etc.

By theoretical and practical reasons in this publication there is the application of the method b) as the best method between them. In the scientific investigations the method c) was also tested but by this direct method the smoothing of the measured data cannot be solved. Method a) was good for the actual cases, but the tabulated form of the parameters does not support to develop general method of the non-linear simulations. The proposed method b) itself can solve the problem of the smoothing of the measured data and by the calculated parameters it can give inputs for the network computer programs. The accuracy of the method can be increased by increasing the order of the regressions. In this publication there are a first order (linear) regression and a second order (parabolic) regression documented.

The general analytic form of the first order solution is *Eq.* (7):

$$Q(u) = Q_0(U/U_0)^{a_0+a_1(U/U_0)} \tag{7}$$

and the form of the second order solution is *Eq.* (8):

$$Q(u) = Q_0(U/U_0)^{b_0+b_1(U/U_0)+b_2(U/U_0)^2} . \tag{8}$$

In the equations:
U_0 : nominal voltage in kV, and U : actual voltage in kV,
a_0, a_1, b_0, b_1, b_2 : calculated parameters of the regressions.
The basis for estimating the unknown parameters is the criterion of *least squares* (LS). The calculation of unknown parameters is done for $\beta(u)$ function calculated with $U_{\text{ref}} = U_0 = 1.0$ p.u. and voltage range of $\pm 20\%$ of U_0.

6.2. Verification of the Methods

In order to validate, to test the accuracy and the sensitivity of the algorithms applied compared to the actual measured data, an evaluation was made of the deviation ΔQ of the calculated reactive power with constant power exponent (α), with $\beta(u)$ as first order function and with $\beta(u)$ as second order regression function. For this evaluation, large synchronous machine of 259 MVA and distribution transformer of 630 kVA were selected. As base for the evaluation and comparing, Z type load simulation was selected for the calculation with constant power exponent ($\alpha = 2$).

Evaluation of the deviation of the calculated data from the actual measured data was carried by two approaches. The first approach is aimed for compressing the error levels, all data errors are evaluated to their maximum values. The second approach is aimed for zooming the errors in order to show the significance of the proposed method, all the data are evaluated to their actual values.

In the first approach the deviation of the calculated data $\Delta Q\%$ is evaluated by *Eq.* (9):

$$\Delta Q\% = (Q_{act} - Q_{cal}) * 100/Q_{\max} . \tag{9}$$

In the second approach the deviation of the calculated data $\Delta Q\%$ is evaluated by *Eq.* (9):

$$\Delta Q\% = (Q_{act} - Q_{cal}) * 100/Q_{act} . \tag{10}$$

In the equations:

Q_{act} : the actual measured reactive power,
Q_{cal} : the calculated reactive power of ($Q_{const}, Q_{lin}, Q_{2nd}$),
Q_{const} : reactive power calculated with constant $\alpha = 2$ at U_{ref},
Q_{lin} : reactive power calculated with first order regression of $\beta(u)$,
Q_{2nd} : reactive power calculated with second order regression of $\beta(u)$,
$Q_{\max}$: maximum measured reactive power.

The results of the first and second approach for synchronous machine of 259 MVA are plotted in *Figs. 13* and *14*.

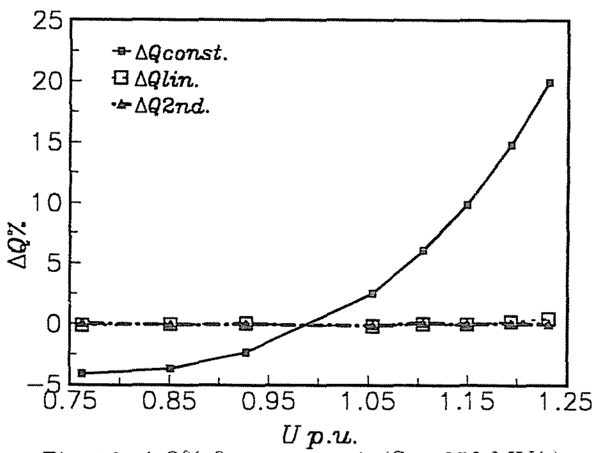

Fig. 13. $\Delta Q\%$ first approach (Syn.259 MVA)

The results of the first and second approach for distribution transformer of 630 kVA $\Delta Q\%$ are shown in *Figs. 15* and *16*.

In *Figs. 13–16*, it is clear that the deviation of the calculated reactive power from the actual data is significant, specially with voltage variation limit of $> \pm 10\%$. The evaluation shows that the Syn.259 MVA and distribution transformers have the same qualitative tendency and are different

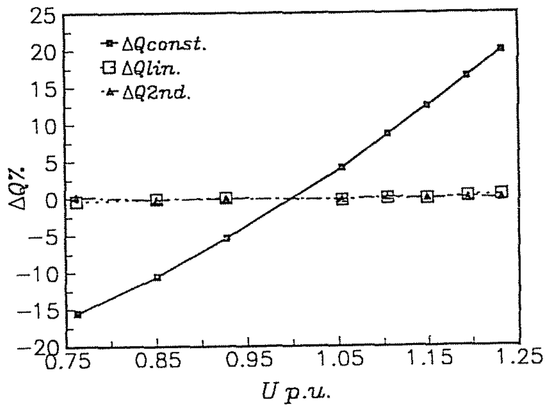

Fig. 14. $\Delta Q\%$ second approach (Syn.259 MVA)

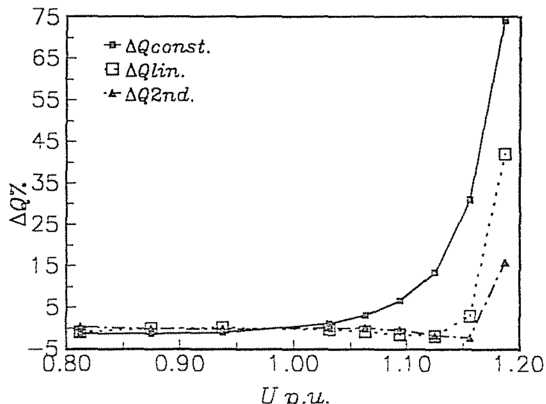

Fig. 15. $\Delta Q\%$ first approach (Dist. tr.)

quantitatively. This is due to the difference in their saturation characteristic as consequence of the different magnetic core design and the material used.

From the studied cases, it seems that the second order regression resulted in less ΔQ deviation in comparison with $Q_{const.}$ and $Q_{lin.}$ So we can conclude that the $\beta(u)$ function can be represented with the second order model with much better accuracy than the constant exponent approximation. The parameters obtained by the second order algorithm produce a good numerical fit to the $\beta(u)$ which will lead to more accurate simulation results if implemented in advanced power system simulation studies. The parameters of this model can be estimated easily and this model requires less computation in simulation. Therefore, the second order model should be sufficient for single inductive load.

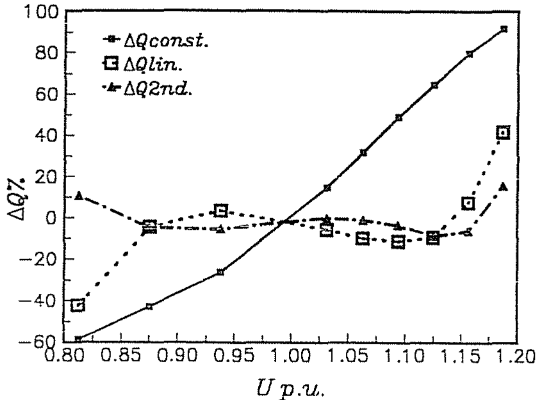

Fig. 16. $\Delta Q\%$ second approach (Dist. tr.)

7. Summary

This paper describes a novel method for the modelling and simulation of non-linear inductive loads like synchronous machines and transformers. The method accepts the generally used exponent (power) type of Q–U characteristics but by the $\beta(u)$ voltage dependent exponent there is the possibility of the more accurate load simulations. By the proposed algorithm there is a closed path of the investigations: from the measured data to calculate some parameters of the regressions and to implement them to advanced network simulation computer program like Electro Dynamic Simulation (EDS).

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