PERIODICA POLYTECHNICA SER. EL. ENG. VOL. 41, NO. 4, PP. 279–286 (1997)

SINGLE-ROW ROUTING PROBLEM WITH ALTERNATIVE TERMINALS

Tibor SZKALICZKI[‡]

Computer and Automation Research Institute
1521 Budapest, Hungary
sztibor@pele.ilab.sztaki.hu
36 1 269-8400/376

Received: July 1, 1997

Abstract

The present article concentrates on the dogleg-free Manhattan model where horizontal and vertical wire segments are positioned on different sides of the board and each net (wire) has at most one horizontal segment. Gallai's classical result on interval packing can be applied in VLSI routing to find, in linear time, a minimum-width dogleg-free routing in the Manhattan model, provided that all the terminals are on one side of a rectangular (single-row routing). We deal with the generalization of this routing problem when we have the possibility to select another terminal from a corresponding set instead of a terminal to be connected. It will be shown that in this case there is no hope to find a polynomial algorithm because this problem is NP-complete. The results on dogleg-free Manhattan routing can be connected with other application areas related to colouring of interval graphs.

In this paper the alternative interval placement problem will be defined. We show that this problem is NP-complete. This implies the NP-completeness of the single-row routing problem with alternative terminals.

Keywords: VLSI, single-row routing, NP-completeness.

1. Introduction

The single-row routing problem is one of the simplest problems in VLSI routing. The description of this problem is the following. Given a channel with length n and width w. The single-row routing is a special case of the channel routing when the terminals appear only on one side of the channel. A net is a collection of terminals. A channel routing problem is a set of pairwise disjoint nets. The solution of a routing problem is a set of subgraphs (wires) where each subgraph connects all the terminals of the corresponding net under the conditions of the wiring model. In the Manhattan model horizontal and vertical wire segments are positioned on different sides of the board.

[‡]This work was supported by the Hungarian National Foundation for Scientific Research (Grant No. OTKA T 017181)

LENGAUER (1990) presents a detailed exposition of the routing in the Manhattan model. The minimum width Manhattan single-row routing is polynomial (GALLAI, 1958; see also RECSKI, 1992).

Now a generalization of the minimum width Manhattan single-row routing will be investigated. It is typical in practice that some terminals are *functionally equivalent*, i.e. it does not matter which of them is contained by a net. The functionally equivalent terminals form *functional classes*. If no other terminal is functionally equivalent with a terminal then this terminal is the only member of its class.These terminals are called *fixed terminals* and the others *alternative terminals*. In this case a net is a collection of classes of terminals. A class may belong to more than one net. The routing problem with alternative terminals is to choose one terminal from each class of each net in such a way that each terminal is contained by at most one resultant net and the problem instance specified by the new nets can be routed optimally. Such a routing with alternative terminals can be regarded as a restricted version of the pin assignment problem (LENGAUER, 1990, pp. 364-366).

Some other forms of the pin assignment problem have already been proved to be $\mathcal{NP}$-complete (ATALLAH and HAMBRUSCH, 1987; CAI and WONG, 1991). We investigate the interval placement at first and introduce the alternative interval placement problem. We show that this problem is $\mathcal{NP}$-complete. This result implies the $\mathcal{NP}$-completeness of the routing problem with alternative terminals in one of the simplest cases, in the single-row routing with minimum width in the Manhattan model. This result holds even if a functional class has at most two members, a net contains at most one class with two members and a class belongs only to one net.

2. Alternative Interval Placement

Problem Given a finite set of intervals on a line, w rows and a set of disjoint pairs of the intervals. If an interval does not belong to any pair then it has to be placed into one of the rows. However, only one member of each pair has to be placed into one of the rows. Two intervals can be placed into the same row if and only if they have no common point. Can the intervals be placed under the prescribed conditions?

Example *Fig. 1a* shows an example of the alternative interval placement. The intervals belonging to the same pair are marked with the same letter. *Fig. 1b* depicts the solution with minimum width (i.e. with the minimum number of the rows). *Fig. 1c* and *Fig. 1d* illustrate the same problem instance using another notation where a rectangle represents an interval.

Definitions An interval is *single* if it does not belong to any pair. A *choice* is a set of the intervals containing each single interval and exactly one interval from each pair.

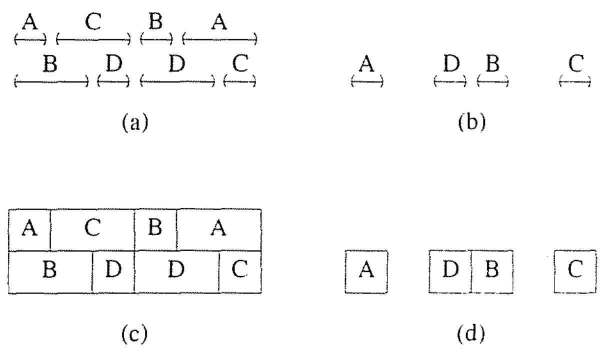


Fig. 1. Example of the alternative interval placement problem. (a) The problem instance. (b) Its solution. (c) The rectangular representation for the problem instance. (d) The rectangular representation for the solution

3. Construction

We shall reduce 3SAT to this problem. We show how to construct an instance of the alternative interval placement problem that can be realized in two rows if and only if the original instance of 3SAT is satisfiable.

3.1. Variable

An interval pair corresponds to each variable. They will be referred to as *variable pairs*. No two intervals in such pairs overlap each other. One interval in a pair corresponds to true and the other to false.

Another interval pair corresponds to each occurrence of each variable. They will be referred to as *literal pairs*. (An occurrence of a variable together with possible negation is called literal.) One member of such a literal pair is located between the boundaries of the member of the variable pair corresponding to true if the occurrence of the variable is negated, otherwise it is between the boundaries of the member of the variable pair corresponding to false. None of the members of literal pairs among the variable pairs overlap each other.

In *Fig. 2*, occurrences of variables are marked with a letter and the corresponding literal pairs are marked with the same letter. In addition, there is a single interval which overlaps each interval belonging to variables.

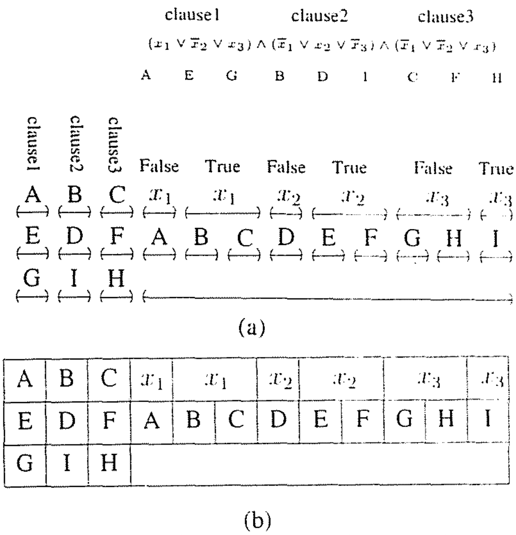


Fig. 2. The problem instance belonging to the formula $(x_1 \vee \overline{x}_2 \vee x_3) \wedge (\overline{x}_1 \vee x_2 \vee \overline{x}_3) \wedge (\overline{x}_1 \vee \overline{x}_2 \vee x_3)$ (a) Interval representation. (b) Rectangular representation

3.2. Clause

The intervals belonging to clauses are separated from the intervals belonging to variables. Three intervals belong to each clause. They are the *clause triplets*. The members of the same clause triplet overlap each other but members of different clause triplets don't. The members of the clause triplet are the other members of the literal pairs already mentioned in Section 3.1 and, if less than three variables occur in the clause, single intervals. The members of the literal pairs in the clause triplet correspond to the variables occurring in the clause.

The line containing the intervals can be divided into variable and clause sections. The *variable section* contains the variable pairs and the *clause section* contains the clause triplets.

Figs. 2, 3 and *4* present instances of the alternative interval placement problem corresponding to Boolean formulas.

3.3. The Proof of the $\mathcal{NP}$-Completeness

Theorem 1 *The alternative interval placement problem is $\mathcal{NP}$-complete.*

Each instance of 3SAT can be translated into an instance of the alternative interval placement problem in polynomial time. We prove that the instance of the alternative interval placement problem can be realized in

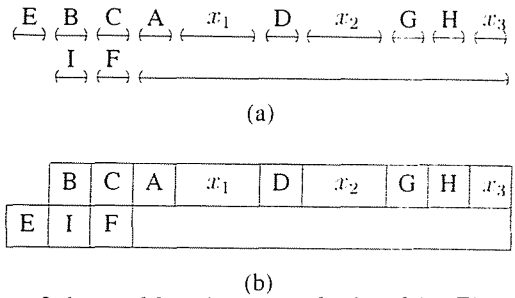


Fig. 3. The solution of the problem instance depicted in *Fig. 2.* (a) Interval representation. (b) Rectangular representation

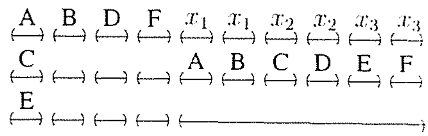
two rows if and only if the original instance of 3SAT is satisfiable. By Gallai's theorem (GALLAI, 1958; see also RECSKI, 1992), the minimum width is equal to the maximum density. In case of the alternative interval placement problem, the density of a point of the line containing the intervals is the number of chosen intervals covering the point. Using Gallai's theorem and the definition of the choice, the previous theorem can be formulated as follows.

Lemma 1 *The instance of the alternative interval placement problem has a choice with maximum density 2 if and only if the original instance of 3SAT is satisfiable.*

Proof: (if) Pick any truth assignment that satisfies the formula. Choose the member corresponding to the truth assignment from each variable pair. Choose the member in the variable section from each literal pair if it is not overlapped by an already chosen variable interval. Otherwise, choose the member in the clause section. In this case if a literal is true then the member on the variable section is chosen, otherwise the member on the clause section. Thus, there is no point covered simultaneously by a member of a variable and a member of a literal pair. Hence at most one interval covers each point on the variable section beside the single interval covering the whole variable section.

If the Boolean formula is satisfiable then at most two literals are false from each clause. Hence, at most two literal intervals are chosen from each clause triplet. Thus, the maximum density is two on the clause section as well.

(only if) There is a choice where width 2 is enough for placing the intervals. Let a variable be true if the interval corresponding to true is chosen from the variable pair and be false if the interval corresponding to false is chosen. No literal interval overlapped by a selected variable interval is chosen, otherwise two rows would not be enough. For this reason, the other member of the literal pair is chosen which is on the clause section. Because of



(a)

A	B	D	F	x_1	x_1	x_2	x_2	x_3	x_3
C				A	B	C	D	E	F
E									

(b)

Fig. 4. The problem instance belonging to the formula $(x_1 \vee x_2 \vee x_3) \wedge \overline{x}_1 \wedge \overline{x}_2 \wedge \overline{x}_3$ (a) Interval representation. (b) Rectangular representation

the construction, these literals are the false ones under the truth assignment. Since two rows are enough, at most two literal intervals are chosen from each clause triplet. Hence at most two literals can be false in each clause and so at least one is true. Thus the entire formula is satisfied. $\square$

4. Single-row Routing Problem Allowing Alternative Terminals is $\mathcal{NP}$-Complete

Now we return to the alternative single-row routing problem in the Manhattan model. In a restricted version of the Manhattan routing each wire could occupy only one horizontal row (track). This model is called the *dogleg-free Manhattan model.* If *doglegs* are permitted then wires can switch from one row to another. We remark that in the single-row routing, a problem instance can be solved using the same width in the dogleg-free Manhattan model as if doglegs are permitted. For this reason, these two variants of the Manhattan model need not be distinguished and we may assume that each wire has only one horizontal segment. The solvability is trivial because any problem instance can be solved in this model if the width is large enough. The interesting problem is to find the minimum width. We use the decision version of this optimization problem which is to decide whether the specification can be realized with width w.

Theorem 2 *The single-row routing with minimum width and alternative terminals is $\mathcal{NP}$-complete in the Manhattan model.*

We reduce the alternative interval placement problem to this problem. Each instance of the alternative interval placement can be transformed into an instance of the single-row routing with minimum width and alternative terminals as follows:

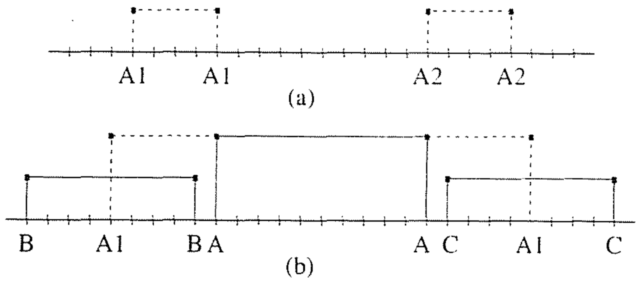


Fig. 5. Conversion of alternative nets into alternative terminals. (a) The original alternative net pair. (b) Its conversion into nets with alternative terminals

Step 1. Modify the interval lengths in such a way that the intervals remain overlapping the same intervals, all intervals become closed and no two intervals have a common boundary. Assign terminals to the boundaries of the closed intervals. Thus, the set of intervals is converted into the set of two-terminal nets and the alternative interval placement problem is reduced to the variant of the single-row routing in the dogleg-free Manhattan model which allows alternative nets. We have already proved that the alternative interval placement problem is $\mathcal{NP}$-complete, hence the single-row routing with minimum width and alternative nets is $\mathcal{NP}$-complete in the Manhattan model. This is true even if the width is 2.

Step 2. Now we reduce the routing problem with alternative nets to the routing problem with alternative terminals. Insert new terminals and replace each alternative net by a net with an alternative terminal pair and two nets without alternative terminals, as shown in *Fig. 5.* The effect of one such replacement is that the density of each vertical line is increased by one. If there are l alternative nets then the original problem instance with alternative nets can be routed using w tracks if and only if the new one with alternative terminals can be routed using $w+l$ tracks. For this reason, the single-row routing with minimum width and alternative terminals is $\mathcal{NP}$-complete in the Manhattan model. Notice that although the routing problem with alternative nets is $\mathcal{NP}$-complete even if the width is fixed, the width does not remain fixed when it is converted into the routing problem with alternative terminals.

5. Conclusions

In this paper we introduced the alternative interval placement problem and proved that it is $\mathcal{NP}$-complete, that is computationally difficult. This result could be applied to other problems. We proved that the routing problem with alternative terminals or nets is $\mathcal{NP}$-complete even in one of the simplest cases in the single-row routing in the Manhattan model.

Acknowledgement

Useful conversations with A. Recski are gratefully acknowledged.

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