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AFA BASED PERIODIC NOISE CANCELLATION

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Abstract

There are many methods for active cancellation of low-frequency periodic noise. Now a new efficient method is investigated. By means of a resonator based observer structure all harmonic components (up to the half of the sampling frequency) can be suppressed. Since the resonator filters out the signal at its own frequency, the rate of suppression is theoretically infinite. Since the system needs a reference signal only to estimate the frequency of the noise to be suppressed, it can be any signal, relatively free of noise, with the same frequency as the primary source. One of the advantages of the method is that the reference source can be either the error sensor, or any other microphone. For the successful noise cancellation the system needs information about the acoustic transfer function, at least its values relatively densely. There was applied a transfer function measurement method based on the resonators. The practical results were obtained by means of a DSP 96002 System Board.

Keywords: active noise control, resonator based observer.

Introduction

The reduction of acoustic noise generated by different machines and equipment is an old problem. There is a traditional solution: using passive sound absorbers placed either around the noise source, or around the space to be controlled. Since the acoustic wavelength is too large at low frequencies, as compared to the thickness of the typical absorber, the passive noise reduction does not provide a proper solution to the problem. Due to this disability of passive sound reduction, active noise control should be used (ELLIOT – NELSON, 1993) working as follows: a 'secondary' noise has to be generated which suppresses the 'primary' (i.e. the original) noise at the properly situated microphones. The first problem is how to arrange the microphones. The second one is how to control the loudspeakers. During the last few years many algorithms have developed and implemented. The basic idea of the control algorithm also depends on the type of noise. In this paper we only consider the control problem (one error microphone and one loudspeaker), and periodic noise. Most of the algorithms need a solution of the identification of the transfer function between the secondary source and the error microphone (or any other transfer functions) (ELLIOT – NELSON, 1993), (DOELMAN, N. J. 1993).

In our method the exact identification problem need not be solved, however, we need the values of the transfer function mentioned above relatively densely. The error of the identification does not affect the rate of the suppression (against other algorithms), only the speed of convergence.

In this paper a unique solution is investigated. The resonator based observer structure proves to be an appropriate tool for periodic noise cancellation since it contains a periodic signal model. This structure was first published as a recursive Fourier transformer (PÉCELI, 1986). Later several papers have been published in this field. With suitable modifications the structure can be completed to form digital filters with excellent features (PÉCELI), 1989). The signal model and the observer in our case can be described as follows:

$$y_n = \mathbf{r}_n^T \mathbf{x}_n, \tag{1}$$

$$\mathbf{r}_n = [r_{n,k}] = e^{j2\pi f k n}, \qquad k = -L..L, \qquad L = \mathrm{int}(0.5/f), \tag{2}$$

$$\hat{\mathbf{x}}_{n+1} = \hat{\mathbf{x}}_n + \frac{1}{N}\overline{\mathbf{r}}_n(y_n - \mathbf{r}_n^T \hat{\mathbf{x}}_n), \tag{3}$$

where $\mathbf{x}_n$ is the state vector of the signal model, y_n is its output and the input of the observer, $\hat{\mathbf{x}}_n$ is the estimated state vector, $\mathbf{r}_n$ represents the basis of the transformation, and $\bar{\ }$ denotes the complex conjugate operator. If the input signal is periodic consisting of only resonator frequencies, then the input of resonators (i. e. the error signal) is equal to zero. Furthermore, in this case the corresponding state variables (as a complex vector) do not change. However, if the input signal changes, the state vector will rotate. The speed of this rotation at each resonator is proportional to the corresponding frequency deviation. This is the basic idea for the frequency adaptation in AFA (NAGY, 1993). The exact formula is the following:

$$f_{n+1} = f_n + \frac{1}{2\pi N}\mathrm{angle}(\hat{x}_{n+1,L+2}, \hat{x}_{n,L+2}), \tag{4}$$

where $\hat{x}_{i,L+2}$ is the state variable belonging to the positive fundamental frequency.

The Proposed Structure

As mentioned in the introduction, if the resonators are set to harmonic components of the input signal, then the input of resonators – that is

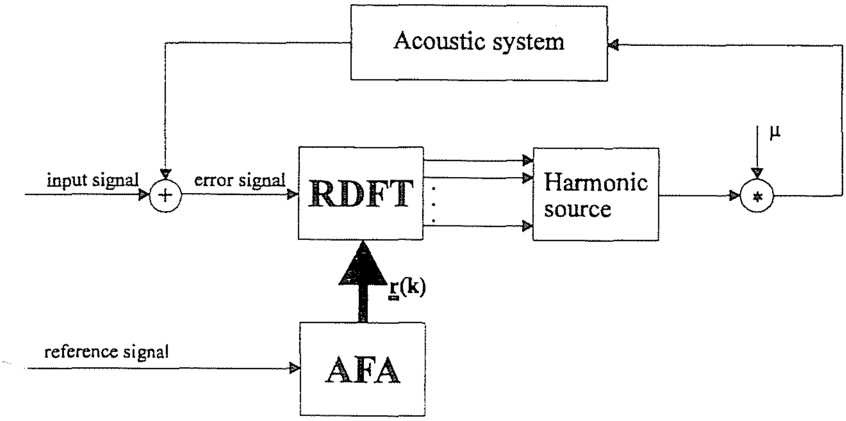

Fig. 1. The proposed structure

the error signal – will be equal to zero. This feature will be used for noise suppression. Let us multiply the outputs of resonators by weighting coefficients w_i^{-1} so that:

$$w_i = W(z_i), \tag{5}$$

where $W(z)$ is the transfer function of the acoustic system, and it has to be evaluated in each resonator frequency. Furthermore, let us take the acoustic system to the feedback path. (The aim of the method developed was that any IIR system can be used in the feedback path.) However, instead of negative static feedback we have used a dynamic system. Obviously, there will be some stability problems. Decreasing the loop gain some results can be reached, but these are not satisfactory. Indeed, we need changes in the structure. The next (and the proposed) structure can be found in *Fig. 1.* It can be seen that there are *two* feedback paths in the figure. The system can be described as follows:

$$\begin{bmatrix} \mathbf{x}_{n+1}^1 \\ \mathbf{x}_{n+1}^2 \\ \mathbf{x}_{n+1}^a \end{bmatrix} = \begin{bmatrix} <\mathbf{z}> -\mathbf{g}\mathbf{c}^T & -\mu d_a \mathbf{g}\mathbf{w}^T & -\mathbf{g}\mathbf{c}_a^T \\ <\mathbf{g}> & <\mathbf{z}> & 0 \\ 0 & \mu \mathbf{b}_a \mathbf{w}^T & \mathbf{A}_a \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x}_n^1 \\ \mathbf{x}_n^2 \\ \mathbf{x}_n^a \end{bmatrix} + \begin{bmatrix} \mathbf{g} \\ 0 \\ 0 \end{bmatrix} \cdot s_n, \tag{6}$$

$$y_n = \begin{bmatrix} 0 & \mu \mathbf{w}^T & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x}_n^1 \\ \mathbf{x}_n^2 \\ \mathbf{x}_n^a \end{bmatrix}, \tag{7}$$

where: $\mathbf{x}$ vectors mean the state vector of the RDFT structure, the harmonic source, and the acoustic system, respectively. $\mathbf{g} = \frac{1}{n}\overline{\mathbf{r}}$;

$\mathbf{c}^T = [1, 1..1]; \mathbf{w}^T$ is the weighting vector, s_n the input signal, and y_n is the output of the system. The acoustic system can be described by means of following equations:

$$\mathbf{x}_{n+1}^a = \mathbf{A}_a \mathbf{x}_n^a + \mathbf{b}_a y_n, \qquad y_n^a = \mathbf{c}^T \mathbf{x}_n^a + d_a y_n, \tag{8.a, b}$$

where y^a is the output of the acoustic system. The error signal is analysed by means of an ordinary resonator based recursive Fourier transformer, and the state variables – that is the Fourier coefficients – are integrated. This integration performs a 'harmonic source' that can be seen in *Fig. 2.* The operation of this method is obvious: if the error signal is equal to zero then the Fourier coefficients are zeros. Consequently there is no input to the integrators, so the output signal does not change. Naturally, we also need a loop gain factor (μ) in this structure. The measure of this gain depends on the transfer function in the feedback path. This μ depends on the acoustic transfer function.

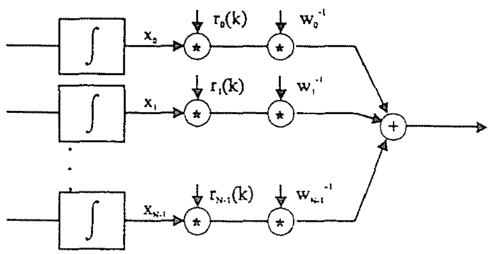

Fig. 2. The harmonic source

It is clear that the advantageous features mentioned above are valid only if the resonators exactly coincide with harmonic components of the input signal. That is why we need a reference channel for frequency adaptation. An Adaptive Fourier Analyser (AFA) finds the right frequency, and gives the corresponding data to any parts of the system.

As mentioned in the introduction, the reference signal can be taken from the suppressed one. To achieve this let us modify (4) so that:

$$f_{n+1} = f_n + \frac{1}{2\pi N} \text{abs}(\hat{x}_{n+1,L+2}) \text{angle}(\hat{x}_{n+1,L+2}, \hat{x}_{n,L+2}). \tag{9}$$

Using the suppressed signal as reference, the bottom part of *Fig. 1* can be ignored.

Resonator Based Transfer Function Measurement

It was mentioned in the introduction that the values of the acoustic transfer function have to be known densely enough. Our task was to obtain these (complex) values. It is already well known that the state variables of the resonator based structure give the Fourier coefficients of the input signal. However, the phase of these vectors depends on the initialisation of the system. If the phase angles of the rotating vectors are zeros, then by starting the system in phase with the input signal we achieve that the angles of the state variables give the phase angles of the input components. If there is only one sinusoid signal, only two complex conjugate resonators are needed.

So the measuring procedure is very simple. Using a sinusoid signal at the loudspeaker (it will be the secondary source) and analysing the signal of the error microphone – as described above –, the corresponding amplitude and phase can be obtained. It is clear that there is no information about the 2π multiple part of the phase shift caused by the delay. An exponential averaging was used to suppress the measurement noise, and in order to eliminate the DC component of the signal taken from the microphone, a DC resonator was used.

The Experimental Set-up

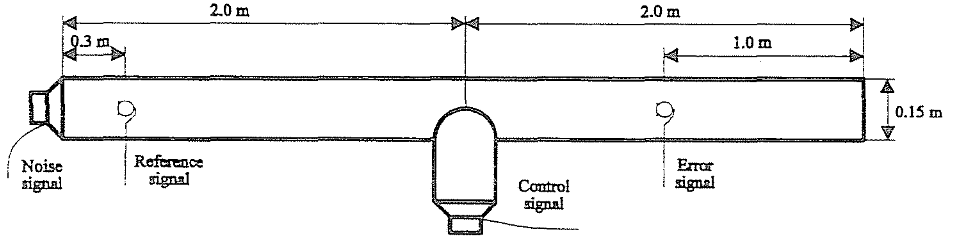

Fig. 3. The experimental set-up

The examined set-up (*Fig. 3*) is a simple model of a ventilation duct. It is a circular pipe with an attached loudspeaker for the simulation of the ventilator noise and another one for the secondary source. There are two built-in microphones, one for the reference input, one for the error signal. The used microphones are quite common, their characteristics are not sensitive to the input sound direction. The reference microphone is situated close to the noise source and the error microphone is placed beyond the secondary source. The reference signal can be taken from the reference microphone or from an external reference source. The set-up is quite a good model for a ventilation duct except for the air-flow present in a real ventilating system and causing difficulties in form of measurement noise.

For measurement and control purposes we have used a DSP board from the Loughborough Sound Images. It is a PC card with one Motorola DSP96002 signal processor and two analog input and output channels. The minimum sampling frequency of the on-board sigma-delta AD converter is 8 kHz, but at this signal rate the input to output delay of the board is approximately 6.4 ms, which is too much especially in case of stochastic primary noise. The ADs on the input and the DAs on the output have 16 bit resolution till a frequency of 96 kHz and only 12 bit above.

All the programs for the DSP are written in assembly language. We used the DPS96000CLASA package of Motorola for the compilation and Loughborough's MS-Windows based monitor for debugging. The results were evauated by a Hewlett Packard Spectrum Analyser and a simple storage oscilloscope.

Simulations, Results

A relatively universal program was developed in MATLAB in order to examine the structure. As mentioned above. our aim was to make a robust system which is stable at any arbitrarily chosen transfer function, especially at acoustic one. So the input parameters are:

1. the numerator and the denominator of the acoustic transfer function.
2. the loop gain factor μ.

The first practical question was the evaluation of the transfer function. Since the resonator frequencies are as dense as possible, we had to calculate the corresponding *ws* in each step of simulation. It makes the program much slower, and using this method is impossible on the hardware. Another possibility is to calculate these *ws* 'densely enough', and use the nearest ones. However, if (5) is not exactly satisfied, the system will be slower, therefore to achieve a fast control, the transfer function has to be known more densely. Apparently, since a high order system has zeros close to each other, so the phase of its transfer function changes relatively fast.

The outputs of the program can be seen in *Fig. 4.a,b,c,d.* Instead of the simulation of the acoustic system here a simple example is presented. The transfer function was a simple linear phase FIR filter with extra delays. It has 8 coefficients and 4 extra delays (i.e. 7.5 delays). The exact specification of this filter is not interesting, although the magnitude response of the filter is shown in *Fig. 4c*. It can be seen that there is a relatively wide range where the magnitude is at most −40 dB. We have chosen μ=0.07. The input signal was a 'band limited' triangular signal made of three sinusoids. The frequency of the signal was set to 0.04 (if the sampling frequency is 2 kHz, it means 80 Hz). It can be seen as the excitation in *Fig. 4a*. There are 12 harmonic components according to the frequency of the excitation. *Figs. 4b* and *d* show the transient process of the reference channel and the error channel, respectively. In this simulation, the reference was taken from the error signal, that is why its settling is not complete (namely, the error signal changes). The settling of the error channel is a typical exponential process.

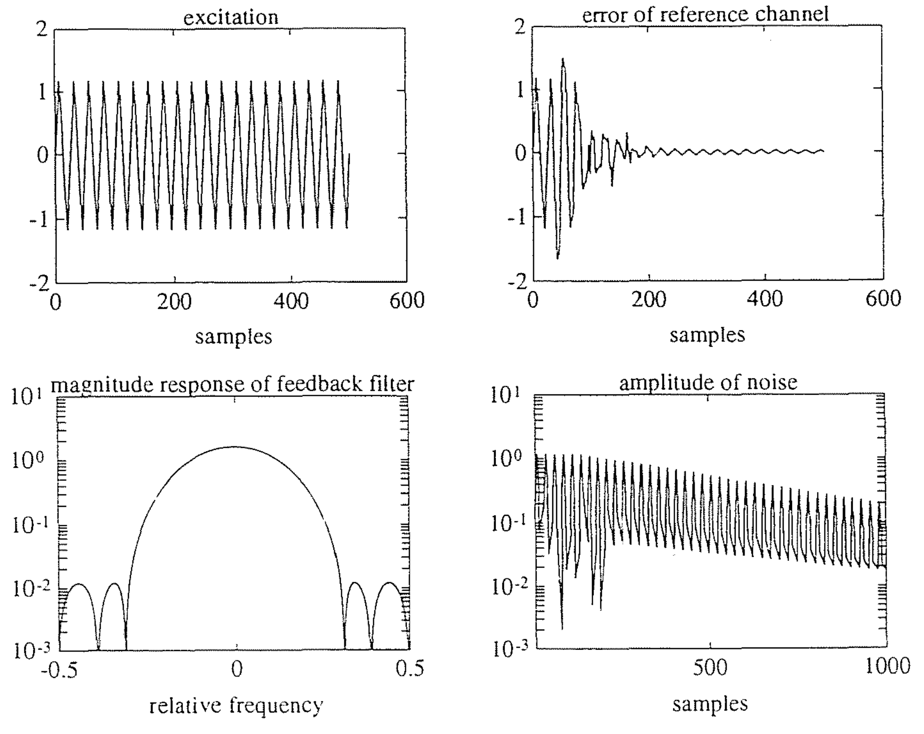

Fig. 4. a,b,c,d. Outputs of the MATLAB simulation

The on-line results were obtained on the hardware mentioned above. To avoid the large delay at A/Ds and D/As, the real sampling frequency was 96 kHz, but we used only every 48th sample, so mathematically the sampling frequency was 2 kHz. The speed of the program would allow a higher sampling frequency, but it is not useful for us, due to the large suppressions in the transfer function above 1 kHz. The frequency of the first harmonic component was approximately 100 Hz. So we used 8 resonators, that is we can suppress 8 harmonic components. The transfer values of the acoustic system were given in 200 points.

Various situations were examined. The real situation can be modelled very well by a tachometer generator or any sensor producing a small rectangular reference signal. In this case we have reached approximately 45 dB suppression at the fundamental frequency. At any other frequencies the suppression was smaller, due to their smaller amplitude. If the input signal is noisy, the noise is only suppressed in the vicinity of the resonator frequencies, but it does not disturb the quality of the suppression. A further advantage of the adaptation of the algorithm described in (9) is that if the reference signal leaves off, the frequency freezes but the system remains stable.

Conclusions

By means of the proposed structure better results could be reached than with other methods, because of the special periodic model built in the control algorithm. However, if the acoustic noise involves also stochastic noise, it cannot be suppressed, only near to the resonator frequencies. Since the exact identification of the acoustic system need not be solved, the algorithm requires smaller computational capacity. We have good experiencie, so our method can be proposed for periodic noise suppression.

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