# ACTIVE NOISE CONTROL – SIMULATION IN MATLAB

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### Abstract

The authors have developed a new method for periodic acoustic noise cancellation which is already proved to work efficiently in the practice. A simulation had preceded the implementation, in order to have experiments with different types of acoustic systems. The simulations were carried out in MATLAB which is widely used for such investigations. The aim of the paper is to summarize the conditions, the theoretical and the computational details of this work.

Keywords: periodic noise control, resonator based observer, MATLAB.

## Introduction

Nowadays the idea of active compensation of low-frequency acoustic noise or vibration is very popular (ELLIOT – NELSON, 1993). Although the basic ideas were published in the thirties, it became possible to realize them only in the last decade by the usage of signal processors. The active noise cancellation is based on the phenomenon of the destructive interference. A 'secondary' noise has to be generated, which suppresses the 'primary' (i.e. the original) noise at the properly situated microphones. The whole problem has two different branches: acoustics and control. The first problem is, how the microphones and the loudspeakers have to be arranged. The second one is, how the loudspeakers have to be controlled. In the last years many algorithms have been developed and implemented, and they also have two groups, depending on the type of the noise: algorithms for cancellation of stochastic or periodic noise.

In the last two years we have developed an algorithm for the cancellation of periodic noise. The novelty of our resonator based compensator is that it utilizes a so-called periodic signal model (PÉCELI 1986), which provides the total suppression of periodic components. The algorithm does not need to solve any identification problem exactly, it needs only the complex values of the acoustic transfer function 'dense enough'. For the estimation of the fundamental frequency of the primary noise the Adaptive Fourier Analyzer (AFA, NAGY, 1993) is used. The controller is a resonator based observer, in the feedback path of which is the acoustic system. The output of the resonators is connected to the loudspeaker(s) (to the secondary source); the signal(s) of the error microphone(s) (the difference of the primary and the secondary signals) is the input of the resonators. There are two versions of the controller: the first one is an ordinary resonator structure (*Fig. 2*), the second one uses the RDFT (Recursive Discrete Fourier Transform) of the error signal (*Fig. 4*). Both of them use complex multipliers at each resonator, in order to set the stability and also the speed of the whole system. Actually, these parameters are the inverses of the acoustic transfer function at the corresponding resonator frequency. The algorithm proved to work efficiently in the practice as well (SUJBERT – DUNAY, 1995). The investigation of the new method was carried out in MATLAB which is widely used for the simulation of signal processing or control problems.

# The Systems to be Investigated

As it was mentioned in the introduction, the novelty of the method is that it uses a so-called periodic signal model and the Adaptive Fourier Analyzer. Now, at first these systems will be introduced (PÉCELI, 1986; NAGY, 1993). The signal model and the observer in our case can be described as follows:

$$y_n = \mathbf{r}_n^T \mathbf{x}_n,\tag{1}$$

$$\mathbf{r}_n = [r_{n,k}] = e^{j2\pi f_1 k n}, \quad k = -L \dots L, \quad N = 2L + 1.$$
 (2)

$$\hat{\mathbf{x}}_{n+1} = \hat{\mathbf{x}}_n + \frac{1}{N} \bar{\mathbf{r}}_n (y_n - \mathbf{r}_n^T \hat{\mathbf{x}}_n),$$
(3)

$$Lf_1 < 0.5 < (L+1)f_1, \tag{4}$$

where  $\mathbf{x}_n$  is the state vector of the signal model,  $y_n$  is its output and the input of the observer,  $\hat{\mathbf{x}}_n$  is the estimated state vector,  $\mathbf{r}_n$  represents the basis of the transformation,  $f_1$  is the fundamental frequency and  $\bar{}$  denotes the complex conjugate operator.

The observer described in (3) can be seen in Fig. 1. Each channel contains an integrator, the outputs of which are the state variables. The channels are called resonators, because each channel has a pole on the unit circle (See Eq. (6)). The angles of the poles belong to the frequencies of the basis functions. If the input signal is periodic, consisting only of components of resonator frequencies then the input of the resonators (i.e. the



Fig. 1. Observer for periodic signals



Fig. 2. The first version of the compensator

feedback error) equals zero. Furthermore, in this case the corresponding state variables (as a complex vector) do not change. However, if the frequency of the input signal changes, the components of the state vector will rotate. The speed of this rotation at each resonator is proportional to the corresponding frequency difference. This is the basic idea for the frequency adaptation in the AFA (NAGY, 1993). The exact formula is the following:

$$f_{1,n+1} = f_{1,n} + \frac{1}{2\pi N} \text{angle}(\hat{x}_{1,n+1}, \hat{x}_{1,n}),$$
(5)

where  $\hat{x}_{i,1}$  is the state variable belonging to the positive fundamental frequency, and 'angle' gives the angle between two complex numbers.

The noise controller is a resonator based observer (*Fig. 2*). The Signal Generator on the figure is the feedforward path in the observer (*Fig. 1*). The input of resonator channels comes from an AD converter, the sum of

the channels multiplied by  $-\mu$  is connected to a DA converter. The D/A is connected to a loudspeaker (it will be the secondary source), and the A/D is connected to a microphone. The simplified arrangement can be seen in Fig. 3 where G(z) represents the acoustics and  $\sum H_i(z)$  is the controller. The **r** vector is taken from an AFA. The reference signal for AFA can be any signal, relatively free of noise, with the same frequency as the primary source.

All observers contain a feedback to set their dynamic features. The resonator based observer has a negative feedback path also, but it contains a dynamic system (acoustics) which can cause stability problems.



Fig. 3. Dynamics in the feedback path

See Fig. 3! The  $\mu$  in the figure is a positive scalar parameter which the loop gain can be modified with. Let us suppose that the dynamics is stable and it contains no resonators. The open loop transfer function of a resonator channel is:

$$H_i(z) = \frac{\mu}{N} \frac{z_i}{z - z_i},\tag{6}$$

where  $z_i$  is the resonator pole. For the whole system the open loop transfer function is:

$$W(z) = G(z) \sum_{i=1}^{N} H_i(z),$$
(7)

where G(z) is the transfer function of the system which is in the feedback path. To achieve stability, each resonator output is multiplied with the complex value  $w_i$ , where:

$$w_i = G^{-1}(z_i), \tag{8}$$

where  $z_i$  is the resonator pole. It is already proved (SUJBERT – DUNAY, 1995) that for the stability only the phases of w values are important. Though the amplitude of w values influences the control speed: the system will be the fastest (at a given  $\mu$ ) in case of the settings in (8). For the investigations regarding the control speed the state matrix of the system will be important. For a given G(z) it is the following:

$$\begin{bmatrix} \mathbf{x}_1(k+1) \\ \mathbf{x}_g(k+1) \end{bmatrix} = \begin{bmatrix} \langle \mathbf{z} \rangle - \mu d_2 \mathbf{g} \mathbf{w} & -\mathbf{g} \mathbf{c}_g \\ \mu \mathbf{b}_g \mathbf{w} & \mathbf{A}_g \end{bmatrix} \begin{bmatrix} \mathbf{x}_1(k) \\ \mathbf{x}_g(k) \end{bmatrix},$$
(9)

where  $x_1(k)$  is the state vector of the controller, z is a vector containing the resonator poles and  $\mathbf{g} = 1/N\mathbf{z}$ , w is a vector with components  $w_i$ , and the dynamic system G(z) can be described with its state equations:

$$\mathbf{x}_g(k+1) = \mathbf{A}_g \mathbf{x}_g(k) + \mathbf{b}_g s(k), \tag{10}$$

$$y(k) = \mathbf{c}_g \mathbf{x}_g(k) + d_g s(k), \tag{11}$$

where y(k) is the output, and s(k) is the excitation.

This structure works well, but it is sensitive to the  $\mu$  parameter. It means that there is only a small interval of  $\mu$  where the system is stable, and it is difficult to set such a  $\mu$  value by which the system has an acceptable speed. (See also the Example!) Due to this disadvantage of the structure, some modifications are needed. The proposed structure can be seen in *Fig. 4.* An Adaptive Fourier Analyzer finds the fundamental frequency of the signal to be suppressed, and gives the corresponding **r** vector to the compensator. In this compensator the state variables of the signal model are modified by the transformed error signal. The statements remain valid regarding the **w** vector. The state-space description of the whole system:



Fig. 4. The proposed version of the compensator

 $\begin{bmatrix} \mathbf{x}_{1}(k+1) \\ \mathbf{x}_{2}(k+1) \\ \mathbf{x}_{g}(k+1) \end{bmatrix} = \begin{bmatrix} \langle \mathbf{z} \rangle - \mathbf{g} \mathbf{c}_{1} & -\mu d_{g} \mathbf{g} \mathbf{w} & -\mathbf{g} \mathbf{c}_{g} \\ \langle \mathbf{g} \rangle - \langle \frac{1}{N} \rangle \mathbf{g} \mathbf{c}_{1} & \langle \mathbf{z} \rangle - \mu d_{g} \langle \frac{1}{N} \rangle \mathbf{g} \mathbf{w} & -\langle \frac{1}{N} \rangle \mathbf{g} \mathbf{c}_{g} \\ \mathbf{h} \mathbf{b}_{g} \mathbf{w} & \mathbf{A}_{g} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1}(k) \\ \mathbf{x}_{2}(k) \\ \mathbf{x}_{g}(k) \end{bmatrix},$ (12) where  $\mathbf{x}_{2}(k)$  is the state vector of the signal generator,  $\mathbf{c}_{1} = [1, 1, ... 1]$ . For

the other variables see (9), (10) and (11).
 All of the systems described above are Single Input - Single Output (SISO) systems. If there are more microphones and more loudspeakers,

the Multi Input – Multi Output (MIMO) system has to be constructed. In this case G(z) will be a matrix at a certain frequency:

$$\mathbf{e}(z) = \mathbf{G}(z)\mathbf{y}(z),\tag{13}$$

where  $\mathbf{e}(z)$  is the input vector (from the error microphones),  $\mathbf{y}(z)$  is the output vector of the controller. A signal generator is connected to each output, and the number of subtractors equals the number of inputs. The w parameters have to be used between each subtractor and resonator input. Let us cut the loop at the inputs of the resonator channels. In the SISO system the open loop gain of each channel was set to unity. It was the best way as well for stability as for the speed of control. In the MIMO system it is also necessary, hence the w parameters (it is a  $\mathbf{W}$  matrix at a certain frequency) have to be calculated in the following way:

$$\mathbf{W}(z_i) = \mathbf{G}^{\#}(z_i),\tag{14}$$

where # denotes the pseudo- (or Moore-Penrose) inverse. If the number of inputs equals the number of outputs, the ordinary inverse can be used. An example can be seen in *Fig. 5.* 



Fig. 5. Compensator for multidimensional case

#### Simulation

The program written had to satisfy some requirements regarding the controller and the acoustics.

- The acoustics can be considered as a linear system (DOELMAN, 1993), which can be described by a rational discrete transfer function. It is a high-order transfer function containing large delay (e.g. the order of the denominator equals 30, the order of the numerator equals 90, in terms of  $z^{-1}$ ). This transfer function (if it is needed) is a solution of an identification problem, and it is affected with considerable measurement error. In addition, this transfer function is changing, due to the physical circumstances. Fortunately the acoustic frequency response can be measured accurately enough in many points even in the practice. In practical cases G(z) is not analytically known. It means that w values cannot be calculated for an arbitrarily chosen frequency, and, in addition, this calculation would be impossible to realize on-line. However, it is proved (SUJBERT – DUNAY, 1995) that the phase shift at a resonator can change in the range of  $(-\pi/2, \pi/2)$ . If the complex transfer values of G(z) are known (e.g. they are measured) 'dense enough' in the whole frequency range, this set can be used to calculate the actual w vector. It depends on G(z), what 'dense enough' means. If the phase changes rapidly, lots of measurement points are needed. The calculation of the actual w could be very simple: for a certain frequency the nearest measurement result could be used.

- The AFA is an independent system, the input of which is either the reference signal or the error signal. Since it is a structurally adaptive system which gives the frequency information to the other parts of the system, the behavior of the whole system will be nonlinear. Its stability can be investigated only by simulation in the time-domain. While the frequency is being adapted, the state variables change quasi randomly.
- The stability of the system has to be investigated at a constant fundamental frequency, i.e. the system is linear. The state variables of the controller contain random data after the frequency estimation. A related feature is the speed of the control which has to be examined also.
- The system to be analyzed consists of individual components which cannot be found in any tool-box of MATLAB.

According to the above requirements, two groups of programs were written. The first one simulates the working of the algorithm in the discrete time-domain. The programs in the first group differ from each other in the type of the algorithm (with or without RDFT), and in the number of the channels (SISO or MIMO case). Their main features are the following:

- The input of the system (as well for primary noise as for reference signal) is uploaded from a file. The acoustics can be described by its numerator and denominator. There is also a positive scalar input parameter which the loop gain can be modified with.
- Although the method does not need the numerator and the denominator of the acoustic transfer function, they are necessary to simulate the transients and test the stability. In the initialization phase the transfer values are calculated and during the simulation the nearest ones are used.
- The outputs of the program are different diagrams: the excitation, the magnitude response of the acoustics, the feedback error of the

AFA and the signal of the microphone. The resonator poles are also represented.

In order to investigate the speed of the control, a different approach is used. By means of the exact acoustic transfer function the state-space description can be used. It is proved that the system is the fastest if the absolute value of its maximal eigenvalue is minimal. The input parameters of these programs were the  $\mathbf{w}$  vector and the loop gain. The outputs are in this case also diagrams: a curve consisting of the greatest eigenvalues as a function of the loop gain, with a certain set of  $\mathbf{w}$ . The minima of the curves give the appropriate loop gain, and it can be checked visually in the time-domain whether this value is accurate or not. In this phase of the research it is not mathematically proven that the above setting of  $\mathbf{w}$  is the best regarding the control speed. Adding a small random vector to the  $\mathbf{w}$ vector, the new curve can be calculated. Such experiments prove for the practice the truth of the above conjecture about  $\mathbf{w}$ .

Due to the last condition mentioned above we have used neither any tool-box of MATLAB nor higher-level tools (e.g. SIMULINK). In our special case, where the structure is given, this solution seems better, regarding the running time of the program. The speed of the program is important, particularly in the case of the eigenvalue problem. The last versions of the program are written in MATLAB 4.2 and run on Sun Workstations.

# Speed of a Linear System

It was already mentioned that the speed of the control is very important. It is supposed from the simulations that (8) guarantees the fastest system, at least in the case of uniformly placed resonators. These investigations were carried out by means of the state matrices of the systems. The aim of this section is to prove that our method was right.

Now the settling of an IIR linear system will be investigated. An IIR system has definitely infinite impulse response, its settling is asymptotic. However, the settling can be considered complete when the impulse response is smaller than a prescribed small positive constant  $\varepsilon$ . It is well-known that the impulse response consists of exponential functions of the eigenvalues of the state matrix. In this section, the expression *transient* response instead of *impulse* response will be used. In this way it will be emphasized that the settling should be investigated in case of any initial state vector. The following description will precisely show that the transient response can be handled by the greatest eigenvalue of the system, by which a good estimation of the settling time can be given. The transient response is strongly related to the energy accumulated in the system. This is why

we use the Euclidean norm for the investigations. Note that it is not a restriction, because any equivalent norm can be used for the proof. Let us consider a discrete linear invariant system, which has no excitation. It can be described by its state equations:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k), \quad \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k), \quad \mathbf{x}(k) = \mathbf{A}^{k}\mathbf{x}(0), \tag{15}$$

where  $\mathbf{x}(k)$  is the state vector,  $\mathbf{y}(k)$  is the output vector,  $\mathbf{A}$  is the state matrix, and  $\mathbf{C}$  is the output matrix. The matrix  $\mathbf{A}$  is quadratic, its order is N. Let the system be stable,  $|\lambda_i| < 1$ , i = 1..N, i.e. the eigenvalues of the state matrix are inside the unit circle.

**Definition 1.** The transient response of a system is smaller than  $\varepsilon > 0$ , if  $||\mathbf{y}(k)|| < \varepsilon$ ,  $\forall k > k_0(\varepsilon)$ .

Let  $\{\varepsilon_m\}$  be a set of  $\varepsilon_m$ :  $\varepsilon_m \ge \varepsilon_n$  if n > m and  $\lim_{m \to \infty} \varepsilon_m = 0$ .

**Definition 2.** Let us consider the systems  $\Sigma_i = [\mathbf{A}_i, \mathbf{C}_i], i = 1, ...$  and a set of  $\{\varepsilon_m\}$ . There is no faster system than  $\Sigma_L$  if there is no initial state vector of  $\Sigma_L$  at each initial vector of  $\Sigma_i$ :  $\forall m > M ||y_L(k_L)|| > ||y_i(k_i)||$ ;  $k_L > k_{0L}(\varepsilon_m); k_i > k_{0i}(\varepsilon_m); i = 1, ...; i \neq L$ . Remark. The above definition means that the transient responses are compared only if they are already smaller than a common  $\varepsilon$ . Different initial conditions produce different transient responses, regarding the coefficients of the exponentials. This is why the norm is used during this section. If a system has multiple poles, the transient response has to be calculated over the above set of  $\varepsilon$ s.

**Theorem.** Let us consider the systems  $\Sigma_i$ , i = 1, ... There is no faster system than  $\Sigma_L$ , if  $|\lambda_1(\mathbf{A}_L)| = \min_i |\lambda_1(\mathbf{A}_i)|$ , where  $|\lambda_1(\mathbf{A})| = \max_{j=1..N} |\lambda_j(\mathbf{A})|$ .

*Proof:* Let  $\mathbf{T}$  be a transformation so that:

$$\tilde{\mathbf{A}} = \mathbf{T}\mathbf{A}\mathbf{T}^{-1} = \mathbf{J},\tag{16}$$

where  $\mathbf{J}$  is the Jordan form of the matrix  $\mathbf{A}$ . Consequently:

$$\tilde{\mathbf{x}}(k) = \mathbf{T}\mathbf{x}(k), \qquad \tilde{\mathbf{C}} = \mathbf{C}\mathbf{T}^{-1}.$$
 (17)

From (15),(16) and (17):

$$\|\mathbf{y}(k)\| = \|\mathbf{C}\mathbf{T}^{-1}\tilde{\mathbf{A}}^{k}\mathbf{T}\mathbf{x}(0)\|.$$
(18)

Furthermore:

$$\|\mathbf{y}(k)\| \le \|\mathbf{C}\mathbf{T}^{-1}\| \|\tilde{\mathbf{A}}^{k}\| \|\mathbf{T}\| \|\mathbf{x}(0)\|.$$
(19)

**C**, **T** and  $\mathbf{x}(0)$  are constant. With these remarks:

$$\|\mathbf{y}(k)\| \le c \|\tilde{\mathbf{A}}^k\|,\tag{20}$$

where c is a positive scalar number. At first consider the state matrices which have simple structure. It means that  $\tilde{\mathbf{A}} = \langle \lambda_i \rangle$ , i = 1..N.

$$\|\mathbf{y}(k)\| \le c \|\tilde{\mathbf{A}}\|^k = c |\lambda_1|^k, \tag{21}$$

where  $\lambda_1$  is the eigenvalue which has the maximal absolute value, on the other hand  $|\lambda_1| = \sigma_1$  which is the largest singular value of  $\tilde{\mathbf{A}}$ , i.e. its norm. Hereby the **Theorem** for simple matrices is proven.

Now, let us consider the case of the matrices which have no simple structure. It means that J in (16) has diagonal blocks:

$$\mathbf{J} = \langle \mathbf{J}_1, \mathbf{J}_2, ..., \mathbf{J}_M \rangle, \tag{22}$$

where

$$\mathbf{J}_{j} = \begin{bmatrix} \lambda_{i} & 1 & 0 & 0 & \cdots & 0\\ 0 & \lambda_{i} & 1 & 0 & \cdots & 0\\ & \vdots & & & \\ 0 & \cdots & 0 & \lambda_{i} & 1\\ 0 & \cdots & 0 & \lambda_{i} \end{bmatrix} \quad i = 1...N, j = 1...M.$$
(23)

From (15) and (16) it is known that the calculation of  $\tilde{\mathbf{A}}^k$  is necessary. It can be calculated by means of the Jordan blocks (Rózsa, 1991):

$$\tilde{\mathbf{A}}^{k} = \langle \tilde{\mathbf{A}}_{j}^{k} \rangle = \langle \sum_{l=0}^{p_{j}-1} \frac{k^{l}}{l!} \lambda_{j}^{k-l} \mathbf{J}_{j}^{l} \rangle, \qquad (24)$$

where j = 1...M and  $p_j$  is the size of the *j*-th Jordan-block. For example, let us consider a  $3 \times 3$  Jordan block ( $p_j = 3$ ). It means that the state matrix has an eigenvalue at least triple. (The system has a triple pole.) So the Jordan block:

$$\tilde{\mathbf{A}}_{j}^{k} = \begin{bmatrix} \lambda_{j}^{k} & k\lambda_{j}^{k-1} & \frac{k^{2}}{2}\lambda^{k-2} \\ 0 & \lambda_{j}^{k} & k\lambda_{j}^{k-1} \\ 0 & 0 & \lambda_{j}^{k} \end{bmatrix}.$$
(25)

It is clear that the norm of the matrix  $\tilde{\mathbf{A}}^k$  is the maximum of the norms of the blocks  $\tilde{\mathbf{A}}_i^k$ . Furthermore:

$$\tilde{\mathbf{A}}^{k} = \langle \lambda_{j}^{k-p+1} \sum_{l=0}^{p_{j}-1} \frac{k^{l}}{l!} \lambda_{j}^{l+p_{j}-1} \mathbf{J}_{j}^{l} \rangle = \langle \lambda_{j}^{k-p_{j}+1} \mathbf{P}_{j}(k) \rangle,$$
(26)

where  $\mathbf{P}_{j}(k)$  is a matrix the elements of which are polynomials of k. See (26) and the example (25)! It is easy to imagine that the norm of the matrix  $\mathbf{P}_{j}(k)$  for large ks is approximately  $[\mathbf{P}_{j}(k)]_{1,p_{i}}$ . Thus:

$$\lim_{k \to \infty} (\|\mathbf{P}_j(k)\| - [\mathbf{P}_j(k)]_{1,p_j}) = 0.$$
(27)

In the example (25) this estimated norm is  $k^2/2$ . The last equation means that:

$$\|\tilde{\mathbf{A}}_{j}^{k}\| \approx \lambda_{j}^{k-p_{j}+1} \frac{k^{p_{j}-1}}{(p_{j}-1)!}.$$
(28)

Let  $\mu_j$  be a positive scalar number so that:

$$\mu_j > 1 , \ |\lambda_j \mu_j| < 1.$$
 (29)

Let us write  $\mu_j^{k-p_j+1}$  in (28) instead of the polynomial. It is true that:

$$\lambda_j^{k-p_j+1} \frac{k^{p_j-1}}{(p_j-1)!} < (\lambda_j \mu_j)^{k-p_j+1},$$
(30)

if  $k > k_0(\mu_j)$ . Now, let  $\mu_j = 1 + \eta_j$ , where  $\eta_j$  is a small positive scalar. In this case for large ks each Jordan block can be represented by diagonal blocks the element of which is  $\lambda_j(1+\eta_j)$ . These elements are approximately the same as the corresponding eigenvalue. Summarising the results form (21) it can be written:

$$\|\mathbf{y}(k)\| \le c \max_{j=1...M} |\lambda_j|^{k-p_j+1} \le c |\lambda_1|^k,$$
(31)

if  $k > k_0$ . Thus the **Theorem** is already proven.

At last a practical and simple estimation of the settling of a linear system can be given. The transient response of a system is less than  $\varepsilon$ , if:

$$k > \frac{\log \varepsilon}{\log \lambda}.$$
(32)

#### An Example

Instead of the simulation of an acoustic system, a simple example is presented here. The controller is the one shown in Fig. 4. See Fig. 6 ! G(z) is a linear phase FIR filter, but it has a relatively wide range where the magnitude is at most -40 dB. The filter has 8 coefficients. The input signal was a 'band-limited' triangular signal made of three sinusoids. The frequency of the signal was set to 0.05. (If the sampling frequency is 2 kHz, it means 100 Hz.) The figures on the right side show the transient process of the reference channel and the suppressed signal. In this case  $\mu = 0.3$  was chosen. It can be seen clearly that in the first time steps the frequency is not estimated, because the error of the reference channel does not equal zero. During this frequency estimation the signal to be suppressed does not change. too. The exponential settling starts only after the complete settling of the reference channel.



Fig. 6. The result of time-domain simulation

Fig. 7 is even more interesting: the two curves show the greatest eigenvalues of the whole control system in the case of the above FIR filter as a function of  $\mu$  ( $0 < \mu < 1$ ) in 100 points. Both of them has an interval where the greatest eigenvalue is less than unity, but in case of the upper curve it is very small, and the minimum is 0.998. This curve belongs to the simpler system which can be seen in Fig. 2. The lower curve belongs to the system analyzed above (Fig. 4), here the stability interval is broader, and the minimum is smaller which means that this system is faster. Such figures helped us to choose the second structure (*Fig.* 4) for the implementation. The sensitivity to the  $\mu$  parameter is very important, because there is no analytical information about the acoustics in the practice.



Fig. 7. Eigenvalues of different compensators

### Conclusion

Our noise control algorithm is already implemented on different DSP cards and works on different hardwares. The experiences verify our expectations based on the simulation which means that the simulation was also successful. First there was no theoretical result about the stability of the proposed method. By means of the simulation we were able to prove the statements for the practice. We hope that this simulational environment will help us to present new results in this field.

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