SPACE PHASOR TRANSIENT ANALYSIS OF ASYMMETRICAL A.C. MACHINES WITH SPACE HARMONICS

Luis Fernando Mantilla PEÑALBA and Miguel Angel Rodriguez POZUETA

Universidad de Cantabria
Escuela Técnica Superior de Ingenieros Industriales y de Telecomunicación
Avda. los Castros, s/n. 39005. Santander, Spain
Fax: +34 42 201-873. E-mail: mantillf@ccaix3.unican.es

Received: January 9, 1996

Abstract

The paper develops the Space Phasor Theory for obtaining the harmonic phasor equations of a balanced winding of m asymmetrical phases.

Some Equivalence Principles are stated and some equivalent harmonic space phasors of e.m.f. and voltage are defined. This enables us to analyse the effect of all harmonics on a winding using few equations.

The authors give a physical interpretation of the harmonic phasor equations and represent them by means of space phasor diagrams.

A phasor study of an asynchronous machine with harmonics of both the winding distribution and air-gap irregularities is also given.

List of Symbols

D	Air-gap diameter
e_k	Instantaneous e.m.f. of phase k
i_k	Instantaneous current of phase k
J	Moment of inertia
j	Imaginary unit
L_σ	Leakage inductance
l	Air-gap length
R	Resistance
T	Electromagnetic torque
T_L	Load torque
t	Time
u_k	Instantaneous voltage of phase k
Z	Number of conductors
δ	Air-gap
$\overline{\Phi}_{\mathrm{cor,v,maq}}$	v-th harmonic space phasor of the total yoke flux
x	Cross vectorial product
$*$	Complex conjugate

1. Introduction

Studies of a.c. electrical machines transients usually assume a sinusoidal distribution of air-gap magnetic field. This is accurate enough in the majority of cases. However, in certain studies (such as asynchronous machine starting or speed variation by pole amplitude modulation (PAM)) the influence of the magnetic field space harmonics must be taken into account. In some of these cases, such as PAM, asymmetrical windings are involved.

Some authors have analysed electrical machines with space harmonics in different ways. Most of them have used the General Machine Theory ([1], [4], [5]) or the Vector Spatial Theory, extending its definition given by KOVÁCZ and RÁCZ ([15], [16], [17]).

Another way to do this analysis was first presented by STEPINA ([8], [9], [10]) by means of the Space Phasors Method. These phasors represent internal machine quantities, so they have an immediate physical interpretation, and their calculation for each machine conductor effect enables asymmetrical machines to be studied easily.

Certainly, [8], [9], [10] and [18] had studied asymmetrical and, even unbalanced, electrical machines by space phasors. However, these papers do not use real phasor methods because they only apply space phasors to obtain the flux linkages of each phase winding of the machine. Furthermore, they do not operate with phasor equations but with electrical equations of each phase.

A real phasor analysis of an electrical machine with space harmonics requires the previous definition of some harmonic space phasors of electric voltage and establishes the harmonics space equations of each winding of the machine.

SERRANO gave the definition of the space phasors of electric voltage, resistive voltage drop and inductive voltage drop in [12], [13] and [14]. These definitions are general and useful for electrical machines with space harmonics. Moreover, he obtained the formulae to calculate these phasors and the phasor equation that relates them when the machine has no space harmonics. He also got in this case the phasor diagram that graphically represents the phasor equation.

In this paper the authors perform a real phasor study of a balanced and asymmetrical electrical machine with both e.m.f. and air-gap permeance harmonics. The use of the Symmetrical Component Method with the space harmonics of each phase winding distribution could probably extend this study to unbalanced and asymmetrical machines.

Here, each space phasor represents an internal magnitude of the machines and thus they have physical meaning (although some space phasors refer to the situation when the magnetic field acting on the winding is not the real field but an 'equivalent' field). In this analysis, the formulae that give the relationships among phase magnitudes of a winding and their space phasors and vice versa are stated. Later, the authors establish the phasor equations for each 'basic harmonics' of every winding and graphically represent them by their corresponding harmonic phasor diagrams.

Finally, this paper gives the phasor model of an asynchronous machine with magnetic field space harmonics, whose rotor could have either a winding or a squirrel cage. (In squirrel cage windings we define each phase as consisting of two successive bars and two end ring segments between them [1]).

2. Simplifying Assumptions, Complex Winding Factor

– This study is developed under the following assumptions:

a) All the machine windings are balanced, but they can be asymmetrical. Their phase number could be higher than three.
b) The winding conductors have a negligible cross-section.
c) The magnetic circuits are linear and the iron losses are not taken into account.
d) All the machine cross-sections are equivalent, thus, the analysis is bidimensional.
e) The leakage flux of a phase is proportional to its current.

– The phases of the machine windings that we are going to analyse could be *asymmetrical.*

These phases have all their conductors connected in series and their distribution has p identical groups. Thus, the Fourier series of the magnetic potential difference F generated by one of these phases has only harmonics whose absolute orders v (the absolute order v of a harmonic is its pole pairs number) are integer multiples of p. Both even and odd multiples of p harmonic orders could exist.

In some machines, some of these phases are connected parallel to a phase of an electrical network. However, we do not regard this parallel group as one phase but rather as several phases in parallel connection and, thus, with the same terminal voltage.

This study admits that windings of the same machine could have different values of p.

– It is useful to use *complex winding factors* (STEPINA [8]) to analyse asymmetrical phases.

A phase k, with Z_k conductors in series, that has $Z_{y,k}$ conductors inside the slot y, and which is in the position given by the geometrical angle α_y, has the following complex factor for the v-th harmonic:

$$\overline{\xi}'_{v,k} = \frac{\sum_y \pm Z_{y,k} \cdot e^{jv\alpha_y}}{Z_k} \ . \tag{1}$$

In the expression the $\pm$ sign corresponds to the winding direction of the phase k conductors in each slot.

If the phase is symmetrical, the module of $\overline{\xi}'_{v,k}$ is equal to the absolute value of the classical winding factor.

- A polyphase *balanced* winding has m identical phases which have a separation of $\frac{2\pi}{mp}$ radians between two successive phases.

Thus, if A is the first of the phases of a polyphase and balanced winding from *Eq.* (1) follows that the complex winding factor of the phase k is:

$$\overline{\xi}'_{v,k} = \overline{\xi}'_{v,A} \cdot e^{jv(k-1)\frac{2\pi}{mp}} \ . \tag{2}$$

- We are only using geometrical angles in this paper.

3. Harmonic Current Space Phasors

According to SERRANO ([11], [13]), the harmonic current space phasor $\overline{i}_v$ of a winding represents the v-th harmonic of the current sheet generated by this winding. This formula calculates it

$$\overline{i}_v = \sum_{k=1}^{m} \frac{2Z_k}{\pi D} \overline{\xi}'_{v,k} \cdot i_k \ . \tag{3}$$

For balanced windings, *Eqs.* (2) and (3) yield:

$$\overline{i}_v = \frac{2Z_k}{\pi D} \cdot \overline{\xi}'_{v,A} \sum_{k=1}^{m} i_k \cdot e^{jv(k-1)\frac{2\pi}{mp}} \ . \tag{4}$$

In a specific moment, when the phase currents of the winding are $i_A, i_B, \ldots, i_K, \ldots, i_M$, the result of *Eq.* (4) could only take a few values (which corresponds with the Instantaneous Symmetrical Components – according to Lyon's definition – of phase currents).

Thus, all harmonics of a balanced winding could be grouped in sets. Two current phasors, $\overline{i}_v$ and $\overline{i}_{v'}$, of the winding, corresponding to the harmonics v and v' (which belong to the same set of harmonics), fulfil some of these two *Currents Equivalence Principles*, based on those obtained by STEPINA and VAN DER MERWE [15]:

Currents Equivalence Principle I:

If

$$v = kmp + v' \ ; \quad k = 0, 1, 2, \ldots \tag{5.1}$$

it follows that

$$\frac{\bar{i}_v}{\bar{\xi}'_{v,A}} = \left(\frac{\bar{i}'_v}{\bar{\xi}'_{v',A}} \right) . \tag{6.1}$$

Currents Equivalence Principle II:

If

$$v = kmp - v' \ ; \quad k = 0, 1, 2, \ldots \tag{5.2}$$

it follows that

$$\frac{\bar{i}_v}{\bar{\xi}'_{v,A}} = \left(\frac{\bar{i}'_v}{\bar{\xi}'_{v',A}} \right)^* . \tag{6.2}$$

Each set is characterized by its *basic harmonic* v', that is, the harmonic whose order is the lowest and whose winding factor value is not zero. A harmonic v is related to its set basic harmonic v' by one of the two relations (5.1) and (5.2).

Notice that, with *Eqs.* (6.1) and (6.2), you can obtain all the harmonic current space phasors of the same set, knowing only the basic harmonic current phasor $\bar{i}_{v'}$.

– An interesting harmonic set of a winding is the one whose basic harmonic is $m \cdot p$. Its members are the harmonics which are multiples of $m \cdot p$. Hence all its harmonic current space phasors simultaneously fulfil the two Equivalence Principles (6.1) and (6.2), and each one of these phasors has the same direction as their corresponding complex winding factor and

$$\begin{aligned} \bar{\mathbf{i}}_v &= \frac{2Z}{\pi D} \bar{\xi}'_{v,A} \cdot (i_A + i_B + \ldots + i_M) \\ v &= kmp \quad k = 1, 2, 3, \ldots \ . \end{aligned} \tag{7}$$

Thus, these current phasors only exist if the phase currents of the winding have zero-sequence component.

– Another interesting harmonic set of a winding is the one whose basic harmonic is $\frac{mp}{2}$. This set only exists if m is even, its members are odd multiples of $\frac{mp}{2}$, each of its harmonic current space phasors has the same direction as its corresponding complex factor winding and:

$$\bar{\mathbf{i}}_v = \frac{2Z}{\pi D}\bar{\xi}'_{v,A} \cdot (i_A - i_B + i_C - i_D \dots) = \frac{2Z}{\pi D}\bar{\xi}'_{v,A} \cdot \left(\sum_{k\text{odd}} i_k - \sum_{k\text{even}} i_k \right) \quad (8)$$

$$v = k\frac{mp}{2} \quad k = 1, 3, 5, \dots \ .$$

– Following from *Eqs.* (4), (6.1) and (6.2), we get a *Current Correlation Theorem* valid for all balanced windings:

$$i_k = \frac{\pi D}{Zm} \sum_v \frac{1}{G_v} \cdot \mathrm{Re} \left\{ \frac{\bar{\mathbf{i}}_v}{\bar{\xi}'_{v,A}} \cdot e^{-jv(k-1)\frac{2\pi}{mp}} \right\} \ . \quad (9)$$

In this formula, the sum includes these members:

– If m is *odd*:

$$v = p, 2p, 3p, \dots, \frac{m-1}{2}p, mp \ .$$

– If m is *even*:

$$v = p, 2p, 3p, \dots, \frac{m}{2}p, mp \ .$$

When a harmonic v of the *Eq.* (9) sum has a winding factor whose value is zero, this harmonic must be replaced with another of its same set using the *Equivalence Principles* (6.1) and (6.2).

G_v is a coefficient whose value is 2 for the harmonics multiple of mp and $\frac{mp}{2}$. Its value is 1 in any other case.

– If a winding has *symmetrical phases*, all harmonics that are even multiples of p have winding factors whose values are zero and p is equal to the number of pole pairs of the winding. This does not cause problems when we try to use the Theorem (9) for symmetrical windings with odd number of phases because every set contains both odd and even multiples of p harmonics. Thus, it is always possible to replace an even multiple of the p harmonic with one odd multiple of p harmonic of the same set in *Eq.* (9).

However, if m is even, there are some harmonics of the sum of *Eq.* (9) which cannot be replaced with odd multiples of p harmonics and, thus, *Eq.* (9) cannot be used.

This is because in these windings the influence of two diametrical phases, k and L ($L = k + \frac{m}{2}$), on the harmonic current space phasors $\bar{i}_v$ is always proportional to the difference $i_k - i_L$. Hence, with the current phasors $\bar{i}_v$ we can only obtain the value of $i_k - i_L$ but not each phase current value.

When $i_k - i_L$ is known, each phase current value can be obtained by means of the following additional equation

$$u_k + u_L = R(i_k + i_L) + L_\sigma \frac{d}{dt}(i_k + i_L) \ , \qquad \left(L = k + \frac{m}{2} \right) \ . \quad (10)$$

Fig. 1 summarizes how to use the Current Correlation Theorem in symmetrical windings.

4. Monoharmonic Phasor Equations for Balanced Short-Circuited Windings

- The electrical equation of the phase k of a balanced short-circuited winding is

$$e_k = R \cdot i_k + L_\sigma \cdot \frac{di_k}{dt} \ . \tag{11}$$

The e.m.f. e_k, induced on the phase k, could be given by the First General Correlation Theorem ([8], [9]):

$$e_k = \sum_{v=p}^{\infty} e_{v,k} \ , \tag{12}$$

$$e_{v,k} = Z \cdot Re\left\{\overline{e}_v \cdot \overline{\xi}'_{v,k}\right\} = Z \cdot Re\left\{\overline{e}_v \cdot \overline{\xi}'_{v,A} \cdot e^{-jv(k-1)\frac{2\pi}{mp}}\right\} \ .$$

- $i_{v,k}$ is the current in the phase k of a balanced short-circuited winding when only the v-th order of the magnetic field acts on it and, thus, the e.m.f. induced in it is $e_{v,k}$.

$$e_{v,k} = R \cdot i_{v,k} + L_\sigma \cdot \frac{di_{v,k}}{dt} \ . \tag{13}$$

Following from *Eqs.* (12) and (13), we get the *Current Correlation Theorem for balanced short-circuited windings*:

$$i_k = \sum_{v=p}^{\infty} i_{v,k} \ ,$$

$$i_{v,k} = \frac{\pi D}{mZ\left|\overline{\xi}'_{v,A}\right|^2} \cdot Re\left\{\overline{\mathbf{i}}^v_v \cdot \overline{\xi}'^{\,*}_{v,k}\right\} = \tag{14}$$

$$= \frac{\pi D}{mZ\left|\overline{\xi}'_{v,A}\right|^2} \cdot Re\left\{\overline{\mathbf{i}}^v_v \cdot \overline{\xi}'^{\,*}_{v,A} \cdot e^{-jv(k-1)\frac{2\pi}{mp}}\right\} \ ,$$

where $\overline{i}^v_v$ is the current space phasor that represents the v-th harmonic of the current sheet generated by the short-circuited winding when its current phases are $i_{v,A}, i_{v,B}, \ldots$ that is to say, when only the v-th harmonic of the machine magnetic field effect is taken into account.

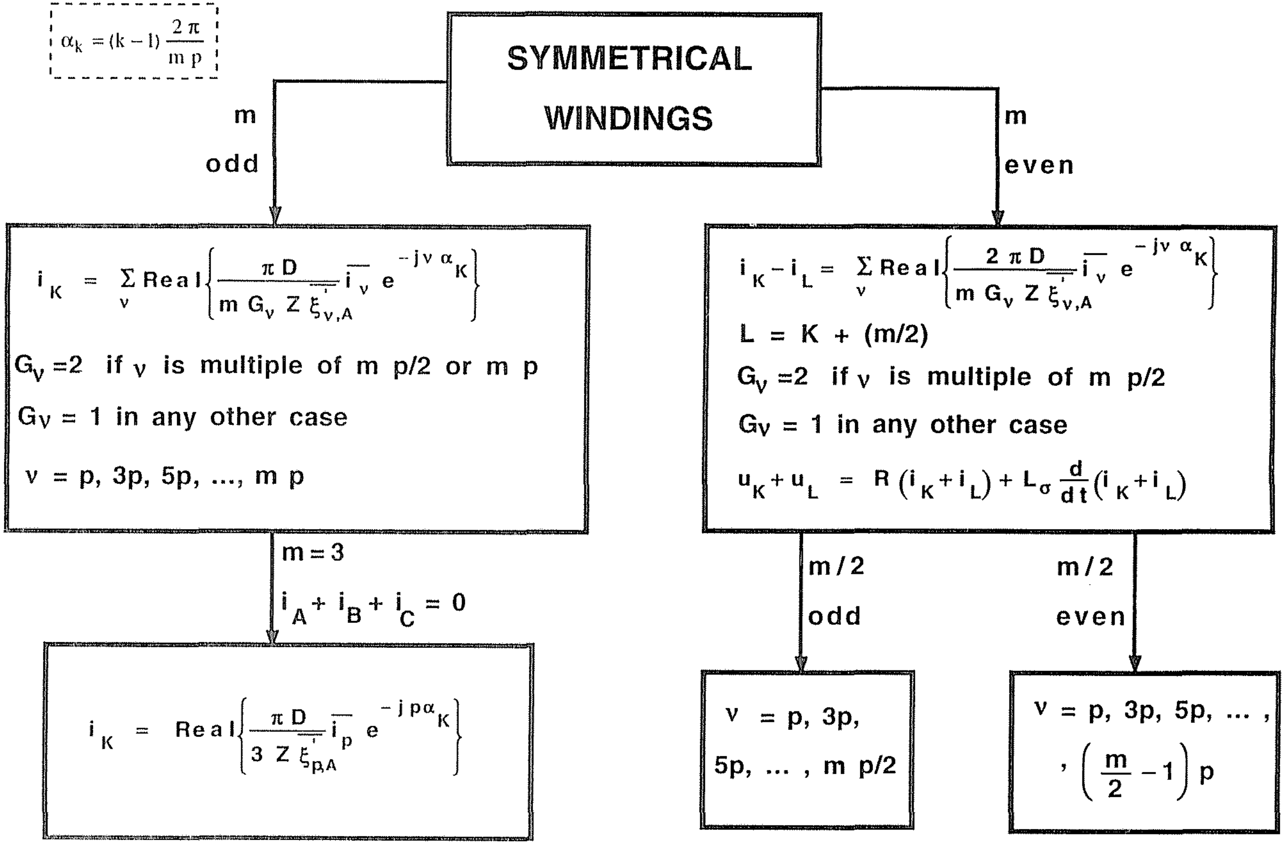

Fig. 1. Currents Correlation Theorem for symmetrical windings

– Following from *Eqs.* (12), (13) and (14), we get the monoharmonic phasor equations of a balanced short-circuited winding [7]. One of these equations, which corresponds to the v-th harmonic of the magnetic field, is:

$$\overline{e}_v = r_v \cdot \overline{\mathbf{i}}_v^v + l_{\sigma,v} \cdot \frac{d\overline{\mathbf{i}}_v'}{dt} \ , \tag{15}$$

where

– For a balanced winding with m phases:

$$r_v = \frac{\pi D R}{mG_v \left(Z \left| \overline{\xi}_{v,A}' \right|^2 \right)} \qquad l_{\sigma,v} = \frac{\pi D L_\sigma}{mG_v \left(Z \left| \overline{\xi}_{v,A}' \right|^2 \right)} \ . \tag{16}$$

– For a squirrel cage with m bars:

$$\begin{aligned} r_v &= \frac{\pi D}{mG_v} \left(R_{\text{bar}} + \frac{R_{\text{ring}}}{2m\text{sen}^2\left(\frac{v\pi}{m}\right)} \right) \ , \\ l_{\sigma,v} &= \frac{\pi D}{mG_v} \left(L_{\sigma,\text{bar}} + \frac{L_{\sigma,\text{ring}}}{2m\text{sen}^2\left(\frac{v\pi}{m}\right)} \right) \ . \end{aligned} \tag{17}$$

– When the short-circuited winding has star connection with isolated neutral point, its phase currents do not have zero-sequence component. Thus, from *Eq.* (7) follows that the harmonics whose order is a multiple of $m \cdot p$ do not fulfil *Eq.* (15), but, rather, this equation:

$$\sum_{k=1}^{m} i_k = 0 \quad \Rightarrow \quad \overline{\mathbf{i}}_v^v = 0 \ , \qquad v = kmp \ , \quad k = 1, 2, 3, \dots \ . \tag{18}$$

– If we compare the v-th monoharmonic phasor equation (15) with the one obtained by SERRANO in [12] and [13] taking only the fundamental magnetic field harmonic into account, we obtain that the terms $r_v \overline{i}_v^v$ and $l_{\sigma,v}\frac{di_v^v}{dt}$ are, respectively, the space phasors of resistive and inductive voltage drops of the short-circuited winding when the v-th harmonic of the machine magnetic field is only taken into account. That is to say, these terms represent the internal quantities of resistive and inductive voltage drops, respectively, which could correspond to the short-circuited winding if these magnitudes had a perfectly sinusoidal space distribution with v pole pairs.

5. Harmonic Phasor Equations for Balanced Short-Circuited Windings

– Suppose a harmonic set of a balanced short-circuited winding whose basic harmonic is v' and which contains the harmonic v. From *Eqs.* (12), (13) and (14) follows that the phase currents $i_{v,A}, i_{v,B}, \dots i_{v,M}$ (originated from the e.m.f.s. represented by the space phasor $\overline{\mathbf{e}}_v$) and $i_{v',A}, i_{v',B}, \dots, i_{v',M}$ (originated from $\overline{\mathbf{e}}_{v'}$) generate current sheets with the same harmonic orders (which belong to the same set as v and v') and the same proportion among them. Thus, for this winding, the v-th harmonic e.m.f. space phasor $\overline{\mathbf{e}}_v$ can be replaced with another equivalent phasor of order v', $\overline{\mathbf{e}}^{v}_{eq,v'}$. Then both $\overline{\mathbf{e}}_v$ and $\overline{\mathbf{e}}^{v}_{eq,v'}$ generate the same phase e.m.f.s. $(e_{v,A}, e_{v,B}, \dots, e_{v,M})$, and, thus, the same current sheet.

From the previous paragraph and the *Eq.* (12) follows that there are some *E.m.f.s. Equivalence Principles.*

E.m.f.s. Equivalence Principle I

If $v = kmp + v'$, $k = 0, 1, 2, \dots$ it follows that

$$\overline{\mathbf{e}}^{v}_{eq,v'} = \left(\frac{\overline{\xi}'_{v,A}}{\overline{\xi}'_{v',A}} \right)^* \cdot \overline{\mathbf{e}}_v \ . \tag{19.1}$$

E.m.f.s. Equivalence Principle II

If $v = kmp - v'$, $k = 1, 2, \dots$ it follows that

$$\overline{\mathbf{e}}^{v}_{eq,v'} = \frac{\overline{\xi}'_{v,A}}{\overline{\xi}'_{v',A}} \cdot \overline{\mathbf{e}}_v^{\,*} \ . \tag{19.2}$$

Thus, the effect of all e.m.f. space phasors of a harmonic set is the same as the effect of one e.m.f. space phasor of order v', $\overline{\mathbf{e}}_{eq,v'}$

$$\overline{\mathbf{e}}_{eq,v'} = \sum_{v=v'}^{\infty} \overline{\mathbf{e}}^{v}_{eq,v'} \Big|_{v=kmp\pm v'} \ . \tag{20}$$

There are no harmonics that simultaneously belong to two sets, and each e.m.f. space phasor of a set originates a current sheet (in a short-circuited winding), whose harmonics only belong to the same set as the e.m.f. phasor.

Thus, the equivalent e.m.f. space phasors $(\overline{\mathbf{e}}_{eq,v'})$ act independently: There are no harmonic current space phasors $(\overline{\mathbf{i}}_v)$ of the short-circuited winding that are produced by the simultaneous effect of two or more equivalent e.m.f. phasors $(\overline{\mathbf{e}}_{eq,v'})$.

– It follows from the monoharmonic phasor equations (15) of a balanced short-circuited winding which correspond to all harmonics of one set that the *short-circuited winding harmonic phasor equation* of the set basic harmonic v' is

$$\overline{\mathbf{e}}_{eq,v'} = r_{v'} \cdot \overline{\mathbf{i}}_{v'} + l_{\sigma,v'} \cdot \frac{d\overline{\mathbf{i}}_{v'}}{dt} \ . \tag{21}$$

When we use the equivalent e.m.f. space phasors, we suppose that it is not the real machine magnetic field (which has infinite harmonics) that acts on the balanced short-circuited winding, but an equivalent field that only has the basic harmonics (v') and which generates the $\overline{\mathbf{e}}_{eq,v'}$ phasors. In both situations, with the real or the equivalent fields, the short-circuited winding originates the same current sheet.

– When the short-circuited winding has star connection with isolated neutral point, the harmonics whose order is a multiple of mp fulfil this equation

$$\sum_{k=1}^{m} i_k = 0 \quad \Rightarrow \quad \overline{\mathbf{i}}_{v'} = 0 \ , \tag{22}$$
$$v' = kmp \ , \quad k = 1, 2, 3, \ldots$$

rather than *Eq.* (21).

6. Harmonic Phasor Equations for Balanced Windings

– Now we consider a balanced winding with these terminal phase voltages: $u_A, u_B, \ldots, u_M$.

Choosing the load sign criteria, the electrical equation of the phase k is

$$u_k + e_k = R \cdot i_k + L_\sigma \cdot \frac{di_k}{dt} \ . \tag{23}$$

From all phase equations follows that the v-th harmonic phasor equation of the balanced winding is

$$\overline{\mathbf{u}}_{eq,v'} + \overline{\mathbf{e}}_{eq,v'} = r_v \cdot \overline{\mathbf{i}}_{v'} + l_{\sigma,v'} \cdot \frac{d\overline{\mathbf{i}}_{v'}}{dt} \ . \tag{24}$$

There are as many harmonic phasor equations of a winding as harmonic sets it has: one equation per each basic harmonic.

If the winding has star connection with isolated neutral point, *Eq.* (22) must be used instead of *Eq.* (24) for harmonics whose order is multiple of $m \cdot p$.

– The equivalent voltage space phasor $\overline{\mathbf{u}}_{eq,v'}$ is obtained by means of this equation:

$$\overline{\mathbf{u}}_{eq,v'} = \frac{2}{mG_{v'} Z \overline{\xi}'_{v,A}}^{*} \sum_{k=1}^{m} u_k \cdot e^{jv'(k-1)\frac{2\pi}{mp}} \ . \tag{25}$$

Notice that the sum of *Eq.* (25) adopts the values of the instantaneous symmetrical components of the phase voltages.

The $\overline{\mathbf{u}}_{eq,v'}$ phasors fulfil this *Voltages Correlation Theorem*

$$u_k = Z \cdot \sum_{v'} Re \left\{ \overline{\mathbf{u}}_{eq,v'} \cdot \overline{\xi}'_{v',k}{}^{*} \right\} \ , \tag{26}$$

where the sum only includes the basic harmonics of the winding.

– From *Eqs.* (21) and (24) follows that the behaviour of the winding when it has the phase voltages $u_A, u_B, \ldots, u_M$ is identical to the behaviour it could have if is was short-circuited and, besides the real magnetic field, an additional fictitious magnetic field acts on it. This fictitious field generates an e.m.f.s represented by a phasor identical to $\overline{\mathbf{u}}_{eq,v'}$. In both cases, real and fictitious, the winding creates the same current sheet.

– SERRANO ([12], [13]) introduced the voltage internal quantities of a winding in this way:

Suppose an electrical machine with Z_{aA} conductors of the winding phase A, Z_{aB} conductors of the phase B, etc. embedded in one slot placed at the position α_a. The internal voltage quantities are:

- Average value at a given instant $t, u(\alpha, t)$ of the instantaneous voltages of all the winding's conductors placed in the position α.
- Average value at a given instant $t, \Delta u_{\text{res}}(\alpha, t)$ of the instantaneous resistive voltage drops of all the winding's conductors placed in the position α.
- The same as in the preceding paragraph, but related to the inductive voltage drop, $\Delta u_{\text{ind}}(\alpha, t)$, due to the leakage flux.

Serrano has developed expressions to calculate the space phasors related to these internal quantities when the machine does not have space harmonics.

Taking Serrano's definitions and formulae into account we obtain an interpretation of the space phasors $\overline{\mathbf{u}}_{eq,v'}$, $r_{v'}\overline{i}_{v'}$ and $l_{\sigma,v'}\frac{d\overline{i}_{v'}}{dt}$. They represent, respectively, Serrano's internal quantities of the balanced winding terminal voltage, and resistive and inductive voltage drops corresponding to this winding situation:

- it has the phase voltages $u_A, u_B, \ldots, u_M$,
- it is under the action of the equivalent magnetic field (which generates the e.m.f.s defined by $\overline{\mathbf{e}}_{eq,v'}$) instead of the real field.
- only the v'-th harmonic is taken into account. (That is to say, when it is assumed that all internal quantities are sinusoidal with v' pole pairs).

– Thus, we can extend Serrano's work to the fundamental harmonic and graphically represent each harmonic phasor equation (24) by means of *harmonic phasor diagrams* of the balanced winding (*Fig. 2*):

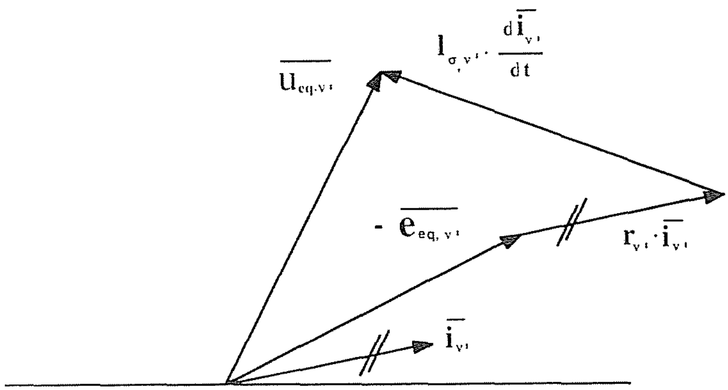

Fig. 2. v-th harmonic phasor diagram of a balanced winding

7. Phasor Model of an Asynchronous Machine with Space Harmonics

– The study of any balanced electrical machine with space harmonics can be accomplished by means of the harmonic phasor equations (24) of all its windings for all its basic harmonics and the mechanical equation.

This section presents, as an example, the way to analyse an asynchronous machine with only either one winding or one squirrel cage in the rotor.

This analysis follows the steps detailed below:

a) Identify the basic harmonics of the stator and rotor windings.
b) Choose the harmonics that are to be taken into account. These harmonics must include at least all basic harmonics of the stator and rotor windings.

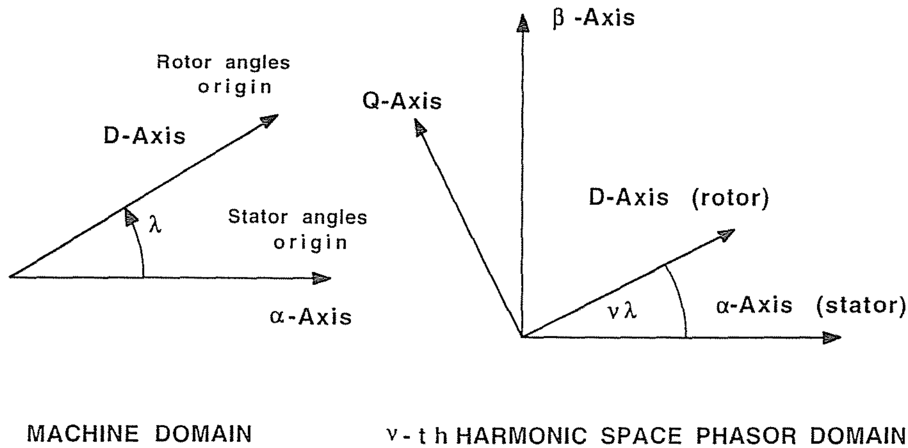

Fig. 3. Coordinate systems

c) Choose the stator and rotor angles origins that define the real axis of the stator and rotor coordinate systems.

As *Fig. 3* shows, the angle λ between these axes in the machine domain is converted into the angle $v \cdot \lambda$ in the v-th harmonic space phasor domain ([11], [13]). It is clear that the machine speed Ω is

$$\Omega = \frac{d\lambda}{dt} . \tag{27}$$

d) Calculate, by means of formula (1), the complex winding factors of each winding, expressed in its own natural reference frames, and for all harmonics which are to be taken into account.
Moreover, the expression (25) will be used to obtain the equivalent voltage harmonic space phasors of both windings expressed in their own natural reference frames.
If the rotor is short-circuited, every space phasor $\overline{u}_{eq,v',R}$ has zero value.

e) Choose a common reference frame that all space phasors will be referred to. Here we are going to use the stator reference frame, so the rotor space phasors will have to be changed to the stator coordinate system.
A rotor space phasor $\overline{\mathbf{X}}_{v,R}$ in rotor coordinates is changed to the stator coordinates in this way:

$$\overline{\mathbf{X}}^{S}_{v,R} = \overline{\mathbf{X}}_{v,R} \cdot \mathrm{e}^{jv\lambda} . \tag{28}$$

When the Equivalence Principles (6) and (19) are used with rotor phasors in stator coordinates, the rotor complex winding factors must also be in the stator coordinates.

f) The machine phasor model has the following equations:
   - Stator harmonic phasor equations:

$$\overline{\mathbf{u}}_{eq,v',S} = -\overline{\mathbf{e}}_{eq,v',S} + r_{v',S} \cdot \overline{\mathbf{i}}_{v',S} + l_{\sigma,v',S} \cdot \frac{d\overline{\mathbf{i}}_{v',S}}{dt} \ . \tag{29}$$

   - Rotor harmonic phasor equations:

$$\overline{\mathbf{u}}^{S}_{eq,v',R} = -\overline{\mathbf{e}}^{S}_{eq,v',R} + r_{v',R} \cdot \overline{\mathbf{i}}^{S}_{v',R} + l_{\sigma,v',R} \left(\frac{d\overline{\mathbf{i}}^{S}_{v',R}}{dt} - jv'\Omega\overline{\mathbf{i}}^{S}_{v',R} \right) \ . \tag{30}$$

   - Mechanical equation

$$T = \sum_{v} \frac{\mu_0 \pi l D^3}{8\delta v} \left[\overline{\mathbf{i}}^{S}_{v',R} \times \overline{\mathbf{i}}_{v',S} \right] = T_L + J\frac{d\Omega}{dt} \ . \tag{31}$$

The mechanical equation includes all harmonics whose effects we want to take into account. Moreover, in this equation the current harmonic space phasors can be expressed as a function of the basic harmonics current phasors by means of the Principles (6).

g) Modify the phasor equations (29) and (30) expressing the equivalent e.m.f. harmonic space phasors as a function of the stator and rotor basic harmonics phasors of current.
For this purpose the *Eqs.* (19), (20), (27) and (28) must be used. Moreover, we must take into account that, according to [11]:

$$\begin{aligned} \overline{\mathbf{e}}_{v,S} &= -\frac{d\overline{\Phi}^{S}_{\mathrm{cor},v,\mathrm{maq}}}{dt} \ , \\ \overline{\mathbf{e}}^{S}_{v,R} &= -\left[\frac{d\overline{\Phi}^{S}_{\mathrm{cor},v,\mathrm{maq}}}{dt} - jv\Omega\overline{\Phi}^{S}_{\mathrm{cor},v,\mathrm{maq}} \right] \ , \end{aligned} \tag{32}$$

where

g.1) If there are air-gap permeance space harmonics, STEPINA gives in [8] and [10] this expression:

$$\overline{\Phi}^{S}_{\mathrm{cor},v,\mathrm{maq}} = \frac{\mu_0 l D^2}{8v} \left[\sum_{v_i} \frac{\overline{\Lambda}^{S}_{(v-v_i)}}{v_i} \overline{\mathbf{i}}_{v_i,\mathrm{maq}^S} + \right.$$

$$+\sum_{v_i\geq v}\frac{\overline{\Lambda}^S_{(v_i-v)}}{v_i}\overline{\mathbf{i}}^S_{v_i,\mathrm{maq}}-\sum_{v_i<v}\frac{\overline{\Lambda}^S_{(v_i+v)}}{v_i}\overline{\mathbf{i}}^*_{v_i,\mathrm{maq}^S}\Bigg] \tag{33}$$

$$\overline{\mathbf{i}}^S_{v_i,\mathrm{maq}}=[\overline{\mathbf{i}}_{v_i,S}+\overline{\mathbf{i}}^S_{v_i,R}]\ . \tag{34}$$

$\overline{\Lambda}^S_{(v-v_i)}$ is the space phasor that represents the $(v-v_i)$-th harmonic of the air-gap permeance wave expressed in the stator reference frame.

VAN DER MERWE criticizes these expressions in [17].

g.2) If the air gap is uniform, according to [11], it follows that

$$\overline{\Phi}^S_{\mathrm{cor},v,\mathrm{maq}}=\frac{\mu_0 l D^2}{4v^2\delta}\overline{\mathbf{i}}^S_{v,\mathrm{maq}}\ . \tag{35}$$

Thus, when the air gap is uniform, the rotor phasor equation (30) for the basic harmonic v' is converted into

$$\overline{\mathbf{u}}^S_{eq,v',R}=\overline{\mathbf{u}}_{eq,v',R}-e^{jv'\lambda}=r_{v',R}\ \overline{\mathbf{i}}^S_{v',R}$$

$$+\left\{\frac{\mu_0 l D^2}{4\delta}-\sum_{v_{1,2}}\left(\frac{-\overline{\xi}'_{1,2},AR-}{v_{1,2}-\overline{\xi}'_{v'},AR-}\right)^2+l_{\sigma,v',R}\right\}-\left[\frac{d\overline{\mathbf{i}}^S_{v',R}}{dt}-j\Omega v'\overline{\mathbf{i}}^S_{v',R}\right]+$$

$$+\frac{\mu_0 l D^2}{4\delta\overline{\xi}'^{*}_{v',AR}}\left\{\sum_{v_1}\frac{\overline{\xi}'^{*}_{v_1,AR}}{v_1^2}-e^{j(v'-v_1)\lambda}\left[\frac{d\overline{\mathbf{i}}^S_{v_1,S}}{dt}-j\Omega v_1\overline{\mathbf{i}}_{v_1,S}\right]+\right.$$

$$\left.+\sum_{v_2}\frac{\overline{\xi}'^{*}_{v_2,AR}}{v_2^2}-e^{j(v'+v_2)\lambda}-\left[\frac{d\overline{\mathbf{i}}_{v_2,S}{}^*}{dt}+j\Omega v_2\overline{\mathbf{i}}_{v_2,S}{}^*\right]\right\}\ , \tag{36}$$

where

$$v_1=k_1\ m_R\ p_R+v'\ ,\qquad k_1=0,1,2,\dots\ ,$$
$$v_2=k_2\ m_R\ p_R-v'\ ,\qquad k_2=1,2,3,\dots\ ,$$
$$v_{1,2}=v_1\ ;v_2\ .$$

Formula (36) could be simplified expressing the stator current harmonic space phasors as a function of the current space phasors of the stator basic harmonics by means of *Eq.* (6).

The stator phasor equation for the basic harmonic v' has the same form as *Eq.* (36) if λ and Ω have zero value and if R and S subscripts are interchanged.

h) Using the equation mentioned above and the convenient boundary conditions, it is possible to obtain the current basic harmonic space phasors of stator and rotor. All harmonic current phasors can be calculated from the basic harmonic ones by means of *Eqs.* (6), (9) and (14).
In the harmonic phasor equations in compact form ((29), (30)) only the equivalent e.m.f. space phasors depend on the number of harmonics taken into account.

– If a three-phase winding does not have a zero-sequence component of currents, it only has one basic harmonic and thus we only need one harmonic phasor equation for it.

8. Conclusions

This paper extends Stepina and Serrano's work to study an electrical machine with asymmetrical and balanced polyphase windings and with both m.m.f. and air-gap permeance harmonics by means of a real phasor method.

Here the harmonic phasor equations of a balanced winding are obtained using equivalent harmonic space phasors of e.m.f. and voltage that have received a physical interpretation. These harmonic phasor equations represent, in a compact way, the electromagnetic behaviour of a winding and they can be graphically represented by means of their corresponding harmonic phasor diagrams.

Some Equivalence Principles and Correlation Theorems have been established that enable us to analyse the behaviour of a balanced winding using only few (basic) harmonics.

Finally, the phasor analysis of an asynchronous machine (whose rotor could have either a winding or a squirrel cage) with space harmonics is presented.

Acknowledgement

The authors wish to acknowledge the cooperation received from Mr. M. Dominguez, Lecturer at the University of Cantabria.

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