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EDDY CURRENTS IN FERROMAGNETIC SHEETS TAKING MAGNETIC HYSTERESIS NONLINEARITIES INTO ACCOUNT

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Abstract

The paper deals with the numerical solution of the diffusion problem getting for the time varying electromagnetic field in ferromagnetic and conductive media. It presents a method based on the scalar Preisach model describing local magnetic behaviour of material (the static hysteresis characteristic) and on the pseudo-transient method for integration of the partial differential equation system formulated by finite difference. It enables simulation of dynamic operation demanding experimental data obtained by static measurements only. Numerical results are presented for different frequencies and wave forms of the source field applied at the surface of a ferromagnetic sheet.

Keywords: dynamic hysteresis modelling, finite difference method, electromagnetic field in non-linear media, pseudo-transient method of integration, eddy currents, skin effect.

1. Introduction

The aim of this paper is to elaborate a dynamic hysteresis model applicable for any frequency and wave form of the source field at which magnetic viscosity can be ignored. The model is based on the static hysteresis characteristic of the given ferrous material described by the scalar Preisach model [1] on one hand and numerical computation of the electromagnetic field diffusion in the given geometry obtained by means of the pseudo-transient method for integration of the partial differential equation system formulated by finite difference [2] on the other.

The scalar Preisach model provides the relationship between magnetic field intensity and magnetisation or flux density at local level. It is included in the governing equation system of the electromagnetic field, enabling eddy current simulation [3] taking non-linear hysteresis into account.

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The model is implemented for the case of ferromagnetic sheets for both sinusoidal and non-sinusoidal (with zero and non-zero mean value) variation of the source field applied at the surface of the sheet, obtaining field intensity, flux density and eddy current intensity with respect to time and distance from the symmetry plane of the sheet.

Integrating the flux density over the sheet thickness, the relative magnetic flux is obtained at every time step. Plotting it with respect to the source field intensity provides the dynamic characteristic of the given material in the given geometry and operation.

The effective values of magnetic field intensity, flux density and eddy current density with respect to distance from the symmetry plane illustrate the skin effect in cases of sinusoidal source fields at several frequencies.

2. Implementation of the Scalar Preisach Model

The scalar Preisach model [1] has been implemented for the following weighted function [4], [5] for the elementary hysteresis switches density:

$$\mu(\alpha\beta) = e^{-\frac{(\alpha-\beta)^2}{a} - \frac{(\alpha+\beta)^2}{b}}, \tag{1}$$

$$\alpha \in [-1,1]\ ; \quad \beta \in [-1.1]\ ,$$

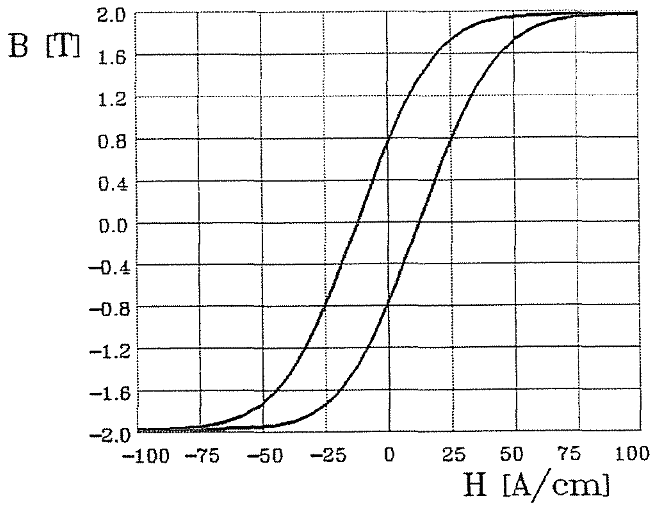

Fig. 1. Major hysteresis loop

which for $a = 0.2$ and $b = 0.2$ yields the major hysteresis loop plotted in *Fig. 1* with the following scales: $sh = 10^4$ A/m for field intensity and

$sm = 705$ A/m for magnetisation. The Preisach triangle has been divided into elementary squares with edge length of 0.01 which implies field intensity steps of $\Delta H = 100$ A/m. Outputs for field intensity values between steps are obtained by linear interpolation between the outputs corresponding to the two neighbouring step values. In order to ensure correct approximation of $\partial M/\partial H$ slopes, two staircase lines [1] are memorised, one trailing and another going ahead the actual field intensity variation, so that both neighbouring values would be available even when field intensity variation changes sign.

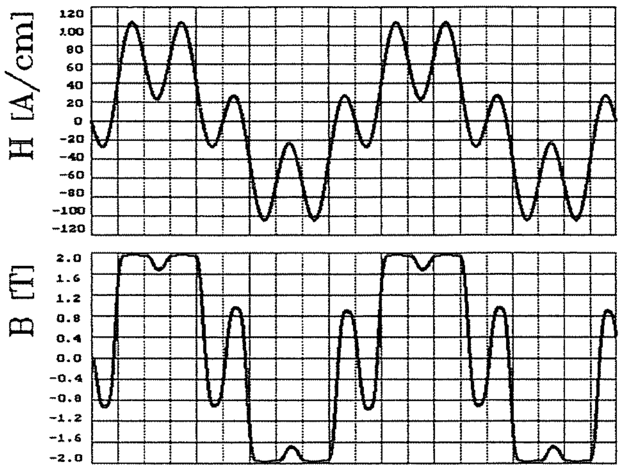

Fig. 2. Magnetic field intensity and flux density versus time (test problem)

The accuracy of the implementation as well as that of the interpolation routine has been tested for the following field intensity function (*Fig. 2*):

$$H_0 = H_{0m}\left(\sin \omega t - \frac{2}{3}\sin 5\omega t\right) , \tag{2}$$

with $H_{0m} = 7000$ A/m and ω so small that the eddy currents do not appear (static magnetisation). The resulting flux density is plotted on *Fig. 2* with respect to time and on *Fig. 3* with respect to the applied field intensity.

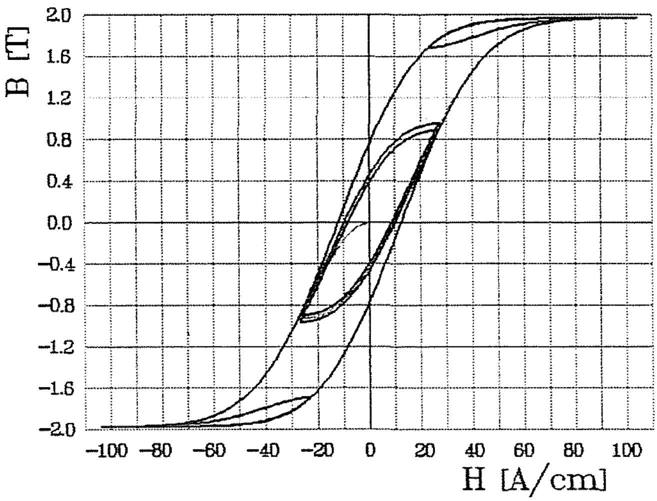

Fig. 3. Magnetic hysteresis characteristic (test problem)

3. The Governing Equations of the Electromagnetic Field

The governing equations of the time-varying electromagnetic field are:

$$\begin{array}{ll} \text{rot } \mathbf{H} = \mathbf{J} & \\ \text{rot } \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} & \mathbf{J} = \sigma \mathbf{E} \\ \text{div } \mathbf{B} = 0 & \mathbf{B} = \mu_0[\mathbf{H} + \mathbf{M}(H)] \end{array} \quad , \tag{3}$$

where $\mathbf{M}$ is the magnetisation vector, parallel to the field intensity vector. Its magnitude results from the scalar Preisach model, taking actual and past values of H into account [1].

Applying to both surfaces of a very wide sheet, with yOz as its symmetry plane (*Fig. 4*) a z-directed external magnetic field results a magnetic field parallel to the Oz axis and current density parallel to the Oy axis in the material:

$$\mathbf{H} = \mathbf{k}H(x,t) \ ; \quad \mathbf{J} = \mathbf{j}J(x,t) \ . \tag{4}$$

Eqs. (1) lead to the following differential equation:

$$\frac{\partial H}{\partial t} = \frac{\frac{\partial^2 H}{\partial x^2}}{\mu_0 \sigma \left(1 + \frac{\partial M}{\partial H}\right)} \ ; \quad x \in [0, a] \ , \quad t \geq 0 \ , \tag{5}$$

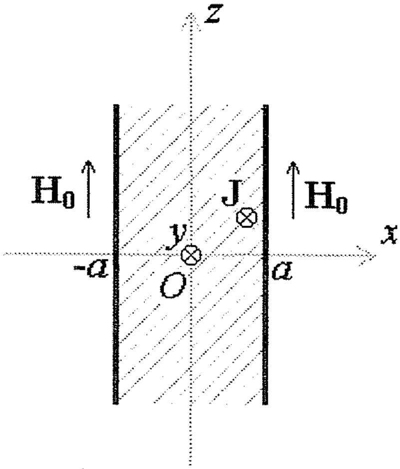

Fig. 4. Geometrical configuration

with the boundary conditions:

$$\left.\begin{array}{l}\dfrac{\partial H}{\partial x}(0,t)=0\\[2ex] H(a,t)=H_{0m}\sin 2\pi f t\end{array}\right\} \quad ; \quad t\geq 0\ . \tag{6}$$

According to the solution in time domain obtained by the pseudo-transient method the initial condition is selected as:

$$H(x,0)=0\ ; \quad x\in[0,a]\ . \tag{7}$$

The condition of convergence (the criteria for stopping computations) is:

$$H(x,t+T)=H(x,t)\ ; \quad x\in[0,a]\ , \tag{8}$$

where T stands for the period of the source field.

4. Finite Difference Formulation and Pseudo-Transient Integration Method

Formulating the solution by finite difference method, discretisation is performed over a grid selected as:

$$\begin{array}{l} x=j\dfrac{a}{n}\ ; \quad j=\overline{0,n}\ ;\\[3ex] t=l\tau\ ; \qquad l\geq 0\ .\end{array} \tag{9}$$

A three time-level discretisation scheme yields the following approximations of the partial differentials:

$$\frac{\partial H}{\partial t}(x,t) \approx \frac{H_j^{l+1} - H_j^l}{\tau} \,,$$

$$\frac{\partial^2 H}{\partial x^2}(x,t) \approx$$

$$\approx \frac{1}{3} \cdot \frac{H_{j+1}^{l+1} + H_{j-1}^{l+1} - 2H_j^{l+1} + H_{j+1}^{l} + H_{j-1}^{l} - 2H_j^{l} + H_{j+1}^{l-1} + H_{j-1}^{l-1} - 2H_j^{l-1}}{h^2} \,, \tag{10}$$

where

$$H_j^l = H(jh, l\tau) \;; \quad h = \frac{a}{n} \,. \tag{11}$$

The reason for selection of a three-level scheme is the presence of the partial derivative of magnetisation versus field intensity in the denominator of the right side of *Eq.* (4), which is evaluated at a central field intensity value in both spatial and time direction, the following way:

– for increasing trend of field intensity:

$$\frac{\partial M}{\partial H} \approx \frac{M\left(\left(\left[\frac{H_j^l}{\Delta H}\right] + 1\right)\Delta H\right) - M\left(\left[\frac{H_j^l}{\Delta H}\right]\Delta H\right)}{\Delta H} \tag{12}$$

– for decreasing trend of field intensity:

$$\frac{\partial M}{\partial H} \approx \frac{M\left(\left[\frac{H_j^l}{\Delta H}\right]\Delta H\right) - M\left(\left(\left[\frac{H_j^l}{\Delta H}\right] + 1\right)\Delta H\right)}{\Delta H} \tag{13}$$

In *Eqs.* (12) and (13) the square brackets stand for the integer value of the quantities inside them and magnetisation is written as a function of the corresponding field intensity. Note that the value of magnetisation depends on history as well (i.e. the past values of field intensity), so that it would have different values for different trends even if the actual values of the arguments in *Eqs.* (12) and (13) are the same.

The system resulted can be solved by Gaussian elimination [2] at every time level.

With the calculated values of the magnetic field intensity the corresponding flux densities can be computed at every grid point by means of the scalar Preisach model and integrated over half of the sheet thickness $[0, a]$

(due to symmetry of geometry and source field) to find the relative magnetic flux (flux per width unit - along the Oy axis) at each temporal grid level

$$\Phi_l(t) = \Phi^l \approx 2h \left(\frac{1}{2} B_0^l + \sum_{j=1}^{n-1} B_j^l + \frac{1}{2} B_n^l \right) . \tag{14}$$

Current density is computed as:

$$\begin{gathered} J_0^l = 0 \; ; \quad J_j^l \approx \frac{H_{j+1}^l - H_{j-1}^l}{2h} \; , \quad j = 1, 2, \ldots , n-1 \; ; \\ J_n^l = \frac{H_n^l - H_{n-1}^l}{h} \; . \end{gathered} \tag{15}$$

5. Numerical Results in Harmonic Operation

A very wide and long ferromagnetic sheet (*Fig. 4*) has been considered with thickness $2a = 0.5$ mm, magnetic characteristic plotted in *Fig. 1* (the major hysteresis loop) and electric conductivity $\sigma = 5 \cdot 10^6$ S/m. The half thickness of the sheet has been divided into $n = 20$ equal parts and the time steps have been selected with respect to the frequency of the source field:

f [Hz]	50	500	2000	10000
τ [s]	$2.5 \cdot 10^{-5}$	$2.5 \cdot 10^{-6}$	$2.5 \cdot 10^{-7}$	$2.5 \cdot 10^{-8}$

The source field magnitude was set: $H_{0m} = 10000$ A/m.

The trace of the operation point on the $\Phi_l - H_0$ (relative flux versus source field intensity) characteristic during the pseudo-transient integration can be seen on *Fig. 5* for the following cases:

- a – static hysteresis characteristic (extremely low variation speed of source field, so that no eddy currents are induced)
- b, c, d and e – dynamic hysteresis characteristics obtained for source field frequencies of 50, 500, 2000 and 10000 Hz.

The steady-state magnetisation loops are plotted on *Fig. 6.* The effective values of magnetic field intensity, flux density and eddy current density are shown on *Fig. 7*, illustrating the frequency-dependent skin effect.

The wave forms of magnetic field intensity, flux density and eddy current intensity at different depths in the considered sheet are plotted on *Figs. 8*, *9* and *10* for source field frequency of 2000 Hz.

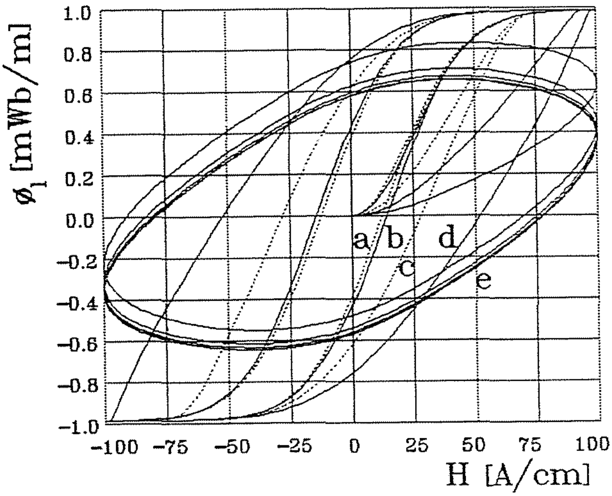

Fig. 5. Pseudo-transient magnetisation loops

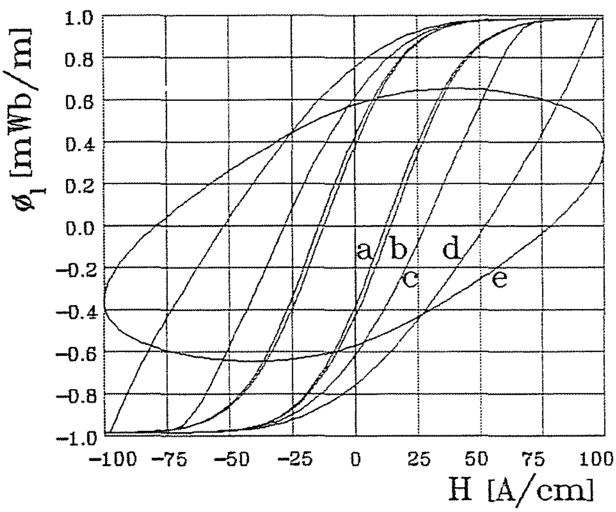

Fig. 6. Steady-state magnetisation loops

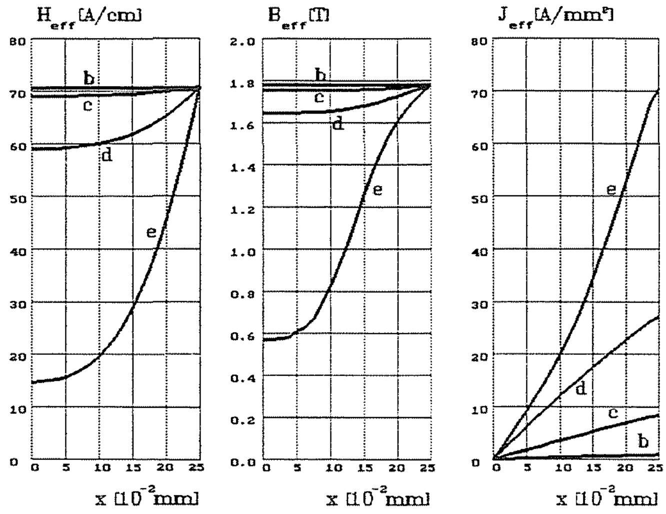

Fig. 7. Effective values of magnetic field intensity, flux density and eddy current intensity with respect to distance from the symmetry plane

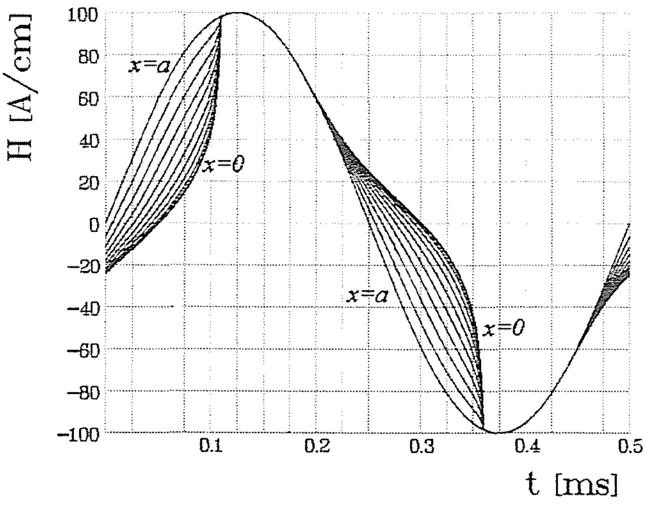

Fig. 8. Magnetic field intensity versus time at different depths in the sheet at 2000 Hz source field frequency

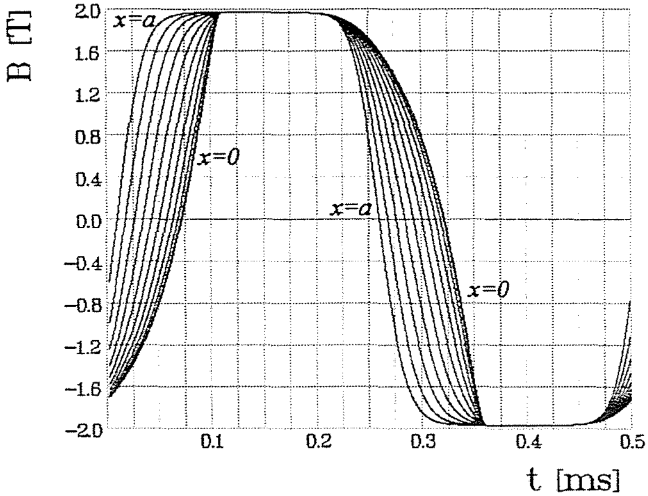

Fig. 9. Magnetic flux density versus time at different depths in the sheet at 2000 Hz source field frequency

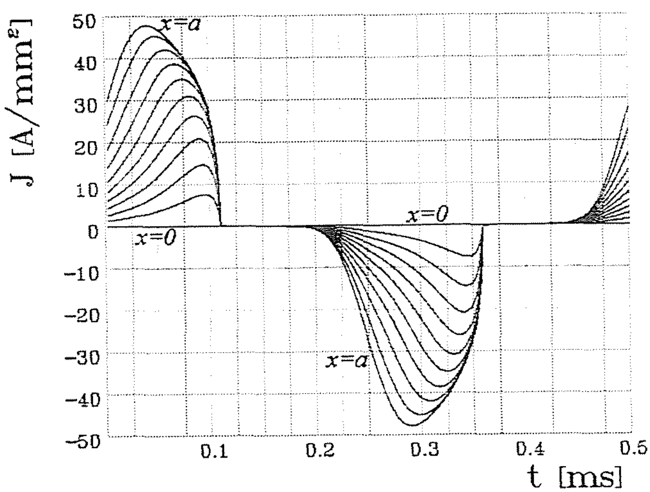

Fig. 10. Eddy current density versus time at different depths in the sheet at 2000 Hz source field frequency

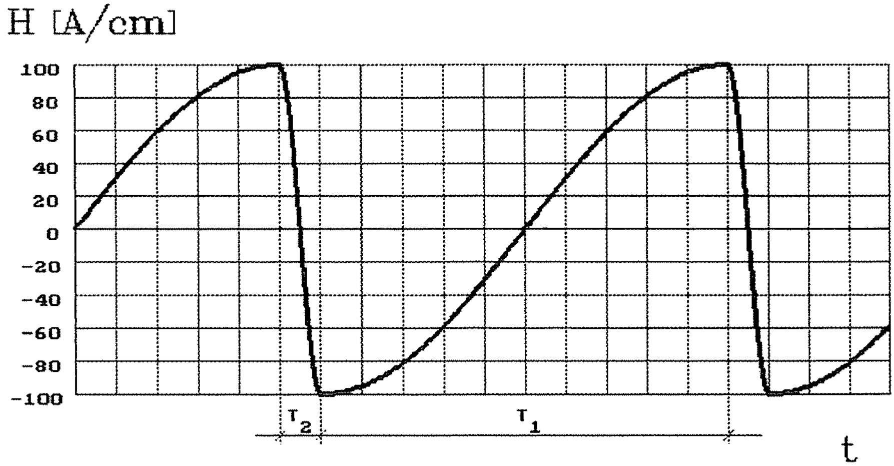

Fig. 11. Non-sinusoidal source field wave form with zero mean value

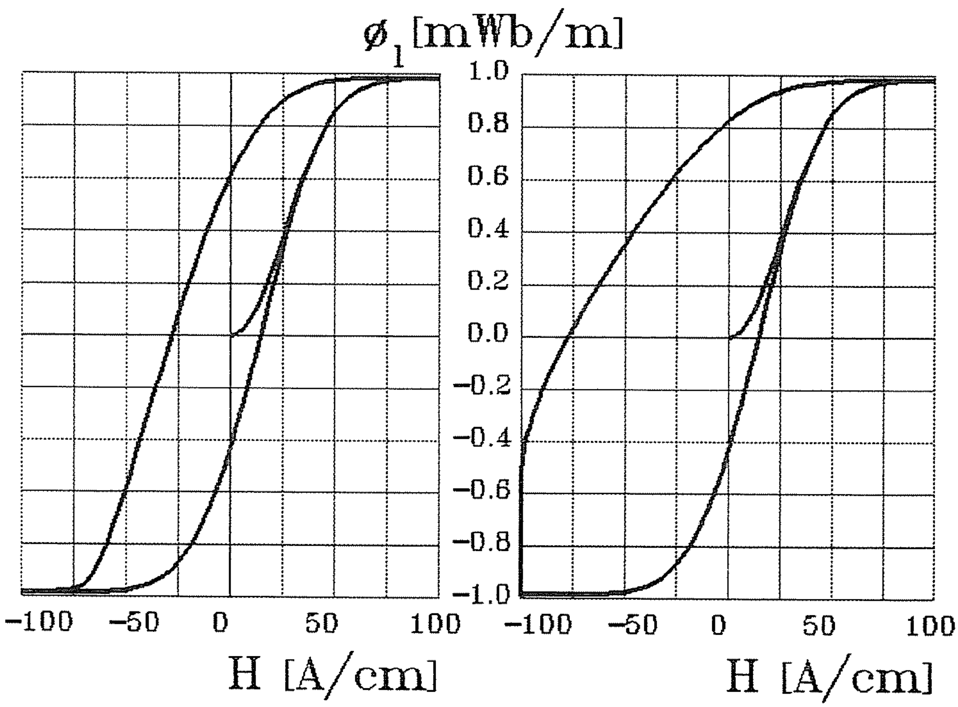

Fig. 12. Hysteresis loops for zero mean value non-sinusoidal source field

6. Non-Harmonic Operation

The presented model has been applied in non-harmonic operation as well. For the source field wave form (with zero mean value) plotted in *Fig. 11* with

[s]	T_1	T_2
a)	10^{-2}	10^{-3}
b)	10^{-2}	10^{-4}

resulted the asymmetric (due to the effect of the eddy currents, depending on the speed of source field variation) hysteresis loops in *Fig. 12a* and *b* for the two studied cases.

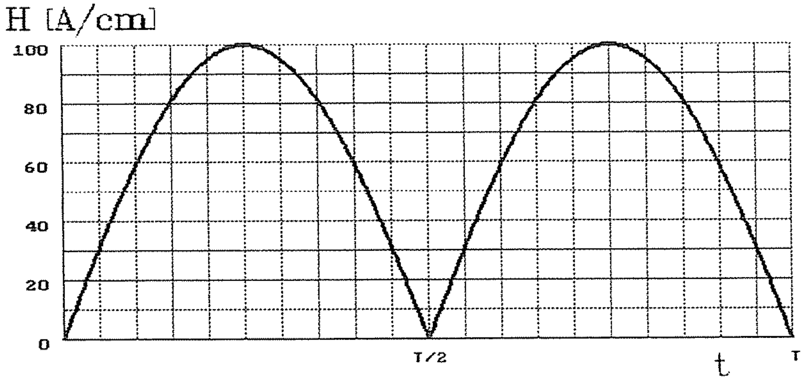

Fig. 13. Source field wave form with non-zero mean value

For the source field (*Fig. 13*) $H_0 = H_{0m}|\sin \omega t|$ with non-zero mean value, the magnetisation curves plotted in *Fig. 14* have been obtained for the same frequencies of the source field as in case of the harmonic operation.

7. Conclusions

The model presented in the paper proved to be suitable for computing time-varying electromagnetic field in ferrous materials. Magnetic cores can be modelled in dynamic operation for different wave-forms of the source field and distribution of electromagnetic field functions obtained at different depths in the sheets. Eddy current simulation can be performed in different dynamic regimes requiring only static measurements regarding the magnetic behaviour of the considered material. Spatial distribution of heat sources can also be computed, thus, combined with an appropriate heating model, the way towards temperature-dependent modelling is prepared.

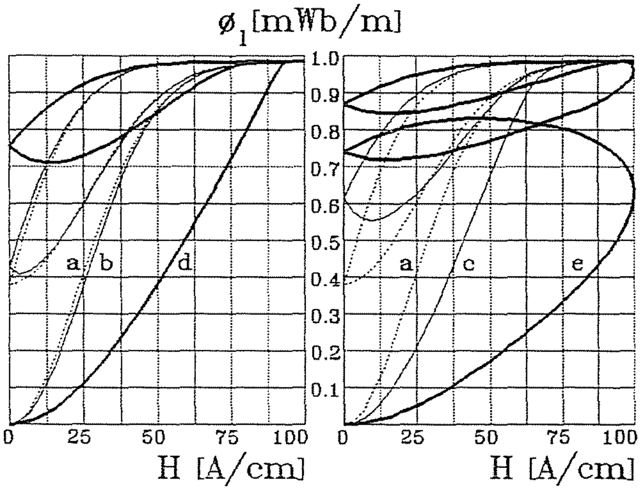

Fig. 14. Magnetisation process for non-zero mean value source field wave form

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