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ESTIMATION OF THE EFFECT OF NONLINEAR HIGH POWER AMPLIFIER IN M-QAM RADIO RELAY SYSTEMS

Nguyen Quoc BINH, József BÉRCES and István FRIGYES

Department of Microwave Telecommunications, TUB
H-1521 Budapest, Hungary

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Abstract

The estimation of the effect of both linear and nonlinear distortions in M-QAM radio systems requires either complicated analytical calculation or very long run of simulation. In this paper a new parameter of nonlinearity is produced and the relationship between this parameter and the signal to noise ratio degradation ($SNRD$) caused by the separated effect of nonlinear HPA (High Power Amplifier) is presented. In addition, the estimation of the simultaneous effect of linear and nonlinear distortions is discussed and a procedure to calculate the upper bound of BER (bit-error ratio) for this case is also presented.

Keywords: High Power Amplifier, nonlinearity, M-QAM, $SNRD$, $OAPS$.

1. Introduction

In the analysis, design and evaluation of M-QAM systems the estimation of BER under the effects of linear and nonlinear distortions is a very important task. The separated effect of linear distortion can be estimated by fast and simple simulation. In this case the calculation of BER is rather fast by applying GQR (Gaussian Quadrature Rule) or other techniques for approximating pdf of ISI (Probability Density Function of InterSymbol Interference) [1, 2, etc.]. The separated effect of nonlinearity caused mainly by HPA at the transmitter has been analysed, estimated and presented in many papers [1, 2, 3, 4, 7, 8, 9, etc.]. In general, the effect of nonlinear HPA to BER performance can be estimated either by analytical calculation of the signals at the output of the system by applying Volterra series expansion, or by using Monte-Carlo simulation for obtaining either BER directly or the empirical pdf of nonlinear ISI and after that calculating BER by the QA (Quasi-Analytical) method. In [7], AMADESI et al. have simulated 16-QAM systems by using six different practical HPAs and produced a formula to calculate the $SNRD$ taken at the BER level of $5 \cdot 10^{-4}$ caused by the separated effect of HPA's nonlinearity. Other effort has been made by SILVANO et al. [4]. It is based on the analytical calculation of the 1st and 2nd order statics of the signals at the output by applying Volterra series and the assumption that the nonlinear ISI can be well approximated as a Gaussian noise. BER can therefore be calculated quasi-analytically. These estimations of BER or $SNRD$ (at some level of BER), however, are not enough under the separated effect of only linear or only nonlinear distortion, for evaluating the systems and used only for estimating roughly the performance of the systems. In more details, the results obtained from such estimations can be used only as the lower bound of BER or $SNRD$, depending on what distortion (linear or nonlinear) dominates.

In practice, the nonlinearity of HPA is often described by 3rd or higher-order polynomials and linearised by applying predistorter or/and appropriate back-off (BO). In the case when no predistorter is used the 3rd-order term of the polynomial is often much higher than the 5th-order one. The 5th and higher-order terms can therefore be neglected in the calculations. A practical analog predistorter is a cubic predistorter which, in cascade with HPA, eliminates effectively the cubic term of the overall response. When a cubic analog predistorter is used, not only the 3rd-order term, but at least the 5th-order term of the overall response polynomial have also to be taken into account in the calculations.

Taking into account not only nonlinear distortion but also the linear one and the effect of the predistorter, the analytical method, and then the method given by SILVANO requires an unpractical number of complicated computations [4, 5]. The Monte-Carlo method or the QA method based on Monte-Carlo simulation, however, requires very long runs of simulation, especially if the total ISI is rather high. In addition, such a complicated simulation program is not available everywhere.

In this paper, at first, as an effort to estimate the effect of nonlinear HPA, a new parameter of nonlinearity used for the HPAs in M-QAM systems will be produced. This parameter can be defined easily from the characteristics of HPA and can be used to calculate approximately the $SNRD$ caused by the separated effect of HPA. The relationship between this parameter and the $SNRD$ can be obtained by simulating systems with different nonlinearities. An empirical formula describing the relationship between the new parameter and $SNRD$ caused by the separated effect of HPA in 64-QAM radio relay systems will be presented. The $SNRD$ value obtained by applying this formula can be used together with the results obtained by simulating the system under either only the effect of linear distortion or only the effect of nonlinear one to define the lower limit of the system performance.

For estimating the simultaneous effects of linear and nonlinear distortions on the system performance, in order to avoid the very complicated computations or the very long runs of simulation, a rather simple procedure will be presented to calculate the upper bound of BER. This procedure is based on the analysis of a hypothesis system which has the $SNRD$ caused by HPA higher than one of the actual system but its BER can be calculated rather simply by combining with the result of simulating the linear system.

2. Separated Effect of Nonlinear HPA

2.1 The Effects of the Nonlinear HPA in M-QAM Radio Systems

In general, the M-QAM radio link can be modelled by the block diagram depicted in *Fig. 1a.* The dashed boxes present operations that may or may not be included in the design. In *Fig. 1a,* AGC (Automatical Gain Controller), carrier regenerator and clock regenerator are not depicted and are assumed to be ideal. The impairment sources of such a system are the linear distortions caused by the filters and the selective fading and the nonlinear distortion caused mainly by the nonlinear HPA of the transmitter.

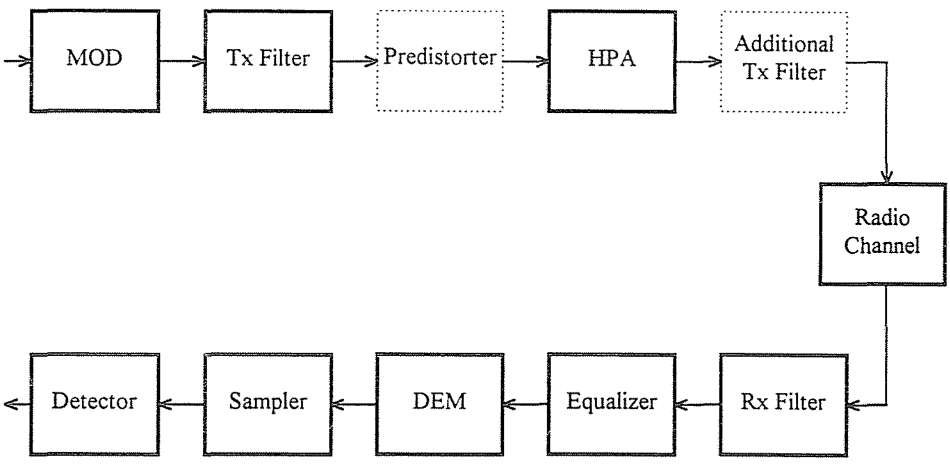

Fig. 1a. The model of M-QAM radio link

When taking only the separated effect of the nonlinear HPA into account the linear distortions are regarded to be zero. The block diagram of the system, in this case, is shown in *Fig. 1b,* where the overall response of the filters is assumed to meet the conditions of the 1st Nyquist criterion. The effects of the nonlinear HPA for this case can be summarised as follows:

- The odd-order nonlinearities of HPA produce inband products, which cause interference to the primary digital signal being transmitted. The intermodulation spectrum generated by a digital signal passing through an amplifier with a 3rd-order nonlinearity is approximately three times as wide as the original signal. This spectrum spreading can cause interference to signals in adjacent channels;
- AM-AM, AM-PM conversions of HPA cause the displacement of the average states of signals in the phase plane. This displacement reduces the distances from the average states of signals to the nearest boundaries and then reduces the attainable BER performance of the system;
- The HPA sandwiched between two filters causes also ISI, we call it as the nonlinear ISI to distinguish from ISI (linear ISI) caused purely by the linear distortions of practical filters and fading. This nonlinear ISI, in essence, is caused by the fact that when the HPA is placed between Tx filter and Rx filter the overall response of them does not longer meet the 1st Nyquist criterion.

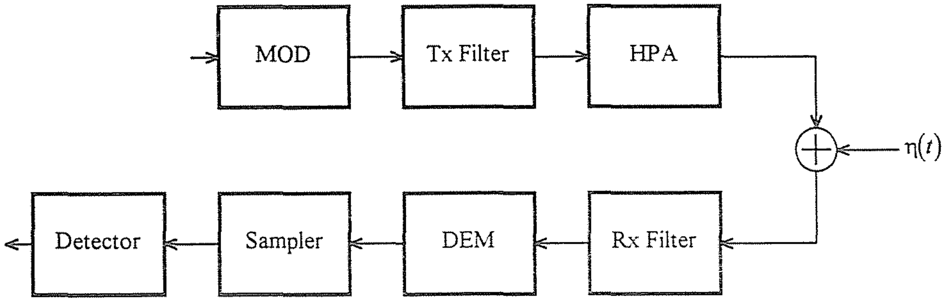

Fig. 1b. Block diagram of the system under the separated effect of HPA

At present, predistorters and high enough back-off are used to linearise the HPA. For many practical cases, the effect of the spectrum spreading can therefore be negligible. In our considerations the additional filter in *Fig. 1a*, which is designed to combat the spectrum spreading, is therefore omitted, not depicted in *Fig. 1b*. The Tx filter is assumed to be a square-root raised cosine roll-off filter followed an $x/\sin x$ corrector, and the Rx filter is assumed to be also a square-root raised cosine roll-off filter with the roll-off factor of the filters α ($\alpha \in [0, 1]$).

Under the effects of the HPA, instead of M discrete points arranged in a regular square grid, the received signals appear in form of M clusters of points in the phase plane. The averages of the individual clusters are shifted from their original states. The constellation of the received signals for a concrete case is shown in *Fig. 2.* The clusters and the warping of constellation show the nonlinear ISI and the displacement of signal states, respectively.

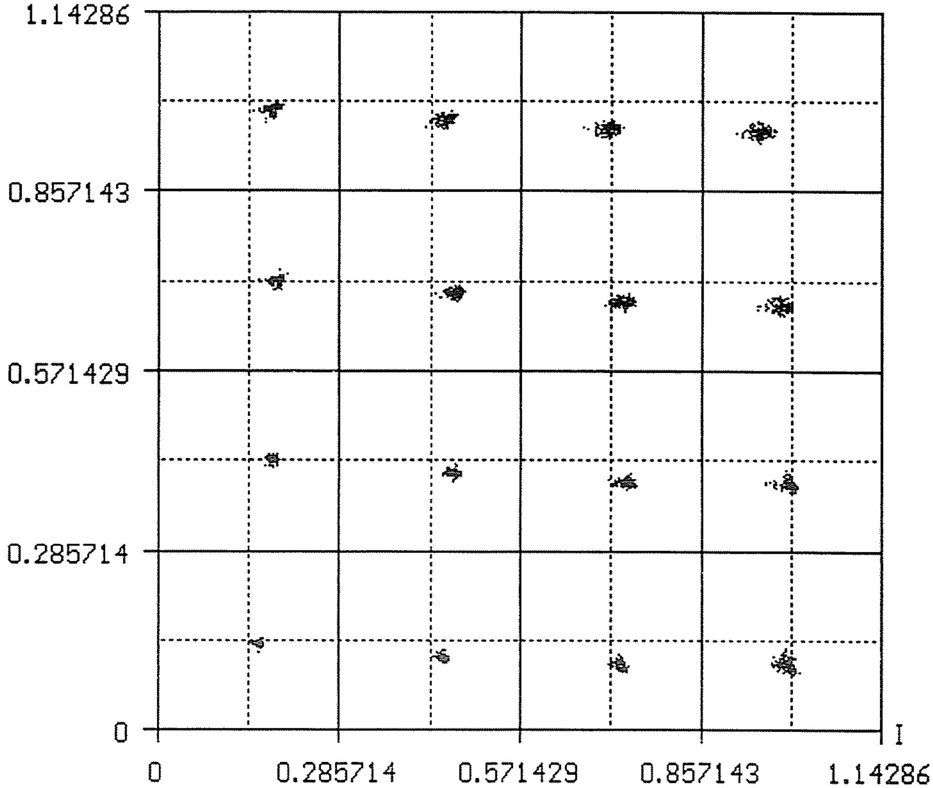

Fig. 2. A quadrant of 64-QAM constellation (HPA given in [4], $BO_p = 3.5\,\text{dB}$; $\alpha = 0.5$)

When the carrier regenerator operates ideally, an average shift of the signal carrier phase caused by AM-PM conversion is compensated by the carrier regenerator. The minimum value of $SNRD$ taken at some level of BER is obtained by suitable setting of the reference carrier phase at the demodulator [2, 7, 8]. The optimum value of this additional shift of the reference carrier phase will be called the optimum additional phase shifting ($OAPS$). It is easy to see that in the case when the system is purely linear the $OAPS$ is zero and of course when the HPA is nonlinear the higher the nonlinearity of HPA the higher is the value of $OAPS$. Similarly, in the case when AGC is ideal, the variations of the received signal amplitude caused by AM-AM conversion can be reduced by the operation of AGC. For example, if the simplest AGC, which controls the receiver gain by the average gain of received signals, is applied the variation of the received signals is reduced by the average value of the signal gain reductions caused by AM-AM conversion of the HPA. In our considerations, these effects of the AGC and the carrier regenerator are taken into account.

2.2 New Parameter of Nonlinearity of the HPA in M-QAM Radio Systems

In practice, at present the HPAs are modelled by many ways [2, 10, etc.] and the nonlinearity of HPA is characterised by many different parameters (by saturation power, by 1-dB compression point, by the 3rd-order intercept point or by AM-AM, AM-PM conversions, etc.). Among these parameters the AM-AM, AM-PM conversions are the most complete ones for SSPA (Solid State Power Amplifier) as well as for TWT (Travelling Wave Tube). These conversions are measured easily and given often by manufacturers in form of the characteristics of ΔG and $\Delta\Phi$ (Gain degradation and Phase distortion of the output signals, respectively) versus the output power of HPA ($\Delta G(P_{out})$ and $\Delta\Phi(P_{out})$ [14]. However, it is difficult to apply all the above parameters for calculating directly the BER of the system or the $SNRD$ caused by the nonlinearity. For calculating easily and directly these parameters of the system performance, a new parameter of nonlinearity of the HPA in M-QAM radio systems should be produced. This new parameter should satisfy the following requirements:

- It should be calculated easily from the characteristics of HPA given by the manufacturers;
- It should characterise the nonlinearity of HPA in some sense;
- By using this parameter some performance parameter of the system should be calculated directly (e.g. the $SNRD$ at some level of BER of the system). In other words, a relationship between this new parameter and some performance parameter of the system should be defined in the form of a simple expression.

It was well known that the higher the nonlinearity of HPA, the higher are the reductions of the distances from the signal states to the nearest boundaries, the higher is the nonlinear ISI and then the higher is the $SNRD$ at some level of BER (or the higher is the BER of the system at some value of the signal-to-noise ratio). Thus, there must be a relationship between them. Either the nonlinear ISI or the reduction of the distances from the signal states to the nearest boundaries can therefore be used as the parameter of the nonlinearity of HPA in M-QAM systems. However, it is very difficult to define the nonlinear ISI from the existing characteristics of HPAs. It was also known that both AM-AM, AM-PM conversions cause the shift of signal states on the phase plane and then reduce the distances from them to the nearest boundaries. These reductions of the distances can easily be calculated for a given pair of $\Delta G(P_{out})$ and $\Delta\Phi(P_{out})$ of HPA and a given peak back-off (the difference in dB of the peak power of the output signal from the saturation output power of HPA) if the input signal is NRZ (NonReturn to Zero). This fact suggests us to take the reductions of these distances or their average value as the parameter of nonlinearity. Since the reductions of the distances from the signal states to the nearest boundaries are not the same, they will obtain many different values, and then it is difficult to apply them in the calculation of the system performance. The average value of them should therefore be used as the parameter of HPA nonlinearity. We call this parameter in terms of the distance degradation (dd).

In M-QAM systems for a given HPA and a given value of peak back-off (BO_p) this parameter (dd) has a unique value and can be calculated simply as follows:

- From the characteristics $\Delta G(P_{out})$, $\Delta\Phi(P_{out})$ given by the manufacturers of HPA and the given BO_p, for each signal taken from the constellation of M-QAM signal (the maximum power signal of which is mapped to the peak output power) we can define ΔG_{ij} and $\Delta\Phi_{ij}$ (for symmetry, only one quadrant of the signal constellation is needed);
- For each signal state $[i, j]$ on the constellation, we can define geometrically the smallest distance from the shifted signal state to the nearest boundary (d_{ij}) by taking ΔG_{ij}, $\Delta\Phi_{ij}$ also the effects of the AGC and the carrier regenerator into account, the distance degradation dd_{ij} of this signal state $[i, j]$ is: $dd_{ij} = 1 - d_{ij}$;

$$dd = \frac{4}{M} \sum_{i,j=1}^{\frac{\sqrt{M}}{2}} dd_{ij} \,. \tag{1}$$

From (1) we can see that dd depends also on M.

Here we should discuss briefly about the possibility of choosing the maximum value of distance degradations $(\max(dd_{ij}))$ as the parameter of nonlinearity. Of course, in many cases the $\max(dd_{ij})$ can characterise the nonlinearity of HPA, but it was not chosen to be the parameter of HPA nonlinearity for two following reasons:

- For most of cases the $\Delta G(P_{out})$, $\Delta\Phi(P_{out})$ characteristics are monotonous functions with P_{out}, but in many cases (e.g. when the predistorter is used or for SSPAs) these $\Delta G(P_{out})$, $\Delta\Phi(P_{out})$ are not monotonous (as shown in [12, 14, etc.]). If the $\max(dd_{ij})$ was chosen to be the parameter of HPA nonlinearity, for the higher BO the nonlinearity of HPA would be able to become higher, this does not reflect the fact;

- If the $\max(dd_{ij})$ was chosen to be the parameter of HPA nonlinearity, for many cases (e.g. for TWTs) the peak power signal would always present the $\max(dd_{ij})$ and this would lead to the fact that for every value of M the nonlinearity of the HPA would be the same, this does not reflect the fact either.

Since the actual signal at the input of the HPA (or at the input of the cubic predistorter when it is applied) (*Fig. 1b*) is not NRZ but is the summation of the responses of the signal sequence at the input of the Tx filter, the actual nonlinearity of HPA depends also on the pulse shaping carried out by the Tx filter. In our case, it means that the actual nonlinearity of HPA depends also on the roll-off factor α of the filters and the above parameter dd does not reflect completely the actual nonlinearity of HPA. In order to avoid the dependence on the roll-off factor, the above parameter dd, however, can still be used as a nominal parameter of the nonlinearity of HPA in M-QAM systems.

2.3 Relationship between dd and Some Parameters of the Systems

The relationship between dd and the $OAPS$ as well as the $SNRD$ at the BER level of 10^{-6} for 64-QAM systems with roll-off factor $\alpha = 0.5$ were obtained by simulating the 64-QAM system shown in *Fig. 1b* with many different nonlinearities. These nonlinearities were obtained by applying the characteristics of 4 practical amplifiers with different values of BO_p. Both SSPA and TWT were taken into simulation. The simulation program is the ASTRAS program package (Analog Simulation of TRAnsmission Systems) [1, 6] with some developments. The number of symbols used in simulation is high enough for obtaining the stable and reliable results. The relationships obtained from the simulation are shown in *Fig. 3a* and *Fig. 3b* and expressed in the following formula:

$$SNRD_{64} = \left(\text{at} \quad BER = 10^{-6}\right) \approx 45dd^2 + 2dd \quad [\text{dB}], \tag{2}$$

$$OAPS_{64} = \left(\text{at} \quad BER = 10^{-6}\right) \approx 11dd \quad [\text{degree}]. \tag{3}$$

Comparing to the formula given by AMADESI et al. [7], the advantage of the formulae of $SNRD$ as shown in (2) is that it can be calculated from the arbitrary practical characteristics of AM-AM, AM-PM conversions of HPA with or without predistorter. The formula given in [7], however, is based on the assumption that the phase distortion of HPA is 3°/dB, this is not exact in many cases, especially for SSPAs. In addition, the formula given in [7] is for $SNRD$ at $BER = 5 \cdot 10^{-4}$ only, and that is not enough for estimating the system in many cases.

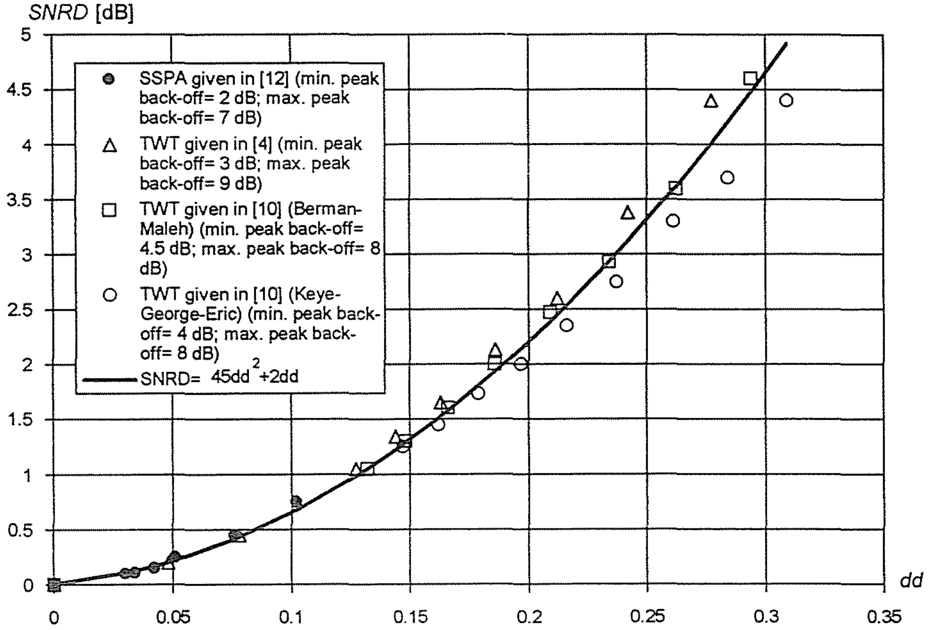

Fig. 3a. Relationship between $SNRD$ (taken at $BER = 10^{-6}$) and dd

3. Estimation of an Upper Bound of BER under the Simultaneous Effects of Linear and Nonlinear Distortions

3.1 The Hypothesis Model Used for Calculation

The model of the M-QAM radio link shown in *Fig. 1a* can be described simply again in *Fig. 4a*, where the predistorter and the HPA are combined in the HPA block, other blocks are not presented here and assumed to be ideal. In the case when HPA is assumed to be linear, the BER of the system can be obtained by simulating the purely linear system, for example by using ASTRAS-QL subprogram (ASTRAS for QAM, Linear systems) [1, 6]. When taking into account the nonlinearity of HPA, the calculation as well as the simulation become very complicated. A hypothesis model of the system should be produced so as to be able to calculate simply and fast, at least, the upper bound of BER of the actual system. Such model is shown in *Fig. 4b*, where the pulse shaping is carried out after the HPA. All

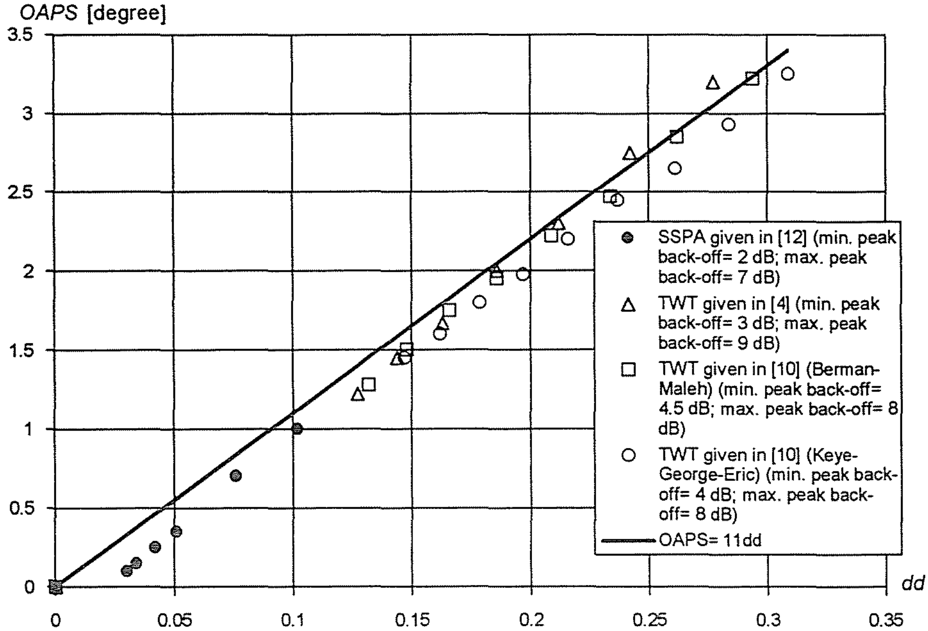

Fig. 3b. Relationship between $OAPS$ (taken at $BER = 10^{-6}$) and dd

blocks of this model have the same characteristics as those of the blocks in the actual model (*Fig. 4a*). In addition, for both HPAs of these models the same peak input back-off is applied. At present, for the economical and technical reasons, a system in which the pulse shaping is carried out after the HPA is not applied but for our purpose it is rather effective.

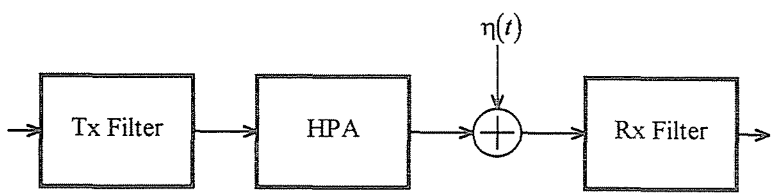

Fig. 4a. HPA is sandwiched between two filters

For comparing the two models of *Fig. 4*, it should be noted that for the 1st model the input signal of the HPA is not NRZ, but that of the 2nd model is NRZ. This fact leads to that with the same input peak back-off of the HPAs, the actual nonlinearity of HPA in the 1st model (*Fig. 4a*) is much less than the one of the 2nd model (*Fig. 4b*), and then the $SNRD$ required

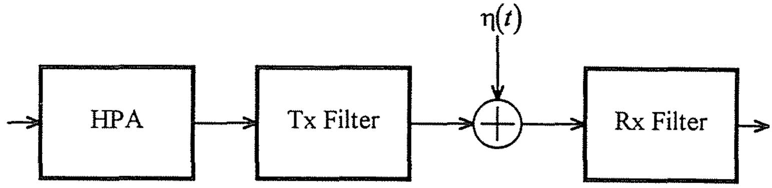

Fig. 4b. The hypothesis model

for obtaining the same level of BER is higher in the 2nd model. This can be explained as follows. It was well known that the actual nonlinearity of HPA and then the performance degradation of the system (for example $SNRD$ or BER) depends on the average power of the input signal of HPA (in other words: depends on the average back-off of HPA defined as the difference in dB between the input saturation power of HPA and the average power of the input signal). The smaller the average input power, the higher the average input back-off is, the smaller the nonlinearity of HPA, and the smaller the $SNRD$ of the system is. Denoting the peak input power of HPA by P_{in}, the peak power signal on the constellation by $\max_m\{\gamma_m\}$, where γ_m is the mth signal vector on the M-QAM signal constellation, $m = 1, 2, \ldots, M$, we have:

$$P_{in} = S_2^2 \max_m \left\{|\gamma_m|^2\right\} = S_1^2 \max_m \left\{|\gamma_m|^2\right\} \max \left[\left(\sum_k |h_T(t-kT)|\right)^2\right] , \tag{4}$$

where S_1, S_2 are the scale factors for the 1st and 2nd model, respectively;
k is the time slot index;
T is the symbol interval;
$h_T(t)$ is the impulse response of the Tx filter.

For the overshooting of the signal passing through the Tx filter, the maximum value of $|h_T(t)|$ (for $k = 0$) is higher than 1. The results of calculation by using ASTRAS-QL subprogram package are shown in *Fig. 5*. The overlapping of the symbols at the output of the Tx filter $\sum_k' |h_T(t-kT)|$ is higher than zero, thus we have $\max\left[\sum_k |h_T(t-kT)|\right]$ (the prime following the symbol $\sum'$ indicates that the term $k = 0$ should be omitted from the summation). We have thus:

$$S_2^2 \max_m \left\{|\gamma_m|^2\right\} > S_1^2 \max_m \left\{|\gamma_m|^2\right\} .$$

It means that the peak power, and then the average power of the NRZ signal (the input signal of both models) of the 1st model is smaller than one of the 2nd model. Taking more into account the power degradation caused by the loss and the selectivity of the practical Tx filter we can see that the average power of the signal at the input of HPA in the 1st model is much smaller than one in the 2nd model. According to [9], the difference between the average powers of the input signal of HPA for two models is always higher than 1 dB. For a concrete case when $\alpha = 0.35$, according to [9] as well as the practical measurement data given in [12] or the simulation result obtained by using the subprogram ASTRAS-NL (ASTRAS for NonLinear Systems) [1, 6], this power difference is about 2 dB (max $\left[\sum_k |h_T(t-kT)|\right] \approx 1.273$). We can therefore conclude that for the same level input peak back-off of HPA, the $SNRD$ required for the same level of BER of the 1st model is smaller than the one of the 2nd model. The results of simulation for some concrete HPAs by using ASTRAS-NL are shown in *Fig. 6.* Thus the BER of the 2nd model can be used as an upper bound of the one of the actual system (i.e. the 1st model).

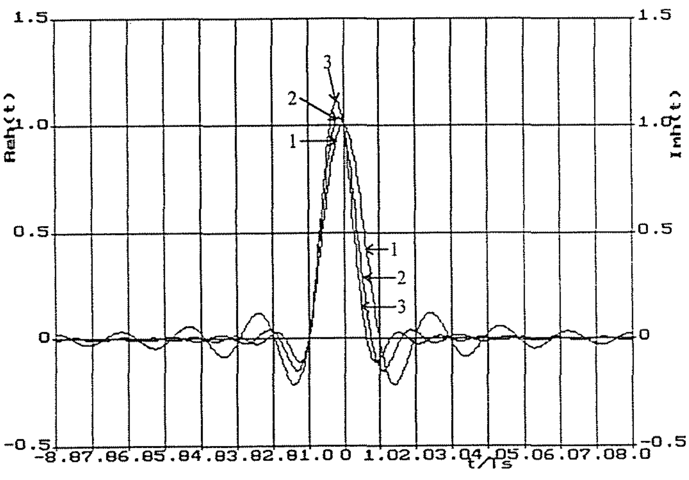

Fig. 5. Impulse response of the signal at the output of the Tx filter. 1: $\alpha = 0.1$; 2: $\alpha = 0.5$; 3: $\alpha = 0.75$

3.2 Analysis of the Second Model

The block diagram of the system for the 2nd model is shown in *Fig. 7.* Input data are the sequence of M-level complex data values $\{C_k\}$, k is

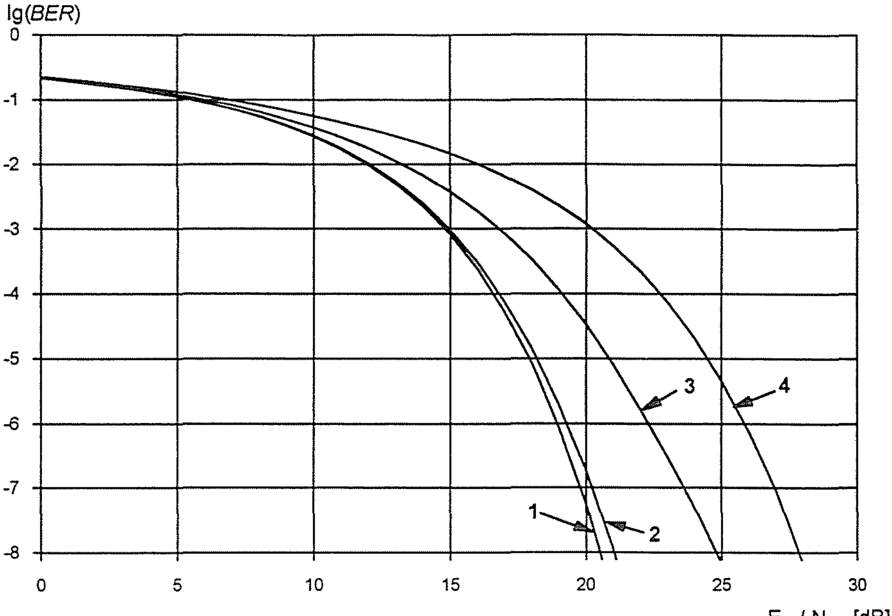

Fig. 6. Comparison of two models. 1: 1st model ($OAPS = 0.4$ deg.), 2: 2nd model ($OAPS = 0.7$ deg.) (HPA given in [12] with predistorter, $BO_p = 2$ dB); 3: 1st model ($OAPS = 2.8$ deg.), 4: 2nd model ($OAPS = 5.3$ deg) (HPA given in [4] without predistorter, $BO_p = 3.5$ dB)

the time slot index. Each C_k is taken from an M-point alphabet $\{\gamma_m\}$ $m = 1, 2, \ldots, M$ (constellation). The symbol C_k can be expressed as: $C_k = a_k + jb_k$ where a_k is the inphase and b_k is the quadrature component on the phase plane. The symbol C_k is also expressed in the vector form as: $\vec{C}_k = \begin{bmatrix} a_k \\ b_k \end{bmatrix}$, $a_k, b_k = \pm 1, \pm 3, \ldots, \pm\left(\sqrt{M} - 1\right)$.

At the sample time, at the output of the sampling device (SD) the received signal is the set $\left\{\tilde{C}_k\right\}$, similarly $\tilde{C}_k = \tilde{a}_k + j\tilde{b}_k$ and $\vec{\tilde{C}}_k = \begin{bmatrix} \tilde{a}_k \\ \tilde{b}_k \end{bmatrix}$.

At the output of the modulator, the signal is:

$$S(t) = \sum_k \left[a_k \varepsilon(t - kT) + jb_k \varepsilon(t - kT)\right] , \tag{5}$$

where T is the symbol interval;

$$\varepsilon(t) = \begin{cases} 1 & t \in \left[-\frac{T}{2}, \frac{T}{2}\right] \\ 0 & \text{elsewhere} \end{cases} .$$

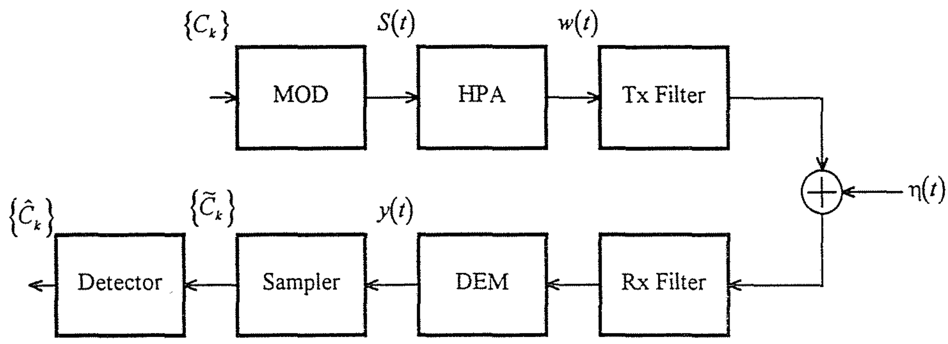

Fig. 7. The hypothesis system

In the vector form, $S(t)$ can be expressed as:

$$S(t) = \begin{bmatrix} S_a \\ S_b \end{bmatrix},$$

where

$$S_a = \sum_k a_k \varepsilon(t - kT),$$

$$S_b = \sum_k b_k \varepsilon(t - kT). \tag{6}$$

Since in the M-QAM radio relay systems the HPA is regarded as a memoryless nonlinear block described by AM-AM and AM-PM conversions, the output signal of the HPA can be expressed by a vector $\begin{bmatrix} W_a \\ W_b \end{bmatrix}$, where:

$$W_a = \sum_k \left[a_k + \Delta_1(a_k)\right] \varepsilon(t - kT),$$

$$W_b = \sum_k \left[b_k + \Delta_2(b_k)\right] \varepsilon(t - kT), \tag{7}$$

where $\Delta_1(a_k)$ and $\Delta_2(b_k)$ are the nonlinear distortions of the signal, caused by AM-AM, AM-PM conversions, $\Delta_1(a_k)$ and $\Delta_2(b_k)$ depends on the AM-AM, AM-PM characteristics of the HPA and on C_k. In practice $|\Delta_1(a_k)| \ll |a_k|$, $|\Delta_2(b_k)| \ll |b_k|$. The Tx and Rx filters are described by the equivalent lowpass transfer functions $H_T(j\omega)$ and $H_R(j\omega)$, respectively. The overall impulse response of them is:

$$h(t) = F^{-1}\left\{H_T(j\omega)H_R(j\omega)\right\} = h_c(t) + h_s(t), \tag{8}$$

where F^{-1} is the inverse Fourier transformation.

The output signal of the demodulator is:

$$y(t) = w(t) * h(t)\,. \tag{9}$$

In form of vector and transfer matrix, $y(t)$ can be expressed:

$$Y(t) = \begin{bmatrix} y_a \\ y_b \end{bmatrix} = \sum_k \mathbf{h}_k \begin{bmatrix} a_k + \Delta_1(a_k) \\ b_k + \Delta_2(b_k) \end{bmatrix} + \begin{bmatrix} n_c \\ n_s \end{bmatrix}. \tag{10}$$

where

$$y(t) = y_a(t) + jy_b(t) = y_a + jy_b\,,$$

$$\mathbf{h}_k = \begin{bmatrix} h_{ck} & h_{sk} \\ h_{sk} & h_{ck} \end{bmatrix}, \qquad \begin{array}{l} h_{ck} \doteq h_c(t-kT) \\ h_{sk} \doteq h_s(t-kT) \end{array},$$

$$n(t) = \eta(t) * h_R(t) = n_c(t) + jn_s(t) = n_c + jn_s\,,$$

$$h_R(t) = F^{-1}\left\{H_R(j\omega)\right\}. \tag{11}$$

At the sampling point $t = t_0 = 0$, the received vector at the output of the SD is:

$$\begin{bmatrix} \tilde{a}_0 \\ \tilde{b}_0 \end{bmatrix} = \mathbf{h}_0(0) \begin{bmatrix} a_0 + \Delta_1(a_0) \\ b_0 + \Delta_2(b_0) \end{bmatrix} + \sum_k{}' \mathbf{h}_k(0) \begin{bmatrix} a_k + \Delta_1(a_k) \\ b_k + \Delta_2(b_k) \end{bmatrix} + \begin{bmatrix} n_c(0) \\ n_s(0) \end{bmatrix} =$$

$$= \mathbf{h}_0(0) \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} + \mathbf{h}_0(0) \begin{bmatrix} \Delta_1(a_0) \\ \Delta_2(b_0) \end{bmatrix} + \sum_k{}' \mathbf{h}_k(0) \begin{bmatrix} a_k + \Delta_1(a_k) \\ b_k + \Delta_2(b_k) \end{bmatrix} + \begin{bmatrix} n_c(0) \\ n_s(0) \end{bmatrix}. \tag{12}$$

In the case of nonexisting linear distortion, $H(j\omega)$ meets the 1st Nyquist criterion and $h(kT) = 0 \quad \forall \quad k \neq 0$ and $h(0) = 1$.

In this case:

$$\mathbf{h}_0(0) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{h}_k(0) \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \forall \quad k \neq 0\,.$$

Thus we have:

$$\begin{bmatrix} \tilde{a}_0 \\ \tilde{b}_0 \end{bmatrix} = \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} + \begin{bmatrix} \Delta_1(a_0) \\ \Delta_2(b_0) \end{bmatrix} + \begin{bmatrix} n_c(0) \\ n_s(0) \end{bmatrix}. \tag{13}$$

According to (13), the received signal in the case of nonexisting linear distortion is the sum of the useful signal vector $\begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$, the nonlinear distortion vector $\begin{bmatrix} \Delta_1(a_0) \\ \Delta_2(b_0) \end{bmatrix}$ and the noise vector. On the phase plane, the received signal constellation is the set of M points which are obtained by shifting the original signal points under the effects of AM-AM, AM-PM conversions. No ISI appears. We call the vector $\begin{bmatrix} \Delta_1(a_0) \\ \Delta_2(b_0) \end{bmatrix}$ in terms of the signal point shifting vector.

In the case of existing linear distortion, in general we have: $h(kT) \neq 0 \quad \forall \quad k$, $h_c(0) \neq 1$, $h_s(0) \neq 0$.

At the output of the SD, at the sampling point, the signal vector is:

$$\begin{bmatrix} \tilde{a}_0 \\ \tilde{b}_0 \end{bmatrix} = \begin{bmatrix} h_c(0) & -h_s(0) \\ h_s(0) & h_c(0) \end{bmatrix} \begin{bmatrix} a_0 + \Delta_1(a_0) \\ b_0 + \Delta_2(b_0) \end{bmatrix} +$$

$$+ \sum_k{}' \mathbf{h}_k(0) \begin{bmatrix} a_k + \Delta_1(a_k) \\ b_k + \Delta_2(b_k) \end{bmatrix} + \begin{bmatrix} n_c(0) \\ n_s(0) \end{bmatrix} =$$

$$= \begin{bmatrix} 1 + \Delta h_c(0) & -h_s(0) \\ h_s(0) & 1 + \Delta h_c(0) \end{bmatrix} \begin{bmatrix} a_0 + \Delta_1(a_0) \\ b_0 + \Delta_2(b_0) \end{bmatrix} +$$

$$+ \sum_k{}' \mathbf{h}_k(0) \begin{bmatrix} a_k + \Delta_1(a_k) \\ b_k + \Delta_2(b_k) \end{bmatrix} + \begin{bmatrix} n_c(0) \\ n_s(0) \end{bmatrix}, \tag{14}$$

where: $1 + \Delta h_c(0) = h_c(0)$.

Thus, we have:

$$\begin{bmatrix} \tilde{a}_0 \\ \tilde{b}_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_0 + \Delta_1(a_0) \\ b_0 + \Delta_2(b_0) \end{bmatrix} + \begin{bmatrix} \Delta h_c(0) & -h_s(0) \\ h_s(0) & \Delta h_c(0) \end{bmatrix} \begin{bmatrix} a_0 + \Delta_1(a_0) \\ b_0 + \Delta_2(b_0) \end{bmatrix} +$$

$$+ \sum_k{}' \mathbf{h}_k(0) \begin{bmatrix} a_k + \Delta_1(a_k) \\ b_k + \Delta_2(b_k) \end{bmatrix} + \begin{bmatrix} n_c(0) \\ n_s(0) \end{bmatrix} =$$

$$= \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} + \begin{bmatrix} \Delta_1(a_0) \\ \Delta_2(b_0) \end{bmatrix} + \begin{bmatrix} \Delta h_c(0) & -h_s(0) \\ h_s(0) & \Delta h_c(0) \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} +$$

$$+ \begin{bmatrix} \Delta h_c(0) & -h_s(0) \\ h_s(0) & \Delta h_c(0) \end{bmatrix} \begin{bmatrix} \Delta_1(a_0) \\ \Delta_2(b_0) \end{bmatrix} +$$

$$+ \sum_k{}' \mathbf{h}_k(0) \begin{bmatrix} a_k \\ b_k \end{bmatrix} + \sum_k{}' \mathbf{h}_k(0) \begin{bmatrix} \Delta_1(a_k) \\ \Delta_2(b_k) \end{bmatrix} + \begin{bmatrix} n_c(0) \\ n_s(0) \end{bmatrix}, \tag{15}$$

where

$$A = \begin{bmatrix} \Delta_1(a_0) \\ \Delta_2(b_0) \end{bmatrix},$$

$$B = \begin{bmatrix} \Delta h_c(0) & -h_s(0) \\ h_s(0) & \Delta h_c(0) \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix},$$

$$C = \begin{bmatrix} \Delta h_c(0) & -h_s(0) \\ h_s(0) & \Delta h_c(0) \end{bmatrix} \begin{bmatrix} \Delta_1(a_0) \\ \Delta_2(b_0) \end{bmatrix},$$

$$D = \sum_k' \mathbf{h}_k(0) \begin{bmatrix} a_k \\ b_k \end{bmatrix},$$

$$E = \sum_k' \mathbf{h}_k(0) \begin{bmatrix} \Delta_1(a_k) \\ \Delta_2(b_k) \end{bmatrix},$$

$$F = \begin{bmatrix} n_c(0) \\ n_s(0) \end{bmatrix}.$$

The sum $B + D$ is known as the linear ISI. We call the sum $C + E$ in terms of the nonlinear ISI. F is the noise vector.

Thus, in general, the received signal vector consists of five parts: the useful signal vector, the signal point shifting vector (A), the linear ISI vector ($B + D$), the nonlinear ISI vector ($C + E$), and the noise vector (F).

According to [13], the linear ISI vector can be well approximated by a uniform-distributed, zero-mean random vector. The characteristics of the nonlinear ISI, however, have not been yet reported.

In practice, the effect of this nonlinear ISI is rather small and negligible. This can be proved as follows.

The linear ISI can be expressed as:

$$X_I(0) = \sum_k' a_k h_c(-kT) - \sum_k b_k h_s(-kT), \tag{16a}$$

$$X_Q(0) = \sum_k' b_k h_c(-kT) + \sum_k a_k h_s(-kT), \tag{16b}$$

where X_I, X_Q are the inphase and quadrature components of the linear ISI, respectively.

The peak perturbation of the I-component can be estimated as:

$$|X_I(0)| = \left| \sum_k' a_k h_c(-kT) - \sum_k b_k h_s(-kT) \right| \leq$$

$$\leq \sum_k' |a_k h_c(-kT)| + \sum_k |b_k h_s(-kT)| \leq$$

$$\leq \max\{|a_k|\} \sum_k^{\prime} |h_c(-kT)| + \max\{|b_k|\} \sum_k |h_s(-kT)| =$$

$$= \left(\sqrt{M}-1\right) \left\{ \sum_k^{\prime} |h_c(-kT)| + \sum_k |h_s(-kT)| \right\} = PD_I\,, \tag{17}$$

where PD_I is the I-component peak distortion.

In the case when the linear distortion is not dramatically high (when the eye-pattern is open):

$$PD_I < 1 < |a_0| \quad \forall \quad a_0\,. \tag{18}$$

For the nonlinear ISI, similarly, the maximum nonlinear perturbation can be expressed and estimated:

$$Y_I = \sum_k^{\prime} \Delta_1(a_k) h_c(-kT) - \sum_k \Delta_2(b_k) h_s(-kT)\,, \tag{19a}$$

$$Y_Q = \sum_k^{\prime} \Delta_2(b_k) h_c(-kT) + \sum_k \Delta_1(a_k) h_s(-kT)\,. \tag{19b}$$

and

$$|Y_I| = \left| \sum_k^{\prime} \Delta_1(a_k) h_c(-kT) - \sum_k \Delta_2(b_k) h_s(-kT) \right| \leq$$

$$\leq \sum_k^{\prime} |\Delta_1(a_k) h_c(-kT)| + \sum_k |\Delta_2(b_k) h_s(-kT)| \leq$$

$$\leq \max\{|\Delta_1(a_k)|\} \sum_k^{\prime} |h_c(-kT)| + \max\{|\Delta_2(b_k)|\} \sum_k |h_s(-kT)| \leq$$

$$\leq \max\left\{ \left|\Delta_1\left(\sqrt{M}-1\right)\right|, \left|\Delta_2\left(\sqrt{M}-1\right)\right| \right\}$$

$$\left\{ \sum_k^{\prime} |h_c(-kT)| + \sum_k |h_s(-kT)| \right\} = PND_I\,, \tag{20}$$

where PND_I is the I-component peak nonlinear distortion.

In practice, since the nonlinear part of the signal at the output of HPA is much smaller than the linear one, we have $|\Delta_1(x)| \ll |x|$ and $|\Delta_2(x)| \ll |x|$.

Thus we have:

$$\max\left\{ \left|\Delta_1\left(\sqrt{M}-1\right)\right|, \left|\Delta_2\left(\sqrt{M}-1\right)\right| \right\} \ll \left(\sqrt{M}-1\right)\,. \tag{21}$$

Combining (17), (18), (20), (21) we have:

$$PND_I \ll PD_I < |a_0| \,. \tag{22}$$

Similarly,

$$PND_Q \ll PD_Q < |b_0| \,. \tag{23}$$

Thus,

$$\|[C+E]\| < \left\| \begin{bmatrix} PND_I \\ PND_Q \end{bmatrix} \right\| \ll \left\| \begin{bmatrix} PD_I \\ PD_Q \end{bmatrix} \right\| < \left\| \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} \right\| . \tag{24}$$

In practice:

$$\|[C+E]\| <<< \left\| \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} \right\| . \tag{25}$$

In (25), the symbol $<<<$ means that the length of the nonlinear ISI vector $[C+E]$ is very much smaller than the one of the useful signal vector $\begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$. The effects of the nonlinear ISI vector can thus be neglected. For example, the results are for a concrete 64-QAM system where the linear ISI is caused purposely by choosing the roll-off factor α_1 of the Tx filter to be 0.5 but the α_2 of the Rx filter to be 0.3, by using the ASTRAS program package :

- The eye-opening: 74%
- The average deviations of linear ISI taken on all signal states:

  $$\sigma_{Iavg} \approx 0.0077 \,, \qquad \sigma_{Qavg} \approx 0.0077 \,.$$

  By applying the HPA given in [4] (without predistortion) with the peak back-off $BO_p = 3.5\,\mathrm{dB}$, the simulation result is:
- The average deviation of the total ISI (including the linear and nonlinear ISI):

  $$\sigma_{Iavg} \approx 0.0075 \,, \qquad \sigma_{Qavg} \approx 0.0073 \,.$$

  From the obtained results, the differences between the deviations in two cases are very small and negligible.

Thus, we can conclude that for the hypothesis system the received signal vector is the sum of the useful signal vector, the signal point shifting vector, the uniform-distributed, zero-mean linear ISI vector, the negligible nonlinear ISI vector, and the noise vector. The signal point shifting vector can be calculated geometrically from the $\Delta G(P_{out})$, $\Delta\Phi(P_{out})$ characteristics of HPA with a given peak back-off. The characteristics of the linear ISI can be obtained by simulating simply and fast the purely linear system (for example, by using the ASTRAS-QL subprogram). The BER of the hypothesis system can therefore be computed simply if the noise is assumed to be the additive white Gaussian noise.

3.3 A Procedure for Estimating the Effects of the Linear and Nonlinear Distortions

According to the above conclusions, we would like to produce a procedure for estimating the effects of both linear and nonlinear distortions in M-QAM radio systems as follows:

- By simulating the purely linear system we can estimate the $SNRD$ caused by only the linear distortion;
- By simulating the nonlinear system but the Tx and Rx filters are assumed to satisfy the 1st Nyquist criterion (in practice, the overall response of them is the one of the raised cosine roll-off filter), or by applying such a formula as presented in expression (2) we can obtain the $SNRD$ caused by only the nonlinear distortion. It should be noted here that before the simulation of the nonlinear system, the $OAPS$ has to be defined. This $OAPS$ can be calculated by a formula presented in expression (3);
- Depending on which distortion (linear or nonlinear) dominates (what $SNRD$ is higher), we can define the lower bound of the $SNRD$ (or BER) of the system;
- By calculating the shifted states of signals on phase plane caused by AM-AM, AM-PM conversions (calculating the signal point shifting vectors) we can obtain the warped constellation of the received signal of the hypothesis system. It should be noted that in these calculations the effects of the AGC and the carrier regenerator as well as the $OAPS$ should be taken into account;
- By combining with the characteristics of the linear ISI obtained from the simulation of the purely linear system, the upper bound of BER of the actual system can be defined simply by the BER of the hypothesis system.

As an example of applying this procedure, the upper bound of BER for the 64-QAM system used for transmitting an STM-1 payload (Synchronous Transport Module-1) over the IFP (Interleaved Frequency Plan) 29.65 MHz separated channels [11] is calculated. The result is shown in *Fig. 8.* The HPA applied in this calculation is the SSPA with predistorter given in [12]. The peak back-off is 2 dB.

4. Conclusions

In this paper, a new nominal parameter of the nonlinearity of HPA in M-QAM systems is produced as an effort to estimate the nonlinearity of

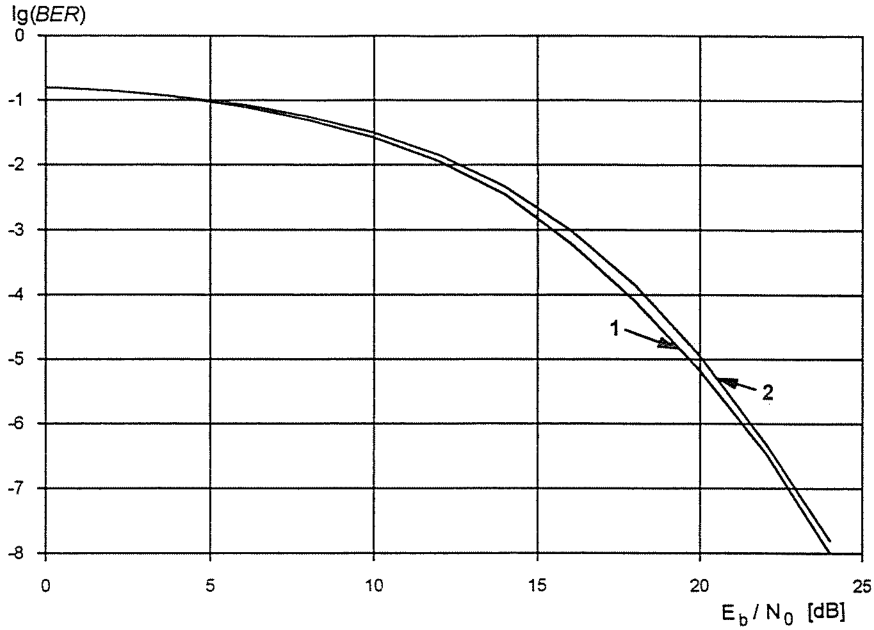

Fig. 8. Lower bound and upper bound of BER for the 64-QAM given in [11]. HPA is given in [12] with predistorter, $BO_p = 2\,\text{dB}$. 1: result of ASTRAS-QL for the purely linear system; 2: result of the simple procedure

HPA. This parameter can be calculated simply from the characteristics of HPA given by the manufacturers and can be used effectively to characterise the nonlinearity of HPA in M-QAM systems. As a proof of the effectiveness of this parameter, the way to define the formulae of $SNRD$ of the system caused by the nonlinear HPA is pointed out and a concrete formula of $SNRD$ for 64-QAM systems is also defined. This formula can be used effectively to calculate simply and fast the $SNRD$ caused by the nonlinearity of HPA in 64-QAM systems and the result can be used in some sense as a lower bound of $SNRD$ of the system. In the M-QAM systems with nonlinear HPA the existing of $OAPS$ has been known for long time but only applying the new parameter dd, this $OAPS$ value is calculated rather exactly and fast.

The simultaneous effects of both linear and nonlinear distortions can be estimated fast and simply by applying a simple procedure presented in this paper to estimate the upper bound of BER. This procedure is based on the introduction of a hypothesis model and on the careful analysis of this model. This procedure can be used]effectively in design and evaluation of M-QAM systems, especially when the very complicated simulation program is not available.

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