PERIODICA POLYTECHNICA SER. EL. ENG. VOL. 38, NO. 1, PP. 5–16 (1994)

PROBLEMS OF PREISACH MODEL APPLYING IN FINITE ELEMENT METHOD

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Received: March 2, 1994

Abstract

The paper deals with determination of magnetic field, calculated by finite element method with regard to magnetic nonlinearity. The hysteresis loop of the material is modelled by the scalar Preisach model. A hysteresis motor problem was selected for the demonstration of the hysteresis effect.

Keywords: finite elements, Preisach model, time solution of magnetic field, hysteresis motor.

1. Introduction

Nowadays in presence of linear materials many numerical methods can be selected for determination of electromagnetic field. Nevertheless, in the case of nonlinear material the solution of two or three dimensional problems applying scalar or vector potentials results in significant rise of computation time. In ferromagnetic material the nonlinear relation between the field quantities yields hysteresis character. Development of the Preisach model holds the possibility to describe a real ferromagnetic material, to simulate the $B - H$ characteristic of hysteresis loop.

The main purpose of this work is to examine the problems of the Preisach model in the framework of the finite element method. In order to analyze the problems arisen in application of the Preisach model, a two-dimensional field problem is developed for hysteresis motor with determination of inhomogeneous magnetic field of the machine in presence of the ferromagnetic material.

2. Model of the Hysteresis Machine

The armature of the hysteresis motor is built up from a ferromagnetic ring combined with non-magnetic and soft-magnetic materials. Neglecting the variation along the z-axis ($\partial/\partial z = 0$), the field problem yields a 2-D model (*Fig.* 1).

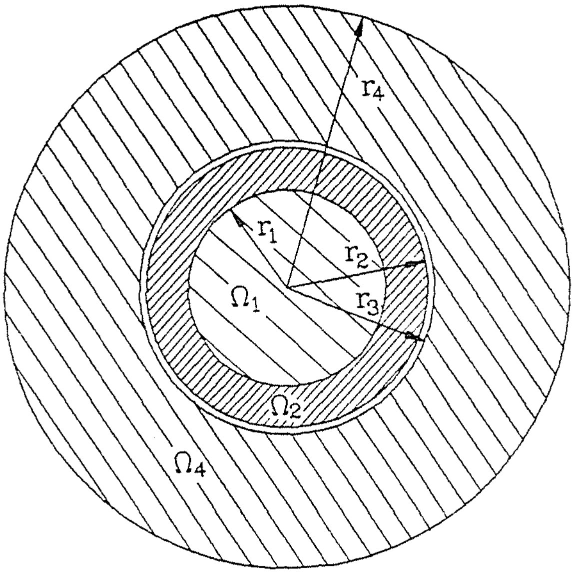

Fig. 1. Cross-section of the hysteresis motor, model for calculation

For the field calculation the model of the motor can be separated into four regions. The internal region ($\Omega_1 = (r \leq r_1)$) of the armature and the air gap of the motor ($\Omega_3(r_2 \leq r \leq r_3)$) is filled with non-magnetic material of $\mu = \mu_0$. The region ($\Omega_2(r_1 \leq r \leq r_2)$) is the ferromagnetic ring with a wide hystersis loop. The stator of the motor is modelled with the region ($\Omega_4(r_3 \leq r \leq r_4)$) of soft-magnetic material with linear characteristic, μ_{r4} [2]. The boundary surface and the interfaces between the regions are Γ_1, Γ_2, Γ_3 and Γ_4, respecting to the radius of the surfaces r_1, r_2, r_3 and r_4. The excitation of the motor is modelled with an ideal coil along the surface of the stator, resulting surface current density of sinusoidal distribution along Γ_3.

$$\mathbb{K} = \frac{I_p}{2\pi r_3} \sin \vartheta e_z \ . \tag{1}$$

3. Governing Equations

In the static magnetic field neglecting the eddy current and introducing the magnetization vector $\mathbf{M}$, the field equations have the form [1]

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J}\delta(r_3) \,, \\ \nabla \cdot \mathbf{B} &= 0 \\ \mathbf{B} &= \mu \mathbf{H} + \mathbf{M} \,, \end{aligned} \tag{2}$$

where $\mathbf{B}$ is the flux density, $\mathbf{H}$ is the magnetic field intensity and μ is the permeability of the non-magnetic and the soft-magnetic materials. $\mathbf{M}$ is the magnetization vector generated by Preisach model in the nonlinear magnetic material of wide hysteresis loop. Taking into account the linear behaviour of the materials in regions Ω_1, Ω_3 and Ω_4, the magnetization vector has zero value $\mathbf{M} = \mathbf{0}$ and $\mu_i = \mu_0 \mu_{ri}$, $i = 1, 2, 3$. Introducing the z-directed magnetic vector potential $\mathbf{A} = A(x, y)\mathbf{e}_Z$ as $\mathbf{B} = \nabla \times \mathbf{A}$, and the Dirac distribution $\delta(r)$ for modelling the surface current density, the field problem yields the solution of the differential equation in the regions Ω_i, $i = 1, 2, 3, 4$

$$\nabla \times \left(\frac{1}{\mu} \left(\nabla \times \mathbf{A} - \mathbf{M}(\mathbf{H}) \right) \right) = \mathbf{J}\delta(r_3) \,. \tag{3}$$

The boundary conditions are as follows:

Along the interfaces Γ_1, Γ_2 and Γ_3 between the regions Ω_1, Ω_2, Ω_3 and Ω_4 the normal components of the flux densities have to be continuous. The tangential components of the magnetic field intensities are continuous as well along the surfaces of Γ_1 and Γ_2, while along the interface Γ_3 it is prescribed by the surface current density, $\mathbf{K}$. Introducing the radius directed normal vector $\mathbf{n} = \mathbf{e}_r$ the interface conditions for the magnetic vector potentials can be formulated as

$$\begin{aligned} &\mathbf{A} \cdot \mathbf{n}_{|\Gamma_i \in \Omega_i} - \mathbf{A} \cdot \mathbf{n}_{|\Gamma_i \in \Omega_{i+1}} = 0 \,, \quad i = 1, 2, 3, \\ &- \nabla \times \mathbf{A} \times \mathbf{n}_{|\Gamma_i \in \Omega_i} + \nabla \times \mathbf{A} \times \mathbf{n}_{|\Gamma_i \in \Omega_{i+1}} = \begin{cases} \mathbf{0}, & \text{if} \quad i = 1, 2, \\ \mathbf{K}, & \text{if} \quad i = 3 \end{cases} \end{aligned} \tag{4}$$

The condition that no flux lines leave the external surface of the electrical machine results in a constant value for the vector potential along the surface Γ_4. Selecting this constant to be zero, the boundary condition is prescribed as homogeneous Dirichlet boundary condition along Γ_4

$$\mathbf{A}_{|\Gamma_4} = \mathbf{0} \,. \tag{5}$$

4. Formulation by Finite Element Method

Taking into account that the problem is two-dimensional, the vector potential can be determined from the condition that the first variation of the energy related functional must disappear

$$W(\mathbf{A}) = \int_{\Omega} \nabla \times \mathbf{A} \frac{\nabla \times \mathbf{A} - 2\mathbf{M}}{\mu} \mathrm{d}\Omega - \int_{\Gamma} 2\mathbf{A}\mathbf{K}\delta(r_3)\mathrm{d}\Gamma \,, \tag{6}$$

where $\Omega = \bigcup_{i=1}^{4} \Omega_i$ is the cross-section of the motor and $\Gamma = \Gamma_3$ according to the excitation.

The solution of the problem is generated by the finite element method [2], introducing triangular grids for the arrangement as it is plotted in Cartesian coordinate plane (*Fig.2*). On the ith triangular element the shape function is selected as a linear one.

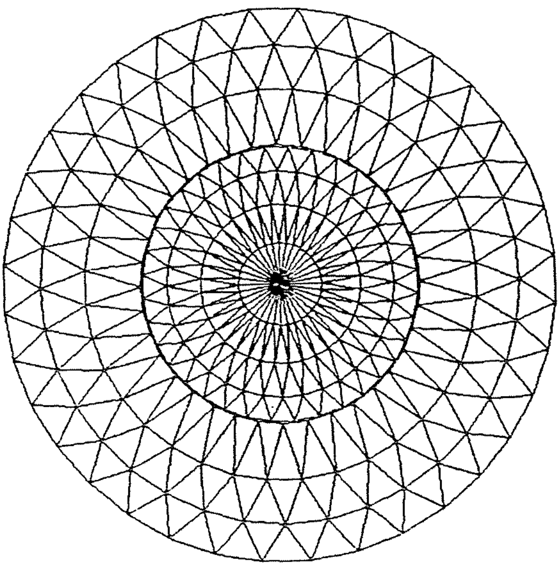

Fig. 2. Mesh and grid points of triangular finite elements

$$\mathbf{A}^i = F_i^T A_i \,. \tag{7}$$

Introducing the coincidence matrix C as $A_i = CA$ for declaration of the correlation between the local and the global numbering of the nodes, the extreme value of the energy related functional (6) yields a non-linear set of equations for the magnetic vector potentials of the nodes

$$KA = G \,, \tag{8}$$

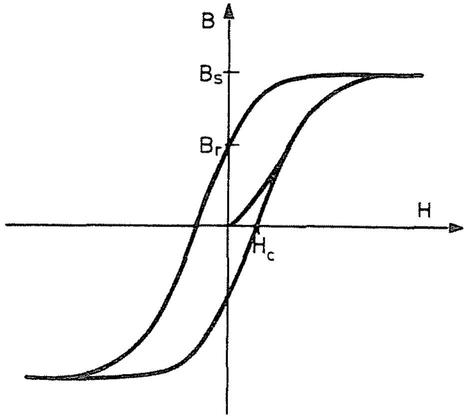

Fig. 3. The hysteresis curve

where the K and G matrices are the following

$$K = C^T \int_{\Omega_i} \frac{1}{\mu_i} \nabla \times F_i \nabla \times F_i^T \mathrm{d}\Omega C \; ,$$

$$G = C^T \int_{\Omega_i} \frac{1}{\mu_i} \nabla \times F_i \mathbf{M}_i(\mathbf{H}) \mathrm{d}\Omega - C^T \int_{\Gamma_i} F_i \mathbf{K}_i \delta(r_3) \mathrm{d}\Gamma \; . \tag{9}$$

For the solution of the nonlinear system of *Eq.* (8) first the magnetization vector is supposed to be zero. The prescribed boundary conditions yield the first approximation for the magnetic vector potential by the solution of the *Eq.* (8). Further approximation can be carried out determining the magnetization from the previous vector potential on the basis of Preisach model. The error estimation is evaluated by the aid of the everage value of the magnetic energy.

5. Preisach Model

During the years the Preisach model has been developed. In order to describe the nonlinearity of the magnetic materials, the scalar Preisach model is introduced [3]. By the superposition of scalar model the magnetic vector field can be modelled. This is attractive for Computer Aided Analyses because of its simplicity. Here a short summary of the Preisach model and some remarks for application are given.

For simulation of a hysteresis curve (*Fig. 3*) the characteristic points like remanent induction, $\mathbf{B}_r$ the coercitive field, $\mathbf{H}_c$ and the field $\mathbf{H}_s$ with flux density $\mathbf{B}_s$, at saturation of a real curve have to be known from experiments. From these parameters our scalar model can be built up.

The Preisach model regards a piece of magnetic material as a collection of elementary hysteresis transducer with different rectangular loops. The collection of elementary hysteresis loops has a global memory. This system can produce a shape like a hysteresis curve and it has to be fitted onto a real curve by using distribution function, constants and feedback. So our Preisach model generates the magnetization M to the field H as follows

$$\begin{aligned} M &= \int_{H_a}\int_{H_b} \mu(H_a, H_b, H)\gamma(H_a, H_b)\mathrm{d}H_a\mathrm{d}H_b \\ &= \iint_{T+} \mu(H_a, H_b)\mathrm{d}H_a\mathrm{d}H_b - \iint_{T-} \mu(H_a, H_b)\mathrm{d}H_a\mathrm{d}H_b \ , \end{aligned} \tag{10}$$

where a Gaussian distribution function $\mu(H_a, H_b)$ is used as a weighted function, and $\gamma(H_a, H_b)$ is an elementary hysteresis operator (*Fig. 4*). The Preisach diagram (*Fig. 5*) shows the integration territory. Introducing a suitable discretization, the integrated value of the weighted function on a finite square element can be calculated and stored preliminary.

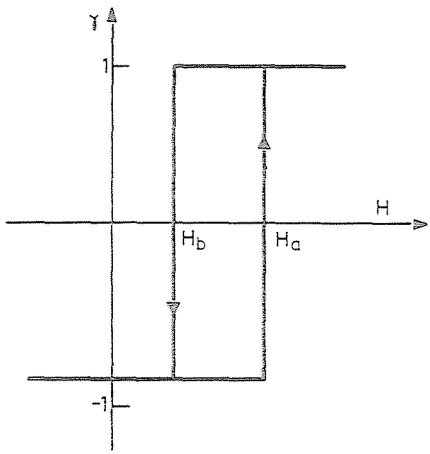

Fig. 4. Elementary hysteresis operator

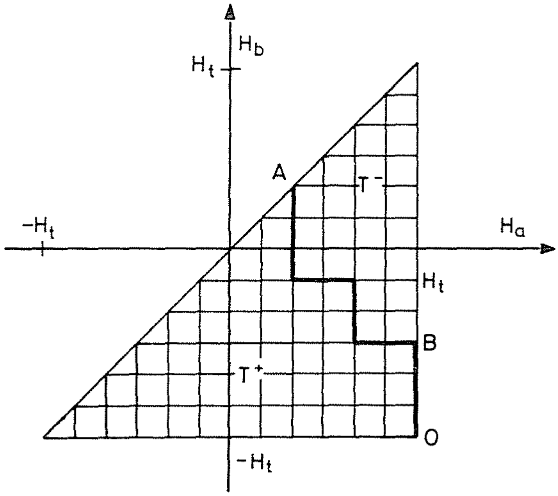

Fig. 5. Preisach diagram

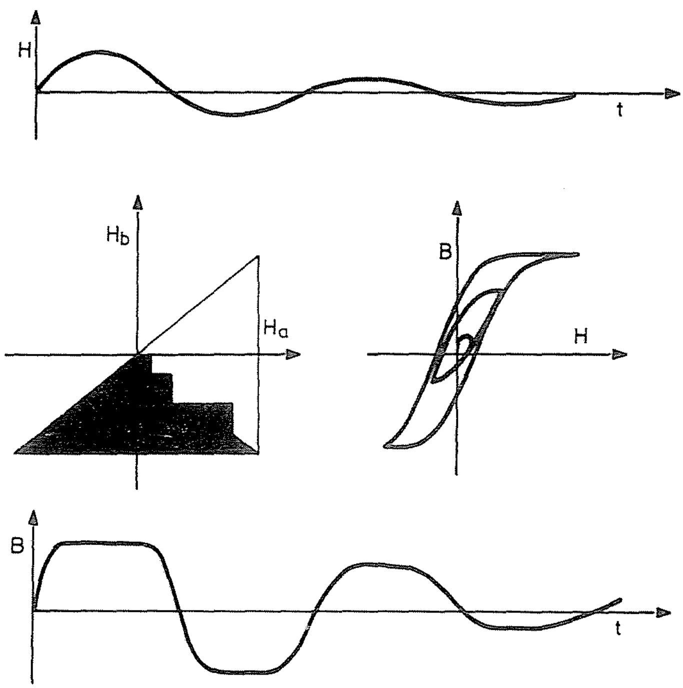

Fig. 6. Application of the Preisach model

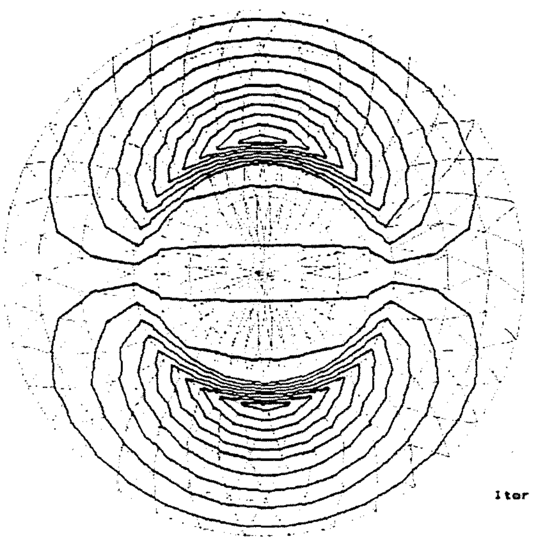

Fig. 7a. Flux lines distribution in a two-pole hysteresis motor a) at $\vartheta = 0$,

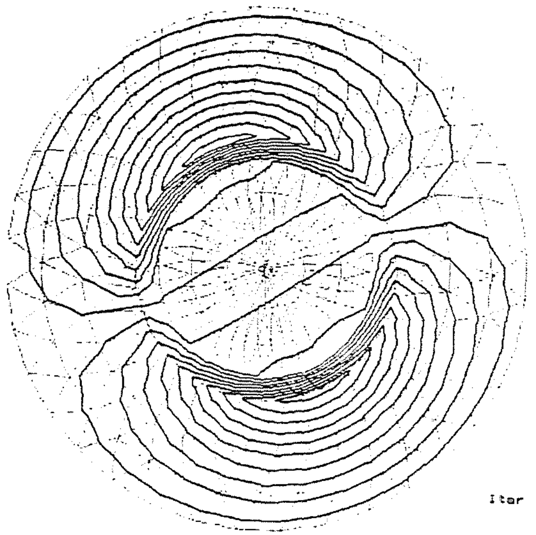

Fig. 7b. Flux lines distribution in a two-pole hysteresis motor b) at $\vartheta = 30°$

For computer simulation one of the end points (A) of the staircase line L_i, subdividing the Preisach diagram into two parts (T^+, T^-) is constantly attached to the line $H_a = H_b$. Considering the other end point (0), not the moving point (B), the subdivider line L_i can be referred as a memory vector coded by the previous state of the magnetic material. The number of its digits gives the length of a staircase line. The digit will be true if on

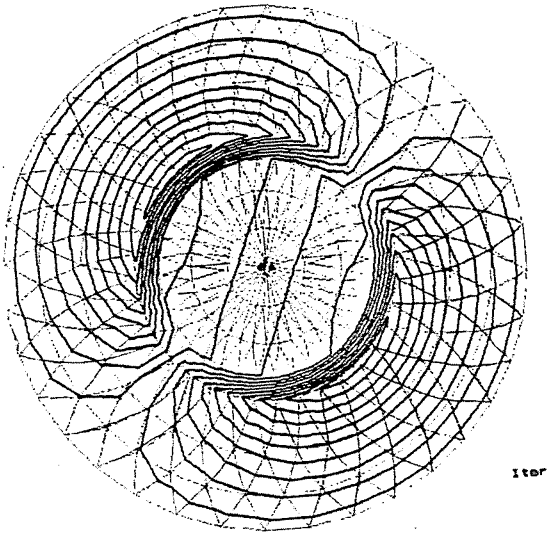

Fig. 7c. Flux lines distribution in a two-pole hysteresis motor c) at $\vartheta = 60°$ rotation of the excitation.

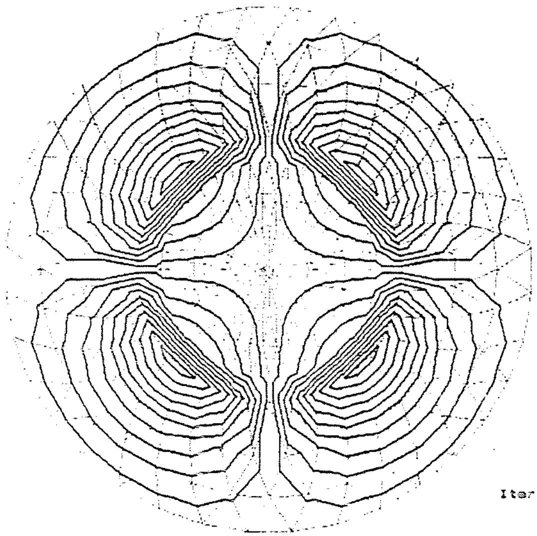

Fig. 8a. Flux lines distribution in a four-pole hysteresis motor a) at $\vartheta = 0$,

an elementary length according to the discretization the staircase line is horizontal and false if it is vertical. This way, in knowledge of the previous vector L_i and the magnetization M_i only the difference in the magnetization has to be calculated [4]. For application of the model a B-H characteristic of a material is plotted in *Fig. 6*.

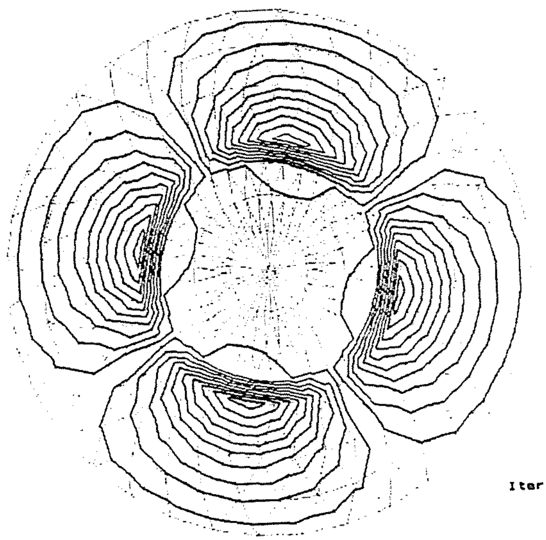

Fig. 8b. Flux lines distribution in a four-poles hysteresis motor b) at $\vartheta = 30°$

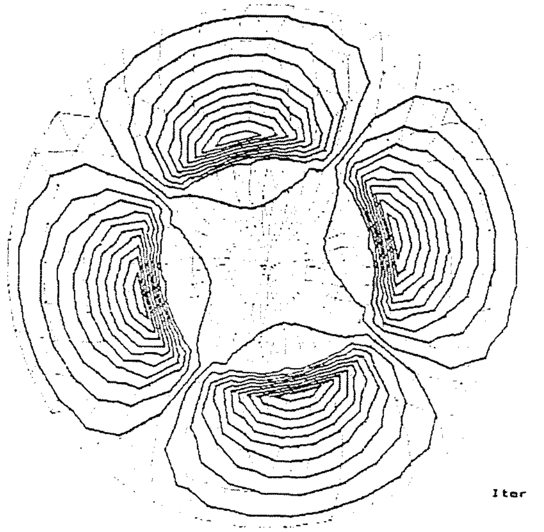

Fig. 8c. Flux lines distribution in a four-poles hysteresis motor c) at $\vartheta = 60°$ rotation of the excitation.

6. Numerical Realization

For numerical investigation the magnetic field in a hysteresis motor is determined. The normed values of the radii are $r_1/r_2 = 0.82$, $r_3/r_2 = 1.01$ and $r_4/r_2 = 2.00$. In the region Ω_4 the relative permeability of the soft magnetic material is selected as $\mu_{r4} = 1000$. Taking into account the excitation of the hysteresis motor at the air-gap (Γ_3), the maximum value of the surface current is prescribed as $I_p = 1500$ A [5].

For the finite element realization the number of the triangular elements generated by $N_{\text{nodal points}} = 289$ are $N_\triangle = 540$.

The calculation is evaluated for two-pole winging configurations at $\vartheta = 0$, 30 and 60 degrees turn in the magnetization current, and the field lines are plotted in *Fig. 7a-c*, while for four-pole configuration the distribution of the flux lines are plotted in *Fig.* 8 a-c for the same ϑ values.

From the figures the delay of the flux comparing to the rotating excitation can be seen in the hysteresis motor at fixed rotor position.

7. Conclusion

According to the nonlinear behaviour of the magnetic material in the hysteresis motor the solution of the field equations have to be generated in time domain.

The numerical treatments prove that despite the fact that in the arrangement the amplitude of the excitation does not vary with time, the total magnitude of the excitation cannot be applied from the first moment because increasing the source current or field from zero to the maximum value within an infinitely short time results in a local saturation in the material and yields a non-stabil oscillation. To avoid this phenomena the excitation is increasing step by step with stabilization in grid-points. This way the convergency of the solution can be ensured.

The next problem comes from the huge number of the unknowns, the iterations and the time-steps resulting long *CPU* time. The system of the equations has the form $KA = G$, where K depends on the geometry of the arrangements and the shape function of the finite elements G holds the excitation, the current and the magnetization, generated by the previous stabil iteration. Forming the inverse of matrix K the iteration became faster according to the fact that for determination of the next step only the new value of the vector G has to be determined.

To increase the accuracy of the solution the discretization in the finite elements and in the Preisach model cannot be selected independently.

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